

Asymmetry $A_{LL}^{\cos 2\phi}$ in the polarized proton-proton Drell-Yan process within TMD factorization*

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Abstract: We study $\cos 2\phi$ azimuthal asymmetry in doubly longitudinally polarized proton-proton Drell-Yan collisions within the transverse momentum dependent factorization framework. The asymmetry arises from the convolution of the longitudinal transversity distribution $h_{1L}^{\perp}$ for both protons. Using the Bacchetta-Delcarro-Pisano-Radici-Signori parameterization for the nonperturbative Sudakov form factor and the Wandzura-Wilczek approximation for collinear $h_{1L}^{\perp}$, we predict the double spin asymmetry $A_{LL}^{\cos 2\phi}$ at RHIC and NICA kinematics. Our results demonstrate sensitivity to sea quark distributions, with the asymmetry reaching up to 25% for maximal sea quark contributions. These predictions highlight the potential of polarized Drell-Yan measurements to probe sea quark dynamics and advance our understanding of nucleon structures.

Keywords: transverse momentum dependent factorization, longitudinal transversity distribution, Relativistic Heavy Ion Collider, Nuclotron-based Ion Collider Facility

DOI: 10.1088/1674-1137/add5c7 **CSTR:** 32044.14.ChinesePhysicsC.49083101

I. INTRODUCTION

The three-dimensional tomography of nucleons represents a fundamental frontier in quantum chromodynamics (QCD). While collinear parton distribution functions (PDFs) provide longitudinal momentum information, transverse momentum dependent parton distribution functions (TMDPDFs) [1–4] offer crucial insights into the transverse momentum ($\mathbf{k}_{\perp}$) structure, enabling a complete momentum-space description of nucleon constituents. Formally, the quark-quark correlator for a nucleon with momentum P and spin vector S can be expressed as

$$\Phi_{ij}(x, \mathbf{k}_{\perp}) = \int \frac{d\xi^- d^2\xi_{\perp}}{(2\pi)^3} e^{ik^+\xi^-} \langle P, S | \bar{\psi}_j(0) \mathcal{U}_{[0, \xi]} \psi_i(\xi) | P, S \rangle \Big|_{\xi^+=0}, \quad (1)$$

where $\mathcal{U}_{[0, \xi]}$ denotes the gauge link ensuring color gauge invariance. Through Lorentz structure decomposition [4, 5], eight leading-twist TMDPDFs emerge, characterized by distinct spin-momentum correlations. Three TMDPD-

Fs, the unpolarized distribution $f_1(x, \mathbf{k}_{\perp})$, helicity distribution $g_1(x, \mathbf{k}_{\perp})$, and transversity distribution $h_1(x, \mathbf{k}_{\perp})$, reduce to the corresponding collinear PDFs when the transverse momentum is integrated over. The remaining five TMDPDFs, depending on the spin of the parent proton or the quarks, encode genuine transverse momentum effects.

Among the TMDPDFs, the distribution $h_{1L}^{\perp}$ is arguably the least explored. It quantifies the correlation between a longitudinally polarized nucleon and transversely polarized quarks through the matrix element:

$$h_{1L}^{\perp} \propto \epsilon_T^{\rho\sigma} k_{T\rho} S_{L\sigma} \langle \bar{\psi} \gamma^+ \psi \rangle, \quad (2)$$

where $\epsilon_T^{\rho\sigma}$ is the transverse Levi-Civita tensor, and S_L is the longitudinal spin vector. Therefore, $h_{1L}^{\perp}$ is often called longitudinal transversity (abbreviated as longi-transversity). Another peculiarity of $h_{1L}^{\perp}$ is its chiral-odd nature, necessitating partner chiral-odd functions (e.g., Collins fragmentation function [6, 7]) for experimental accessibility via single spin asymmetry $A_{UL}^{\sin 2\phi}$ in semi-inclusive

Received 24 March 2025; Accepted 7 May 2025; Published online 8 May 2025

* Hui Li is supported by Fundamental Research Program of Shanxi Province (202203021222224) and Natural Science Foundation of Shanxi Normal University (JCYJ2023021). Xiao-Yu Wang is supported by the Natural Science Foundation of Henan Province (242300421377, 232300421140)

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deep inelastic scattering (SIDIS). Predictions for this asymmetry, as reported in Refs. [8–16], have been found to be approximately at the few-percent level, although systematic uncertainties remain significant.

The distribution h_{1L}^+ can also be probed through the Drell-Yan process. As demonstrated in Refs. [17, 18], the combination of h_{1L}^+ contributions from both colliding protons can generate a distinctive $\cos 2\phi$ azimuthal asymmetry in doubly longitudinally polarized Drell-Yan process, where ϕ denotes the azimuthal angle of the dilepton system with respect to the hadronic plane. Typically, the double spin asymmetry $A_{LL}^{\cos 2\phi}$ in the proton-proton Drell-Yan process is expected to be small as it is proportional to the product of the polarized quark and antiquark distributions. However, the polarized $p^\rightarrow p^\rightarrow$ Drell-Yan process provides unique sensitivity to sea quark dynamics within the proton. Notably, despite its theoretical significance, no systematic phenomenological investigation of this asymmetry has been reported in existing literature. This motivates our primary objective: a comprehensive analysis of $A_{LL}^{\cos 2\phi}$ in the polarized Drell-Yan process.

In this paper, we present a detailed phenomenological study of the double longitudinal spin asymmetry $A_{LL}^{\cos 2\phi}$ in the polarized pp Drell-Yan process within the TMD factorization framework [19–31]. Our predictions are computed for kinematic configurations accessible at both the Relativistic Heavy Ion Collider (RHIC) and Nuclotron-based Ion Collider Facility (NICA) [32]. Utilizing the TMD factorization, we provide the expressions of the spin-dependent cross section for $p^\rightarrow p^\rightarrow \rightarrow l^+ l^- X$ alongside the unpolarized one, with the asymmetry defined as the ratio of these two cross sections.

Over the past two decades, TMD factorization has emerged as an effective tool for exploring the three-dimensional structure of the nucleon, and it has been widely applied in various high-energy processes. The TMD factorization theorem expresses the differential cross section in the small transverse momentum region of the lepton ($q_\perp \ll Q$, where Q is the invariant mass of the dilepton pair and $q_\perp$ is the transverse momentum of the dilepton pair) as a convolution of two contributions: one representing hard scattering factors at short distances, and the other accounting for coherent long-distance interactions, described in terms of well-defined TMDs. In our analysis, $\cos 2\phi$ asymmetry results from the convolution of h_{1L}^+ (for both protons) with the hard scattering factors. Additionally, the TMD formalism encodes the evolution of TMDs, governed by the Collins-Soper (CS) equation [29–31, 33]. The solution to this equation is typically expressed as an exponential form of the Sudakov-like form factor [19, 30, 31, 34], which describes the transformation of TMDs from an initial scale to another. The Sudakov factor decomposes into perturbative and nonperturbative components: while the perturbative part admits a universal operator product expansion, the nonperturbat-

ive contribution requires phenomenological parameterization constrained by experimental data. Several nonperturbative components of the Sudakov form factor for TMDs have been extracted from experimental data in the literature [19, 21, 23, 24, 30, 31, 35–48]. For our analysis, we adopt the Bacchetta-Delcarro-Pisano-Radici-Signori (BDPRS) parameterization [24] for the h_{1L}^+ -associated nonperturbative Sudakov factor, building upon established extractions from deep inelastic scattering and Drell-Yan data [19, 21, 23, 24].

The remaining content of the paper is organized as follows. In Sec. II, we present the formalism of the asymmetry $A_{LL}^{\cos 2\phi}$ in the process $p^\rightarrow p^\rightarrow \rightarrow l^+ l^- X$ within the TMD factorization. In Sec. III, we investigate the evolution effect of the distribution h_{1L}^+ . Particularly, we discuss the parameterization of the non-perturbative Sudakov form factors associated with the h_{1L}^+ in detail. In Sec. IV, we present the phenomenological predictions for the asymmetry $A_{LL}^{\cos 2\phi}$ at RHIC and NICA. We conclude our paper in Sec. V.

II. FORMALISM OF THE ASYMMETRY $A_{LL}^{\cos 2\phi}$ IN THE DOUBLY POLARIZED PROTON-PROTON DRELL-YAN PROCESS

In this section, we present the theoretical framework for calculating the double longitudinal spin asymmetry $A_{LL}^{\cos 2\phi}$ in polarized proton-proton Drell-Yan collisions within the TMD factorization formalism [31], incorporating the scale evolution of TMDs. The asymmetry originates from the convolution of the h_{1L}^+ distributions of both colliding protons at leading twist. The specific process under study is described as follows [18]:

$$p^\rightarrow(P_1, S_1) + p^\rightarrow(P_2, S_2) \rightarrow \gamma^*(q) + X \rightarrow l^+(\ell) + l^-(\ell') + X, \quad (3)$$

where $P_{1/2}, S_{1/2}$ denote the four-momenta and spin vectors of the incoming protons, respectively. The virtual photon momentum q is time-like, distinguishing this process from semi-inclusive deep inelastic scattering (SIDIS). Here, $Q^2 = q^2$ represents the invariant mass squared of the lepton pair, and the notation $\rightarrow$ indicates longitudinal polarization of the protons.

The following kinematic variables are frequently introduced to characterize the experimental observables:

$$\begin{aligned} s &= (P_1 + P_2)^2, & x_1 &= \frac{Q^2}{2P_1 \cdot q}, & x_2 &= \frac{Q^2}{2P_2 \cdot q}, \\ x_F &= 2q_L/s = x_1 - x_2, & \tau &= Q^2/s = x_1 x_2, \\ y &= \frac{1}{2} \ln \frac{q^+}{q^-} = \frac{1}{2} \ln \frac{x_1}{x_2}, \end{aligned} \quad (4)$$

where s represents the total center of the mass energy squared. The variables $x_{1/2}$ denote the longitudinal momentum fraction. q_L is the longitudinal momentum of the virtual photon, whereas x_F is the Feynman x variable. The variable y corresponds to the rapidity of the dilepton. Furthermore, $x_{1/2}$ can be expressed as functions of x_F , τ and y , τ , respectively, as follows:

$$x_{1/2} = \frac{\pm x_F + \sqrt{x_F^2 + 4\tau}}{2}, \quad x_{1/2} = \sqrt{\tau} e^{\pm y}. \quad (5)$$

In Drell-Yan processes, when the transverse momentum $\mathbf{q}_\perp$ of the dilepton pair is measured, the TMD factorization framework becomes applicable in the kinematic regime $\mathbf{q}_\perp \ll Q$ [19–28]. Within this framework, the differential cross section at leading twist can be expressed as [17]

$$\begin{aligned} & \frac{d\sigma}{dx_1 dx_2 d^2\mathbf{q}_\perp d\Omega} \\ &= \frac{\alpha_{\text{em}}^2}{4Q^2} \left\{ (1 + \cos^2 \theta) F_{UU} + S_{1L} S_{2L} \sin^2 \theta \cos 2\phi F_{LL}^{\cos 2\phi} \right. \\ & \quad + |S_{1T}| S_{2L} (1 + \cos^2 \theta) \cos \phi_1 F_{TL}^{\cos \phi_1} \\ & \quad + S_{1L} |S_{2T}| \sin^2 \theta \left[\cos(2\phi + \phi_2) F_{LT}^{\cos(2\phi + \phi_2)} \right. \\ & \quad \left. + \cos(2\phi - \phi_2) F_{LT}^{\cos(2\phi - \phi_2)} \right] \\ & \quad + |S_{1T}| |S_{2T}| (1 + \cos^2 \theta) \left[(\cos(\phi_1 + \phi_2) F_{TT}^{\cos(\phi_1 + \phi_2)} \right. \\ & \quad \left. + \cos(\phi_1 - \phi_2) F_{TT}^{\cos(\phi_1 - \phi_2)}) \right] + \dots \left. \right\}, \quad (6) \end{aligned}$$

where α_{em} is the fine-structure constant, $\phi_{1/2}$ denotes the azimuthal angle of the transverse spin vector $S_{(1/2)T}$ with respect to the lepton plane, and ϕ and θ represent the azimuthal and polar angles of the lepton momentum in the CS frame, respectively. The solid angle Ω specifies the orientation of the dilepton system.

Furthermore, $F_P^{f[\phi, \phi_{1/2}]}$ denotes the structure functions with a specific modulation $f[\phi, \phi_{1/2}]$, with $P = UU$, $P = LL$, $P = TL$, $P = LT$, $P = TT$ denoting the polarization of the incoming protons (U for unpolarized, T for transversely polarized, and L for longitudinally polarized). In this study, the relevant structure functions are F_{UU}^1 , representing the unpolarized structure function, and $F_{LL}^{\cos 2\phi}$, corresponding to the spin-dependent structure function. The ratio of these two structure functions defines the $\cos 2\phi$ azimuthal asymmetry:

$$A_{LL}^{\cos 2\phi} = \frac{F_{LL}^{\cos 2\phi}}{F_{UU}}, \quad (7)$$

which can be measured in double-longitudinally polarized Drell-Yan processes.

The spin-averaged structure function F_{UU} can be expressed as the convolution of the unpolarized distribution functions from each proton [17, 18]:

$$F_{UU} = C \left[f_1^{q/p} f_1^{\bar{q}/p} \right], \quad (8)$$

whereas the spin-dependent structure function $F_{LL}^{\cos 2\phi}$ is expressed as the convolution of the longi-transversity distributions [17, 18]:

$$F_{LL}^{\cos 2\phi} = C \left[\frac{2 (\hat{\mathbf{h}} \cdot \mathbf{k}_{1\perp}) (\hat{\mathbf{h}} \cdot \mathbf{k}_{2\perp}) - \mathbf{k}_{1\perp} \cdot \mathbf{k}_{2\perp}}{M^2} h_{1L}^{\perp, q/p} h_{1L}^{\perp, \bar{q}/p} \right], \quad (9)$$

where the unit vector $\mathbf{h}$ is defined as $\mathbf{h} \equiv \mathbf{q}_\perp / |\mathbf{q}_\perp|$, and M represents the mass of the proton [18, 49]. $\mathbf{k}_{1\perp}$ and $\mathbf{k}_{2\perp}$ denote the transverse momenta of the quark and anti-quark in the incoming protons, respectively. The convolution of TMDPDFs in the transverse momentum space is defined as follows [18]:

$$\begin{aligned} & C \left[w(\mathbf{k}_{1\perp}, \mathbf{k}_{2\perp}) f_1 f_2 \right] \\ &= \frac{1}{N_c} \sum_q e_q^2 \int d^2\mathbf{k}_{1\perp} d^2\mathbf{k}_{2\perp} \delta^{(2)}(\mathbf{q}_\perp - \mathbf{k}_{1\perp} - \mathbf{k}_{2\perp}) \\ & \quad \times \omega(\mathbf{k}_{1\perp}, \mathbf{k}_{2\perp}) f_1^q(x_1, \mathbf{k}_{1\perp}^2) f_2^{\bar{q}}(x_2, \mathbf{k}_{2\perp}^2), \quad (10) \end{aligned}$$

with $N_c = 3$ representing the number of colors, $\mathbf{q}_\perp$ denoting the transverse momentum of the lepton pair. Finally, $\omega(\mathbf{k}_{1\perp}, \mathbf{k}_{2\perp})$ is a function of $\mathbf{k}_{1\perp}$ and $\mathbf{k}_{2\perp}$.

Generally, a more feasible approach to study the structure function is the $b_\perp$ -space framework, where the convolution of TMDs can be simplified into a product of $b_\perp$ -dependent TMDs. Subsequently, the physical observables can be derived via Fourier transformation from the $b_\perp$ -space to the $k_\perp$ -space.

By using the Fourier transformation of the δ function

$$\delta^2(\mathbf{q}_\perp - \mathbf{k}_{1\perp} - \mathbf{k}_{2\perp}) = \frac{1}{(2\pi)^2} \int d^2\mathbf{b}_\perp e^{i\mathbf{b}_\perp \cdot (\mathbf{q}_\perp - \mathbf{k}_{1\perp} - \mathbf{k}_{2\perp})}, \quad (11)$$

we can obtain the spin-dependent structure function $F_{LL}^{\cos 2\phi}$, which is given by

$$\begin{aligned} F_{LL}^{\cos 2\phi} &= \frac{1}{N_c} \sum_q e_q^2 \int d^2\mathbf{k}_{1\perp} d^2\mathbf{k}_{2\perp} \int \frac{d^2\mathbf{b}_\perp}{(2\pi)^2} e^{i(\mathbf{q}_\perp - \mathbf{k}_{1\perp} - \mathbf{k}_{2\perp}) \cdot \mathbf{b}_\perp} \\ & \quad \times \left[\frac{2(\hat{\mathbf{h}} \cdot \mathbf{k}_{1\perp})(\hat{\mathbf{h}} \cdot \mathbf{k}_{2\perp}) - (\mathbf{k}_{1\perp} \cdot \mathbf{k}_{2\perp})}{M^2} \right] \\ & \quad \times h_{1L}^{\perp, q/p}(x_1, \mathbf{k}_{1\perp}^2, Q) h_{1L}^{\perp, \bar{q}/p}(x_2, \mathbf{k}_{2\perp}^2, Q) \\ &= \frac{1}{N_c} \sum_q e_q^2 \int \frac{d^2\mathbf{b}_\perp}{(2\pi)^2} e^{i\mathbf{q}_\perp \cdot \mathbf{b}_\perp} (2\mathbf{h}_\alpha \cdot \mathbf{h}_\beta - g_{\alpha\beta}) \\ & \quad \cdot \tilde{h}_{1L}^{\perp, q/p, \alpha}(x_1, \mathbf{b}_\perp, Q) \tilde{h}_{1L}^{\perp, \bar{q}/p, \beta}(x_2, \mathbf{b}_\perp, Q). \quad (12) \end{aligned}$$

Here, the tilde terms represent the ones in the $b_\perp$ -space. The longi-transversity of the incoming protons in the $b_\perp$ -space is defined as follows:

$$\tilde{h}_{1L}^{\perp q/p, \alpha}(x_1, \mathbf{b}_\perp; Q) = \int d^2 \mathbf{k}_{1\perp} e^{-i \mathbf{k}_{1\perp} \cdot \mathbf{b}_\perp} \frac{\mathbf{k}_{1\perp}^\alpha}{M} h_{1L}^{\perp q/p}(x_1, \mathbf{k}_{1\perp}^2; Q), \quad (13)$$

$$\tilde{h}_{1L}^{\perp \bar{q}/p, \beta}(x_2, \mathbf{b}_\perp; Q) = \int d^2 \mathbf{k}_{2\perp} e^{-i \mathbf{k}_{2\perp} \cdot \mathbf{b}_\perp} \frac{\mathbf{k}_{2\perp}^\beta}{M} h_{1L}^{\perp \bar{q}/p}(x_2, \mathbf{k}_{2\perp}^2; Q). \quad (14)$$

Similarly, the spin-averaged structure function F_{UU} can be expressed as

$$\begin{aligned} F_{UU} &= \frac{1}{N_c} \sum_q e_q^2 \int d^2 \mathbf{k}_{1\perp} d^2 \mathbf{k}_{2\perp} \int \frac{d^2 \mathbf{b}_\perp}{(2\pi)^2} e^{i(\mathbf{q}_\perp - \mathbf{k}_{1\perp} - \mathbf{k}_{2\perp}) \cdot \mathbf{b}_\perp} \\ &\quad \times f_1^{q/p}(x_1, \mathbf{k}_{1\perp}^2, Q) f_1^{\bar{q}/p}(x_2, \mathbf{k}_{2\perp}^2, Q) \\ &= \frac{1}{N_c} \sum_q e_q^2 \int \frac{d^2 \mathbf{b}_\perp}{(2\pi)^2} e^{i \mathbf{q}_\perp \cdot \mathbf{b}_\perp} \tilde{f}_1^{q/p}(x_1, \mathbf{b}_\perp, Q) \tilde{f}_1^{\bar{q}/p}(x_2, \mathbf{b}_\perp, Q), \end{aligned} \quad (15)$$

where the unpolarized distribution functions from each proton in the $b_\perp$ -space are defined as follows:

$$\tilde{f}_1^{q/p}(x_1, \mathbf{b}_\perp, Q) = \int d^2 \mathbf{k}_{1\perp} e^{-i \mathbf{k}_{1\perp} \cdot \mathbf{b}_\perp} f_1^{q/p}(x_1, \mathbf{k}_{1\perp}^2; Q), \quad (16)$$

$$\tilde{f}_1^{\bar{q}/p}(x_2, \mathbf{b}_\perp, Q) = \int d^2 \mathbf{k}_{2\perp} e^{-i \mathbf{k}_{2\perp} \cdot \mathbf{b}_\perp} f_1^{\bar{q}/p}(x_2, \mathbf{k}_{2\perp}^2; Q). \quad (17)$$

III. TMD EVOLUTION OF DISTRIBUTION FUNCTIONS

In this section, we systematically review the energy evolution of both unpolarized (f_1) and longi-transversity ($h_{1L}^\perp$) distributions within the TMD factorization framework. Our analysis incorporates recent theoretical developments in TMD resummation techniques and nonperturbative parameterizations.

Based on the TMD factorization theorem, as formulated in various systems (such as CS-81 [29], JMY [50, 51], and Collins-11 [31]), the distribution function $\tilde{F}(x, b; \mu, \zeta_F)$ in b -space depends on two energy scales: the renormalization scale μ , associated with the corresponding collinear distribution functions, and the energy scale ζ_F , which regularizes the light-cone singularity in the operator definition of TMDs. The ζ_F dependence of TMDs is governed by the CS equation [29] ($b = |\mathbf{b}_\perp|$):

$$\frac{\partial \ln \tilde{F}(x, b; \mu, \zeta_F)}{\partial \sqrt{\zeta_F}} = \tilde{K}(b; \mu), \quad (18)$$

where $\tilde{K}$ represents the CS evolution kernel, which can

be computed perturbatively at a small b region. The result up to order α_s has the form

$$\tilde{K}(b; \mu) = -\frac{\alpha_s C_F}{\pi} [\ln(\mu^2 b^2) - \ln 4 + 2\gamma_E] + O(\alpha_s^2), \quad (19)$$

where $\gamma_E \approx 0.577$ is Euler's constant.

The μ dependence of the TMDs is derived from the renormalization group equation

$$\frac{d\tilde{K}}{d \ln \mu} = -\gamma_K(\alpha_s(\mu)), \quad (20)$$

$$\frac{d \ln \tilde{F}(x, b; \mu, \zeta_F)}{d \ln \mu} = \gamma_F\left(\alpha_s(\mu); \frac{\zeta_F^2}{\mu^2}\right), \quad (21)$$

where γ_K and γ_F are the anomalous dimensions of $\tilde{K}$ and $\tilde{F}$, respectively,

$$\gamma_K = 2\frac{\alpha_s C_F}{\pi} + O(\alpha_s^2), \quad (22)$$

$$\gamma_F = \alpha_s \frac{C_F}{\pi} \left(\frac{3}{2} - \ln \left(\frac{\zeta_F}{\mu^2} \right) \right) + O(\alpha_s^2). \quad (23)$$

By solving Eqs. (18), (20), and (21), we can obtain the general solution for the energy dependence of TMDs:

$$\tilde{F}(x, b; Q) = \mathcal{F} \times e^{-S(Q, b)} \times \tilde{F}(x, b; \mu), \quad (24)$$

where $\mathcal{F}$ is the hard scattering factor, and $S(Q, b)$ is the Sudakov-like form factor. Here, we have set $\mu = \sqrt{\zeta_F} = Q$, allowing us to simplify $\tilde{F}(x, b; \mu, \zeta_F)$ as $\tilde{F}(x, b; Q)$. Eq. (24) demonstrates that the energy evolution of TMDs from the initial energy μ to another energy scale Q can be achieved through the Sudakov form factor $S(Q, b)$ using the exponential form, $e^{-S(Q, b)}$.

The exponential $\exp(-S(Q, b))$ for $\tilde{F}$ can be expressed as [16, 52]

$$\begin{aligned} \exp(-S(Q, b)) &= \exp \left\{ \ln \frac{Q}{\mu} \tilde{K}(b_*; \mu) + \int_{\mu}^Q \frac{d\bar{\mu}}{\bar{\mu}} \right. \\ &\quad \times \left[\gamma_F(g(\bar{\mu}); 1) - \ln \frac{Q}{\bar{\mu}} \gamma_K(g(\bar{\mu})) \right] \Big\} \\ &\quad \times \exp \left\{ g_{i/p}(x, b) + g_K(b) \ln \frac{Q}{Q_0} \right\}. \end{aligned} \quad (25)$$

where the first exponential term results from the perturbative region ($b \ll 1/\Lambda$), containing the CS evolution kernel $\tilde{K}(b_*; \mu)$ within the perturbative region, specifically in the small b region where $b \ll 1/\Lambda$, and the anomalous dimensions γ_F and γ_K . The second exponential term ac-

counts for nonperturbative effects in the large b region, where $\tilde{K}(b;\mu)$ cannot be calculated perturbatively. Here, $g_{i/p}(x, b)$ parameterizes the intrinsic nonperturbative behavior of parton i within the proton, whereas $g_K(b)$ describes the nonperturbative behavior of the evolution kernel $\tilde{K}(b;\mu)$.

To ensure a smooth transition between the perturbative and nonperturbative regions, we introduce a boundary parameter $b_{\max} \approx 1 \text{ GeV}^{-1}$, along with a b -dependent function $b_*(b)$. This function is designed to have the properties $b_* \approx b$ for a small b value and $b_* \approx b_{\max}$ for a large b value. In Refs. [24, 30, 53], several different expressions for $b_*(b)$ have been proposed. A commonly used prescription is the Collins-Soper-Sterman (CSS) form [30]:

$$b_*(b) = b / \sqrt{1 + b^2/b_{\max}^2}, \quad b_{\max} < 1/\Lambda_{\text{QCD}}. \quad (26)$$

Therefore, the Sudakov form factor $S(Q, b)$ appearing in Eq. (24) can be decomposed into a perturbative part $S_P(Q, b_*)$ and a nonperturbative part $S_{NP}(Q, b)$ as follows:

$$S(Q, b) = S_P(Q, b_*) + S_{NP}(Q, b), \quad (27)$$

with the boundary of two parts set by the $b_{\max}$. According to the intensive studies in Refs. [23, 35, 36, 40, 54], the perturbative part can be expanded as an α_s/π series:

$$S_P(Q, b_*) = \int_{\mu^2}^{Q^2} \frac{d\bar{\mu}^2}{\bar{\mu}^2} \left[\mathcal{A}(\alpha_s(\bar{\mu})) \ln \frac{Q^2}{\bar{\mu}^2} + \mathcal{B}(\alpha_s(\bar{\mu})) \right], \quad (28)$$

where the coefficients A and B are given by

$$\mathcal{A} = \sum_{n=1}^{\infty} A^{(n)} \left(\frac{\alpha_s}{\pi} \right)^n, \quad (29)$$

$$\mathcal{B} = \sum_{n=1}^{\infty} B^{(n)} \left(\frac{\alpha_s}{\pi} \right)^n. \quad (30)$$

In our calculations, we take $A^{(n)}$ up to $A^{(2)}$ and $B^{(n)}$ up to $B^{(1)}$ in the accuracy of next-to-leading logarithmic (NLL) order [19, 30, 35, 37, 54, 55]:

$$\begin{aligned} A^{(1)} &= C_F, \\ A^{(2)} &= \frac{C_F}{2} \left[C_A \left(\frac{67}{18} - \frac{\pi^2}{6} \right) - \frac{10}{9} T_R n_f \right], \\ B^{(1)} &= -\frac{3}{2} C_F, \end{aligned} \quad (31)$$

where $C_F = \frac{4}{3}$, $C_A = 3$, $T_R = \frac{1}{2}$, and $n_f = 5$.

The non-perturbative part S_{NP} in Eq. (27) cannot be calculated perturbatively; it is frequently extracted from experimental data. Several authors have performed different types of extractions for S_{NP} [19, 21, 23, 24, 30, 31, 35–48]. In our study, we employ the BDPRS parameterization for the unpolarized TMDs [24]:

$$S_{NP}^{q/p} = S_{NP}^{\bar{q}/p} = -\frac{1}{2} g_K(b) \ln(Q^2/Q_0^2) - \ln(\tilde{f}_{1NP}^p(x, b^2)), \quad (32)$$

where $g_K(b) = -g_2 b^2/2$, following the traditional choice in Refs. [37, 43, 45], with g_2 being a free parameter. Moreover, the intrinsic nonperturbative part $\tilde{f}_{1NP}^p(x, b^2)$ of the TMDs can be parameterized in the following form:

$$\tilde{f}_{1NP}^a(x, b^2) = \frac{1}{2\pi} e^{-g_1 \frac{b^2}{4}} \left(1 - \frac{\lambda g_1^2}{1 + \lambda g_1} \frac{b^2}{4} \right), \quad (33)$$

with

$$g_1(x) = N_1 \frac{(1-x)^\alpha x^\sigma}{(1-\hat{x})^\alpha \hat{x}^\sigma}, \quad (34)$$

Here, $\hat{x}$ is fixed as $\hat{x} = 0.1$, and α , σ , and $N_1 \equiv g_1(\hat{x})$ are free parameters fitted to the available data from SIDIS, Drell–Yan, and W/Z boson production processes. In Ref. [24], a new $b_*(b)$ prescription different from Eq. (26) was proposed as

$$b_*(b) = b_{\max} \left(\frac{1 - e^{-b^4/b_{\max}^4}}{1 - e^{-b^4/b_{\min}^4}} \right)^{1/4}. \quad (35)$$

Again, $b_{\max}$ serves as the boundary separating the nonperturbative and perturbative regions in the b space. It is assigned a fixed value of $b_{\max} = 2e^{-\gamma_E} \text{ GeV}^{-1} \approx 1.123 \text{ GeV}^{-1}$. Moreover, the authors in Ref. [24] choose to saturate b_* at the minimum value, where $b_{\min} \propto 2e^{-\gamma_E}/Q$.

In addition to the Sudakov form factor in Eq. (24), another crucial element within Eq. (24) is the TMDPDFs at a fixed scale μ . In the small b region, $\tilde{F}(x, b; \mu)$ at μ can be expressed as a convolution of the perturbatively calculable coefficients C and the corresponding collinear TMDs [29, 56]:

$$\tilde{F}(x, b; \mu) = \sum_i C_{q \leftarrow i} \otimes F_{i/H}(x, \mu), \quad (36)$$

where $\sum_i$ runs over both quark and antiquark flavors, and $\otimes$ denotes the convolution in the longitudinal momentum fraction x :

$$C_{q \leftarrow i} \otimes F_{i/H}(x, \mu) \equiv \int_x^1 \frac{d\xi}{\xi} C_{q \leftarrow i}(x/\xi, b; \mu) F_{i/H}(\xi, \mu). \quad (37)$$

Here, $F_{i/H}(\xi, \mu)$ is the corresponding collinear TMDs of flavor i in hadron H at the scale μ , which can be a dynamical scale related to b_* by $\mu = c_0/b_*$, with $c_0 = 2e^{-\gamma_E}$ [29].

Therefore, with the TMD evolution, the scale-dependent TMDs $\tilde{F}_{q/H}(x, b; Q)$ can be expressed as

$$\tilde{F}_{q/H}(x, b; Q) = e^{-\frac{1}{2}S_P(Q, b_*) - S_{NP}^{F_{q/H}}(Q, b)} \mathcal{F}(\alpha_s(Q)) \times \sum_i C_{q \leftarrow i}^F \otimes F_{i/H}(x, \mu), \quad (38)$$

where the factor $1/2$ preceding S_P results from the fact that S_P of quarks and antiquarks satisfies the following relation [57]:

$$S_P^q(Q, b_*) = S_P^{\bar{q}}(Q, b_*) = S(Q, b_*)/2. \quad (39)$$

In addition, the hard coefficients C , $\mathcal{F}$ for f_1 and $h_{1L}^\perp$ have been calculated up to next-to-leading order (NLO). Nevertheless, only the first term of the $h_{1L}^\perp$ result in Eq. (60) of Ref. [58] (specifically, the $\tilde{h}(x)$ term) dominates. In this study, we need not consider the contribution of $\tilde{T}_F^{(\sigma)}$ because it is beyond the Wandzura-Wilczek (WW)-approximation. For consistency, here, we adopt the leading-order results for the hard coefficients C and $\mathcal{F}$ for f_1 and $h_{1L}^\perp$, where $\mathcal{F} = 1$ and $C_{q \leftarrow i}^0 = \delta_{iq}\delta(1-x)$.

Using the above choices, the expression for the unpolarized TMD of the proton $\tilde{f}_1^{q/p}$ in b -space reduces to [24]

$$\tilde{f}_1^{q/p}(x, b; Q) = e^{-\frac{1}{2}S_P(Q, b_*) - S_{NP}^{f_1^{q/p}}(Q, b)} \tilde{f}_1^{q/p}(x, \mu). \quad (40)$$

By performing the Fourier transformation, we can convert the function $\tilde{f}_1(x, b; Q)$ into the transverse momentum space:

$$f_1^{q/p}(x, k_\perp; Q) = \int_0^\infty \frac{dbb}{2\pi} J_0(|\mathbf{k}_\perp|b) e^{-\frac{1}{2}S_P(Q, b_*) - S_{NP}^{f_1^{q/p}}(Q, b)} f_{1,q/p}(x, \mu), \quad (41)$$

where J_0 is the Bessel function of the first kind.

According to Eqs. (36) and (37), in the small b region, we can express the longi-transversity of the incoming protons at scale μ in terms of the perturbatively calculable coefficients and the corresponding collinear correlation function as follows [59, 60]:

$$\tilde{h}_{1L}^{\perp(\alpha)q/p}(x, b; \mu) = ib^\alpha M h_{1L}^{\perp(1)}(x, \mu). \quad (42)$$

Here, the superscript (1) denotes the first transverse moment of the longi-transversity $h_{1L}^\perp$, which is defined as [59, 60]

$$h_{1L}^{\perp(1)}(x, \mu) = \int d^2\mathbf{k}_\perp \frac{\mathbf{k}_\perp^2}{2M^2} h_{1L}^\perp(x, \mathbf{k}_\perp^2; \mu). \quad (43)$$

Regarding the nonperturbative part of the Sudakov form factor associated with the longi-transversity, the information still remains unknown. In practical calculations, we assume that it is the same as that for the unpolarized distribution function $S_{NP}^{f_1^{q/p}}$. Thus, we can obtain the longi-transversity in b -space as

$$\tilde{h}_{1L}^{\perp(\alpha)q/p}(x, b; Q) = ib^\alpha M e^{-\frac{1}{2}S_P(Q, b_*) - S_{NP}^{f_1^{q/p}}(Q, b)} h_{1L}^{\perp(1)}(x, \mu). \quad (44)$$

After the Fourier transformation is performed, the longi-transversity in the transverse momentum space is given by

$$\frac{k_\perp^\alpha}{M} h_{1L}^{\perp(\alpha)q/p}(x, k_\perp; Q) = M \int_0^\infty \frac{dbb^2}{2\pi} J_1(|\mathbf{k}_\perp|b) e^{-\frac{1}{2}S_P(Q, b_*) - S_{NP}^{f_1^{q/p}}(Q, b)} \times h_{1L}^{\perp(1)}(x, \mu). \quad (45)$$

With all ingredients described above, Eqs. (12) and (15) can be rewritten in the following forms:

$$F_{LL}^{\cos 2\phi} = \frac{1}{N_c} \sum_q e_q^2 \int_0^\infty \frac{dbb^3}{2\pi} J_2(|\mathbf{q}_\perp|b) M^2 h_{1L}^{\perp(1),q/p}(x_1, \mu) \times h_{1L}^{\perp(1),\bar{q}/p}(x_2, \mu) \times e^{-\left(S_{NP}^{f_1^{q/p}} + S_{NP}^{f_1^{\bar{q}/p}} + S_P\right)}, \quad (46)$$

$$F_{UU}^1 = \frac{1}{N_c} \sum_q e_q^2 \int_0^\infty \frac{dbb}{2\pi} J_0(|\mathbf{q}_\perp|b) f_1^{q/p}(x_1, \mu) f_1^{\bar{q}/p}(x_2, \mu) \times e^{-\left(S_{NP}^{f_1^{q/p}} + S_{NP}^{f_1^{\bar{q}/p}} + S_P\right)}. \quad (47)$$

IV. NUMERICAL CALCULATION

In this section, we present a comprehensive numerical analysis of $\cos 2\phi$ azimuthal asymmetry in double-longitudinally polarized proton-proton Drell-Yan collisions utilizing the theoretical framework established in previous sections. Our calculations are performed for the kinematic regimes accessible at both RHIC and NICA, with particular emphasis on the role of sea quark contributions.

The calculation requires the collinear functions $h_{1L}^{\perp(1)}(x, \mu)$ and $f_1(x, \mu)$ as inputs to Eqs. (46) and (47). For the unpolarized distribution $f_1(x, \mu)$, we employ the NLO CT10 parameterization (central PDF set) from Ref. [61]. For the longitudinally polarized transversity distribution

$h_{1L}^{\perp(1)}(x, \mu)$, we adopt the WW approximation [59, 62]:

$$h_{1L}^{\perp(1)}(x) \stackrel{\text{WW-type}}{\approx} -x^2 \int_x^1 \frac{dy}{y^2} h_1(y), \quad (48)$$

where h_1 is the transversity distribution. For the valence quark component of the transversity distribution, we select the parameterization extracted from Ref. [28], which employed the TMD evolution formalism. At the initial scale $Q_0 = \sqrt{2.4}$ GeV, it is given by

$$h_1^q(x, Q_0) = N_q^h x^{a_q} (1-x)^{b_q} \frac{(a_q + b_q)^{a_q + b_q}}{a_q^{a_q} b_q^{b_q}} \times \frac{1}{2} (f_1^q(x, Q_0) + g_1^q(x, Q_0)). \quad (49)$$

Here, g_1^q is the helicity distribution function [63], and f_1^q is the unpolarized distribution function. For the sea quark component of the transversity distribution, we make the following assumption:

$$h_1^{\text{sea}}(x, Q_0) = N \frac{1}{2} (f_1^q(x, Q_0) + g_1^q(x, Q_0)), \quad (50)$$

where N represents the proportion of the contribution of sea quarks in the proton. In this calculation, we consider three scenarios for the contribution ratio of sea quarks in the proton: $N = 0$, $N = 0.5$, and $N = 1$.

We employ the QCDNUM package [64] for the evolution of f_1 from the initial scale $Q_0 = \sqrt{2.4}$ GeV to another energy level. Concerning the energy evolution of $h_{1L}^{\perp(1)}$, the effect has been studied in Refs. [26, 28, 65, 66]. For simplicity, we consider only the homogenous terms in the evolution kernel:

$$P_{qq}^h = C_F \left[\frac{2x}{(1-x)_+} + 2\delta(1-x) \right] - \frac{C_F}{2} \frac{2x}{(1-x)}. \quad (51)$$

Drell-Yan experiment at RHIC is proposed to use two

proton beams colliding at either $\sqrt{s} = 200$ GeV or 500 GeV [67]. In this study, we estimate the $A_{LL}^{\cos 2\phi}$ asymmetry at the kinematical regions of RHIC [32]:

$$4 \text{ GeV} < Q < 9 \text{ GeV}, \quad 0 < q_T < 1 \text{ GeV}, \quad 1 < y < 2.$$

We also estimate the $A_{LL}^{\cos 2\phi}$ asymmetry at the kinematical regions of NICA [32]:

$$\begin{aligned} \sqrt{s} &= 27 \text{ GeV}, & 4 \text{ GeV} < Q < 9 \text{ GeV}, \\ 0 < q_T < 1 \text{ GeV}, & 0.1 < x < 0.8. \end{aligned}$$

Figures 1 and 2 present our numerical predictions for the $\cos 2\phi$ azimuthal asymmetry in pp Drell-Yan collisions at RHIC kinematics. The left, middle, and right panels display the rapidity (y), transverse momentum ($q_\perp$), and invariant mass (Q) dependencies of the asymmetry, respectively. Figure 1 corresponds to $\sqrt{s} = 200$ GeV, whereas Fig. 2 shows results for $\sqrt{s} = 500$ GeV. The dashed, solid, and dotted lines represent the asymmetry predictions for sea quark contribution parameters $N = 0$, $N = 0.5$, and $N = 1$, respectively.

As shown in Fig. 1, in all the cases except $N = 0$ (where the asymmetry is zero), the $\cos 2\phi$ azimuthal asymmetry in the longitudinally polarized proton-proton Drell-Yan process from our calculation is positive. Our estimates also show that the asymmetry changes slightly with the change in y . For $N = 0.5$, the magnitude of the y -dependent asymmetries is in the range of 2% to 5%, whereas for $N = 1$, it is in the range of 8% to 10%. For $q_\perp$ asymmetry, we find a strong growth with $q_\perp$ for $N = 1$, reaching 17.5% at $q_\perp \approx 1$ GeV. A moderate increase in the asymmetry with increasing $q_\perp$ is found for $N = 0.5$, peaking at 5%. For the Q -dependent asymmetry, $A_{LL}^{\cos 2\phi}$ moderately decreases from 12% to 5% as Q increases from 4 to 9 GeV when $N = 1$. Weaker Q dependence is observed for $N = 0.5$ (2%–4%). The same tendency is observed in Fig. 2, but with larger magnitudes (not more

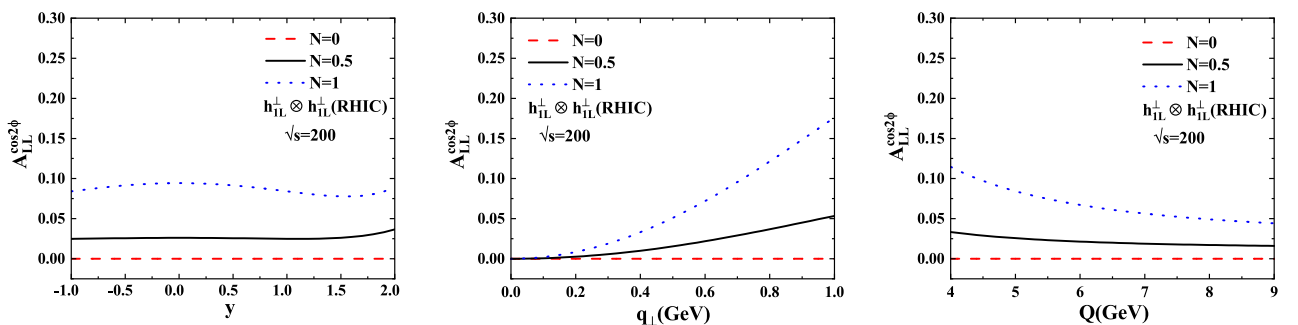


Fig. 1. (color online) Asymmetry $A_{LL}^{\cos(2\phi)}$ in the double-longitudinally polarized pp Drell-Yan process at the kinematic range of RHIC with $\sqrt{s} = 200$ GeV as functions of y (left panel), $q_\perp$ (middle panel), and Q (right panel). The dashed, solid, and dotted lines depict the asymmetry for $N = 0, 0.5, 1$, respectively.

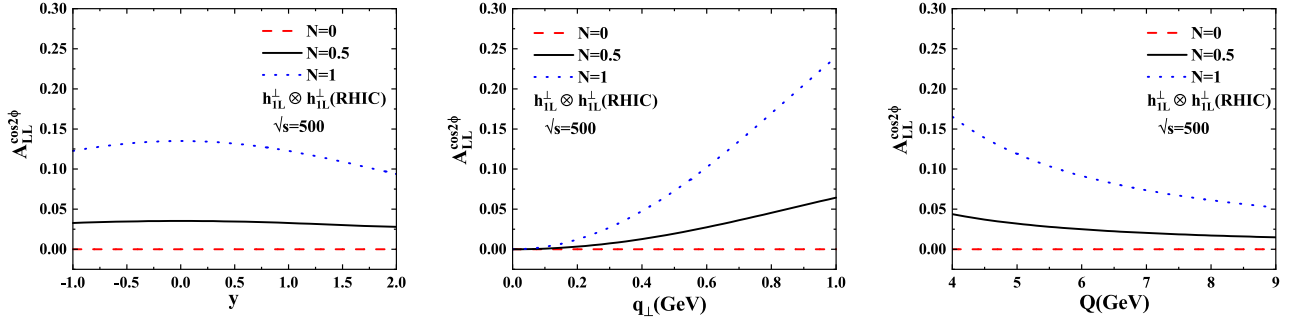


Fig. 2. (color online) Similar to Fig. 1, but for the asymmetry at the kinematic range of RHIC with $\sqrt{s} = 500$ GeV.

than 25%).

The NICA predictions ($\sqrt{s} = 27$ GeV) in Fig. 3 exhibit a qualitative behavior similar to that of the RHIC results but with different quantitative features. For the Bjorken- x dependence, we observe non-monotonic behavior in the asymmetry for $N = 1$, with a maximum value reaching 22%. The asymmetry initially decreases in the range $0.1 < x < 0.2$, followed by a gradual increase up to $x \approx 0.7$. This behavior reflects the complex interplay between valence and sea quark contributions in the longitudinally polarized proton. In terms of TMD dependence, the $q_\perp$ behavior at NICA shows similar trends to RHIC but with reduced magnitudes. The maximum asymmetry reaches 17% for $N = 1$, demonstrating the sensitivity of the measurement to the transverse momentum distribution of quarks within the proton. The Q -dependence at NICA exhibits a behavior comparable to that of the RHIC results, with the asymmetry exhibiting a gradual decrease as Q increases. The peak asymmetry reaches 15% for $N = 1$, consistent with expectations from TMD factorization. These results highlight the importance of measuring the asymmetry across different kinematic regimes to fully constrain the sea quark contributions and their dependence on the hard scale Q .

We also provide an estimate of the Drell-Yan event rate at NICA. The projected luminosity at NICA is $\mathcal{L} = 10^{32} \text{ cm}^{-2} \text{ s}^{-1}$ [68], with an annual operation time of $3 \times 10^7 \text{ s}$; the integrated luminosity can be obtained as

$$\mathcal{L}_{\text{int}} = \mathcal{L} \times t \approx 3 \times 10^{39} \text{ cm}^{-2}. \quad (52)$$

The unpolarized cross section σ^{DY} calculated in our framework (Eq. (6)) is initially in natural units (GeV^{-2}). Converting it to experimental units cm^2 via the relation $1 \text{ GeV}^{-2} \approx 3.894 \times 10^{-28} \text{ cm}^2$, we obtain $\sigma^{\text{DY}} \approx 5 \times 10^{-37} \text{ cm}^2$. The expected number of events per year can be estimated as

$$N_{\text{events}} = \sigma^{\text{DY}} \times \mathcal{L}_{\text{int}} \approx 1600. \quad (53)$$

This yield suggests that the proposed asymmetry $A_{LL}^{\cos 2\phi}$ could be experimentally accessible at NICA.

In summary, as demonstrated in Figs. 1–3, the $\cos 2\phi$ azimuthal asymmetries in longitudinally polarized proton-proton Drell-Yan collisions are consistently positive for all cases except $N = 0$, where the asymmetry vanishes. Specifically, the asymmetry reaches a maximum of 25% for $N = 1$ and up to 10% for $N = 0.5$, highlighting a strong dependence on the longitudinally polarized transversity distribution of sea quarks. These results underscore the potential of future precision measurements at RHIC and NICA to significantly enhance our understanding of nucleon structure and QCD dynamics. Moreover, the doubly polarized $p^\rightarrow p^\rightarrow$ Drell-Yan process emerges as a powerful probe for elucidating the role of sea quarks in the proton's internal structure, providing crucial insights into the nonperturbative regime of strong interactions.

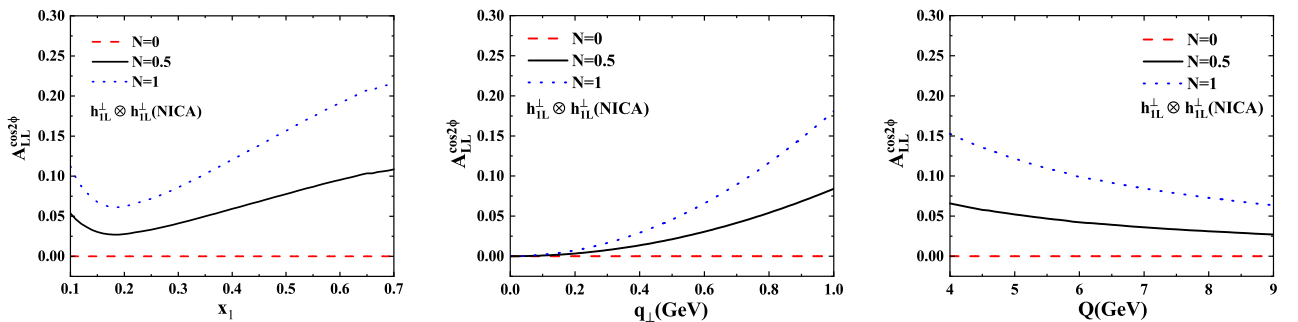


Fig. 3. (color online) Similar to Fig. 1 but for the asymmetry at the kinematic range of NICA.

V. CONCLUSION

In this study, we employ the TMD factorization framework to explore the $\cos 2\phi$ azimuthal asymmetry in double-longitudinally polarized proton-proton Drell-Yan collisions at the kinematic regimes relevant to RHIC and NICA. This asymmetry originates from the coupling of the longi-transversity distributions h_{1L}^+ from both proton beams. To incorporate the scale evolution of TMDs, we introduce the Sudakov form factor for h_{1L}^+ , which consists of perturbative and nonperturbative components. For the perturbative part, we adopt the result up to the NLL accuracy, whereas the nonperturbative part is modeled using the BDPRS parameterization. The hard coefficients associated with the corresponding collinear functions are retained at leading-order precision.

For the longi-transversity distribution of the proton,

we adopt the WW approximation. Using this framework, we compute the $\cos 2\phi$ azimuthal asymmetry in the p^+p^+ Drell-Yan process at RHIC and NICA. Our results reveal a sensitivity of the asymmetry to the sea quark contributions, parameterized by N . Specifically, for $N = 1$, the asymmetry reaches a maximum value of approximately 25%; for $N = 0.5$, it decreases to approximately 10%. This indicates that the sea quark contribution plays an important role in shaping the observed effect.

Our analysis demonstrates that the study of $\cos 2\phi$ asymmetry can provide quantitative constraints on the longi-transversity distribution within the proton. Future high-precision measurements of this asymmetry at RHIC and NICA could provide valuable information about valence and sea quark distributions, further enhancing our understanding of proton structure.

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