

Signature splitting in three-quasineutron rotational band $3/2[521]_v \otimes 1/2[660]_v \otimes 1/2[660]_v$ of $^{155}\text{Dy}^*$

Manpreet Kaur¹  Sushil Kumar¹ Sukhjeet Singh^{1†}  A. K. Jain^{1,2}

¹Department of Physics, Akal University Talwandi Sabo, Bathinda, Punjab-151302, India

²AINST, Amity University, Noida- 201313, India

Abstract: The signature splitting observed in the $3/2[521]_v \uparrow \otimes 1/2[660]_v \uparrow \otimes 1/2[660]_v \uparrow$ three-quasineutron rotational band of ^{155}Dy is examined within the framework of axially symmetric three-quasiparticle plus axially symmetric rotor model. The experimental level energies and magnitudes of observed splitting are well-reproduced with RMS deviations of 68.13 keV and 0.58 keV, respectively. The major contributing bands in the observed splitting are $K^\pi=7/2^-: 5/2[512]_v \uparrow \otimes 3/2[651]_v \uparrow \otimes 1/2[660]_v \downarrow$, $K^\pi=5/2^-: 5/2[512]_v \uparrow \otimes 3/2[651]_v \uparrow \otimes 3/2[651]_v \downarrow$, $K^\pi=1/2^-: 3/2[521]_v \uparrow \otimes 1/2[660]_v \uparrow \otimes 3/2[651]_v \downarrow$, $K^\pi=1/2^-: 3/2[521]_v \downarrow \otimes 1/2[660]_v \uparrow \otimes 3/2[651]_v \uparrow$, $K^\pi=5/2^-: 3/2[521]_v \uparrow \otimes 3/2[651]_v \uparrow \otimes 1/2[660]_v \downarrow$, and $K^\pi=3/2^-: 3/2[521]_v \uparrow \otimes 3/2[651]_v \uparrow \otimes 3/2[651]_v \downarrow$, which mix through rotor-particle ($\Delta K=1$) and particle-particle ($\Delta K=0$) couplings among the bands comprising the given basis space. The observed signature splitting is also well-reproduced by the superposition of calculated energy staggering of the strongly interacting bands, which further strengthens the validity of present particle rotor model calculations. Based on the present calculations, we assign the bandhead spin $K^\pi=3/2^-$ to the band under discussion. Additionally, the locations of 13 low-lying band members in the spin range $I^\pi=3/2^-$ to $23/2^-$ and at $27/2^-$ and $31/2^-$ are predicted, which will be useful for future experimental investigations.

Keywords: signature splitting, rotor-particle coupling, particle-particle coupling, three-quasiparticle rotational bands, three quasiparticle plus rotor model

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I. INTRODUCTION

Due to the invariance of nuclear wave functions under rotation by an angle π about an axis perpendicular to the symmetry axis, the two $\Delta I=2$ rotational branches of a given band are observed instead of one $\Delta I=1$ rotational branch. This splitting of the $\alpha = +1/2$ signature branch with respect to the $\alpha = -1/2$ branch of the same band is known as signature splitting and observed in the rotational bands composed of neutrons and/or protons occupying high- j orbitals. Over the last three decades, considerable efforts have been made to explain the physical mechanisms facilitating the observed signature splitting and signature inversion in one-quasiparticle (1qp) and two-quasiparticle (2qp) rotational bands observed in odd-A [1–3], odd-odd [4–9], and even-even [10, 11] nuclei. These investigations revealed various possible causes for the observed odd-even staggering in 1qp and 2qp rotational

bands, such as γ -deformation/triaxiality, residual n - p interactions, and contribution of rotor-particle and particle-particle couplings. Very limited efforts have been made to investigate the signature effects of three-quasiparticle (3qp) rotational bands observed in odd- A nuclei. Matsuzaki *et al.* [12, 13] proposed a microscopic model that includes both the static and dynamical triaxial deformation in the total Hamiltonian and studied these effects in 3qp rotational bands observed in ^{157}Ho , ^{159}Tm , and $^{161,165}\text{Lu}$ nuclides. They suggested that an increase in signature splitting is a consequence of an increase in deformation parameter with neutron number. It was emphasized that more accurate and systematic data of signature splitting and $B(M1)$ and $B(E2)$ ratios are required to perform more realistic calculations. Ikeda *et al.* [14] adopted a particle plus symmetric rotor model with γ -vibrational degree of freedom and suggested that the γ -dependency of the moment of inertia can influence the signature splitting and

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[†] E-mail: sukhjeet_phy@auts.ac.in

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signature inversion decisively. They attempted to explain the signature inversion observed in ^{157}Ho , ^{163}Lu , and ^{165}Lu nuclides, but the same calculations became less consistent in the case of ^{159}Tm , ^{161}Lu , and ^{155}Ho nuclides [14].

In the present paper, we perform axially symmetric 3qp plus rotor model (3QPPRM) [15, 16] calculations for theoretical explanation of signature splitting observed in a three-quasineutron rotational band based on the $3/2[521]_v \otimes 1/2[660]_v \otimes 1/2[660]_v$ configuration of ^{155}Dy (Band Nos. 5 & 7 of [17]). The particle rotor model calculations have an advantage over other calculations as this approach is in terms of angular momentum—a physical observable in the experiments; hence, a direct comparison with the experimental data can be made. We successfully reproduced the experimental level energies, phase, and magnitude of experimentally observed staggering pattern, and also propose the bandhead assignment as $K^\pi=3/2^-$ for the band under discussion. The underlying reasons for the observed signature splitting are understood in terms of rotor-particle and particle-particle couplings among various rotational bands participating in the given basis space. The remainder of this article is organized as follows. Sec. II briefly describes the methodology of 3QPPRM, Sec. III presents detailed discussion of the results, and the main conclusions of this study are summarized in Sec. IV.

II. METHODOLOGY: THREE-QUASIPARTICLE PLUS ROTOR MODEL

In present calculations, we used an axially symmetric 3QPPRM, which incorporates first- and higher-order Coriolis mixing of various 3qp rotational bands comprising the basis space designed for the band under discussion. In this approach, the total Hamiltonian (H) is divided into following two parts [15, 16]:

$$H = H_{\text{intrinsic}} + H_{\text{collective}} \quad (1)$$

where the first term $H_{\text{intrinsic}}$ consists of a Hamiltonian pertaining to a deformed axially symmetric mean field potential (H_{av}), pairing Hamiltonian (H_{pair}), and residual neutron-proton interactions (H_{res}). The collective part ($H_{\text{collective}}$) can be further divided into H_{rot}^o (a purely rotational contribution), H_{irrot} (irrotational component), H_{ppc} (contribution corresponding to the couplings of particles among themselves), H_{rpc} (contribution from the coupling of particles with even-even core), and H_{vib} (corresponding to vibrational interactions). Due to the weak coupling among the rotational and vibrational degrees of freedom, the vibrational interaction is not considered in the present model formulation. Therefore, the total Hamiltonian can be written as

$$H = H_{\text{av}} + H_{\text{pair}} + H_{\text{rot}}^o + H_{\text{irrot}} + H_{\text{ppc}} + H_{\text{pc}} + H_{\text{res}}. \quad (2)$$

H_{av} is the Nilsson model Hamiltonian with modified harmonic oscillator potential [18]. The eigenvalues and the single particle wavefunctions are calculated using potential parameters (k_p, μ_p for protons and k_n, μ_n for neutrons) adopted from Jain *et al.* [19] and deformation parameters ($\varepsilon_2, \varepsilon_4$) adopted from Moller *et al.* [20]. The contribution of the pairing Hamiltonian (H_{pair}) is estimated using finite difference mass formulae [21] using binding energies from the latest mass evaluation [22]. The analytical forms of the various terms that contribute to the total Hamiltonian are as follows:

$$H_{\text{rot}}^o = \frac{\hbar^2}{2\mathfrak{I}} [I^2 - I_z^2], \quad (3)$$

$$H_{\text{irrot}} = \frac{\hbar^2}{2\mathfrak{I}} [(j_1^2 - j_{1z}^2) + (j_2^2 - j_{2z}^2) + (j_3^2 - j_{3z}^2)], \quad (4)$$

$$H_{\text{ppc}} = \frac{\hbar^2}{2\mathfrak{I}} [(j_{1+}j_{2-} + j_{1-}j_{2+}) + (j_{2+}j_{3-} + j_{2-}j_{3+}) + (j_{1+}j_{3-} + j_{1-}j_{3+})], \quad (5)$$

$$H_{\text{rpc}} = -\frac{\hbar^2}{2\mathfrak{I}} [I_+J_- + I_-J_+], \quad (6)$$

where $\mathfrak{I}$ is the moment of inertia with respect to the rotation axis, subscripts (1, 2, and 3) represent particles 1, 2, and 3 constituting a given 3qp state, respectively, $I = R + J$ is the total angular momentum, R is the angular momentum of an even-even core, and $J = j_1 + j_2 + j_3$ is the total intrinsic angular momentum. $I_\pm = I_x \pm iI_y$, $I_\pm = I_{1\pm} + I_{2\pm} + I_{3\pm}$, $J_\pm = J_x \pm iJ_y$, $J_\pm = j_{1\pm} + j_{2\pm} + j_{3\pm}$ are raising and lowering operators. The matrix elements corresponding to the above Hamiltonians in the $|IMK\alpha\rangle$ basis are given as follows [15, 16]:

$$\begin{aligned} \langle IMK'\alpha' | H_{\text{rot}}^o | IMK\alpha \rangle &= \langle IMK'\alpha' | \frac{\hbar^2}{2\mathfrak{I}} [I^2 - I_z^2] | IMK\alpha \rangle \\ &= \frac{\hbar^2}{2\mathfrak{I}} [I(I+1) - K^2] \delta_{K'K} \delta_{\alpha'\alpha}, \end{aligned} \quad (7)$$

$$\begin{aligned} \langle IMK'\alpha' | H_{\text{irrot}} | IMK\alpha \rangle &= \frac{\hbar^2}{2\mathfrak{I}} \left[\left(\sum_{j_1} |C_{k_1}^{j_1}|^2 j_1(j_1+1) - k_1^2 \right) + \left(\sum_{j_2} |C_{k_2}^{j_2}|^2 j_2(j_2+1) - k_2^2 \right) + \left(\sum_{j_3} |C_{k_3}^{j_3}|^2 j_3(j_3+1) - k_3^2 \right) \right] \delta_{K'K} \delta_{\alpha'\alpha}, \end{aligned} \quad (8)$$

$$\langle IMK'\alpha' | H_{ppc} | IMK\alpha \rangle = \frac{\hbar^2}{2\mathfrak{I}} \left[\begin{aligned} & (\langle k'_1 \rho'_1 | j_{1+} | k_1 \rho_1 \rangle \langle k'_2 \rho'_2 | j_{2-} | k_2 \rho_2 \rangle + \langle k'_1 \rho'_1 | j_{1-} | k_1 \rho_1 \rangle \langle k'_2 \rho'_2 | j_{2+} | k_2 \rho_2 \rangle) \delta_{k'_3 k_3} \delta_{\rho'_3 \rho_3} \\ & + (\langle k'_2 \rho'_2 | j_{2+} | k_2 \rho_2 \rangle \langle k'_3 \rho'_3 | j_{3-} | k_3 \rho_3 \rangle + \langle k'_2 \rho'_2 | j_{2-} | k_2 \rho_2 \rangle \langle k'_3 \rho'_3 | j_{3+} | k_3 \rho_3 \rangle) \delta_{k'_1 k_1} \delta_{\rho'_1 \rho_1} \\ & + (\langle k'_1 \rho'_1 | j_{1+} | k_1 \rho_1 \rangle \langle k'_3 \rho'_3 | j_{3-} | k_3 \rho_3 \rangle + \langle k'_1 \rho'_1 | j_{1-} | k_1 \rho_1 \rangle \langle k'_3 \rho'_3 | j_{3+} | k_3 \rho_3 \rangle) \delta_{k'_2 k_2} \delta_{\rho'_2 \rho_2} \end{aligned} \right] \delta_{K'K}, \quad (9)$$

$$\langle IMK'\alpha' | H_{rpc} | IMK\alpha \rangle = -\frac{\hbar^2}{2\mathfrak{I}} \left[\begin{aligned} & \left(\sqrt{(I+K)(I-K+1)} \langle k'_1 \rho'_1 k'_2 \rho'_2 k'_3 \rho'_3 | j_{1-} + j_{2-} + j_{3-} | k_1 \rho_1 k_2 \rho_2 k_3 \rho_3 \rangle \right) \delta_{K',K-1} \\ & + \left(\sqrt{(I-K)(I+K+1)} \langle k'_1 \rho'_1 k'_2 \rho'_2 k'_3 \rho'_3 | j_{1+} + j_{2+} + j_{3+} | k_1 \rho_1 k_2 \rho_2 k_3 \rho_3 \rangle \right) \delta_{K',K+1} \\ & + (-1)^{I+K} \left(\sqrt{(I+K)(I-K+1)} \langle k'_1 \rho'_1 k'_2 \rho'_2 k'_3 \rho'_3 | (j_{1+} + j_{2+} + j_{3+}) R_x | k_1 \rho_1 k_2 \rho_2 k_3 \rho_3 \rangle \right) \delta_{K',-K+1} \end{aligned} \right]. \quad (10)$$

H_{res} is the Hamiltonian corresponding to residual n - p interactions [15] and breaks degeneracy among various members of a given 3qp quadruplet and can be estimated using empirical data of Gallagher-Moszkowski (GM) splitting [23, 24] and Newby shift energies [25]. The wave functions corresponding to the total Hamiltonian can be written as the product of Wigner D-functions (D'_{MK}) and the intrinsic wave functions $|K\alpha\rangle$ [15, 16]:

$$|IMK\alpha\rangle = \sqrt{\frac{2I+1}{16\pi^2}} [D'_{MK}\langle K\alpha\rangle + (-1)^{I+K} D'_{M-K}\langle K\alpha\rangle R_x(\pi)] \quad (11)$$

where the index α characterizes the configuration ($\alpha = \rho_1 \rho_2 \rho_3$) of the unpaired particles such that $H_{av}|K\alpha\rangle = (\varepsilon_1 \rho_1 + \varepsilon_2 \rho_2 + \varepsilon_3 \rho_3)|K\alpha\rangle$, and $\varepsilon_1 \rho_1$, $\varepsilon_2 \rho_2$, and $\varepsilon_3 \rho_3$ are the quasiparticle energies of the quasiparticle states ρ_1 , ρ_2 , and ρ_3 respectively.

For a given 3qp quadruplet, various types of couplings of intrinsic angular momenta gives four bandheads, namely, $K_1 = |k_1 + k_2 + k_3|$, $K_2 = |k_1 + k_2 - k_3|$, $K_3 = |k_1 - k_2 + k_3|$, and $K_4 = |-k_1 + k_2 + k_3|$; hence, the basis space comprising the 3qp quadruplets involved in Coriolis mixing calculations becomes large and complex compared to the basis space involved in one- and two-quasiparticle Coriolis mixing calculations. Additionally, the limited availability of the required input data (GM splitting and Newby shift energies) for the calculations of bandhead energies of the bands taking part in the given basis space restricts the more appropriate choice of the basis space. Therefore, special care is required to design the basis space, including all the possible bands interacting through rotor-particle ($\Delta K=1$) and particle-particle couplings ($\Delta K=0$), and the resultant matrix is diagonalized to obtain energy eigenvalues and eigenvectors.

III. RESULTS AND DISCUSSION

In the present work, we examined the signature splitting observed in the 3qp rotational band based on the

$3/2[521]_v \otimes 1/2[660]_v \otimes 1/2[660]_v$ configuration of ^{155}Dy (Band Nos. 5 & 7 of Ref. [17]) using 3QPPRM. This band is observed in the spin range $I^\pi=25/2^-$ to $93/2^-$ with a band crossing at $I^\pi=73/2^-$, where it changes its character from 3qp to five-quasiparticle (5qp) rotational structure [17]. Additionally, the low-lying members (below $I^\pi=25/2^-$) of the band under discussion could not identified in the available experimental investigations; hence, predictions of these members in the present calculations will guide future experimental investigations of these lower spin levels. To explain the phase as well as magnitude of experimentally observed signature splitting, the basis space consisting of 48 3qp rotational bands stemming from the combination of $5/2[512]$, $3/2[521]$, $1/2[530]$, $1/2[660]$, and $3/2[651]$ quasineutrons is built. Each 3qp rotational band participating in the given basis space is characterized by three parameters, namely, band-head energy (E_a), inertia parameter ($\hbar^2/2\mathfrak{I}$) and Newby shift energy (E_N). To estimate the bandhead energies of all bands comprising the given basis space of the present calculations, we use an improved version of the semi-empirical formulation [26, 27]. The input values of inertia parameters ($\hbar^2/2\mathfrak{I}$) are estimated using experimental data of the first two levels of a given 3qp rotational band, if available; otherwise, it is calculated using inertia parameters of valence nucleons of neighboring odd- A and even-even nuclei as $\mathfrak{I}_{3qp} = \mathfrak{I}_{(1)} + \mathfrak{I}_{(2)} + \mathfrak{I}_{(3)} - \mathfrak{I}_{\text{even-even}}$, where $\mathfrak{I}_{(1)}$, $\mathfrak{I}_{(2)}$, $\mathfrak{I}_{(3)}$ are odd nucleon moments of inertia and $\mathfrak{I}_{\text{even-even}}$ is the moment of inertia of the even-even core. The matrix elements and single particle wavefunctions taking part in the present 3QPPRM calculations are obtained using the Nilsson model [18], with deformation parameters $\varepsilon_2=0.210$, $\varepsilon_4=-0.02$ [20] and potential parameters $\kappa_n=0.0636$, $\mu_n=0.393$ [19].

In Fig. 1, we present the variation of signature splitting ($E(I)-E(I-1)/2I$) with spin ($\hbar$) for the rotational band built over the $3/2[521]_v \otimes 1/2[660]_v \otimes 1/2[660]_v$ configuration observed in ^{155}Dy (Band Nos. 5 & 7 of Ref. [17]). The magnitude of signature splitting within the spin range

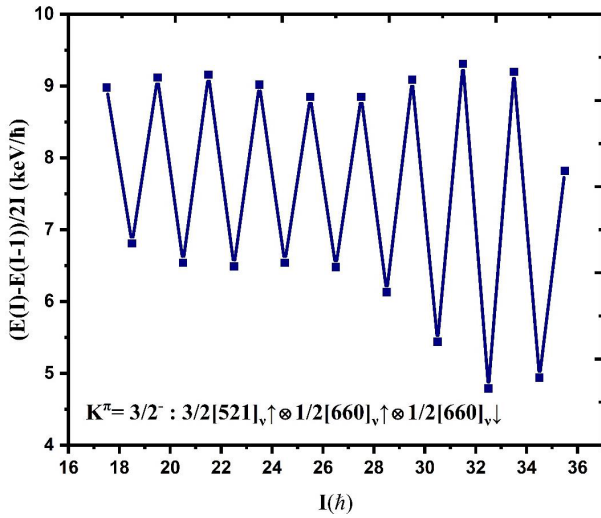


Fig. 1. (color online) Experimentally observed signature splitting in three-quasineutron rotational band $3/2[521]_v \otimes 1/2[660]_v \otimes 1/2[660]_v$ of ^{155}Dy (Band Nos. 5 & 7 of [17]).

of $I^\pi=25/2^- \hbar$ to $71/2^- \hbar$ varies from 4.79 to 9.31 keV.

To explain the observed signature splitting, we designed a structured basis space comprising 48 3qp rotational bands taking part in the present Coriolis mixing calculations. The values of the input parameters (E_α , $\hbar^2/2\mathfrak{I}$, and E_N) are varied within physical limits by minimizing χ^2 deviation [28] among calculated level energies and experimental data. The optimized set of values of the input parameters pertaining to the strongly in-

teracting rotational bands are listed in Table 1. In Fig. 2, we present a comparison of experimental and calculated energies; from this figure, it is clear that level energies of the three-quasineutron band under discussion are well-reproduced with a reasonable RMS deviation of 68.13 keV. These level energies along with calculated and experimental values of $E_\gamma(M1)$ and $E_\gamma(E2)$ are listed in Table 2.

In Fig. 3, we present a comparison of calculated and experimentally observed odd-even staggering patterns with spin. From this figure, it is clear that the present model calculations successfully reproduced the phase as well as magnitude of observed signature splitting with RMS deviation of 0.58 keV. Based on this good agreement among calculated level energies with experimental data, we suggest bandhead spin $K^\pi=3/2^-$ for the rotational band under investigation. We also confirm the three-quasineutron configuration assignment as $3/2[521]_v \uparrow \otimes 1/2[660]_v \uparrow \otimes 1/2[660]_v \downarrow$, which could not be confirmed from the available experimental data. Additionally, the prediction of lower-lying members will be useful for future experimental investigations. Because the band under discussion is the only experimentally observed band among the 48 three-quasineutron rotational bands comprising the given basis space, comparison of calculated level energies of the remaining 47 rotational bands of the given basis space with experimental data is not possible. To further strengthen the validity of our 3QPPRM calculations, we extracted the calculated stag-

Table 1. Optimized set of parameters (bandhead energies (E_α), inertia parameter ($\hbar^2/\text{keV}^{-1}$), Newby shift, and matrix elements) of the strongly interacting rotational bands of the given basis space of the present 3QPPRM calculations. The GM splitting and Newby shift energies used in the present calculations are adopted from Kondev [29] and Singh *et al.* [16], respectively. The Newby shift energies contributing to $K=0$ bands are given in parentheses along with the GM splitting energies. The optimized values of matrix elements are given in parentheses and listed in the lower part of this table.

S.No	Configuration $\Omega[Nn_z\Lambda]\Sigma$	K^π	E_α/keV	Inertia/ $(\hbar^2/\text{keV}^{-1})$	GM Splitting / (Newby shift)/keV		
					(1,2)	(2,3)	(3,1)
1.	$5/2[512]_v \uparrow \otimes 3/2[651]_v \uparrow \otimes 1/2[660]_v \downarrow$	$7/2^-$	2202.1	8.27	368	400	368
2.	$5/2[512]_v \uparrow \otimes 3/2[651]_v \uparrow \otimes 3/2[651]_v \downarrow$	$5/2^-$	2080.1	8.27	368	400 (-70)	368
3.	$3/2[521]_v \uparrow \otimes 1/2[660]_v \uparrow \otimes 1/2[660]_v \downarrow$	$3/2^-$	1000.0	8.61	368	400 (66.15)	368
4.	$3/2[521]_v \uparrow \otimes 1/2[660]_v \uparrow \otimes 3/2[651]_v \downarrow$	$1/2^-$	1505.0	8.27	368	400	368 (200)
5.	$3/2[521]_v \downarrow \otimes 1/2[660]_v \uparrow \otimes 3/2[651]_v \uparrow$	$1/2^-$	1537.0	8.27	368	400	368 (200)
6.	$3/2[521]_v \uparrow \otimes 3/2[651]_v \uparrow \otimes 1/2[660]_v \downarrow$	$5/2^-$	1305.0	8.27	368 (200)	400	368
7.	$3/2[521]_v \uparrow \otimes 3/2[651]_v \uparrow \otimes 3/2[651]_v \downarrow$	$3/2^-$	1382.9	8.27	368 (200)	400 (-70)	368 (200)
Matrix elements among quasineutron states							
	$\langle 1/2[530] J_+ 1/2[530] \rangle$				-0.971 (2.854)		
	$\langle 1/2[660] J_+ 1/2[660] \rangle$				0.069 (1.082)		
	$\langle 3/2[651] J_+ 1/2[660] \rangle$				0.923 (2.558)		
	$\langle 1/2[660] J_+ 1/2[660] \rangle$				0.069 (0.174)		
	$\langle 3/2[651] J_+ 1/2[660] \rangle$				0.923 (1.546)		

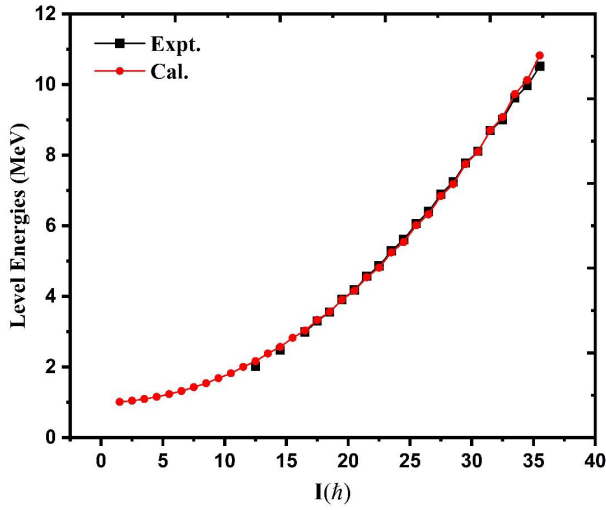


Fig. 2. (color online) Comparison of calculated level energies with the experimental data of three-quasineutron $K^\pi = 3/2^-$: $3/2[521]_{v\uparrow} \otimes 1/2[660]_{v\uparrow} \otimes 1/2[660]_{v\downarrow}$ band observed in ^{155}Dy (Band Nos. 5 & 7 of [17]).

gering patterns of the bands interacting strongly through $\Delta K=0$ (particle-particle) and $\Delta K=1$ (rotor-particle) couplings, as presented in Fig. 4.

In the top panel of this figure, we present the superposition of energy staggering calculated for the $K^\pi=7/2^-$: $5/2[512]_{v\uparrow} \otimes 3/2[651]_{v\uparrow} \otimes 1/2[660]_{v\downarrow}$, $K^\pi=5/2^-$: $5/2[512]_{v\uparrow} \otimes 3/2[651]_{v\uparrow} \otimes 3/2[651]_{v\downarrow}$, $K^\pi=3/2^-$: $3/2[521]_{v\uparrow} \otimes 1/2[660]_{v\uparrow} \otimes 1/2[660]_{v\downarrow}$, $K^\pi=1/2^-$: $3/2[521]_{v\uparrow} \otimes 1/2[660]_{v\uparrow} \otimes 3/2[651]_{v\downarrow}$, $K^\pi=1/2^-$: $3/2[521]_{v\downarrow} \otimes 1/2[660]_{v\uparrow} \otimes 3/2[651]_{v\uparrow}$, $K^\pi=5/2^-$: $3/2[521]_{v\uparrow} \otimes 3/2[651]_{v\uparrow} \otimes 1/2[660]_{v\downarrow}$, and $K^\pi=3/2^-$: $3/2[521]_{v\uparrow} \otimes 3/2[651]_{v\uparrow} \otimes 3/2[651]_{v\downarrow}$ bands, and we compare the superimposed staggering pattern with the experimental data (as shown in top panel of Fig. 4) gives us good confidence in our calculated energies of various bands taking part in the present Coriolis mixing calculations. The staggering amplitude parameters obtained in the above superposition are 0.011, -0.0093 , 0.963, 0.0073, 0.0010, 0.0146, and -0.0065 for the above-mentioned bands, respectively. The observed irregularities in the staggering patterns of the dominant bands can be attributed to differences in how strongly each band couples with the constituent bands that form the basis space.

Based on the present calculations, we assign a band-head spin of $K^\pi=3/2^-$ to the three-quasineutron $3/2[521]_{v\uparrow} \otimes 1/2[660]_{v\uparrow} \otimes 1/2[660]_{v\downarrow}$ rotational band under discussion. Additionally, we predict the location of 13 lower-lying members ($I^\pi=3/2^-$ to $23/2^-$, $27/2^-$, and $31/2^-$) of this band, which will be useful for future experimental investigations. In Fig. 5(a,b), we also present the variation of amplitude of the mixed wave functions ($|\psi|^2$), with spin of the bands interacting strongly through Coriolis and particle-particle couplings.

Table 2. Comparison of experimental and calculated level energies and transition energies.

$E_{\text{level}}^{\text{expt.}}$ /keV	Initial State I_i^π	Final State I_f^π	$E_\gamma^{\text{expt.}}$ /keV	$E_{\text{level}}^{\text{theo.}}$ /keV	$E_\gamma^{\text{theo.}}$ /keV
2012.3	25/2 ⁻	—	—	2165.8	—
—	27/2 ⁻	—	—	2383.3	—
2475.6	29/2 ⁻	25/2 ⁻	463.3	2569.3	403.5
—	31/2 ⁻	—	—	2827.2	—
2990.2	33/2 ⁻	29/2 ⁻	514.6	3035.2	465.9
3304.4	35/2 ⁻	—	—	3335.1	—
3556.3	37/2 ⁻	33/2 ⁻	566.1	3565.0	529.8
3912.1	39/2 ⁻	35/2 ⁻	607.7	3907.4	572.3
4180.2	41/2 ⁻	37/2 ⁻	623.9	4158.5	593.5
4573.9	43/2 ⁻	39/2 ⁻	661.8	4544.4	637.0
4865.8	45/2 ⁻	41/2 ⁻	685.6	4816.4	657.9
5289.7	47/2 ⁻	43/2 ⁻	715.8	5246.5	702.1
5610.2	49/2 ⁻	45/2 ⁻	744.4	5539.2	722.8
6061.8	51/2 ⁻	47/2 ⁻	772.1	6013.6	767.1
6405.2	53/2 ⁻	49/2 ⁻	795.0	6327.0	787.8
6892.2	55/2 ⁻	51/2 ⁻	830.4	6846.1	832.5
7241.4	57/2 ⁻	53/2 ⁻	836.2	7180.1	853.1
7778.0	59/2 ⁻	55/2 ⁻	885.8	7743.8	897.7
8109.7	61/2 ⁻	57/2 ⁻	868.3	8099.5	919.4
8696.4	63/2 ⁻	59/2 ⁻	918.4	8707.0	963.2
9008.0	65/2 ⁻	61/2 ⁻	898.3	9082.9	983.4
9624.4	67/2 ⁻	63/2 ⁻	928.0	9735.3	1028.3
9965.3	69/2 ⁻	65/2 ⁻	957.3	10132.6	1049.7
10520.6	71/2 ⁻	67/2 ⁻	896.2	10826.5	1091.2

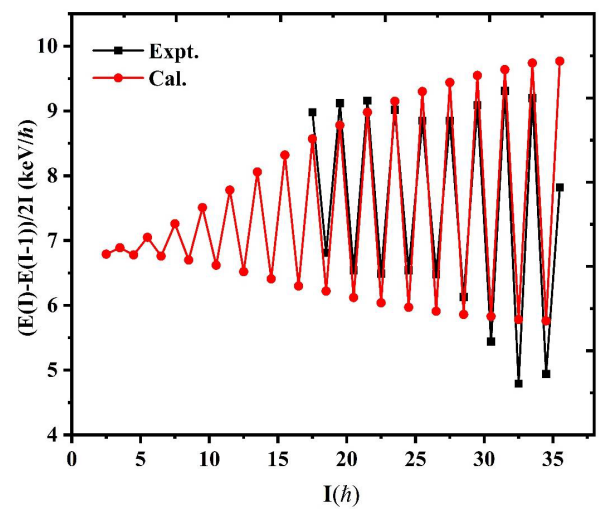


Fig. 3. (color online) Comparison of calculated and experimental staggering pattern exhibited by three-quasineutron rotational band based on $K^\pi=3/2^-$: $3/2[521]_{v\uparrow} \otimes 1/2[660]_{v\uparrow} \otimes 1/2[660]_{v\downarrow}$ configuration of ^{155}Dy .

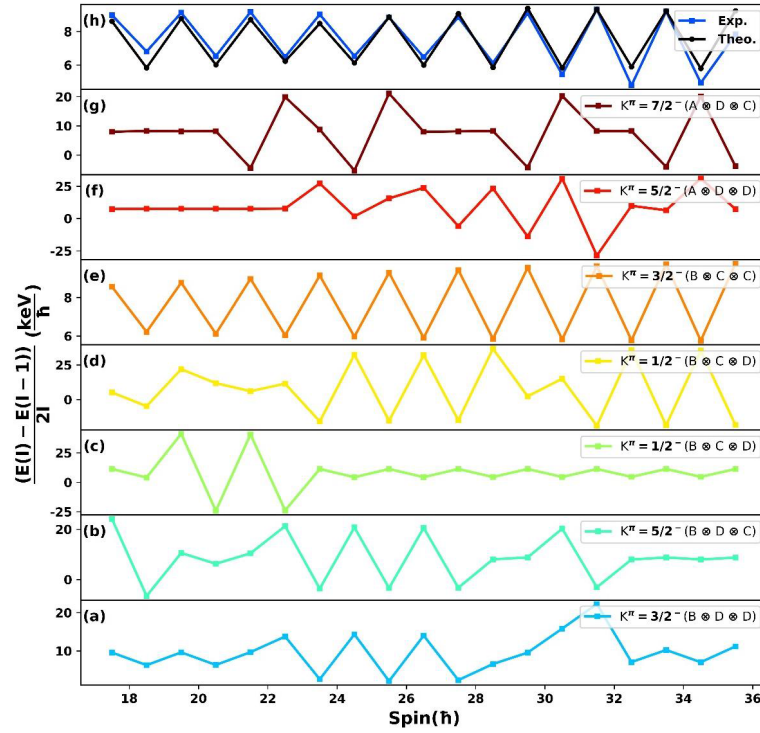


Fig. 4. (color online) Signature splitting of strongly interacting rotational bands obtained in present 3QPPRM rotor model calculations. In the top panel, the results of the superimposed staggered pattern are compared with the experimental data, where A: 5/2[512], B: 3/2[521], C: 1/2[660], and D: 3/2[651] represent quasineutron states.

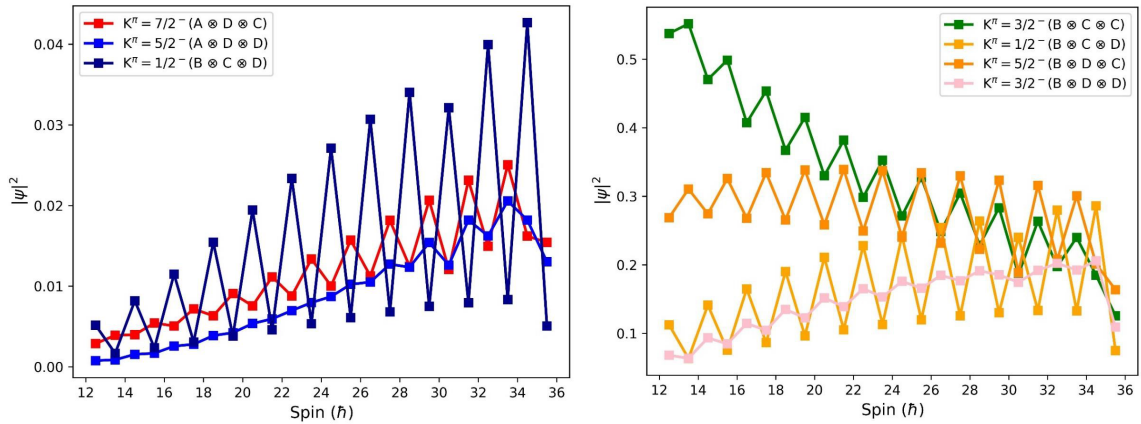


Fig. 5. (color online) (a): Variation of amplitude ($|\psi|^2$) of mixed wave functions with spin of the strongly interacting rotational bands participating in the basis space. (b): Same as (a) but for another set of rotational bands contributing substantially in the band under discussion, where B: 3/2[521], C: 1/2[660], and D: 3/2[651] represent quasineutron states.

IV. CONCLUSION

Three quasiparticle plus rotor model calculations were performed for the theoretical explanation of observed signature splitting in the three-quasineutron $3/2[521]_v \otimes 1/2[660]_v \otimes 1/2[660]_v$ rotational band of ^{155}Dy . The experimentally deduced level energies as well as the magnitude of observed signature splitting in the spin range of $I^\pi = 25/2^- \hbar$ to $71/2^- \hbar$ was successfully reproduced with reasonable RMS deviations of 68.13 keV and 0.58 keV, respectively. The bands contributing substantially to the

above staggering were $K^\pi = 7/2^-$: $5/2[512]_v \uparrow \otimes 3/2[651]_v \uparrow \otimes 1/2[660]_v \downarrow$, $K^\pi = 5/2^-$: $5/2[512]_v \uparrow \otimes 3/2[651]_v \uparrow \otimes 3/2[651]_v \downarrow$, $K^\pi = 1/2^-$: $3/2[521]_v \uparrow \otimes 1/2[660]_v \uparrow \otimes 3/2[651]_v \downarrow$, $K^\pi = 1/2^-$: $3/2[521]_v \downarrow \otimes 1/2[660]_v \uparrow \otimes 3/2[651]_v \uparrow$, $K^\pi = 5/2^-$: $3/2[521]_v \uparrow \otimes 3/2[651]_v \uparrow \otimes 1/2[660]_v \downarrow$, and $K^\pi = 3/2^-$: $3/2[521]_v \uparrow \otimes 3/2[651]_v \uparrow \otimes 3/2[651]_v \downarrow$, which interact strongly through Coriolis ($\Delta K = 0$) and particle-particle couplings ($\Delta K = 1$). The amplitude ($|\psi|^2$) of the mixed wavefunctions of the strongly interacting bands also exhibits a staggering pattern, which is an unusual feature. The observed signature

splitting was also well-reproduced by the superposition of energy staggering calculated for the strongly interacting bands, which further strengthened the validity of our calculations. Based on the present calculations, we assigned the bandhead spin $K^\pi=3/2^-$: $3/2[521]_{\nu\uparrow}\otimes 1/2[660]_{\nu\uparrow}\otimes 1/2[660]_{\nu\downarrow}$ to the band under discussion. We also predicted the locations of 13 lower-lying members corresponding to spin $I^\pi=3/2^-$ to $23/2^-$, $27/2^-$, and $31/2^-$ of the band under discussion, which will be useful for future experimental investigations.

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