Time-reversal asymmetries in $\Lambda_b \to \Lambda(\to p\pi^-)\ell^+\ell^-$*

Chao-Qiang Geng (耿朝强)[1,2,3] Chia-Wei Liu (刘佳韦)[1,2,3] Zheng-Yi Wei (魏正乙)[1,2,3†]

[1]School of Fundamental Physics and Mathematical Sciences, Hangzhou Institute for Advanced Study, UCAS, Hangzhou 310024, China
[2]University of Chinese Academy of Sciences, Beijing 100190, China
[3]CAS Key Laboratory of Theoretical Physics, Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100190, China

Abstract: We study the decays of $\Lambda_b \to \Lambda(\to p\pi^-)\ell^+\ell^-$ with $\ell = (e,\mu,\tau)$. We examine the full angular distributions with polarized Λ_b, where the T-odd observables are identified. We discuss the possible effects of new physics (NP) and find that the T-odd observables are sensitive to them as they vanish in the standard model. Special attention is given to the interference of (pseudo)scalar operators with (axial)vector operators in polarized $\Lambda_b \to \Lambda(\to p\pi^-)\tau^+\tau^-$, which are studied for the first time. Their effects are proportional to the lepton masses and therefore may evade the constraint from $\Lambda_b \to \Lambda(\to p\pi^-)\mu^+\mu^-$ at the LHCb naturally. As $\Lambda_b \to \Lambda(\to p\pi^-)\tau^+\tau^-$ is uncontaminated by the charmonia resonance, it provides a clean background to probe NP. In addition, we show that the experimental central value of K_{10} in $\Lambda_b \to \Lambda(\to p\pi^-)\mu^+\mu^-$ at the LHCb can be explained by the NP case, which couples to the right-handed quarks and leptons. The polarization fraction of Λ_b at the LHCb is found to be consistent with zero regardless of the NP scenarios.

Keywords: time reversal asymmetry, angular distribution, new physics, T-odd correlation

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I. INTRODUCTION

The CP violating observables in $b \to s\ell^+\ell^-$ with $\ell = (e,\mu,\tau)$ play important roles in probing new physics (NP) as they are highly suppressed in the standard model (SM) [1−7]. In recent years, special attention has been given to $B \to K^{(*)}\mu^+\mu^-$ and $B_s \to \phi\mu^+\mu^-$ [8−10]. Precise measurements of angular observables are now accessible owing to experimental developments [11−19]. They are useful in disentangling helicities, providing reliable methods to probe the Lorentz structure of NP [20−27]. Besides, the ratios of $R_{K^{(*)}} \equiv \Gamma(B \to K^{(*)}\mu^+\mu^-)/\Gamma(B \to K^{(*)}e^+e^-)$ were measured, where discrepancies against the SM were found. In particular, 3.1σ and 2.5σ deviations have been found in $R_K(1.1\mathrm{GeV}^2 \le q^2 \le 6.0\mathrm{GeV}^2)$ and $R_{K^*}(0.045\mathrm{GeV}^2 \le q^2 \le 6.0\mathrm{GeV}^2)$ [28, 29], showing that the lepton universality may be violated by NP. Very recently, a global fit of $b \to s\ell^+\ell^-$ with B meson experiments was performed [30], and the experimental data permitted the large complex Wilson coefficients beyond the SM.

The baryonic decays of $\Lambda_b \to \Lambda(\to p\pi^-)\ell^+\ell^-$ are interesting for several reasons. For polarized Λ_b, the decays provide a couple dozen angular observables, which are three times more than those in $B \to K\mu^+\mu^-$. The polarization fraction (P_b) of Λ_b is reported as $(6\pm7)\%$ at the center of mass energy of 7 TeV of pp collisions [31]. The full angular distribution of $\Lambda_b \to \Lambda(\to p\pi^-)\mu^+\mu^-$ has been measured at the LHCb [19]. Notably, the experiment obtains that

$$K_{10} = -0.045 \pm 0.037 \pm 0.006, \tag{1}$$

deviating from the SM prediction of $K_{10} \approx 0$ by 1.2σ. It is reasonable to expect that the precision will be improved in the forthcoming update. In this study, we explicitly show that K_{10} is a T-odd quantity, which can be sizable in the presence of NP.

Theoretically, the angular distributions of $\Lambda_b \to \Lambda\mu^+\mu^-$ have been studied intensively [26, 27, 32]. In particular, an analysis of NP with real Wilson coefficients has been performed in Ref. [33], where they found $P_b = (0\pm5)\%$ at 1σ confidence level. In this study, we focus on the time-reversal (T) violating observables induced by the complex NP Wilson coefficients. Unlike the direct CP

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† E-mail: weizhengyi21@mails.ucas.ac.cn

asymmetries in decay widths, T violation does not require a CP-conserving phase. This feature is very useful in leptonic decays, as strong phases are often negligible. The roles of the (pseudo)scalar operators in $\Lambda_b\to\Lambda\tau^+\tau^-$ are studied for the first time. Their effects are enhanced by m_τ and play an important role in examining NP.

This paper is organized as follows. In Sec. II, we decompose $\Lambda_b\to\Lambda\ell^+\ell^-$ into products of two-body decays. In Sec. III, we construct the T-odd observables. In Sec. IV, we briefly review the angular distributions of $\Lambda_b\to\Lambda(\to p\pi^-)\ell^+\ell^-$ and identify the T-odd observables. In Sec. V, we estimate the effects of the (pseudo)scalar operators on the T-odd observables. We conclude the study in Sec. VI.

II. HELICITY AMPLITUDES

The amplitudes of $\Lambda_b\to\Lambda\ell^+\ell^-$, induced by the $b\to s\ell^+\ell^-$ transitions at the quark level, are given as [34]:

$$\frac{G_F}{\sqrt{2}}\frac{\alpha V_{ts}^*V_{tb}}{2\pi}(\langle\Lambda|\bar{s}j_V^\mu b|\Lambda_b\rangle\bar{\ell}\gamma_\mu\ell+\langle\Lambda|\bar{s}j_A^\mu b|\Lambda_b\rangle\bar{\ell}\gamma_\mu\gamma_5\ell$$
$$+\langle\Lambda|\bar{s}j_S b|\Lambda_b\rangle\bar{\ell}\ell+\langle\Lambda|\bar{s}j_P b|\Lambda_b\rangle\bar{\ell}\gamma_5\ell), \tag{2}$$

where G_F is the Fermi constant, $V_{tq}(q=s,b)$ are the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements,

$$\begin{aligned} j_V^\mu &= (C_9^{\rm eff}+C_9^{\rm NP})L^\mu-\frac{2m_b}{q^2}C_{7\gamma}^{\rm eff}i\sigma^{\mu q}(1+\gamma_5)+(C_L+C_R)R^\mu,\\ j_A^\mu &= (C_{10}+C_{10}^{\rm NP})L^\mu+(C_R-C_L)R^\mu,\\ j_S &= C_S(1-\gamma_5),\quad j_P=C_P(1-\gamma_5), \end{aligned} \tag{3}$$

$C^{(\rm eff)}$ represents the (effective) Wilson coefficients, $\sigma^{\mu q}=i(\gamma^\mu\gamma^\nu-\gamma^\nu\gamma^\mu)q_\nu/2$ with $q=(q^0,\vec{q})$ being the four-momentum of $\ell^+\ell^-$, $L^\mu=\gamma^\mu(1-\gamma_5)$, $R^\mu=\gamma^\mu(1+\gamma_5)$, and m_q is for the quark mass. We only consider the NP operators with the right-handed strange quark in $j_{S,P}$, as the heavy quark mass would suppress the effects of the left-handed ones.

The first (second) term in Eq. (2) can be interpreted as $\Lambda_b\to\Lambda j_{\rm eff}^{V(A)}$ followed by $j_{\rm eff}^{V(A)}\to\ell^+\ell^-$, where $j_{\rm eff}^{V(A)}$ is an effective off-shell (axial) vector boson, conserving the parity in its cascade decays, and $j_{V,A}^\mu$ are the couplings of $b-s-j_{\rm eff}^{V,A}$. Alternatively, the interpretation can be rephrased as $\Lambda_b\to\Lambda j_{\rm eff}^{R,L}\to\Lambda\ell^+\ell^-$ by

$$\begin{aligned} &\langle\Lambda|\bar{s}j_V^\mu b|\Lambda_b\rangle\bar{\ell}\gamma_\mu\ell+\langle\Lambda|\bar{s}j_A^\mu b|\Lambda_b\rangle\bar{\ell}\gamma_\mu\gamma_5\ell\\ =&\langle\Lambda|\bar{s}j_R^\mu b|\Lambda_b\rangle\bar{\ell}R_\mu\ell+\langle\Lambda|\bar{s}j_L^\mu b|\Lambda_b\rangle\bar{\ell}L_\mu\ell, \end{aligned} \tag{4}$$

where $j_{R(L)}^\mu=(j_V^\mu\pm j_A^\mu)/2$ and $j_{\rm eff}^{R(L)}$ clearly couple only to the right-handed (left-handed) leptons. As parity is conserved in $j_{\rm eff}^{V,A}\to\ell^+\ell^-$, it is easier to obtain the angular distributions with the $j_{\rm eff}^{V,A}$ interpretation. In this study, the angular distributions are obtained using $j_{\rm eff}^{V,A}$, while for NP, $j_{\rm eff}^{R,L}$ is used. Similarly, the third (fourth) term describes $\Lambda_b\to\Lambda j_{\rm eff}^{S(P)}$, with $j_{\rm eff}^{S(P)}$, the (pseudo)scalar boson, decaying to $\ell^+\ell^-$ subsequently.

In the SM, $C_{9,10}^{\rm NP}=C_{S,P,L,R}=0$ and the others are [26, 35]

$$\begin{aligned} C_{7\gamma}^{\rm eff}=&-0.313,\\ C_9^{\rm eff}=&C_9+h\left(\frac{m_c}{m_b},\frac{q^2}{m_b^2}\right)-\frac{1}{2}h\left(1,\frac{q^2}{m_b^2}\right)(4C_3+4C_4+3C_5+C_6)\\ &-\frac{1}{2}h\left(0,\frac{q^2}{m_b^2}\right)(C_3+3C_4)+\frac{2}{9}(3C_3+C_4+3C_5+C_6), \end{aligned} \tag{5}$$

where

$$\begin{aligned} h\left(\frac{m_c}{m_b},\frac{q^2}{m_b}\right)&=-\frac{8}{9}\ln\frac{m_c}{m_b}+\frac{8}{27}+\frac{4}{9}x-\frac{2}{9}(2+x)\\ &\times|1-x|^{1/2}\begin{cases}\left(\ln\left|\dfrac{\sqrt{1-x}+1}{\sqrt{1-x}-1}\right|-i\pi\right), & \text{for } x<1,\\ 2\arctan\dfrac{1}{\sqrt{x-1}}, & \text{for } x>1,\end{cases}\\ h\left(0,\frac{q^2}{m_b}\right)&=\frac{8}{27}-\frac{4}{9}\ln\frac{q^2}{m_b}+\frac{4}{9}i\pi, \end{aligned} \tag{6}$$

and $x=4m_c^2/q^2$. Their explicit values are found in Ref. [35].

By decomposing the Minkowski metric as

$$g^{\mu\nu}=\epsilon_t^\mu\epsilon_t^{*\nu}-\sum_{\lambda=0,\pm}\epsilon_\lambda^\mu\epsilon_\lambda^{*\nu}, \tag{7}$$

we arrive at

$$\frac{G_F}{\sqrt{2}}\frac{\alpha V_{ts}^*V_{tb}}{2\pi}\sum_{m=V,A,S,P}\left(L_t^mB_t^m-\sum_{\lambda=0,\pm}L_\lambda^mB_\lambda^m\right), \tag{8}$$

where

$$\begin{aligned} B_{\lambda_m}^{V,A}&=\epsilon_{\lambda_m}^{*\mu}\langle\Lambda|\bar{s}j_{V,A}^\mu b|\Lambda_b\rangle,\quad L_{\lambda_m}^V=\epsilon_{\lambda_m}^\mu\bar{u}_\ell\gamma_\mu v_\ell,\\ L_{\lambda_m}^A&=\epsilon_{\lambda_m}^\mu\bar{u}_\ell\gamma_\mu\gamma_5 v_\ell,\quad B^{S,P}=\langle\Lambda|\bar{s}j_{S,P}b|\Lambda_b\rangle,\\ L_t^S&=\bar{u}_\ell v_\ell,\quad L_t^P=\bar{u}_\ell\gamma_5 v_\ell, \end{aligned} \tag{9}$$

$\lambda_m=(t,0,\pm)$ is the helicity of $j_{\rm eff}^m$, with t indicating spin-0 off-shell contributions, and ϵ are the polarization vectors of $j_{\rm eff}^m$, given as [36]

$$\epsilon_\pm^\mu = \frac{1}{\sqrt{2}}(0,\pm1,\mathrm{i},0)^T\,,$$
$$\epsilon_0^\mu = (0,0,0,-1)^T\,,$$
$$\epsilon_t^\mu = (-1,0,0,0)^T\,, \tag{10}$$

and

$$\epsilon_\pm^\mu = \frac{1}{\sqrt{2}}(0,\mp1,\mathrm{i},0)^T\,,$$
$$\epsilon_0^\mu = \frac{1}{\sqrt{q^2}}(|\vec{q}|,0,0,-q^0)^T\,,$$
$$\epsilon_t^\mu = -\frac{1}{\sqrt{q^2}}q^\mu, \tag{11}$$

in the center of mass (CM) frames of $j^m_{\rm eff}$ and Λ_b, respectively. Note that $L^{S,P}_\lambda = 0$, as they do not contain the spacelike component. In Eq. (8), the amplitudes are decomposed as the products of Lorentz scalars, where B_{λ_m} and L_{λ_m} describe $\Lambda_b \to \Lambda j^m_{\rm eff}$ and $j^m_{\rm eff} \to \ell^+\ell^-$, respectively, reducing the three-body problems to two-body ones.

To deal with the spins, we adopt the helicity approach. The projection operators in the $SO(3)$ rotational ($SO(3)_R$) group are given by

$$|J,M\rangle\langle J,N| = \frac{2J+1}{8\pi^2}\int {\rm d}\phi {\rm d}\cos\theta {\rm d}\psi R_z(\phi) \times R_y(\theta)R_z(\psi)D^{J\dagger}(\phi,\theta,\psi)^N{}_M\,, \tag{12}$$

where N and M are the angular momenta toward the $\hat{z}$ direction, the Wigner-D matrices are defined by

$$D^J(\phi,\theta,\psi)^M{}_N\langle J,N|J,M\rangle = \langle J,N\left|R_z(\phi)R_y(\theta)R_z(\psi)\right|J,N\rangle\,, \tag{13}$$

and $R_{y(z)}$ are the rotation operators pointing toward $\hat{y}(\hat{z})$. Notably, it is important for Eq. (12) to be a linear superposition of $R_{y,z}$, which commutes with scalar operators. In the following, we take the shorthand of $D^J(\phi,\theta) \equiv D^J(\phi,\theta,0)$.

The simplest two-particle state with a nonzero momentum is defined by

$$|p\hat{z},\lambda_1,\lambda_2\rangle \equiv L_z|\vec{p}=0,J_z=\lambda_1\rangle_1 \otimes L_z'|\vec{p}=0,J_z=-\lambda_2\rangle_2\,, \tag{14}$$

where $\lambda_{1,2}$ are the helicities, the subscript denotes the particles, and $L_z^{(\prime)}$ is the Lorentz boost, which brings the first (second) particle to $(-)p\hat{z}$. As $L_z^{(\prime)}$ commutes with R_z, the state defined by Eq. (14) is an eigenstate of $J_z = \lambda_1 - \lambda_2$. Plugging Eq. (12) into Eq. (14) with $N = \lambda_1 - \lambda_2$, we arrive at

$$|\vec{p}^{\,2},\lambda_1,\lambda_2;J,J_z\rangle = \frac{2J+1}{4\pi}\int {\rm d}\phi {\rm d}\cos\theta R_z(\phi)R_y(\theta)|p\hat{z},\lambda_1,\lambda_2\rangle_{1,2}D^{J*}(\phi,\theta)^{J_z}{}_N\,, \tag{15}$$

which expresses the angular momentum eigenstate as the linear superpositions of the three-momentum eigenstate. Conversely, we have

$$|p\hat{z},\lambda_1,\lambda_2\rangle = \sum_J |\vec{p}^{\,2},\lambda_1,\lambda_2;J,N\rangle\,. \tag{16}$$

Note that the identities of Eqs. (15) and (16) are purely obtained from the mathematical consideration. The simplification occurs when the angular momentum conservation is considered. At the CM frames of Λ_b and $j^m_{\rm eff}$, it is clear that only $J=1/2$ and $J=(0,1)$ need to be considered for the $\Lambda j^m_{\rm eff}$ and $\ell^+\ell^-$ systems, respectively.

Utilizing Eq. (16), we find that

$$\langle\vec{p}^{\,2},\lambda_1,\lambda_2;J,N|\mathcal{S}|J,J_z;i\rangle = \langle p\hat{z},\lambda_1,\lambda_2|\mathcal{S}|J,J_z;i\rangle\,, \tag{17}$$

where $\mathcal{S}$ is an arbitrary scalar operator, and $|J,J_z;i\rangle$ stands for an arbitrary initial state. In Eq. (17), the final state on the left side possesses a definite angular momentum, which is irreducible in the $SO(3)_R$ group, *i.e.,* it contains only the dynamical details. On the contrary, the one on the right side is a three-momentum eigenstate, containing fewer physical insights but providing a way to compute the helicity amplitude.

Let us return to $\Lambda_b \to \Lambda j^m_{\rm eff}$ and $j^m_{\rm eff} \to \ell^+\ell^-$. We take the uppercase and lowercase of H and h for the helicity amplitudes of $\Lambda_b \to \Lambda j^m_{\rm eff}$ and $j^m_{\rm eff} \to \ell^+\ell^-$, respectively. To be explicit, we have

$$H^m_{\lambda_\Lambda\lambda_m} = B^m_{\lambda_m}\left(\lambda_{\Lambda_b} = \lambda_\Lambda - \lambda_m, \lambda_\Lambda, \vec{p}_\Lambda = -\vec{q} = |\vec{p}_\Lambda|\hat{z}\right)\,,$$
$$h^m_{0,\lambda_+\lambda_-} = L^m_t(\lambda_+,\lambda_-,\vec{q}=0,\vec{p}_+ = -\vec{p}_- = |\vec{p}_+|\hat{z}),$$
$$h^m_{1,\lambda_+\lambda_-} = L^m_{\lambda_+-\lambda_-}(\lambda_+,\lambda_-,\vec{q}=0,\vec{p}_+ = -\vec{p}_- = |\vec{p}_+|\hat{z})\,, \tag{18}$$

where $\lambda_{\Lambda_b}(\lambda_\Lambda,\lambda_\pm)$ corresponds to the angular momentum (helicities) of $\Lambda_b\,(\Lambda,\ell^\pm)$, and $\vec{p}_\Lambda$ ($\vec{p}_\pm$) is the three-momentum of Λ ($\ell^\pm$) in the CM frame of $\Lambda_b(j^m_{\rm eff})$. Theoretically, the dynamical parts of the amplitudes are extracted by Eq. (17), whereas the kinematic dependencies are governed by D^J.

For compactness, we take the abbreviations

$$|a^m_\pm\rangle = |\Lambda j^m_{\rm eff}, \pm 1/2, 0\rangle, \quad |b^m_\pm\rangle = |\Lambda j^m_{\rm eff}, \mp 1/2, \mp 1\rangle,$$
$$|c^m_\pm\rangle = |\Lambda j^m_{\rm eff}, \pm 1/2, t\rangle, \quad a^m_\pm = H^m_{\pm\frac{1}{2}0} = \langle a^m_\pm|\mathcal{S}^m_{\rm eff}|\Lambda_b\rangle,$$
$$b^m_\pm = H^m_{\mp\frac{1}{2}\mp1} = \langle a^m_\pm|\mathcal{S}^m_{\rm eff}|\Lambda_b\rangle, \quad c^m_\pm = H^m_{\pm\frac{1}{2}t} = \langle c^m_\pm|\mathcal{S}^m_{\rm eff}|\Lambda_b\rangle, \quad (19)$$

where $\mathcal{S}^m_{\rm eff}$ is the transition operator responsible for $\Lambda_b \to \Lambda j^m_{\rm eff}$, and J_z is not written down explicitly. The artificial $\mathcal{S}^m_{\rm eff}$ is needed to interpret $\Lambda_b \to \Lambda \ell^+\ell^-$ as products of two-body ones. For the $\Lambda_b \to \Lambda j^{R,L}_{\rm eff}$ interpretation, the helicity amplitudes are

$$a^R_\pm = \frac{1}{\sqrt{2}}(a^V_\pm + a^A_\pm), \quad a^L_\pm = \frac{1}{\sqrt{2}}(a^V_\pm - a^A_\pm),$$
$$b^R_\pm = \frac{1}{\sqrt{2}}(b^V_\pm + b^A_\pm), \quad b^L_\pm = \frac{1}{\sqrt{2}}(b^V_\pm - b^A_\pm),$$
$$c^R_\pm = \frac{1}{\sqrt{2}}(c^V_\pm + c^A_\pm), \quad c^L_\pm = \frac{1}{\sqrt{2}}(c^V_\pm - c^A_\pm). \quad (20)$$

III. *T*-ODD OBSERVABLES

From Eq. (3), we see that the NP contributions are absorbed into the couplings of $b-s-j^m_{\rm eff}$, whereas the Lorentz structures of $j^m_{\rm eff} \to \ell^+\ell^-$ are plain. Thus, to discuss the NP effects, it is sufficient to study $\Lambda_b \to \Lambda j^m_{\rm eff}$.

The most simple T-odd operator in $\Lambda_b \to \Lambda j^m_{eff}$ is defined as [37]

$$\hat{T} = (\vec{s}_\Lambda \times \vec{s}_m)\cdot \hat{p}_\Lambda, \quad (21)$$

$\vec{s}_\Lambda$ and $\vec{s}_m$ are the spin operators of Λ and $j^m_{\rm eff}$, respectively, and $\hat{p}_\Lambda$ is the unit vector of $\vec{p}_\Lambda$. The spin operators can only be defined for the massive objects, given as

$$M\vec{s} = P^0\vec{J} - \vec{p}\times\vec{K} - \frac{1}{P^0+M}\vec{p}(\vec{p}\cdot\vec{J}), \quad (22)$$

where M is the particle mass, and P^0, $\vec{p}$, $\vec{J}$, and $\vec{K}$ are the time translation, space translation, rotation, and Lorentz boost generators, respectively. Note that $(\vec{p},\vec{J})$ and $\vec{s}$ are T-odd, whereas $\vec{K}$ is T-odd. In addition, $\vec{s}$ satisfies the relations

$$\vec{s}\cdot\vec{p} = \vec{J}\cdot\vec{p}, \quad [s_i, s_j] = {\rm i}\epsilon^{ijk}\epsilon_k, \quad [s_i, p_j] = 0,$$
$$\vec{s}\exp({\rm i}\vec{K}\cdot\vec{\omega})|\vec{p}=0, J_z = M\rangle = \exp({\rm i}\vec{K}\cdot\vec{\omega})\vec{J}|\vec{p}=0, J_z=M\rangle, \quad (23)$$

with arbitrary $\vec{\omega}$. The key to solving the eigenstates of $\hat{T}$ relies on $\hat{T}$ being a scalar operator. Thus, we have

$$\hat{T}|\vec{p}^{\,2}, \lambda_1, \lambda_2; J, J_z\rangle$$
$$= \frac{2J+1}{4\pi}\int {\rm d}\phi {\rm d}\cos\theta R_z(\phi)R_y(\theta)\hat{T}|p\hat{z}, \lambda_1, \lambda_2\rangle_{1,2}$$
$$\times D^{J*}(\phi,\theta)^{J_z}{}_{\lambda_1-\lambda_2}, \quad (24)$$

and

$$\hat{T}|p\hat{z}, \lambda_1, \lambda_2\rangle = \frac{\rm i}{2}(s^+_\Lambda s^-_m - s^-_\Lambda s^+_m)|p\hat{z}, \lambda_1, \lambda_2\rangle, \quad (25)$$

with $s^\pm = s_x \pm {\rm i}s_y$. It is then straightforward to show that

$$\hat{T}|a^m_\pm\rangle = \pm\frac{\rm i}{\sqrt{2}}|b^m_\pm\rangle, \quad \hat{T}|b^m_\pm\rangle = \mp\frac{\rm i}{\sqrt{2}}|a^m_\pm\rangle, \quad \hat{T}|c^m_\pm\rangle = |0\rangle, \quad (26)$$

resulting in the eigenstates

$$|\lambda^m_T = \pm\frac{1}{\sqrt{2}}, \lambda_{\rm tot} = \frac{1}{2}\rangle = \frac{1}{\sqrt{2}}(|a^m_+\rangle \mp {\rm i}|b^m_+\rangle),$$
$$|\lambda^m_T = \pm\frac{1}{\sqrt{2}}, \lambda_{\rm tot} = -\frac{1}{2}\rangle = \frac{1}{\sqrt{2}}(|a^m_-\rangle \pm {\rm i}|b^m_-\rangle), \quad (27)$$

where λ^m_T and $\lambda_{\rm tot}$ are the eigenvalues of $\hat{T}$ and $\vec{J}\cdot\vec{p}$, respectively. They are also the eigenstates of $\vec{J}\cdot\vec{p}$, as $\hat{T}$ commutes with both $\vec{J}$ and $\vec{p}$. Note that $c^m_\pm$ are not involved since they are contributed by spinless $j^m_{\rm eff}$.

Because $\hat{T}$ and $\vec{J}\cdot\vec{p}$ are T-odd and T-even, respectively, we have

$$\mathcal{I}_t|\lambda^m_T, \lambda_{\rm tot}\rangle = e^{{\rm i}\theta_T}|-\lambda^m_T, \lambda_{\rm tot}\rangle, \quad \mathcal{I}_s|\lambda^m_T, \lambda_{\rm tot}\rangle = |-\lambda^m_T, -\lambda_{\rm tot}\rangle, \quad (28)$$

where $\mathcal{I}_{t(s)}$ is the time-reversal (space-inversion) operator, and θ_T depends on the convention. On the other hand, $\mathcal{I}_s$ would interchange $j^R_{\rm eff}$ and $j^L_{\rm eff}$, given as

$$\mathcal{I}_s|\lambda^R_T, \lambda_{\rm tot}\rangle = |-\lambda^L_T, -\lambda_{\rm tot}\rangle, \quad \mathcal{I}_s|\lambda^L_T, \lambda_{\rm tot}\rangle = |-\lambda^R_T, -\lambda_{\rm tot}\rangle, \quad (29)$$

with

$$|\lambda^R_T, \lambda_{\rm tot}\rangle = \frac{1}{\sqrt{2}}\left(|\lambda^V_T, \lambda_{\rm tot}\rangle + |\lambda^A_T, \lambda_{\rm tot}\rangle\right),$$
$$|\lambda^L_T, \lambda_{\rm tot}\rangle = \frac{1}{\sqrt{2}}\left(|\lambda^V_T, \lambda_{\rm tot}\rangle - |\lambda^A_T, \lambda_{\rm tot}\rangle\right), \quad (30)$$

since $j^V_{\rm eff}$ and $j^A_{\rm eff}$ have opposite parity.

For each combination of $\lambda_{\rm tot}$ and $j^m_{\rm eff}$, we define a T-odd quantity

$$\mathcal{T}^m_{\lambda_{\rm tot}} \equiv |\langle \lambda^m_T = 1/\sqrt{2}, \lambda_{\rm tot}|\mathcal{S}_{eff}|\Lambda_b\rangle|^2 - |\langle \lambda^m_T = -1/\sqrt{2}, \lambda_{\rm tot}|\mathcal{S}_{\rm eff}|\Lambda_b\rangle|^2, \quad (31)$$

which vanishes if $\mathcal{S}_{\rm eff}$ is invariant under $\mathcal{I}_t$. Explicitly, we find that

$$\mathcal{T}^m_+ = -2{\rm Im}\left(a^m_+ \overline{b^m_+}\right), \quad \mathcal{T}^m_- = 2{\rm Im}\left(a^m_- \overline{b^m_-}\right), \quad (32)$$

which are proportional to the relative complex phase. They are called T-odd quantities, as $\mathcal{I}_t$ interchanges the final states of the two terms in Eq. (31).

The operator of $\hat{T}$ contains $\vec{s}_\Lambda$, which is difficult to measure directly. To probe the spin of Λ, it is plausible to study the cascade decays of $\Lambda \to p\pi^-$. Subsequently, the final states involve four particles $p\pi^-\ell^+\ell^-$, containing three independent three-momenta. It is then possible to observe the triple product, given by

$$\alpha(\vec{p}_+ \times \vec{p}_p)\cdot \vec{p}_\Lambda, \quad (33)$$

where α is the polarization asymmetry in $\Lambda \to p\pi^-$, and $\vec{p}_p$ is the three momentum of the proton. Notice that α is a necessary component in Eq. (33), as $\vec{s}_\Lambda$ does not affect $\vec{p}_p$ if $\alpha = 0$. As the product in Eq. (33) is P-even, we have to construct P-even observables from Eq. (32). From the transformation rules, it is easy to see that

$$\mathcal{T}^1 \equiv \mathcal{T}^R_- - \mathcal{T}^L_+, \quad \mathcal{T}^2 \equiv \mathcal{T}^L_- - \mathcal{T}^R_+, \quad (34)$$

are both T-odd and P-even.

IV. ANGULAR DISTRIBUTIONS

The lepton helicity amplitudes are calculated as

$$\begin{aligned} &h^V_{0,++} = 0, \quad h^V_{1,++} = 2M_\ell, \\ &h^A_{0,++} = 2M_\ell, \quad h^A_{1,++} = 0, \\ &h^V_{1,+-} = -\sqrt{2q^2}, \quad h^A_{1,+-} = \sqrt{2q^2(1-\delta_\ell)}, \\ &h^S_{0,++} = \sqrt{2q^2(1-\delta_\ell)}, \quad h^P_{0,++} = \sqrt{2q^2}, \end{aligned} \quad (35)$$

where $\delta_\ell = 4M^2_\ell/q^2$ with M_ℓ being the lepton mass. In contrast, the baryon matrix elements are conventionally parameterized by the form factors, given by

$$\begin{aligned} \langle\Lambda|\bar{s}\gamma^\mu b|\Lambda_b\rangle &= \bar{u}_\Lambda\left[f^V_1(q^2)\gamma^\mu - f^V_2(q^2){\rm i}\sigma^{\mu\nu}\frac{q_\nu}{M_{\Lambda_b}} + f^V_3(q^2)\frac{q^\mu}{M_{\Lambda_b}}\right]u_{\Lambda_b}, \\ \langle\Lambda|\bar{s}\gamma^\mu\gamma_5 b|\Lambda_b\rangle &= \bar{u}_\Lambda\left[f^A_1(q^2)\gamma^\mu - f^A_2(q^2){\rm i}\sigma^{\mu\nu}\frac{q_\nu}{M_{\Lambda_b}} + f^A_3(q^2)\frac{q^\mu}{M_{\Lambda_b}}\right]\gamma_5 u_{\Lambda_b}, \\ \langle\Lambda|\bar{s}{\rm i}\sigma^{\mu q} b|\Lambda_b\rangle &= \bar{u}_\Lambda\left[\frac{f^{TV}_1(q^2)}{M_{\Lambda_b}}\left(\gamma^\mu q^2 - q^\mu \not{q}\right) - f^{TV}_2(q^2){\rm i}\sigma^{\mu q}\right]u_{\Lambda_b}, \\ \langle\Lambda|\bar{s}{\rm i}\sigma^{\mu q}\gamma_5 b|\Lambda_b\rangle &= \bar{u}_\Lambda\left[\frac{f^{TA}_1(q^2)}{M_{\Lambda_b}}\left(\gamma^\mu q^2 - q^\mu \not{q}\right) - f^{TA}_2(q^2){\rm i}\sigma^{\mu q}\right]\gamma_5 u_{\Lambda_b}, \\ \langle\Lambda|\bar{s}(1-\gamma_5) b|\Lambda_b\rangle &= \bar{u}_\Lambda(f_s - g_a\gamma_5)u_{\Lambda_b}, \end{aligned} \quad (36)$$

where $u_{\Lambda_{(b)}}$ and $M_{\Lambda_{(b)}}$ are the Dirac spinor and mass of $\Lambda_{(b)}$, respectively. In turn, we find that

$$H^{Vm}_{\frac{1}{2},0} = \sqrt{\frac{Q_-}{q^2}}\left[M_+ F^{Vm}_1(q^2) + \frac{q^2}{M_{\Lambda_b}}F^{Vm}_2(q^2)\right], \quad (37)$$

$$H^{Vm}_{\frac{1}{2},1} = \sqrt{2Q_-}\left[F^{Vm}_1(q^2) + \frac{M_+}{M_{\Lambda_b}}F^{Vm}_2(q^2)\right], \quad (38)$$

$$H^{Vm}_{\frac{1}{2},t} = \sqrt{\frac{Q_+}{q^2}}\left[M_- F^{Vm}_1(q^2) + \frac{q^2}{M_{\Lambda_b}}F^{Vm}_3(q^2)\right], \quad (39)$$

$$H^{Am}_{\frac{1}{2},0} = \sqrt{\frac{Q_+}{q^2}}\left[M_- F^{Am}_1(q^2) - \frac{q^2}{M_{\Lambda_b}}F^{Am}_2(q^2)\right], \quad (40)$$

$$H^{Am}_{\frac{1}{2},1} = \sqrt{2Q_+}\left[F^{Am}_1(q^2) + \frac{M_-}{M_{\Lambda_b}}F^{Am}_2(q^2)\right], \quad (41)$$

$$H^{Am}_{\frac{1}{2},t} = \sqrt{\frac{Q_-}{q^2}}\left[M_+ F^{Am}_1(q^2) - \frac{q^2}{M_{\Lambda_b}}F^{Am}_3(q^2)\right], \quad (42)$$

$$H^S_{\pm\frac{1}{2},t} = C_S\left[f_s\frac{\sqrt{Q_+q^2}}{M_{\Lambda_b}} \pm g_a\frac{\sqrt{Q_-q^2}}{M_{\Lambda_b}}\right], \quad (43)$$

$$H^P_{\pm\frac{1}{2},t} = C_P\left[f_s\frac{\sqrt{Q_+q^2}}{M_{\Lambda_b}} \pm g_a\frac{\sqrt{Q_-q^2}}{M_{\Lambda_b}}\right], \quad (44)$$

where $M_\pm = M_{\Lambda_b} \pm M_\Lambda$, $Q_\pm = (M_\pm)^2 - q^2$, and

$$F_1^{VV}(q^2) = [C_9^{\rm eff} + C_9^{\rm NP} + (C_L + C_R)] f_1^V(q^2) - \frac{2m_b}{M_{\Lambda_b}} C_{7\gamma}^{\rm eff} f_1^{TV}(q^2), \quad (45)$$

$$F_2^{VV}(q^2) = [C_9^{\rm eff} + C_9^{\rm NP} + (C_L + C_R))] f_2^V(q^2) - \frac{2m_b M_{\Lambda_b}}{q^2} C_{7\gamma}^{\rm eff} f_2^{TV}(q^2), \quad (46)$$

$$F_3^{VV}(q^2) = [C_9^{\rm eff} + C_9^{\rm NP} + (C_L + C_R)] f_3^V(q^2) + \frac{2m_b M_-}{q^2} C_{7\gamma}^{\rm eff} f_1^{TV}(q^2), \quad (47)$$

$$F_1^{AV}(q^2) = [C_9^{\rm eff} + C_9^{\rm NP} - (C_L + C_R)] f_1^A(q^2) + \frac{2m_b}{M_{\Lambda_b}} C_{7\gamma}^{\rm eff} f_1^{TA}(q^2), \quad (48)$$

$$F_2^{AV}(q^2) = [C_9^{\rm eff} + C_9^{\rm NP} - (C_L + C_R))] f_2^A(q^2) + \frac{2m_b M_{\Lambda_b}}{q^2} C_{7\gamma}^{\rm eff} f_2^{TA}(q^2), \quad (49)$$

$$F_3^{AV}(q^2) = [C_9^{eff} + C_9^{\rm NP} - (C_L + C_R)] f_3^A(q^2) + \frac{2m_b M_+}{q^2} C_{7\gamma}^{eff} f_1^{TA}(q^2), \quad (50)$$

$$F_i^{VA}(q^2) = [C_{10} + C_{10}^{\rm NP} + (C_R - C_L)] f_i^V(q^2), \quad (51)$$

$$F_i^{AA}(q^2) = [C_{10} + C_{10}^{\rm NP} - (C_R - C_L)] f_i^A(q^2), \quad (52)$$

with $i = (1,2,3)$. Combining the relations

$$H^m_{\lambda_\Lambda\lambda_m} = H^{Vm}_{\lambda_\Lambda\lambda_m} - H^{Am}_{\lambda_\Lambda\lambda_m}, \quad H^{Vm}_{-\lambda_\Lambda,-\lambda_m} = H^{Vm}_{\lambda_\Lambda,\lambda_m}, \quad H^{Am}_{-\lambda_\Lambda,-\lambda_m} = -H^{Am}_{\lambda_\Lambda,\lambda_m},$$

the evaluations of H are completed once the form factors are given.

The angular distributions of $\Lambda_b \to \Lambda(\to p\pi^-)\ell^+\ell^-$, related to the kinematic part, are given by piling D^J, read as

$$\mathcal{D}(q^2,\vec{\Omega}) \equiv \frac{\partial^6\Gamma(\Lambda_b \to \Lambda(\to p\pi^-)\ell^+\ell^-)}{\partial q^2 \partial\cos\theta \partial\cos\theta_b \partial\cos\theta_\ell \partial\phi_b \partial\phi_\ell} = \mathcal{B}(\Lambda \to p\pi^-)\frac{\zeta(q^2)}{32\pi^2} \sum_{\lambda_p,\lambda_\pm,\lambda_b} \rho_{\lambda_b\lambda_b} \left|A_{\lambda_p}\right|^2$$
$$\times \left| \sum_m \sum_{\lambda_m,\lambda_\Lambda} (-1)^{J_m} H^m_{\lambda_\Lambda\lambda_m} D^{\frac{1}{2}*}(0,\theta)^{\lambda_b}{}_{\lambda_\Lambda-\lambda_m} D^{\frac{1}{2}*}(\phi_b,\theta_b)^{\lambda_\Lambda}{}_{\lambda_p} h^m_{J_m,\lambda_+\lambda_-} D^{J_m*}(\phi_\ell,\theta_\ell)^{\lambda_m}{}_{\lambda_+-\lambda_-} \right|^2,$$
$$\zeta(q^2) = \frac{\alpha^2 G_F^2 |V_{ts}^\dagger V_{tb}|^2}{32\pi^5} \frac{q^2|\vec{p}_\Lambda|}{24M_{\Lambda_b}^2} \sqrt{1-\delta_\ell}, \quad (53)$$

where $\rho_{\pm,\pm} = (1 \pm P_b)/2$, $|A_\pm|^2 = (1 \pm \alpha)/2$, $\lambda_p = \pm 1/2$, $|\vec{p}_\Lambda| = \sqrt{Q_+Q_-}/2M_{\Lambda_b}$, and $J_m = 0\,(1)$ for $\lambda_m = t\,(\pm,0)$. The angles are defined in Fig. 1, where θ, θ_b and θ_ℓ are defined in the CM frames of Λ_b, Λ and $\ell^+\ell^-$, respectively, and $\phi_{b,\ell}$ are the azimuthal angles between the decay planes.

The breakdown of the physical meaning of Eq. (53) is as follows:

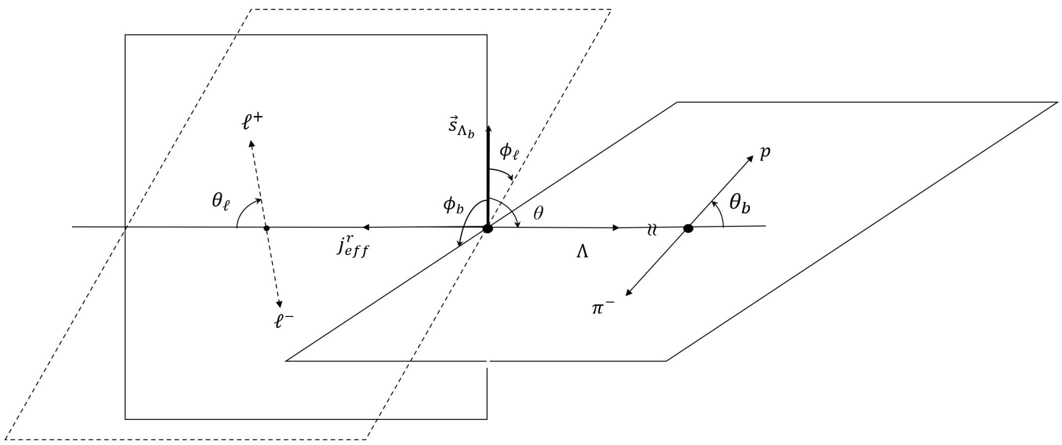

Fig. 1. Definitions of the angles.

• $H^m_{\lambda_\Lambda \lambda_m} D^{\frac{1}{2}*}(0,\theta)^{\lambda_b}{}_{\lambda_\Lambda - \lambda_m}$ is responsible for $\Lambda_b \to \Lambda j^m_{eff}$, where H and D describe the dynamical and kinematic parts of the amplitudes.

• The kinematic part of $\Lambda \to p\pi^-$ is described by $D^{\frac{1}{2}*}(\phi_b,\theta_b)^{\lambda_\Lambda}{}_{\lambda_p}$ and the dynamical part by$|A_{\lambda_p}|$.

• $h^m_{J_m,\lambda_+\lambda_-}$ and $D^{J_m*}(\phi_\ell,\theta_\ell)^{\lambda_m}{}_{\lambda_+-\lambda_-}$ describe the dynamical and kinematic parts of $j^m_{\rm eff} \to \ell^+\ell^-$, respectively.

The derivation is similar to those in the appendices in Ref. [36]. We cross-check our result of $\mathcal{D}(\vec\Omega)$ with Ref. [32] and find that they match. For practical purposes, $\mathcal{D}(\vec\Omega)$ is expanded as follows [19]:

$$
\begin{aligned}
\mathcal{D}(q^2,\vec\Omega) =& \frac{3}{32\pi^2}\Big(\left(K_1 \sin^2\theta_l + K_2\cos^2\theta_l + K_3\cos\theta_l\right) + \left(K_4\sin^2\theta_l + K_5\cos^2\theta_l + K_6\cos\theta_l\right)\cos\theta_b \\
&+ (K_7 \sin\theta_l\cos\theta_l + K_8\sin\theta_l)\sin\theta_b\cos(\phi_b+\phi_l) + (K_9\sin\theta_l\cos\theta_l + K_{10}\sin\theta_l)\sin\theta_b\sin(\phi_b+\phi_l) \\
&+ \left(K_{11}\sin^2\theta_l + K_{12}\cos^2\theta_l + K_{13}\cos\theta_l\right)\cos\theta + \left(K_{14}\sin^2\theta_l + K_{15}\cos^2\theta_l + K_{16}\cos\theta_l\right)\cos\theta_b\cos\theta \\
&+ (K_{17}\sin\theta_l\cos\theta_l + K_{18}\sin\theta_l)\sin\theta_b\cos(\phi_b+\phi_l)\cos\theta + (K_{19}\sin\theta_l\cos\theta_l + K_{20}\sin\theta_l)\sin\theta_b\sin(\phi_b+\phi_l)\cos\theta \\
&+ (K_{21}\cos\theta_l\sin\theta_l + K_{22}\sin\theta_l)\sin\phi_l\sin\theta + (K_{23}\cos\theta_l\sin\theta_l + K_{24}\sin\theta_l)\cos\phi_l\sin\theta \\
&+ (K_{25}\cos\theta_l\sin\theta_l + K_{26}\sin\theta_l)\sin\phi_l\cos\theta_b\sin\theta + (K_{27}\cos\theta_l\sin\theta_l + K_{28}\sin\theta_l)\cos\phi_l\cos\theta_b\sin\theta \\
&+ \left(K_{29}\cos^2\theta_l + K_{30}\sin^2\theta_l\right)\sin\theta_b\sin\phi_b\sin\theta + \left(K_{31}\cos^2\theta_l + K_{32}\sin^2\theta_l\right)\sin\theta_b\cos\phi_b\sin\theta \\
&+ \left(K_{33}\sin^2\theta_l\right)\sin\theta_b\cos(2\phi_l+\phi_b)\sin\theta + \left(K_{34}\sin^2\theta_l\right)\sin\theta_b\sin(2\phi_l+\phi_b)\sin\theta\Big) \,, \qquad (54)
\end{aligned}
$$

with

$$K_i(i=1\sim 34) \equiv K_i^{VA} + K_i^{SP}\,, \qquad (55)$$

where the definitions of K_i^{VA} and K_i^{SP} can be found in Appendix A and Appendix B, respectively. We note that $K_{11\sim34}$ are proportional to P_b, imposing difficulties in extracting physical meanings since P_b depends on their production. Interestingly, in the SM, K_9 and K_{10} are found to be

$$
\begin{aligned}
K_9 &= -\frac{\sqrt{2}\alpha(1-\delta_\ell)}{4}\left(\mathcal{T}^1 + \mathcal{T}^2\right)\,,\\
K_{10} &= \frac{\sqrt{2}\alpha\sqrt{1-\delta_\ell}}{4}\left(\mathcal{T}^1 - \mathcal{T}^2\right)\,, \qquad (56)
\end{aligned}
$$

which are T-odd according to Eq. (34). Notably, $K_{19,20}$, $K_{21,22}$, $K_{25,26}$, $K_{29,30}$, and K_{34} are also sensitive to the complex phases of NP, as they are proportional to the imaginary parts of the helicity amplitudes.

V. NUMERICAL RESULTS

After identifying the T-odd observables, we are ready to estimate the NP contributions. If (pseudo)scalar operators from NP are involved, their contributions are divided into two categories: one is from the interference between NP, which scales as $O(C^2_{S,P})$, and the other arises from the interference of (pseudo)scalar operators with the SM, scaling as $O(C_{S,P})$. We focus on the latter, as it is expected to be larger. As the contributions are proportional to the lepton masses, our main concern lies in the decay channel of $\Lambda_b \to \Lambda(\to p\pi^-)\tau^+\tau^-$. Furthermore, this channel is contaminated a little by the charmonia resonance, providing a clean background to probe NP. We take $C_{L,R} = C^{\rm NP}_{9,10} = 0$ in $\Lambda_b \to \Lambda(\to p\pi^-)\tau^+\tau^-$ and notice that K_9 would not be influenced by the (pseudo)scalar operators. However, K_{10} is sensitive to C_S and C_P.

In this work, we evaluated the form factors of $\Lambda_b \to \Lambda$ from the homogeneous bag model, the details of which are given in Appendix C and good accordance to the experiments is found in $\mathcal{B}(\Lambda_b \to \Lambda\mu^+\mu^-)$. The results are plotted in Figs. 2 to 5 with error bands, where $K_{10}=0$ when $C_S = C_P = 0$. In Figs. 2 and 3, $C_{S,P}$ are set to be purely real, whereas in Figs. 4 and 5, they are purely imaginary.

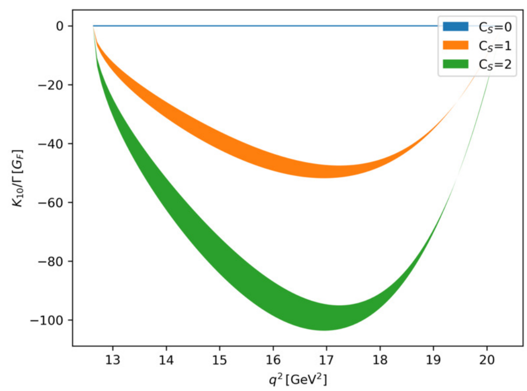

Fig. 2. (color online) q^2-dependencies of K_{10} with different values of C_S ($\ell=\tau, C_P=0$).

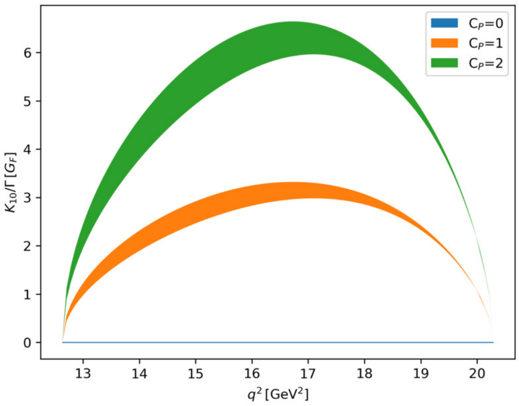

Fig. 3. (color online) q^2-dependencies of K_{10} with different values of C_P ($\ell=\tau, C_S=0$).

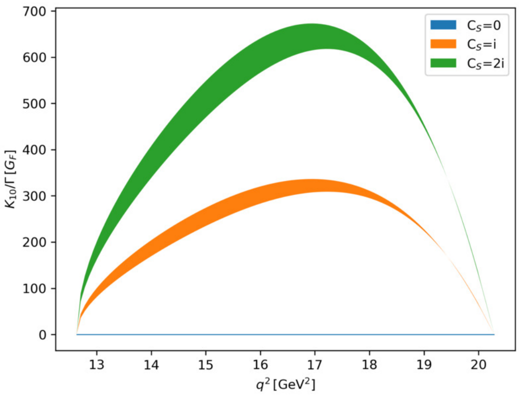

Fig. 4. (color online) q^2-dependencies of K_{10} with different values of C_S ($\ell=\tau, C_P=0$).

We find that K_{10} is more sensitive to the imaginary part of C_S. In the region of $16\,\text{GeV}^2<q^2<18\,\text{GeV}^2$, the interference between (axial)vector and (pseudo)scalar operators can be significantly enhanced. To estimate the experimental results, we consider the integrated K_{10} related to Figs. 2 to 5 in Table 1. The integrated K_i is defined as

$$\langle K_i\rangle=\frac{1}{\Gamma}\int_\kappa^{\kappa'}\zeta K_i\mathrm{d}q^2\,,\quad \Gamma=\int_\kappa^{\kappa'}\zeta(K_1+2K_2)\mathrm{d}q^2\,. \tag{57}$$

where $(\kappa,\kappa')=(4M_\tau^2,(M_{\Lambda_b}-M_\Lambda)^2)$. From the table, we find that the contributions from the imaginary part of C_S are primary and larger than others by one order of magnitude. If we take $N_{\Lambda_b}=5\times10^{12}$ at the LHCb Run3&Run4, a reconstruction efficiency $\epsilon=10^{-4}$, and $\text{Im}(C_S)=1$, we have $\epsilon N_{\Lambda_b}\langle K_{10}\rangle\mathcal{B}\tau\approx20$, which can be measured at the LHCb Run3&Run4.

To consider the NP contributions in $\Lambda_b\to\Lambda(\to p\pi^-)\mu^+\mu^-$, we may ignore the contributions from (pseudo)scalar operators, as they are suppressed by the muon mass. From the global fit in the B meson decays [30], the permitted imaginary parts of the NP Wilson coefficients are found in Table 2 with four different scenarios[1)]. To illustrate this, we calculate $\langle K_j\rangle$ with $K_j\in\{K_9,K_{10},K_{19},K_{30}\}$ and $(\kappa,\kappa')=(15\,\text{GeV}^2,20\,\text{GeV}^2)$ in different scenarios given in Table 2. We fit P_b from the experimental K_{1-34} and find that P_b is consistent with zero regardless of the presence of NP.

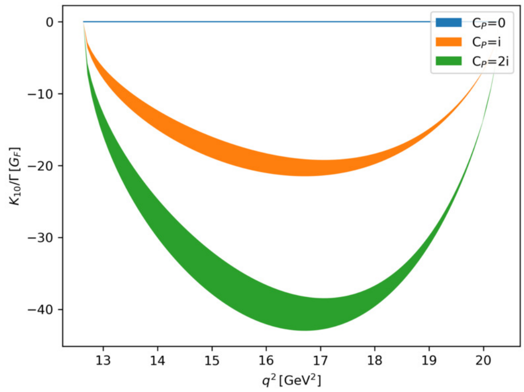

Fig. 5. (color online) q^2-dependencies of K_{10} with different values of C_P ($\ell=\tau, C_S=0$).

Table 1. The (pseudo)scalar Wilson coefficients and $\langle K_{10}\rangle$ with $\ell=\tau$.

Re(C_S)	Re(C_P)	$\langle K_{10}\rangle$	Im(C_S)	Im(C_P)	$\langle K_{10}\rangle$
1	0	−0.0031(2)	1	0	0.0202(10)
2	0	−0.0062(3)	2	0	0.0403(20)
0	1	0.0002	0	1	−0.0014(1)
0	2	0.0004	0	2	−0.0027(2)

In the absence of relative complex phases in the SM, $\langle K_j\rangle$ are found to be less than 10^{-4}. Therefore, they provide excellent opportunities to test the SM. Although K_j are proportional to the imaginary parts of the NP Wilson coefficients, which have not yet been determined, their signs remain unknown. However, nonzero values in the experiments would be a smoking gun in NP, regardless of the signs. Scenario #1 affects little in $\langle K_j\rangle$, and the results are not listed. We find that $\langle K_9\rangle$ is very small in all scenarios, which is consistent with the experiments. Remarkably, the experimental data of $\langle K_{10}\rangle$ can be explained by Scenario #4. On the other hand, K_{19} and K_{30} are highly suppressed by P_b.

Since the CP-conserving phases are absent, the T-odd observables are proportional to the imaginary parts of the Wilson coefficients. Noting that $\overline{\alpha}=-\alpha$, we deduce that $(K_9,K_{10},K_{19},K_{30})$ in $\overline{\Lambda}_b\to\overline{\Lambda}(\to\overline{p}\pi^+)\ell^+\ell^-$ are equal to those in $\Lambda_b\to\Lambda(\to p\pi^-)\ell^+\ell^-$. Here, $\overline{\alpha}$ represents the decay asymmetry of $\overline{\Lambda}\to\overline{p}\pi^+$.

1) See Fig. 1 of Ref. [30]. It is clear that the signs of NP Wilson coefficients are barely determined.

Table 2. The Wilson coefficients and $\langle K_j \rangle$ in units of 10^{-3} in four NP scenarios.

Scenarios	$\mathrm{Im}(C_9^{\mathrm{NP}})$	$\mathrm{Im}(C_{10}^{\mathrm{NP}})$	$\mathrm{Im}(C_L)$	$\mathrm{Im}(C_R)$	K_9	K_{10}	K_{19}	K_{30}	P_b
Scenario #1	± 0.73	0	0	0					
Scenario #2	0	± 1.86	0	0	0	∓ 4	0	0	−0.022(72)
Scenario #3	± 1.66	∓ 1.66	0	0	0	± 3	0	0	−0.021(65)
Scenario #4	± 0.77	0	∓ 0.77	∓ 0.77	∓ 1	∓ 42	∓ 1	0	−0.019(64)

VI. CONCLUSION

We derived the angular distributions of $\Lambda_b \to \Lambda(\to p\pi^-)\ell^+\ell^-$ based on the effective schemes of $\Lambda_b \to \Lambda(\to p\pi^-) j_{\mathrm{eff}}^m(\to \ell^+\ell^-)$, and the results were found to be consistent with those in the literature. By studying the effective two-body decays of $\Lambda_b \to \Lambda j_{\mathrm{eff}}^m$, we identified the T-odd correlations in the form of $(\vec{s}_\Lambda \times \vec{s}_m)\cdot\hat{p}$. We found that K_9 and K_{10} are related to $(\vec{s}_\Lambda \times \vec{s}_m)\cdot\hat{p}_\Lambda$, and K_{10} is sensitive to the complex phases generated by NP. For $\Lambda_b \to \Lambda(\to p\pi^-)\mu^+\mu^-$, we found that $C_R = -0.77\mathrm{i}$ can explain the K_{10} puzzle. We recommend revisiting the experiment, considering K_{10} for a stringent constraint. In $\Lambda_b \to \Lambda(\to p\pi^-)\tau^+\tau^-$, we focused on the effects of the interference of (pseudo)scalar operators with (axial)vector operators and found that the effects of the (pseudo)scalar operators can be largely enhanced in the high q^2 region.

APPENDIX A: ANGULAR OBSERVABLES IN THE SM

Here, all K_i are real. They are given by

$$
\begin{aligned}
K_1^{VA} =& \frac{1}{4}\Big(-\delta_\ell a_+^A\overline{a_+^A} - \delta_\ell a_-^A\overline{a_-^A} + \frac{\delta_\ell b_+^V\overline{b_+^V}}{2} - \frac{\delta_\ell b_+^A\overline{b_+^A}}{2} + \frac{\delta_\ell b_-^V\overline{b_-^V}}{2} - \frac{\delta_\ell b_-^A\overline{b_-^A}}{2} + \delta_\ell c_+^A\overline{c_+^A} \\
& + \delta_\ell c_-^A\overline{c_-^A} + a_+^V\overline{a_+^V} + a_+^A\overline{a_+^A} + a_-^V\overline{a_-^V} + a_-^A\overline{a_-^A} + \frac{b_+^V\overline{b_+^V}}{2} + \frac{b_+^A\overline{b_+^A}}{2} + \frac{b_-^V\overline{b_-^V}}{2} + \frac{b_-^A\overline{b_-^A}}{2}\Big),\\
K_2^{VA} =& \frac{1}{4}\Big(\delta_\ell a_+^V\overline{a_+^V} + \delta_\ell a_-^V\overline{a_-^V} - \delta_\ell b_+^A\overline{b_+^A} - \delta_\ell b_-^A\overline{b_-^A} + \delta_\ell c_+^A\overline{c_+^A} + \delta_\ell c_-^A\overline{c_-^A} + b_+^V\overline{b_+^V} + b_+^A\overline{b_+^A} + b_-^V\overline{b_-^V} + b_-^A\overline{b_-^A}\Big),\\
K_3^{VA} =& -\frac{K_{16}^{VA}}{P_b} = \frac{\sqrt{1-\delta_\ell}}{4}\Big(b_+^V\overline{b_+^A} + b_+^A\overline{b_+^V} - b_-^V\overline{b_-^A} - b_-^A\overline{b_-^V}\Big)\\
K_4^{VA} =& \frac{1}{4}\alpha\Big(-\delta_\ell a_+^A\overline{a_+^A} + \delta_\ell a_-^A\overline{a_-^A} - \frac{\delta_\ell b_+^V\overline{b_+^V}}{2} + \frac{\delta_\ell b_+^A\overline{b_+^A}}{2} + \frac{\delta_\ell b_-^V\overline{b_-^V}}{2} - \frac{\delta_\ell b_-^A\overline{b_-^A}}{2} + \delta_\ell c_+^A\overline{c_+^A} \\
& - \delta_\ell c_-^A\overline{c_-^A} + a_+^V\overline{a_+^V} + a_+^A\overline{a_+^A} - a_-^V\overline{a_-^V} - a_-^A\overline{a_-^A} - \frac{b_+^V\overline{b_+^V}}{2} - \frac{b_+^A\overline{b_+^A}}{2} + \frac{b_-^V\overline{b_-^V}}{2} + \frac{b_-^A\overline{b_-^A}}{2}\Big),\\
K_5^{VA} =& \frac{1}{4}\alpha\Big(\delta_\ell a_+^V\overline{a_+^V} - \delta_\ell a_-^V\overline{a_-^V} + \delta_\ell b_+^A\overline{b_+^A} - \delta_\ell b_-^A\overline{b_-^A} + \delta_\ell c_+^A\overline{c_+^A} - \delta_\ell c_-^A\overline{c_-^A} - b_+^V\overline{b_+^V} - b_+^A\overline{b_+^A} + b_-^V\overline{b_-^V} + b_-^A\overline{b_-^A}\Big),\\
K_6^{VA} =& -\frac{K_{13}^{VA}}{P_b} = \frac{\alpha\sqrt{1-\delta_\ell}}{4}\Big(-b_+^V\overline{b_+^A} - b_+^A\overline{b_+^V} - b_-^V\overline{b_-^A} - b_-^A\overline{b_-^V}\Big),\\
K_7^{VA} - iK_9^{VA} =& \frac{\sqrt{2}\alpha(1-\delta_\ell)}{4}\Big(a_-^V\overline{b_-^V} + a_-^A\overline{b_-^A} - b_+^V\overline{a_+^V} - b_+^A\overline{a_+^A}\Big),\\
K_8^{VA} - iK_{10}^{VA} =& -\frac{\sqrt{2}\alpha\sqrt{1-\delta_\ell}}{4}\Big(a_-^V\overline{b_-^A} + a_-^A\overline{b_-^V} + b_+^V\overline{a_+^A} + b_+^A\overline{a_+^V}\Big),\\
K_{11}^{VA} =& \frac{P_b}{4}\Big(-\delta_\ell a_+^A\overline{a_+^A} + \delta_\ell a_-^A\overline{a_-^A} + \frac{\delta_\ell b_+^V\overline{b_+^V}}{2} - \frac{\delta_\ell b_+^A\overline{b_+^A}}{2} - \frac{\delta_\ell b_-^V\overline{b_-^V}}{2} + \frac{\delta_\ell b_-^A\overline{b_-^A}}{2} + \delta_\ell c_+^A\overline{c_+^A} \\
& - \delta_\ell c_-^A\overline{c_-^A} + a_+^V\overline{a_+^V} + a_+^A\overline{a_+^A} - a_-^V\overline{a_-^V} - a_-^A\overline{a_-^A} + \frac{b_+^V\overline{b_+^V}}{2} + \frac{b_+^A\overline{b_+^A}}{2} - \frac{b_-^V\overline{b_-^V}}{2} - \frac{b_-^A\overline{b_-^A}}{2}\Big),\\
K_{12}^{VA} =& \frac{P_b}{4}\Big(\delta_\ell a_+^V\overline{a_+^V} - \delta_\ell a_-^V\overline{a_-^V} - \delta_\ell b_+^A\overline{b_+^A} + \delta_\ell b_-^A\overline{b_-^A} + \delta_\ell c_+^A\overline{c_+^A} - \delta_\ell c_-^A\overline{c_-^A} + b_+^V\overline{b_+^V} + b_+^A\overline{b_+^A} - b_-^V\overline{b_-^V} - b_-^A\overline{b_-^A}\Big),
\end{aligned} \tag{A1}
$$

$$
\begin{aligned}
K_{14}^{VA} =& \frac{P_b}{4}\alpha\Big(-\delta_\ell a_+^A\overline{a_+^A}-\delta_\ell a_-^A\overline{a_-^A}-\frac{\delta_\ell b_+^V\overline{b_+^V}}{2}+\frac{\delta_\ell b_+^A\overline{b_+^A}}{2}-\frac{\delta_\ell b_-^V\overline{b_-^V}}{2}+\frac{\delta_\ell b_-^A\overline{b_-^A}}{2}+\delta_\ell c_+^A\overline{c_+^A}\\
&+\delta_\ell c_-^A\overline{c_-^A}+a_+^V\overline{a_+^V}+a_+^A\overline{a_+^A}+a_-^V\overline{a_-^V}+a_-^A\overline{a_-^A}-\frac{b_+^V\overline{b_+^V}}{2}-\frac{b_+^A\overline{b_+^A}}{2}-\frac{b_-^V\overline{b_-^V}}{2}-\frac{b_-^A\overline{b_-^A}}{2}\Big),\\
K_{15}^{VA} =& \frac{P_b}{4}\alpha\Big(\delta_\ell a_+^V\,\overline{a_+^V}+\delta_\ell a_-^V\overline{a_-^V}+\delta_\ell b_+^A\overline{b_+^A}+\delta_\ell b_-^A\overline{b_-^A}+\delta_\ell c_+^A\overline{c_+^A}\\
&+\delta_\ell c_-^A\overline{c_-^A}-b_+^V\overline{b_+^V}-b_+^A\overline{b_+^A}-b_-^V\overline{b_-^V}-b_-^A\overline{b_-^A}\Big),\\
K_{17}^{VA}-\mathrm{i}K_{19}^{VA} =& -\frac{\sqrt{2}P_b\alpha(1-\delta_\ell)}{4}\Big(a_-^V\overline{b_-^V}+a_-^A\overline{b_-^A}+b_+^V\overline{a_+^V}+b_+^A\overline{a_+^A}\Big),\\
K_{18}^{VA}-\mathrm{i}K_{20}^{VA} =& -\frac{\sqrt{2}P_b\alpha\sqrt{1-\delta_\ell}}{4}\Big(-a_-^V\overline{b_-^A}-a_-^A\overline{b_-^V}+b_+^V\overline{a_+^A}+b_+^A\overline{a_+^V}\Big),\\
K_{23}^{VA}-\mathrm{i}K_{21}^{VA} =& \frac{P_b\sqrt{2}(1-\delta_\ell)}{4}\Big(b_+^V\overline{a_-^V}-a_+^V\overline{b_-^V}-a_+^A\overline{b_-^A}+b_+^A\overline{a_-^A}\Big),\\
K_{24}^{VA}-\mathrm{i}K_{22}^{VA} =& -\frac{P_b\sqrt{2}\sqrt{(1-\delta_\ell)}}{4}\Big(a_+^V\overline{b_-^A}+a_+^A\overline{b_-^V}+b_+^V\overline{a_-^A}+b_+^A\overline{a_-^V}\Big),\\
K_{27}^{VA}-\mathrm{i}K_{25}^{VA} =& -\frac{P_b\alpha\sqrt{2}(1-\delta_\ell)}{4}\Big(-a_+^V\overline{b_-^V}-a_+^A\overline{b_-^A}-b_+^V\overline{a_-^V}-b_+^A\overline{a_-^A}\Big),\\
K_{28}^{VA}-\mathrm{i}K_{26}^{VA} =& -\frac{P_b\alpha\sqrt{2}\sqrt{(1-\delta_\ell)}}{4}\Big(a_+^V\overline{b_-^A}+a_+^A\overline{b_-^V}-b_+^V\overline{a_-^A}-b_+^A\overline{a_-^V}\Big),\\
K_{31}^{VA}-\mathrm{i}K_{29}^{VA} =& -\frac{P_b\alpha\delta_\ell}{2}\Big(a_-^V\overline{a_+^V}+c_-^A\overline{c_+^A}\Big),\\
K_{32}^{VA}-\mathrm{i}K_{30}^{VA} =& -\frac{P_b\alpha}{2}\Big(-a_-^V\overline{a_+^V}-a_-^A\overline{a_+^A}\Big)+\delta_\ell\Big(a_-^A\overline{a_+^A}-c_-^A\overline{c_+^A}\Big),\\
K_{33}^{VA}-\mathrm{i}K_{34}^{VA} =& \frac{P_b\alpha}{4}b_+^V\overline{b_-^V}.
\end{aligned}
\tag{A2}
$$

APPENDIX B: ANGULAR OBSERVABLES WITH (PSEUDO)SCALAR OPERATORS

For real K_i, we have

$$
\begin{aligned}
K_1^{SP} =& \sqrt{2q^2}M_\ell\Big(\sqrt{1-\delta_\ell}\frac{\alpha c_+^A\overline{c_+^P}}{2}-\frac{\alpha c_+^A\overline{c_+^P}}{2}-\sqrt{1-\delta_\ell}\frac{\alpha c_-^A\overline{c_-^S}}{2}+\frac{\alpha c_-^A\overline{c_-^S}}{2}+\sqrt{1-\delta_\ell}\frac{\alpha c_+^P\overline{c_+^A}}{2}\\
&-\frac{\alpha c_+^P\overline{c_+^A}}{2}-\sqrt{1-\delta_\ell}\frac{\alpha c_-^S\overline{c_-^A}}{2}+\frac{\alpha c_-^S\overline{c_-^A}}{2}+\sqrt{1-\delta_\ell}\frac{c_+^A\overline{c_+^P}}{2}+\frac{3c_+^A\overline{c_+^P}}{2}-\sqrt{1-\delta_\ell}\frac{c_-^A\overline{c_-^S}}{2}\\
&+2c_-^A\overline{c_-^P}+\frac{c_-^A\overline{c_-^S}}{2}+\sqrt{1-\delta_\ell}\frac{c_+^P\overline{c_+^A}}{2}+\frac{3c_+^P\overline{c_+^A}}{2}+2c_-^P\overline{c_-^A}-\sqrt{1-\delta_\ell}\frac{c_-^S\overline{c_-^A}}{2}+\frac{c_-^S\overline{c_-^A}}{2}\Big)+O(C_{S,P}^2)
\end{aligned}
\tag{B1}
$$

$$
\begin{aligned}
K_2^{SP} =& \sqrt{2q^2}M_\ell\Big(\sqrt{1-\delta_\ell}\frac{c_+^A\overline{c_+^P}}{2}+\frac{3c_+^A\overline{c_+^P}}{2}-\sqrt{1-\delta_\ell}\frac{c_-^A\overline{c_-^S}}{2}+2c_-^A\overline{c_-^P}+\frac{c_-^A\overline{c_-^S}}{2}+\sqrt{1-\delta_\ell}\frac{c_+^P\overline{c_+^A}}{2}\\
&+\frac{3c_+^P\overline{c_+^A}}{2}+2c_-^P\overline{c_-^A}-\sqrt{1-\delta_\ell}\frac{c_-^S\overline{c_-^A}}{2}+\frac{c_-^S\overline{c_-^A}}{2}\Big)+O(C_{S,P}^2)
\end{aligned}
\tag{B2}
$$

$$
\begin{aligned}
K_3^{SP} =& \sqrt{2q^2}M_\ell\Big(\sqrt{1-\delta_\ell}\frac{\alpha a_+^V\overline{c_+^P}}{2}-\frac{\alpha a_+^V\overline{c_+^P}}{2}+\sqrt{1-\delta_\ell}\frac{a_+^V\overline{c_+^P}}{2}+2\sqrt{1-\delta_\ell}a_+^V\overline{c_+^S}-\frac{a_+^V\overline{c_+^P}}{2}\\
&-\sqrt{1-\delta_\ell}\frac{\alpha a_-^V\overline{c_-^S}}{2}+\frac{\alpha a_-^V\overline{c_-^S}}{2}+\sqrt{1-\delta_\ell}\frac{3a_-^V\overline{c_-^S}}{2}+\frac{a_-^V\overline{c_-^S}}{2}+\sqrt{1-\delta_\ell}\frac{\alpha c_+^P\overline{a_+^V}}{2}-\frac{\alpha c_+^P\overline{a_+^V}}{2}\\
&-\sqrt{1-\delta_\ell}\frac{\alpha c_-^S\overline{a_-^V}}{2}+\frac{\alpha c_-^S\overline{a_-^V}}{2}+\sqrt{1-\delta_\ell}\frac{c_+^P\overline{a_+^V}}{2}-\frac{c_+^P\overline{a_+^V}}{2}+2\sqrt{1-\delta_\ell}c_+^S\overline{a_+^V}+\sqrt{1-\delta_\ell}\frac{3c_-^S\overline{a_-^V}}{2}+\frac{c_-^S\overline{a_-^V}}{2}\Big)
\end{aligned}
\tag{B3}
$$

$$K_4^{SP} = K_5^{SP} = \sqrt{2q^2}M_\ell\alpha\Big(\sqrt{1-\delta_\ell}c_+^A\overline{c_+^P} + c_+^A\overline{c_+^P} + \sqrt{1-\delta_\ell}c_-^A\overline{c_-^S} - 2c_-^A\overline{c_-^P} - c_-^A\overline{c_-^S} + \sqrt{1-\delta_\ell}c_+^P\overline{c_+^A} + c_+^P\overline{c_+^A} - 2c_-^P\overline{c_-^A} + \sqrt{1-\delta_\ell}c_-^S\overline{c_-^A} - c_-^S\overline{c_-^A}\Big) + O(C_{S,P}^2) \tag{B4}$$

$$K_6^{SP} = \sqrt{2q^2}M_\ell\alpha\Big[(\sqrt{1-\delta_\ell}-1)a_+^V\overline{c_+^P} + 2\sqrt{1-\delta_\ell}a_+^V\overline{c_+^S} - (\sqrt{1-\delta_\ell}+1)a_-^V\overline{c_-^S} + (\sqrt{1-\delta_\ell}-1)c_+^P\overline{a_+^V} + 2\sqrt{1-\delta_\ell}c_+^S\overline{a_+^V} - (\sqrt{1-\delta_\ell}+1)c_-^S\overline{a_-^V}\Big] \tag{B5}$$

$$\begin{aligned}
K_7^{SP} =& K_9^{SP} = 0\\
K_8^{SP} - \mathrm{i}K_{10}^{SP} =& \alpha M_\ell\sqrt{q^2}\left[(\sqrt{1-\delta_\ell}-1)(b_+^V\overline{c_+^P}) + 2\sqrt{1-\delta_\ell}(b_+^V\overline{c_+^S}) - (\sqrt{1-\delta_\ell}+1)(c_-^S\overline{b_-^V})\right]\\
K_{11}^{SP} =& \sqrt{2q^2}M_\ell\Big(\sqrt{1-\delta_\ell}\frac{\alpha c_+^A\overline{c_+^P}}{2} - \frac{\alpha c_+^A\overline{c_+^P}}{2} + \sqrt{1-\delta_\ell}\frac{\alpha c_-^A\overline{c_-^S}}{2} - \frac{\alpha c_-^A\overline{c_-^S}}{2} + \sqrt{1-\delta_\ell}\frac{\alpha c_+^P\overline{c_+^A}}{2}\\
&- \frac{\alpha c_+^P\overline{c_+^A}}{2} + \sqrt{1-\delta_\ell}\frac{\alpha c_-^S\overline{c_-^A}}{2} - \frac{\alpha c_-^S\overline{c_-^A}}{2} + \sqrt{1-\delta_\ell}\frac{c_+^A\overline{c_+^P}}{2} + \frac{3c_+^A\overline{c_+^P}}{2} + \sqrt{1-\delta_\ell}\frac{c_-^A\overline{c_-^S}}{2}\\
&- 2c_-^A\overline{c_-^P} - \frac{c_-^A\overline{c_-^S}}{2} + \frac{c_+^P\sqrt{1-\delta_\ell}\overline{c_+^A}}{2} + \frac{3c_+^P\overline{c_+^A}}{2} - 2c_-^P\overline{c_-^A} + \sqrt{1-\delta_\ell}\frac{c_-^S\overline{c_-^A}}{2} - \frac{c_-^S\overline{c_-^A}}{2}\Big) + O(C_{S,P}^2)
\end{aligned} \tag{B6}$$

$$\begin{aligned}
K_{12}^{SP} =& \sqrt{2q^2}M_\ell\Big(\sqrt{1-\delta_\ell}\frac{\alpha c_+^A\overline{c_+^P}}{2} - \frac{\alpha c_+^A\overline{c_+^P}}{2} + \sqrt{1-\delta_\ell}\frac{\alpha c_-^A\overline{c_-^S}}{2} - \frac{\alpha c_-^A\overline{c_-^S}}{2} + \sqrt{1-\delta_\ell}\frac{\alpha c_+^P\overline{c_+^A}}{2}\\
&- \frac{\alpha c_+^P\overline{c_+^A}}{2} + \sqrt{1-\delta_\ell}\frac{\alpha c_-^S\overline{c_-^A}}{2} - \frac{\alpha c_-^S\overline{c_-^A}}{2} + \sqrt{1-\delta_\ell}\frac{c_+^A\overline{c_+^P}}{2} + \frac{3c_+^A\overline{c_+^P}}{2} + \sqrt{1-\delta_\ell}\frac{c_-^A\overline{c_-^S}}{2}\\
&- 2c_-^A\overline{c_-^P} - \frac{c_-^A\overline{c_-^S}}{2} + \sqrt{1-\delta_\ell}\frac{c_+^P\overline{c_+^A}}{2} + \frac{3c_+^P\overline{c_+^A}}{2} - 2c_-^P\overline{c_-^A} + \sqrt{1-\delta_\ell}\frac{c_-^S\overline{c_-^A}}{2} - \frac{c_-^S\overline{c_-^A}}{2}\Big) + O(C_{S,P}^2)
\end{aligned} \tag{B7}$$

$$\begin{aligned}
K_{13}^{SP} =& \sqrt{2q^2}M_\ell\Big(\sqrt{1-\delta_\ell}\frac{a_+^V\alpha\overline{c_+^P}}{2} - \frac{a_+^V\alpha\overline{c_+^P}}{2} + \sqrt{1-\delta_\ell}\frac{a_+^V\overline{c_+^P}}{2} + 2\sqrt{1-\delta_\ell}a_+^V\overline{c_+^S} - \frac{a_+^V\overline{c_+^P}}{2}\\
&+ \sqrt{1-\delta_\ell}\frac{a_-^V\alpha\overline{c_-^S}}{2} - \frac{a_-^V\alpha\overline{c_-^S}}{2} - \sqrt{1-\delta_\ell}\frac{3a_-^V\overline{c_-^S}}{2} - \frac{a_-^V\overline{c_-^S}}{2} + \sqrt{1-\delta_\ell}\frac{\alpha c_+^P\overline{a_+^V}}{2} - \frac{\alpha c_+^P\overline{a_+^V}}{2}\\
&+ \sqrt{1-\delta_\ell}\frac{\alpha c_-^S\overline{a_-^V}}{2} - \frac{\alpha c_-^S\overline{a_-^V}}{2} + \sqrt{1-\delta_\ell}\frac{c_+^P\overline{a_+^V}}{2} - \frac{c_+^P\overline{a_+^V}}{2} + 2\sqrt{1-\delta_\ell}c_+^S\overline{a_+^V} - \sqrt{1-\delta_\ell}\frac{3c_-^S\overline{a_-^V}}{2} - \frac{c_-^S\overline{a_-^V}}{2}\Big)
\end{aligned} \tag{B8}$$

$$\begin{aligned}
K_{14}^{SP} =& K_{15}^{SP} = \sqrt{2q^2}M_\ell\Big(\sqrt{1-\delta_\ell}c_+^A\overline{c_+^P} + c_+^A\overline{c_+^P} - \sqrt{1-\delta_\ell}c_-^A\overline{c_-^S} + 2c_-^A\overline{c_-^P} + c_-^A\overline{c_-^S}\\
&+ \sqrt{1-\delta_\ell}c_+^P\overline{c_+^A} + c_+^P\overline{c_+^A} + 2c_-^P\overline{c_-^A} - \sqrt{1-\delta_\ell}c_-^S\overline{c_-^A} + c_-^S\overline{c_-^A}\Big) + O(C_{S,P}^2)
\end{aligned} \tag{B9}$$

$$\begin{aligned}
K_{16}^{SP} =& \sqrt{2q^2}M_\ell\Big(\sqrt{1-\delta_\ell}\frac{a_+^V\overline{c_+^P}}{4} + \sqrt{1-\delta_\ell}\frac{a_+^V\overline{c_+^S}}{2} - \frac{a_+^V\overline{c_+^P}}{4} + \sqrt{1-\delta_\ell}\frac{a_-^V\overline{c_-^S}}{4} + \frac{a_-^V\overline{c_-^S}}{4}\\
&+ \sqrt{1-\delta_\ell}\frac{c_+^P\overline{a_+^V}}{4} - \frac{c_+^P\overline{a_+^V}}{4} + \sqrt{1-\delta_\ell}\frac{c_+^S\overline{a_+^V}}{2} + \sqrt{1-\delta_\ell}\frac{c_-^S\overline{a_-^V}}{4} + \frac{c_-^S\overline{a_-^V}}{4}\Big) + O(C_{S,P}^2)
\end{aligned} \tag{B10}$$

$$K_{17}^{SP} = K_{19}^{SP} = 0 \tag{B11}$$

$$K_{18}^{SP} - \mathrm{i}K_{20}^{SP} = M_\ell\sqrt{q^2}(2\sqrt{1-\delta_\ell}b_+^V\overline{c_+^P} + 4\sqrt{1-\delta_\ell}b_+^V\overline{c_+^S} - 2b_+^V\overline{c_+^P} + 2\sqrt{1-\delta_\ell}c_-^S\overline{b_-^V} + 2c_-^S\overline{b_-^V}) \tag{B12}$$

$$K_{23}^{SP} = K_{21}^{SP} = 0 \tag{B13}$$

$$K_{24}^{SP} - \mathrm{i}K_{22}^{SP} = -M_\ell\sqrt{q^2}\Big(\sqrt{1-\delta_\ell}\alpha b_+^V \overline{c_-^S} - \alpha b_+^V \overline{c_-^S} + \sqrt{1-\delta_\ell}\alpha c_+^P \overline{b_-^V} - \alpha c_+^P \overline{b_-^V} - 3\sqrt{1-\delta_\ell} b_+^V \overline{c_-^S} - b_+^V \overline{c_-^S} + \sqrt{1-\delta_\ell} c_+^P \overline{b_-^V} - c_+^P \overline{b_-^V} + 4\sqrt{1-\delta_\ell} c_+^S \overline{b_-^V}\Big) \tag{B14}$$

$$K_{25}^{SP} = K_{27}^{SP} = 0 \tag{B15}$$

$$K_{28}^{SP} - \mathrm{i}K_{26}^{SP} = -2M_\ell\sqrt{q^2}\Big(\sqrt{1-\delta_\ell} b_+^V \overline{c_-^S} + b_+^V \overline{c_-^S} + \sqrt{1-\delta_\ell} c_+^P \overline{b_-^V} - c_+^P \overline{b_-^V} + 2\sqrt{1-\delta_\ell} c_+^S \overline{b_-^V}\Big) \tag{B16}$$

$$K_{31}^{SP} - \mathrm{i}K_{29}^{SP} = K_{32}^{SP} - \mathrm{i}K_{30}^{SP} = \sqrt{2q^2}M_\ell\Big(-c_-^A\sqrt{1-\delta_\ell}\overline{c_+^P} - c_-^A \overline{c_+^P} - 2c_-^P \overline{c_+^A} + c_-^S\sqrt{1-\delta_\ell}\overline{c_+^A} - c_-^S \overline{c_+^A}\Big) + O(C_{S,P}^2) \tag{B17}$$

$$K_{33}^{SP} = K_{34}^{SP} = 0 \tag{B18}$$

APPENDIX C: NUMERICAL RESULTS BASED ON THE HOMOGENEOUS BAG MODEL

In this study, we estimate the form factors using the homogeneous bag model (HBM), where the calculation details are given in Ref. [38]. The bag parameters adopted in this work are given as

$$(m_s, m_b) = (0.28, 4.8)\ \mathrm{GeV}, \quad 0.313\ \mathrm{GeV} < E_{u,d} < 0.368\ \mathrm{GeV}, \tag{C1}$$

where $R = 4.8\ \mathrm{GeV}^{-1}$ is the bag radius, and E_q is the quark energy. Recently, α has been updated by BESIII [39, 40] with remarkable precision. We take $\alpha = 0.732 \pm 0.014$, $M_{\Lambda_b} = 5.6196$ GeV, and the Λ_b lifetime of $\tau_b = 1.471\times 10^{-12}$ s from the particle data group [41]. The main uncertainties of the HBM model are attributed to E_q affecting the form factors largely at the low q^2 region.

The total branching fractions obtained by integrating $\vec{\Omega}$ and q^2 in Eq. (53) are given as

$$\mathcal{B}_\ell = \mathcal{B}(\Lambda_b \to \Lambda \ell^+\ell^-) = \tau_b \int_{4m_\ell^2}^{M_-^2} \zeta(K_1 + 2K_2)\mathrm{d}q^2. \tag{C2}$$

Their computed values and the ones in the literature within the SM are listed in Table C1. In the literature, Refs. [26, 42] consider the covariant quark model (CQM), Refs. [43−45], the light-cone QCD sum rules (LCSR), Ref. [35], the relativistic quark model (RQM), and Ref. [46], the Bethe-Salpeter equation (BSE). Our results of $\mathcal{B}_\ell$ agree with the results of the CQM and RQM as well as current experimental data but are systematically smaller than the results obtained from LCSR. Notably, we find that $\mathcal{B}_e > \mathcal{B}_\mu$ is consistent with Refs. [35] and [43]. Nevertheless, the tendency has not been found in the BSE and CQM. Explicitly, we obtain that $\mathcal{B}_e/\mathcal{B}_\mu =$ 1.15 with a little uncertainty due to the correlations. Future experiments on $\mathcal{B}_e/\mathcal{B}_\mu$ may distinguish between the approaches.

The integrated hadron (lepton) forward-backward asymmetry of A_{FB}^h (A_{FB}^ℓ) is related to $\langle K_i\rangle$ via

$$A_{FB}^h = \langle K_4\rangle + \frac{1}{2}\langle K_5\rangle, \quad A_{FB}^\ell = \frac{3}{2}\langle K_3\rangle, \tag{C3}$$

while

$$A_{FB}^{\ell h} = \frac{3}{4}\langle K_6\rangle, \quad F_L = 2\langle K_1\rangle - \langle K_2\rangle, \tag{C4}$$

are the combined forward-backward asymmetry and longitudinal polarized fraction, respectively. The average decay branching fraction is defined as

$$\left\langle \frac{\partial\mathcal{B}}{\partial q^2}\right\rangle \equiv \frac{\tau_b}{\kappa' - \kappa}\Gamma_\kappa. \tag{C5}$$

Table C1. $\mathcal{B}_\ell$ in units of 10^{-6}.

	HBM	CQM[26]	LCSR[43]	LCSR [44]	BSE[46]	CQM[42]	LCSR[45]	RQM[35]	Exp[41]
$\mathcal{B}_e$	0.91(25)	1.0	4.6(1.6)		0.660 ~ 1.208		$2.03(^{26}_{9})$	1.07	
$\mathcal{B}_\mu$	0.79(18)	1.0	4.0(1.2)	$6.1(^{5.8}_{1.7})$	0.812 ~ 1.445	0.70		1.05	1.08(28)
$\mathcal{B}_\tau$	0.21(2)	0.2	0.8(3)	$2.1(^{2.3}_{0.6})$	0.252 ~ 0.392	0.22		0.26	

Notice that the q^2 region of $[\kappa,\kappa'] = [8,11]$ and $[12.5,15]$ in units of GeV^2 are contaminated largely by the charmonium resonance and are not considered.

The computed results within the HBM are given in Table C2, along with those from the literature and the experimental data [18, 19]. The computed values of $A_{FB}^{h,\ell,h\ell}$ and F_L have little uncertainties as K_i are correlated in the model calculations. In the literature, Ref. [47] employs the lattice QCD, and Ref. [35] includes the contributions from the charmonium resonances. We see that the angular observables in the literature and this study are basically consistent. Our results of $\langle A_{FB}^h\rangle$ and $\langle A_{FB}^{\ell h}\rangle$ are slightly larger than the others owing to the updated α[1)]. Notably, the experimental values of $A_{FB}^{\ell h}$ are nearly twice larger than the theoretical predictions.

Integrating Eq. (54) over $\vec{\Omega}$, we get the differential decay rate as

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}q^2} = 2K_1 + K_2, \tag{C6}$$

Table C2. Decay observables, where $\langle\partial\mathcal{B}/\partial q^2\rangle$ and $\kappa^{(\prime)}$ are in units of 10^{-7} GeV^{-2} and GeV^2, respectively.

	$[\kappa,\kappa']$	HBM	RQM [35]	lattice [47]	LHCb [18, 19]
$\left\langle\frac{\partial\mathcal{B}}{\partial q^2}\right\rangle$	$[0.1,2]$	0.25(11)	0.34	0.25(23)	$0.36(^{14}_{13})$
	$[2,4]$	0.16(7)	0.31	0.18(12)	$0.11(^{12}_{9})$
	$[4,6]$	0.20(8)	0.40	0.23(11)	$0.02(^{9}_{1})$
	$[6,8]$	0.26(9)	0.57	0.307(94)	$0.25(^{13}_{12})$
	$[11,12.5]$	0.44(11)	0.65		0.75(21)
	$[15,16]$	0.61(10)	0.72	0.796(75)	1.12(30)
	$[16,18]$	0.65(8)	0.68	0.827(76)	1.22(29)
	$[1.1,6]$	0.18(7)	0.34	0.20(12)	$0.09(^{6}_{5})$
	$[15,20]$	0.60(6)	0.61	0.756(70)	$1.20(^{26}_{27})$
A_{FB}^{ℓ}	$[0.1,2]$	0.076(0)	0.067	0.095(15)	$0.37(^{37}_{48})$
	$[11,12.5]$	−0.357(6)	−0.35		$0.01(^{20}_{19})$
	$[15,16]$	−0.403(8)	−0.41	−0.374(14)	$-0.10(^{18}_{16})$
	$[16,18]$	−0.396(9)	−0.36	−0.372(13)	$-0.07(^{14}_{13})$
	$[18,20]$	−0.320(9)	−0.32	−0.309(15)	$0.01(^{16}_{15})$
	$[15,20]$	−0.369(7)	−0.33	−0.350(13)	−0.39(4)
A_{FB}^{h}	$[0.1,2]$	−0.294(2)	−0.26	−0.310(18)	$-0.12(^{34}_{32})$
	$[11,12.5]$	−0.408(2)	−0.30		$-0.50(^{11}_{4})$
	$[15,16]$	−0.384(4)	−0.32	−0.3069(83)	$-0.19(^{14}_{16})$
	$[16,18]$	−0.358(6)	−0.31	−0.2891(90)	$-0.44(^{10}_{6})$
	$[18,20]$	−0.275(6)	−0.25	−0.227(10)	$-0.13(^{10}_{12})$
	$[15,20]$	−0.333(4)	−0.29	−0.2710(92)	−0.30(5)
$A_{FB}^{h\ell}$	$[0.1,2]$	−0.028(0)	−0.021	−0.0302(51)	
	$[2,4]$	−0.001(1)	0.010	−0.0169(99)	
	$[4,6]$	0.047(2)	0.045	0.021(13)	
	$[6,8]$	0.084(1)	0.072	0.053(13)	
	$[15,20]$	0.179(1)	0.129	0.1398(43)	0.25(4)
F_L	$[0.1,2]$	0.541(4)	0.66	0.465(84)	$0.56(^{24}_{56})$
	$[11,12.5]$	0.615(0)	0.51		$0.40(^{37}_{36})$
	$[15,16]$	0.507(1)	0.41	0.454(20)	0.49(30)
	$[16,18]$	0.469(0)	0.38	0.417(15)	$0.68(^{15}_{21})$
	$[18,20]$	0.416(1)	0.35	0.3706(79)	$0.62(^{24}_{27})$

1) They used $\alpha = 0.642 \pm 0.013$ [48], in sharp contrast to $\alpha = 0.732 \pm 0.014$ adopted in this work.

we use the differential decay rate to normalize our angular observables. The first ten K observables are accessible even if P_b is zero, i.e., the Λ_b baryon is unpolarized, which is the case that most Λ_b baryons produced at the LHC satisfied.

The completeness relations read as

$$1 = \sum_{J,J_z,\lambda_1,\lambda_2} \frac{4\pi}{2J+1} |J,J_z,\lambda_1,\lambda_2\rangle\langle J,J_z,\lambda_1,\lambda_2|. \quad \text{(C7)}$$

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