Roles of $\Delta(1232)$, $N^*(1520)$, and $N^*(1650)$ resonances in $\gamma p \to \pi^0\pi^0 p$ reaction within an effective Lagrangian approach*

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Abstract: The roles of the $\Delta(1232)$, $N^*(1520)$, and $N^*(1650)$ resonances in the $\gamma p \to \pi^0\pi^0 p$ reaction near threshold is investigated within an effective Lagrangian approach. The differential cross sections of the $\gamma p \to \pi^0\pi^0 p$ reaction was calculated including contributions from the $\Delta(1232)$, $N^*(1520)$, and $N^*(1650)$ intermediate states decaying into $\pi^0 p$ via the *s*-channel nucleon pole and *t*-channel ρ exchange. Current experimental measurements were well reproduced. The production of $\Delta(1232)$ was mainly from the mechanism of the *s*-channel nucleon pole, while the $N^*(1520)$ and $N^*(1650)$ were produced from the mechanism of the *t*-channel ρ exchange. More experimental data on the $\gamma p \to \pi^0\pi^0 p$ reaction could be used to explore the properties of the low-lying excited baryon state.

Keywords: low-lying baryon, photoproduction, effective lagrangian approach

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I. INTRODUCTION

Studying the spectrum of low-lying excited baryons and their decaying properties from available experimental data is one of the most important topics in hadron physics [1−4]. In classical quark models, the baryon resonances are classified in shells according to the energy levels of the harmonic oscillator, and the quark model has achieved significant success in describing the baryon spectrum, particularly for the ground states. These models predict many excited baryons [5−8]. However, only some of them have been identified experimentally [1, 9, 10], with many predicted states lacking eperimental observation; a problem called the ‘missing baryon problem’. Meanwhile, for low-lying excited baryons, the roper resonance $N^*(1440)$ ($J^P = 1/2^+$) belongs to the $N = 2$ shell and is much lower than the first orbitally excited nucleon states, such as $N^*(1535)$ ($J^P = 1/2^-$) and $N^*(1520)$ ($J^P = 3/2^-$) resonances [11, 12]. This is called the ‘mass reversal problem’. Thus, on both the experimental and theoretical sides, research is required to further establish the light-quark baryon spectrum and explore the nature of their excited states.

For instance, the well-established isospin $I = 3/2$ baryon $\Delta(1232)$ with spin-parity $J^P = 3/2^+$ mostly couples to the πN channel, which implies it may have a large absolute value of πN compositeness [13]. For the $N^*(1520)$ resonance, it is the first orbitally excited (quark model prediction) nucleon resonance, and its coupling to the πN channel is strong [1, 12, 14]. However, it has been shown that the $N^*(1650)$ ($J^P = 1/2^-$) resonance has an important contribution to the associate strangeness production reactions [15−17]. Thus, the strangeness component should be further considered when studying the structure of excited baryons.

It should be pointed out that photo-production processes provide a unique site to investigate these intermediate baryon resonances with strangeness component and

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small couplings to πN. For instance, it was shown that the intermediate nucleon and Δ excited states play a crucial role in the two-body reactions of $\gamma p \to p\pi^0$ [18−21], $p\eta$ [21−23], $p\eta'$ [24−26], $p\omega$ [27−30], $p\phi$ [31−36], $K\Lambda$ [37−41], $K\Sigma$ [42, 43], $K^*\Lambda$ [44−49], $K^*\Sigma$ [46, 50−52], $K\Lambda^*(1405)$ [53−56], $K\Sigma^*(1385)$ [53, 57−62], and $K\Lambda^*(1520)$ [63−71]. Detailed investigations of these photo-production reactions have substantially enhanced our understanding of both the reaction mechanisms and properties of intermediate baryon resonances.

Recently, the differential cross sections of the reaction $\gamma p \to \pi^0\pi^0 p$ were first measured using a linearly polarized photon beam with energy from reaction thresholds up to 2.4 GeV by the LEPS2/BGOegg Collaboration [72]. Indeed, the two-pion photo-production processes off proton targets have been measured in numerous experiments [73−77] and theoretically investigated in Refs. [78, 79]. The $\gamma p \to \pi^0\pi^0 p$ reaction provides interesting details because many of its Born terms are strongly suppressed and most nucleon resonances as well as $\rho^0(770)$ cannot directly decay into two neutral pions. Although the main purpose of Ref. [72] was to investigate the nature of $f_0(980)$ decay into $\pi^0\pi^0$ from the $\gamma p \to f_0(980)p \to \pi^0\pi^0 p$ reaction, the uncertainties of the experimental data on the invariant $\pi^0\pi^0$ mass distributions are large and the extracted mass and width of $f_0(980)$ mesons are lower than the averaged values quoted in the Review of Particle Physics (RPP) [1]. The production of the $f_0(980)$ meson in the $\gamma p \to f_0(980)p \to \pi^0\pi^0(K\bar{K})p$ reactions was studied theoretically in Refs. [80−84] within an effective Lagrangian approach. It is worth mentioning that the experimental measurements of Ref. [72] show that the contributions of the $\Delta(1232)$, $N^*(1520)$, and $N^*(1650)$ resonances provide significant contributions to the final $\pi^0 p$ channel, and the resonances have clear peaks in the $\pi^0 p$ invariant mass distributions, which could be used to investigate their roles $\Delta(1232)$, $N^*(1520)$, and $N^*(1650)$ in this process.

In thisy stydy, using new experimental measurements from the LEPS2/BGOegg Collaboration [72], we study the roles of the $\Delta(1232)$, $N^*(1520)$, and $N^*(1650)$ resonances in the $\gamma p \to \pi^0\pi^0 p$ reaction within an effective Lagrangian approach, which is an important theoretical method for describing various processes in the resonance production region [85−115]. For the production of the $\Delta(1232)$, $N^*(1520)$, and $N^*(1650)$ resonances, both s-channel proton pole and t-channel ρ^0 exchange processes are considered. It is shown that the new experimental measurements on the $\pi^0 p$ invariant mass distributions of Ref. [72] can be well reproduced with the contributions of $\Delta(1232)$ in the s-channel process and $N^*(1520)$ and $N^*(1650)$ in the t-channel process. In this respect, this study shows how the new measurements of the $\gamma p \to \pi^0\pi^0 p$ reaction could be used to extract the properties of these baryon resonances decaying into $\pi^0 p$. Note that we will leave the study of scalar meson $f_0(980)$ in the $\gamma p \to \pi^0\pi^0 p$ reaction to further work when more precise experimental data are available.

The article is structuredas follows. Sec. II describes the theoretical formalism employed in the study. Then, the Sec. III provides the numerical results and discussions. A short summary is given in the final section.

II. THEORETICAL FORMALISM

In this section, we introduce the theoretical formalism to calculate the differential scattering cross section for the $\gamma p \to \pi^0 R \to \pi^0\pi^0 p$ [$R \equiv \Delta(1232)$, $N^*(1520)$, or $N^*(1650)$] reaction within the effective Lagrangian approach, which is widely used to investigate the scattering reactions in the resonance production region [68−71, 116, 117].

The basic tree level Feynman diagrams for the $\gamma p \to \pi^0\pi^0 p$ reaction are presented in Fig. 1, which includes the s-channel proton pole [Fig. 1(a)] and t-channel ρ^0 exchange process [Fig. 1(b)]. To compute the contributions of the terms shown in Fig. 1, we used the following effective interaction Lagrangian densities for the γpp and $\gamma\rho\pi$ vertices, as used in Refs. [118−120],

$$\mathcal{L}_{\gamma pp} = -e\bar{p}\left[\not{A} - \frac{\mathcal{K}_p}{2m_N}\sigma^{\alpha\beta}(\partial_\beta A_\alpha)\right]p, \tag{1}$$

$$\mathcal{L}_{\rho\gamma\pi} = \frac{eg_{\rho\gamma\pi}}{m_\rho}\varepsilon^{\mu\nu\alpha\beta}\partial_\mu\rho_\nu\partial_\alpha A_\beta\pi, \tag{2}$$

where $e = \sqrt{4\pi\alpha}$, with $\alpha = 1/137.036$ representing the

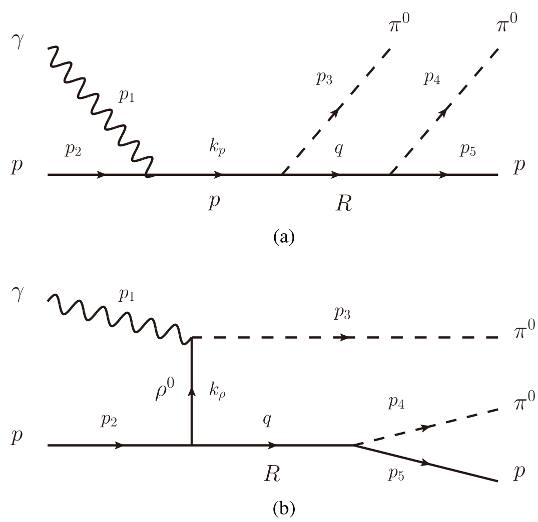

Fig. 1. Feynman diagrams of the reaction $\gamma p \to \pi^0 R \to \pi^0\pi^0 p$ [$R \equiv \Delta(1232)$, $N^*(1520)$, or $N^*(1650)$]. It consists of s-channel nuclepn pole (a) and t-channel ρ^0 meson exchange (b). We also show the definition of the kinematical (p_1, p_2, p_3, p_4, p_5) used in the present calculation. In addition, we use $k_p = p_1 + p_2$, $k_\rho = p_1 - p_3$, and $q = p_4 + p_5$.

electromagnetic fine structure constant. A_β, p, $\bar{p}$, π, and ρ_ν denote the fields of the photon, proton, antiproton, π, and ρ, respectively. The magnetic moment $\mathcal{K}_p=1.5$, which is taken from Ref. [83]. The coupling constant $g_{\rho\gamma\pi}$ can be obtained from the partial decay width:

$$\Gamma_{\rho^0\to\pi^0\gamma}=\frac{e^2g^2_{\rho\gamma\pi}}{96\pi}\frac{(m^2_{\rho^0}-m^2_{\pi^0})^3}{m^5_{\rho^0}}. \tag{3}$$

With $m_{\rho^0}=775.26$ MeV, $m_{\pi^0}=134.98$ MeV, and $\Gamma_{\rho^0\to\pi^0\gamma}=70.08$ keV [1, 113], one can easily get $g_{\rho\gamma\pi}$ = 0.57. Since we do not consider the interference terms between different resonances, the theoretical results will not depend on the sign of the coupling constant $g_{\rho\gamma\pi}$.

For the πNR vertices, we use the effective interaction Lagrangian commonly adopted in Refs. [102, 121, 122]:

$$\mathcal{L}^{\Delta(1232)}_{\pi N}=-\frac{g_{\pi N\Delta}}{m_\pi}\bar{R}_\mu\boldsymbol{\tau}\cdot\partial^\mu\boldsymbol{\pi}N+\text{h.c.}, \tag{4}$$

$$\mathcal{L}^{N^*(1520)}_{\pi N}=-\frac{g_{\pi NN^*(1520)}}{m_\pi}\bar{R}_\mu\gamma_5\boldsymbol{\tau}\cdot\partial^\mu\boldsymbol{\pi}N+\text{h.c.}, \tag{5}$$

$$\mathcal{L}^{N^*(1650)}_{\pi N}=ig_{\pi NN^*(1650)}\bar{R}\boldsymbol{\tau}\cdot\boldsymbol{\pi}N+\text{h.c.}, \tag{6}$$

where $\boldsymbol{\tau}$ is the Pauli matrix and $\boldsymbol{\pi}=(\pi_1,\pi_2,\pi_3)$. We take $\pi^+=(\pi_1-\mathrm{i}\pi_2)/\sqrt{2}$, $\pi^-=(\pi_1+\mathrm{i}\pi_2)/\sqrt{2}$, and $\pi^0=\pi_3$. In addition, the coupling constants $g_{\pi NR}$ are obtained from the following partial decay widths,

$$\Gamma_{\Delta(1232)\to N\pi}=\frac{g^2_{\pi N\Delta(1232)}}{4\pi}\frac{(E_N+m_N)}{m_{\Delta(1232)}m^2_\pi}|\boldsymbol{p}_N|^3, \tag{7}$$

$$\Gamma_{N^*(1520)\to N\pi}=\frac{g^2_{\pi NN^*(1520)}}{4\pi}\frac{(E_N-m_N)}{m_{N^*(1520)}m^2_\pi}|\boldsymbol{p}_N|^3, \tag{8}$$

$$\Gamma_{N^*(1650)\to N\pi}=\frac{3g^2_{\pi NN^*(1650)}}{4\pi}\frac{(E_N+m_N)}{m_{N^*(1650)}}|\boldsymbol{p}_N|, \tag{9}$$

with

$$|\boldsymbol{p}_N|=\frac{\lambda^{\frac{1}{2}}(m^2_R,m^2_\pi,m^2_N)}{2m_R}, \tag{10}$$

$$E_N=\frac{m^2_R+m^2_N-m^2_\pi}{2m_R}, \tag{11}$$

where the Källén function is $\lambda(a,b,c)=a^2+b^2+c^2-2ab-2ac-2bc$, and m_R represents the masses of the $\Delta(1232)$, $N^*(1520)$, or $N^*(1650)$ resonance.

With masses and decay widths of $\Delta(1232)$, $N^*(1520)$, and $N^*(1650)$ resonances to the πN channel [1], the coupling constants $g^2_{\pi NR}$ can be obtained as listed in Table 1.

Table 1. The coupling constants $g^2_{\pi NR}$ used in this work.

State	Mass/MeV	Width/MeV	$\mathcal{B}$	$g^2_{\pi NR}$
$\Delta(1232)$	1210	100	0.994	1.59
$N^*(1520)$	1505	110	0.60	2.55
$N^*(1650)$	1670	110	0.60	0.41

Next, for the ρNR vertices, we use the effective Lagrangians commonly adopted in previous works [121−123],

$$\mathcal{L}^{\Delta(1232)}_{\rho N}=\frac{-\mathrm{i}g_{\rho N\Delta}}{m_\rho}\bar{N}\gamma^\sigma\gamma_5\boldsymbol{\tau}\cdot[\partial_\mu\boldsymbol{\rho}_\sigma-\partial_\sigma\boldsymbol{\rho}_\mu]R^\mu+\text{h.c.}, \tag{12}$$

$$\mathcal{L}^{N^*(1520)}_{\rho N}=g_{\rho NN^*(1520)}\bar{N}\boldsymbol{\tau}\cdot\boldsymbol{\rho}^\mu R_\mu+\text{h.c.}, \tag{13}$$

$$\mathcal{L}^{N^*(1650)}_{\rho N}=\mathrm{i}g_{\rho NN^*(1650)}\bar{N}\gamma_5\left(\gamma_\mu-\frac{q_\mu\not{q}}{q^2}\right)\boldsymbol{\tau}\cdot\boldsymbol{\rho}^\mu R+\text{h.c.}. \tag{14}$$

In general, there are two independent couplings, S-wave and D-wave interactions, for the $N^*(1520)N\rho$ and $N^*(1650)N\rho$ vertexes. To reduce the number of free parameters, we consider only the S-wave coupling for both, and the effective interactions are taken from Ref. [121]. Moreover, since the mass threshold of ρN is higher than the masses of the $\Delta(1232)$, $N^*(1520)$, and $N^*(1650)$ resonances, the coupling constants $g_{\rho NR}$ cannot be extracted from the partial decay widths and must be determined with current experimental data, as cacluated in the following.

Note that the coupling constants $g_{\rho NR}$ have been investigated using the chiral quark model [124], where the relevant coupling constants are expressed in terms of the corresponding vector-coupling constants to nucleons. As an alternative, Ref. [88] proposed obtaining the value of $g_{\rho NR}$ from the $R\to N\rho\to N\pi\pi$ decay or the radiative decay $R\to N\gamma$ using the vector-meson-dominance model.

With the ingredients presented above, the total scattering amplitudes of the $\gamma p\to\pi^0R\to\pi^0\pi^0p$ reaction in the s-channel and t-channel can be written as

$$\mathcal{M}^{\Delta(1232)}_s=\mathrm{i}e\frac{g^2_{\pi N\Delta}}{m^2_\pi}F_p(k^2_p)F_\Delta(q^2)\bar{u}(p_5)G^{\omega\sigma}_{s=\frac{3}{2}}(q)G_{s=\frac{1}{2}}(k_p)\left(\gamma^\mu-\Gamma^\mu_c-\frac{\mathcal{K}_p}{2m_N}\gamma^\mu\not{p}_1\right)u(p_2)\varepsilon_\mu(p_1)p_{3\omega}p_{4\sigma}$$
$$+(\text{exchange terms with } p_3\leftrightarrow p_4), \tag{15}$$

$$\mathcal{M}_s^{N^*(1520)} = \mathrm{ie}\frac{g^2_{\pi NN^*(1520)}}{m_\pi^2} F_p(k_p^2) F_{N^*(1520)}(q^2)\bar{u}(p_5)\gamma_5 G^{\omega\sigma}_{s=\frac{3}{2}}(q)\gamma_5 G_{s=\frac{1}{2}}(k_p)\left(\gamma^\mu - \Gamma_c^\mu - \frac{\mathcal{K}_p}{2m_N}\gamma^\mu \not{p}_1\right) u(p_2)\varepsilon_\mu(p_1) p_{3\omega} p_{4\sigma} + (\text{exchange terms with } p_3 \leftrightarrow p_4), \tag{16}$$

$$\mathcal{M}_s^{N^*(1650)} = \mathrm{ie}g^2_{\pi NN^*(1650)} F_p(k_p^2) F_{N^*(1650)}(q^2)\bar{u}(p_5) G_{s=\frac{1}{2}}(q) G_{s=\frac{1}{2}}(k_p)\left(\gamma^\mu - \Gamma_c^\mu - \frac{\mathcal{K}_p}{2m_N}\gamma^\mu \not{p}_1\right) u(p_2)\varepsilon_\mu(p_1) + \left(\text{exchange terms with } p_3 \leftrightarrow p_4\right), \tag{17}$$

$$\mathcal{M}_t^{N^*(1520)} = \frac{\mathrm{ie}g_{\pi NN^*(1520)} g_{\rho NN^*(1520)} g_{\rho\gamma\pi}}{m_\pi m_\rho} F(k_\rho^2) F_{N^*(1520)}(q^2)\bar{u}(p_5)\gamma_5 p_{4\sigma} G^{\sigma\omega}_{s=\frac{3}{2}}(q) G_{s=1\omega\nu}(k_\rho) k_{\rho\mu} p_{1\alpha}\varepsilon^{\mu\nu\alpha\beta}\varepsilon_\beta(p_1) u(p_2) + (\text{exchange terms with } p_3 \leftrightarrow p_4), \tag{18}$$

$$\mathcal{M}_t^{N^*(1650)} = \frac{\mathrm{e}g_{\pi NN^*(1650)} g_{\rho NN^*(1650)} g_{\rho\gamma\pi}}{m_\rho} F(k_\rho^2) F_{N^*(1650)}(q^2)\bar{u}(p_5) G_{s=\frac{1}{2}}(q)\gamma_5 \left(\gamma^\sigma - \frac{q^\sigma \not{q}}{q^2}\right) G_{s=1\sigma\nu}(k_\rho) k_{\rho\mu} p_{1\alpha}\varepsilon^{\mu\nu\alpha\beta}\varepsilon_\beta(p_1) u(p_2) + (\text{exchange terms with } p_3 \leftrightarrow p_4), \tag{19}$$

where the propagator $G_{s=1}$ for the ρ meson with spin $s=1$ is given by [125]:

$$G^{\mu\nu}_{s=1}(k_\rho) = \mathrm{i}\frac{-g^{\mu\nu} + k_\rho^\mu k_\rho^\nu / k_\rho^2}{k_\rho^2 - m_\rho^2}, \tag{20}$$

the propagator $G_{s=\frac{1}{2}}$ is given by [95, 126],

$$G_{s=\frac{1}{2}}(q) = \frac{\mathrm{i}(\not{q} + m_R)}{q^2 - m_R^2 + i m_R \Gamma_R}, \tag{21}$$

for the nucleon pole and $N^*(1650)$ resonance, and the propagator $G^{\omega\sigma}_{s=\frac{3}{2}}$ is given by Refs. [126, 127]:

$$G^{\omega\sigma}_{s=\frac{3}{2}}(q) = \frac{\mathrm{i}(\not{q} + m_R) P^{\omega\sigma}(q)}{q^2 - m_R^2 + i m_R \Gamma_R}, \tag{22}$$

for the $\Delta(1232)$ and $N^*(1520)$ resonances, where

$$P^{\omega\sigma}(q) = -g^{\omega\sigma} + \frac{1}{3}\gamma^\omega\gamma^\sigma + \frac{1}{3m_R}(\gamma^\omega q^\sigma - \gamma^\sigma q^\omega) + \frac{2}{3m_R^2} q^\omega q^\sigma. \tag{23}$$

In equations above, m_R and Γ_R represent the masses and total widths of the intermediate baryon states, respectively. The four-momenta of the final two pions are represented by p_3 and p_4. In the calculation, we have also included terms where p_3 (p_4) is replaced by p_4 (p_3), ensuring that the resulting scattering amplitudes exhibit symmetry with respect to p_3 and p_4. In this process, we have taken into account the effects of the two identical π^0 particles.

Note that the contact term involving Γ_c^μ is taken into account to keep the scattering amplitudes gauge invariant. By including the following term Γ_c^μ in the scattering amplitudes $\mathcal{M}_s$ [128],

$$\Gamma_c^\mu = \frac{\not{p}_1 p_2^\mu}{p_1 \cdot p_2}, \tag{24}$$

it is easy to show that the total scattering amplitude satisfies gauge invariance

$$p_1 \cdot \mathcal{M}_{\text{total}} = 0, \tag{25}$$

with

$$\mathcal{M}^\mu_{\text{total}}\varepsilon_\mu(p_1) = \mathcal{M}_s^{\Delta(1232)} + \mathcal{M}_s^{N^*(1520)} + \mathcal{M}_s^{N^*(1650)} + \mathcal{M}_t^{N^*(1520)} + \mathcal{M}_t^{N^*(1650)}. \tag{26}$$

In additino, the contribution of $\Delta(1232)$ in the t-channel is zero because the $\Delta N\rho$ coupling vanishes when used in connection with the $\rho\pi\gamma$ vertex to avoid the gauge invariance problem.

Furthermore, because hadrons are not point-like particles, the form factors of hadrons must be considered. The relevant off-shell form factor is also used for the exchanged particles to account for the internal structure of hadrons and off-shell effects [83, 129, 130]. For the ρ

meson exchange, we introduce the form factors used in Refs. [131−133]:

$$F(k_\rho^2)=\frac{\Lambda_\rho^2-m_\rho^2}{\Lambda_\rho^2-k_\rho^2}, \tag{27}$$

where Λ_ρ, m_ρ, and k_ρ are the cutoff parameter, mass, and four-momentum of the exchanged ρ meson, respectively. In this work, we take Λ_ρ = 1.3 GeV [86].

For the intermediate baryon resonances, we adopt the form factor used in Refs. [48, 84, 127, 130],

$$F_R(q^2)=\frac{\Lambda^4}{\Lambda^4+(q^2-m_R^2)^2}, \tag{28}$$

where Λ, m_R, and q are the cutoff parameter, mass, and four-momentum of the exchanged baryon, respectively. In this calculation, the free cutoff parameters are adopted as follows: Λ = 1.0 GeV for Δ(1232), Λ = 2.0 GeV for $N^*(1520)$, and $N^*(1650)$ resonances [87, 122] and Λ = 1.1 GeV for the proton pole [83].

The values of the cutoff parameters can be directly related to the hadron structure. Since the question of hadron structure is still very open, the form factor and value of the cutoff parameter Λ cannot be estimated by first principles. Introducing the form factors has no unique theoretical approach. In practice, its value is usually determined by comparing theoretical calculations with the corresponding experimental measurements. When choosing the cutoff parameters as above, we follow the arguments given in previous studies [83−87, 122, 127], where such values were used.

III. NUMERICAL RESULTS AND DISCUSSIONS

In this section, we will show the numerical results for the $\gamma p\to\pi^0 R\to\pi^0\pi^0 p$ reaction. Considering the three-body phase space of the $\gamma p\to\pi^0\pi^0 p$ reaction (see more details in Fig. 2), one can obtain [1]

$$\frac{d\sigma}{dm_{\pi^0 p}}(W)=\frac{|\boldsymbol{p}_3||p_4^*|}{2^{10}\pi^5 W(W^2-m_p^2)}\int d\Omega_1\int d\Omega_2^* \times\frac{1}{4}\sum_{\text{spins}}\left|\mathcal{M}^{\mu}_{\text{total}}\varepsilon_\mu(p_1)\right|^2, \tag{29}$$

where W is the invariant mass of the γp system. Ω_1 and Ω_2^* are the solid angles in the center-of-mass system of the γp collision and in the center-of-mass system of the two-body $\pi^0 p$ final state, respectively. The $\boldsymbol{p}_3$ and p_4^* are the three-momenta of thr first π^0 (from the electromagnetic vertex) and second π^0 (from baryon resonance decay) in the center-of-mass system of the γp collision and the center-of-mass system of the two-body $\pi^0 p$ final state, respectively, and are given by

$$|\boldsymbol{p}_3|=\frac{\lambda^{\frac{1}{2}}(W^2,m_{\pi^0}^2,m_{\pi^0 p}^2)}{2W}, \tag{30}$$

$$|\boldsymbol{p}_4^*|=\frac{\lambda^{\frac{1}{2}}(m_{\pi^0 p}^2,m_{\pi^0}^2,m_p^2)}{2m_{\pi^0 p}}. \tag{31}$$

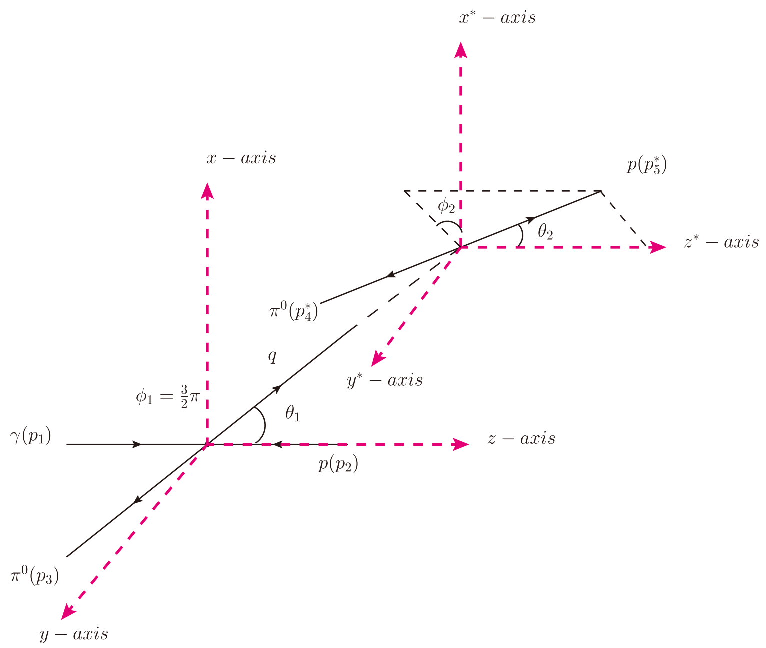

Fig. 2. (color online) Definitions of the variables in the phase space integration of the $\gamma p\to\pi^0\pi^0 p$ reaction.

To compare the theoretical calculations with the experimental measurements from the LEPS2/BGOegg Collaboration in Ref. [72], we calculate the differential cross section $d\sigma/dm_{\pi^0 p}$ by

$$\frac{d\sigma}{dm_{\pi^0 p}} = \frac{\int_{W_{\min}}^{W_{\max}} \frac{d\sigma}{dm_{\pi^0 p}}(W)dW}{W_{\max} - W_{\min}}, \quad (32)$$

with $W_{\min} = 1898$ MeV and $W_{\max} = 2320$ MeV, which are the total energy regions of the experimental measurements given in Ref. [72]. A constant background term is also included. Thus, the $|\mathcal{M}_{\rm total}|^2$ is written as

$$\begin{aligned} \left|\mathcal{M}^{\mu}_{\rm total}\varepsilon_{\mu}(p_1)\right|^2 = c_1 \Big(&\left|\mathcal{M}_s^{\Delta(1232)}\right|^2 + \left|\mathcal{M}_s^{N^*(1520)}\right|^2 \\ &+ \left|\mathcal{M}_s^{N^*(1650)}\right|^2 + \left|\mathcal{M}_t^{N^*(1520)}\right|^2 \\ &+ \left|\mathcal{M}_t^{N^*(1650)}\right|^2 + c_2 \Big), \end{aligned} \quad (33)$$

where the interference terms between different resonances are ignored. 1) In addition, it is found that the contributions of the $N^*(1520)$ and $N^*(1650)$ resonances in the s-channel are rather small and can be also ignored. The factors c_1 and c_2 are introduced to scale the theoretical differential cross sections to match the experimental measurements of the signal yields.

In Eqs. (15)−(17), we added a contact term Γ_c^{μ} to the interaction vertex to keep each of the s-channel scattering amplitudes gauge invariant, which implies that each $|\mathcal{M}_s^R|^2$ is also gauge invariant. Meanwhile, each of the scattering amplitudes for the t-channel are individually gauge invariant, see Eqs. (18) and (19). Thus, each $|\mathcal{M}_t^R|^2$ is also gauge invariant. Finally, although the interference terms are neglected, the sum of $|\mathcal{M}_i^R|^2 (i = s,t)$ in Eq. (33) is still gauge invariant.

We show the theoretical results for the $\pi^0 p$ invariant mass distributions at total energy region of $1898 < W < 2320$ MeV in Fig. 3, where one can see that the experimental data can be well reproduced. The experimental values are represented in blue, while the light blue error band reflects the 15% uncertainty of the experimental data [72]. The three prominent peaks correspond to the $\Delta(1232)$, $N^*(1520)$, and $N^*(1650)$ resonances. The theoretical numerical results are obtained with $c_1 = 1.08\times10^6$, $c_2 = 5.14\times10^2$ and the coupling constants $g_{\rho NN^*(1520)} = 33.34$ and $g_{\rho NN^*(1650)} = 30.37$.

Next we turn to the invariant mass distributions of the $\pi^0\pi^0$ of this process2). It is worth to mention that on the experimental side, to improve the signal-to-noise ratio of the $f_0(980)$ meson, the LEPS2/BGOegg Collaboration screened the events, ultimately leading to the number of events in the sample being 133000. Thus, approximately 27% of the event numbers were lost. In the theoretical calculations for the $\pi^0\pi^0$ invariant mass spectrum, an extra global factor of 73% was taken consdiered to more effectively compare our theoretical results with the number of experimental events. Additionally, based on the experimental data, the area ratio of the high-energy region ($2110 < W < 2320$ MeV) to the low-energy region ($1898 < W < 2110$ MeV) in the spectrum was 0.75 (see more details in Ref. [72]), which is

$$\begin{aligned} &0.73\sum_{m_{\pi^0 p}}\text{Events} = \sum_{m^{\rm low}_{\pi^0\pi^0}}\text{Events} + \sum_{m^{\rm high}_{\pi^0\pi^0}}\text{Events}, \\ &\sum_{m^{\rm low}_{\pi^0\pi^0}}\text{Events}\Big/\sum_{m^{\rm high}_{\pi^0\pi^0}}\text{Events} = 0.75. \end{aligned} \quad (34)$$

Therefore, to achieve a more accurate comparison with experimental results, we introduced two overall factors $C_{\rm low}$ and $C_{\rm high}$ in the low-energy and high-energy regions of the $\pi^0\pi^0$ invariant mass spectrum as follows:

$$\begin{aligned} 0.73\int\frac{d\sigma}{dm_{\pi^0 p}}dm_{\pi^0 p} = &\frac{1}{2}C_{\rm low}\int_{\rm low}\frac{d\sigma}{dm_{\pi^0\pi^0}}dm_{\pi^0\pi^0} \\ &+\frac{1}{2}C_{\rm high}\int_{\rm high}\frac{d\sigma}{dm_{\pi^0\pi^0}}dm_{\pi^0\pi^0}, \\ &\frac{\frac{1}{2}C_{\rm low}\int_{\rm low}\frac{d\sigma}{dm_{\pi^0\pi^0}}dm_{\pi^0\pi^0}}{\frac{1}{2}C_{\rm high}\int_{\rm high}\frac{d\sigma}{dm_{\pi^0\pi^0}}dm_{\pi^0\pi^0}} = 0.75. \end{aligned} \quad (35)$$

Then, we could obtain $C_{\rm low} = 0.68$ and $C_{\rm high} = 0.77$, respectively. With these values, in Fig. 4 and Fig. 5 we show the theoretical results of $d\sigma/dm_{\pi^0\pi^0}$ for the $\pi^0\pi^0$ invariant mass distributions at lower and higher ($1898 < W < 2110$ MeV) total energy regions ($2110 < W < 2320$ MeV) compared with the experimental measurements taken from Ref. [72]. The theoretical calculations can roughly reproduce the experimental data. The contribution of $\Delta(1232)$ provides two bump structures in the invariant $\pi^0\pi^0$ mass distributions.

As discussed in the Introduction the contribution from the $f_0(980)$ meson should be important to the $\gamma p \to \pi^0\pi^0 p$ reaction. However, the current data are limited, and we leave the study of the $f_0(980)$ meson for future studies

1) In this work, we focus on the analysis of the $\pi^0 p$ invariant mass distributions of the $\gamma p \to \pi^0\pi^0 p$ reaction in the energy regions surrounding the excited states of the $\Delta(1232)$, $N^*(1520)$, and $N^*(1650)$. Because these three resonances are well separated in energy, we anticipate that the interference effects will have a negligible impact on the current analysis, especially, the interference terms do not affect the results around the three peaks of the above resonances.

2) Since we donot consider the contributions from $f_0(500)$ and $f_0(980)$ mesons, the theoretical results of $\pi^0\pi^0$ invariant mass distributions can be viewed as the background contributions.

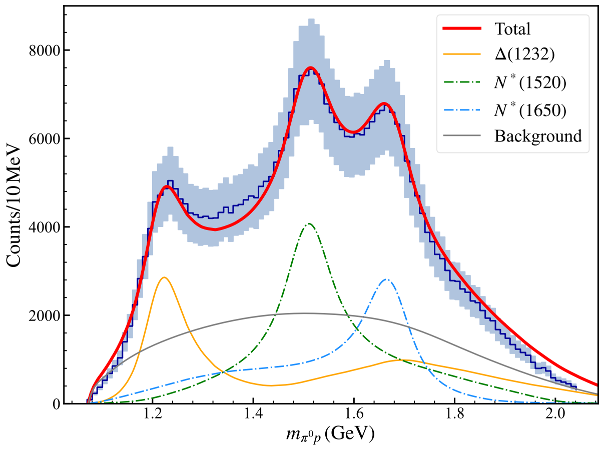

Fig. 3. (color online) Invariant mass distribution of $\pi^0 p$ in the energy range $1898 < W < 2320$ MeV compared with the experimental measurements taken from Ref. [72].

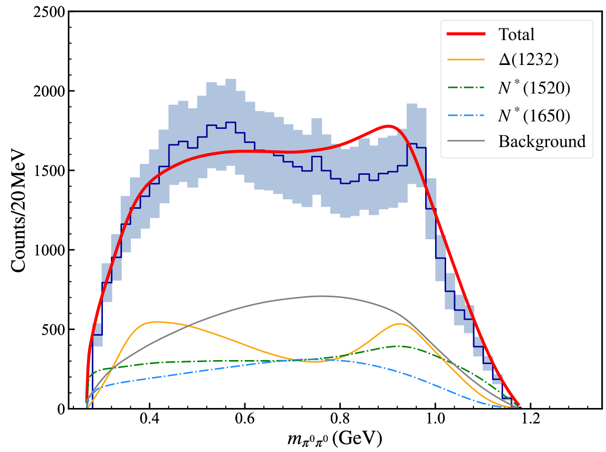

Fig. 4. (color online) Invariant mass spectra for the $\pi^0\pi^0$ pair in the total energy range $1898 < W < 2110$ MeV with experimental data taken from Ref. [72].

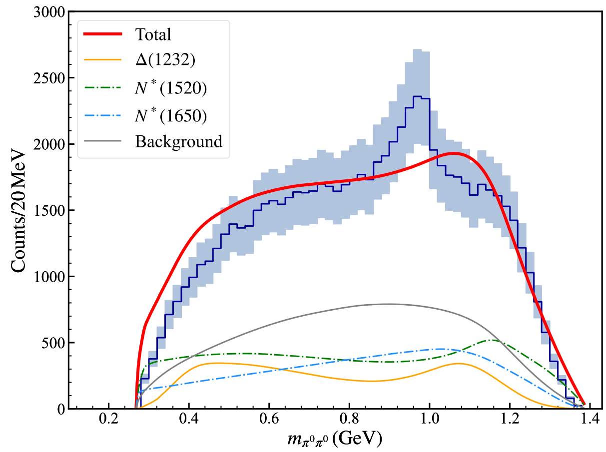

Fig. 5. (color online) Same as Fig. 4 but without invariant $\pi^0\pi^0$ mass distributions in the total energy range $2110 < W < 2320$ MeV.

when more experimental data are available.

IV. SUMMARY

In this work, the experimental data on the differential cross section of the $\gamma p \to \pi^0 R \to \pi^0\pi^0 p$ ($R \equiv \Delta(1232)$, $N^*(1520)$ and $N^*(1650)$) reaction provided by the LEPS2/BGOegg Collaboration [72] are analyzed by using the effective Lagrangian approach, where the tree level diagrams of s-channel nucleon pole and t-channel ρ^0 exchange are considered. We study the $\Delta(1232)$, $N^*(1520)$, and $N^*(1650)$ resonances decay into $\pi^0 p$ through P-wave, D-wave, and S-wave, respectively. In the considered energy region, the contributions of $\Delta(1232)$ in the s-channel and $N^*(1520)$ and $N^*(1650)$ in the t-channel are dominant. Moreover, it is interesting to expect that future measurements on this reaction will offer a further test of the $\rho NN^*(1520)$ and $\rho NN^*(1650)$ couplings and help us better understand the role of vector meson exchange in relevant reactions.

Finally, regarding the contributions of $\Delta(1232)$, $N^*(1520)$, and $N^*(1650)$ resonances in the $\pi^0 p$ invariant mass spectrum, it is hoped that more theoretical work and experimental measurements can be incorporated to further study the properties of the low-lying baryon excited states.

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