

Possibility of experimental study on nonleptonic B_c^* weak decays*

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Abstract: The ground vector B_c^* meson has not yet been experimentally discovered until now. Besides the dominant electromagnetic decays, nonleptonic weak decays provide another choice to search for the mysterious B_c^* mesons. Inspired by the potential prospects of B_c^* mesons in future high-luminosity colliders, nonleptonic B_c^* weak decays induced by bottom and charm quark decays are studied within the SM by using a naive factorization approach. It is found that for $B_c^* \rightarrow B_{s,d}\pi, B_{s,d}^*\pi, B_{s,d}\rho, B_sK, B_s^*K, B_sK^*, \eta_c(1S,2S)\pi, \eta_c(1S,2S)\rho$ and $\psi(1S,2S)\pi$ decays, a few hundred and even thousands of events might be observable in CEPC, FCC-ee and LHCb@HL-LHC experiments.

Keywords: B_c^* meson, weak decay, factorization, branching ratio

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I. INTRODUCTION

According to the $q\bar{q}$ quark model assignments for mesons, bottom-charmed mesons are unique particles consisting of two heavy quarks with different flavors, because it is generally assumed that the top quark being the heaviest element fermion of the standard model (SM) has a very short lifetime and decays before hadronization. The bottom-charmed mesons are isospin singlets, and have nonzero additive quantum numbers, $Q = B = C = \pm 1$, where Q , B and C are respectively electric charge, bottom and charm. The bottom-charmed mesons, analogous with hidden-flavored charmonium $c\bar{c}$ and bottomonium $b\bar{b}$, are often considered as nonrelativistic bound states. Hyperfine splitting interactions divide the ground bottom-charmed states into the spin singlet 1^1S_0 and triplet 1^3S_1 , corresponding to the pseudoscalar B_c meson with the spin-parity $J^P = 0^-$ and vector B_c^* meson with $J^P = 1^-$, respectively.

The pseudoscalar B_c meson was first discovered via $B_c \rightarrow J/\psi\pi$ and $J/\psi\pi\pi^+\pi^-$ decay modes with hadronic Z^0 decay sample data collected by the DELPHI detector at the e^+e^- collider LEP in 1997 [1]. The natural properties

of the B_c meson including its mass, lifetime, and spin-parity have now been well determined [2]. In the meantime, the vector B_c^* meson has not been definitively identified by experimental physicists yet. All the information on the B_c^* meson comes only from theoretical calculation for the moment. Undoubtedly, it is a matter of great urgency to find and identify the B_c^* meson in experiments, which is the basis and prerequisite for obtaining a deeper insight into the desired B_c^* meson, and distinguishing by different theoretical models.

The production probability of bottom-charmed mesons is far less than that of heavy-light $B_{u,d}$ mesons and heavy-heavy charmonium and bottomonium. First, two heavy quark pairs of both $b\bar{b}$ and $c\bar{c}$ should be almost simultaneously produced in a high energy process, and then a b (or \bar{b}) quark from a $b\bar{b}$ pair and \bar{c} (or c) quark from a $c\bar{c}$ pair should be lucky enough to combine and finally form bottom-charmed mesons [3]. In positron-electron collisions, bottom-charmed mesons can be produced via Z^0 boson decay,

$$e^+ + e^- \rightarrow Z^0 \rightarrow (b\bar{c}) + \bar{b} + c, \quad (1)$$

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for example, the observation of B_c mesons by DELPHI [1], ALEPH [4] and OPAL [5] detectors in LEP experiments. In hadron-hadron collisions, the bottom-charmed mesons can be produced via gluon-gluon fusion and quark-antiquark annihilation,

$$g + g \rightarrow Z^0 \rightarrow (b\bar{c}) + \bar{b} + c, \quad (2)$$

$$q + \bar{q} \rightarrow Z^0 \rightarrow (b\bar{c}) + \bar{b} + c, \quad (3)$$

for example, the observation of B_c meson by CDF [6] and D0 [7] at Fermilab Tevatron, by LHCb [8] in LHC experiments. More and more B_c^* mesons are expected to be accessible in the future. Given the branching ratio of Z^0 boson decay $Br(Z^0 \rightarrow b\bar{b}) = 15.12(5)\%$ [2] and the bottom quark fragmentation fraction $f(b \rightarrow B_c^*) \sim 6 \times 10^{-4}$ [9–11], there will be more than 10^8 B_c^* mesons from 10^{12} Z^0 boson decays at the Circular Electron Positron Collider (CEPC) [12], and more than 10^9 B_c^* mesons from 10^{13} Z^0 boson decays at the Future Circular Collider (FCC-ee) [13]. Assuming the B_c^* production cross section is about 100 nb for pp collisions at $\sqrt{s} = 13$ TeV [14], more than 3×10^{10} B_c^* mesons will be available with a total integrated luminosity of 300 fb^{-1} at LHCb of HL-LHC [15]. The huge amount of experimental data provides an excellent opportunity and solid foundation to carefully study the B_c^* mesons.

Because of its nonrelativistic nature, the mass of the B_c^* meson is approximately equal to the sum of the mass of bottom and charm quarks, *i.e.*, $m_{B_c^*} \simeq m_b + m_c$. A recent lattice calculation gives $m_{B_c^*} = 6331(7)$ MeV [16], which agrees basically with many other theoretical estimations with various theoretical models (*e.g.*, references [150–207] of Ref. [17]). Owing to the hierarchical relationships among mesonic masses, $m_{B_c^*} < m_B + m_D$ and $m_{B_c^*} < m_{B_c} + m_\pi$, the B_c^* meson can not decay through the strong interaction. The dominant decay of the B_c^* meson is the radiative transition process, $B_c^* \rightarrow B_c + \gamma$. This partial decay width can be written as [18],

$$\Gamma(B_c^* \rightarrow B_c \gamma) = \frac{4}{3} \alpha_{\text{em}} k_\gamma^3 |\mu_{B_c^* B_c}|^2, \quad (4)$$

where the center-of-mass momentum of a photon in the rest frame of the B_c^* meson and the magnetic dipole (M1) momentum are respectively defined as

$$k_\gamma = \frac{m_{B_c^*}^2 - m_{B_c}^2}{2m_{B_c^*}} \approx 56 \text{ MeV}, \quad (5)$$

$$\mu_{B_c^* B_c} = \langle B_c | \sum_{i=b,c} \frac{Q_i}{2m_i} \sigma_{iz} | B_c^* \rangle = \frac{1}{6} \left(\frac{2}{m_c} - \frac{1}{m_b} \right). \quad (6)$$

It is clear that, kinematically, the parity and angular momentum conservation in electromagnetic interactions require the orbital angular momentum between the B_c meson and photon to be $L = 1$. In addition, the photon is very soft, resulting in a very small phase space. Dynamically, the decay width is proportional to the fine-structure constant α_{em} of the electromagnetic interactions and the module square of the magnetic dipole momentum, while the magnetic dipole momentum is inversely proportional to the mass of heavy quarks. The combined effects of both kinematical and dynamical factors produce a very narrow decay width, $\Gamma(B_c^* \rightarrow B_c \gamma) \approx 60 \text{ eV}$ [17]. A good approximation is the full decay width of the B_c^* meson $\Gamma_{B_c^*} \approx \Gamma(B_c^* \rightarrow B_c \gamma)$. Experimentally, the electromagnetic process $B_c^* \rightarrow B_c \gamma$ with an occurrence probability of almost 100% should be easily detected at the e^+e^- CEPC and FCC-ee colliders, thanks to the excellent photon resolution of the electromagnetic calorimeter and the fine performance in reconstruction technology and methods of the charged particle tracks. Because the masses of B_c^* and B_c mesons are very close, the detection of the photon from the $B_c^* \rightarrow B_c \gamma$ process plays a critical role in distinguishing between the B_c^* and B_c mesons. The identification of the soft photon is very challenging. Moreover, the photon from the M1 transition $B_c^* \rightarrow B_c + \gamma$ decay is bound to be seriously affected by those from bremsstrahlung radiation and chaotic electromagnetic backgrounds, which results in identification inefficiencies. Besides the electromagnetic interactions, the B_c^* meson can also decay through weak interactions in the SM of elementary particles. The narrowness of the full width of the B_c^* meson affords great potential for experimental investigations on B_c^* weak decays. The B_c^* weak decays provide a good opportunity and an important and useful alternative to find and explore the foreseeable B_c^* mesons with some novel methods in future high-luminosity and high-precision experiments.

Compared with electromagnetic B_c^* decays, there are plenty of B_c^* meson weak decay processes. Based on the weak interaction couplings between particles, both component quarks of the B_c^* meson, the heavy bottom and charm quarks can transmit into first and second family quarks lighter than themselves. These B_c^* weak processes can be classified into three types, similar to those for the pseudoscalar B_c meson: (1) the bottom quark decays while the charm quark remain quiescent as a spectator; (2) the charm quark decays while the bottom quark remain inactive as a spectator; (3) the bottom and charm quarks annihilate into a virtual charged W boson. The purely leptonic B_c^* decays belonging to type (3) suffer additional complications from the final neutrinos. The charged hadrons from B_c^* weak decays, such as pions and kaons, are relatively easy to identify in sensitive particle detectors. In this paper, we estimate the branching ratio for $B_c^{*+} \rightarrow BP, BV, B^*P, \psi P, \eta_c P, \eta_c V$ decays arising

from external W emission with the factorization approach, where P (V) denotes the positively charged pion and kaon (ρ^+ and K^{*+}). Some studies [19–22] have shown that these processes in question have relatively large branching ratios between B_c^* meson weak decays and should have priority to be investigated experimentally. Here, we hope to provide a feasibility analysis of searching for the B_c^* meson via some particular nonleptonic weak decays in future high-energy and high-luminosity experiments.

The remainder of this paper is organized as follows. The effective Hamiltonian for the nonleptonic B_c^* weak decays in question is given in Section II. Branching ratios and our comments are presented in Section III. Section IV is devoted to a brief summary. The decay amplitudes are listed in Appendix.

II. THEORETICAL FRAMEWORK

The effective Hamiltonian for the concerned nonleptonic B_c^* decays is written as [23]

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{cb}^* V_{uq_1} \{C_1 O_1^b + C_2 O_2^b\} \quad (7)$$

$$+ \frac{G_F}{\sqrt{2}} V_{cq_2}^* V_{uq_3} \{C_1 O_1^c + C_2 O_2^c\}, \quad (8)$$

where Fermi constant $G_F \approx 1.166 \times 10^{-5} \text{ GeV}^{-2}$ [2] and Wilson coefficients $C_{1,2}$ are process-independent couplings. Eq. (7) and Eq. (8) correspond to type (1) and (2) B_c^* decays, respectively. V_{cq} and V_{uq} are the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements [24, 25], and their magnitudes have been determined experimentally [2].

$$|V_{ud}| = 0.97370(14), \quad (9)$$

$$|V_{us}| = 0.2245(8), \quad (10)$$

$$|V_{cd}| = 0.221(4), \quad (11)$$

$$|V_{cs}| = 0.987(11), \quad (12)$$

$$|V_{cb}| = 0.0410(14). \quad (13)$$

The expressions of effective tetra-quark operators are written as

$$O_1^b = [\bar{b}_\alpha \gamma^\mu (1 - \gamma_5) c_\alpha] [\bar{u}_\beta \gamma_\mu (1 - \gamma_5) q_{1\beta}], \quad (14)$$

$$O_2^b = [\bar{b}_\alpha \gamma^\mu (1 - \gamma_5) c_\beta] [\bar{u}_\beta \gamma_\mu (1 - \gamma_5) q_{1\alpha}], \quad (15)$$

$$O_1^c = [\bar{q}_{2\alpha} \gamma^\mu (1 - \gamma_5) c_\alpha] [\bar{u}_\beta \gamma_\mu (1 - \gamma_5) q_{3\beta}], \quad (16)$$

$$O_2^c = [\bar{q}_{2\alpha} \gamma^\mu (1 - \gamma_5) c_\beta] [\bar{u}_\beta \gamma_\mu (1 - \gamma_5) q_{3\alpha}], \quad (17)$$

where α and β are the color indices, $q_{1,2,3} = d$ and s .

It is clear that the calculation of hadronic matrix elements (HMEs) is the remaining central missions of decay amplitudes. HMEs trap the tetra-quark operators between initial and final mesons, and involve long- and short-distance contributions. Recently, several attractive phenomenological methods based on power counting rules and QCD perturbative calculations have been developed to deal with HMEs, such as the QCD factorization (QCDF) approach built upon the collinear approximation [26–36], and the perturbative QCD (pQCD) approach where contributions from the transverse momentum of valence quarks and Sudakov factors for hadronic wave functions are taken into consideration [37–43]. The theme of both the QCDF and pQCD approaches is to properly factorize the perturbative and nonperturbative contributions contained in HMEs and appropriately evaluate their shares. The main objective of this paper is to determine whether the B_c^* meson can be explored in future experiments, so an approximate estimation of branching ratio is completely sufficient. In this sense, the naive factorization (NF) approach [44] is used and applied to nonleptonic B_c^* meson weak decay in our calculation. The physics picture of the NF approach is very simple and clear. The results from the NF approach can be regarded as the leading order approximation of those from the QCDF approach. What matters is that the NF approach often has good performance for nonleptonic B and D weak decays induced by external W emission interactions. Using the NF approximation as a working hypothesis, it is usually assumed that the final state interactions and annihilation contributions may be disregarded. The product of quark currents in effective tetra-quark operators could be replaced by the product of the corresponding hadronic currents formed by the physical hadrons involved. HMEs of tetra-quark operators are separated into two HMEs of hadronic currents, which are further parameterized by hadronic decay constants and transition form factors.

Due to their relatively large mass, vector mesons decay dominantly through strong and/or electromagnetic interactions within the SM. The weak decays of the vector mesons are rare processes and usually draw less attention. Following the conventions in Refs. [45–49], the hadronic decay constants and transition form factors are defined by

HMEs of color-singlet diquark currents.

$$j_\mu^V = \bar{q}_1 \gamma_\mu q_2, \quad (18)$$

$$j_\mu^A = \bar{q}_1 \gamma_\mu \gamma_5 q_2, \quad (19)$$

$$\langle P(k) | j_\mu^V | 0 \rangle = 0, \quad (20)$$

$$\langle P(k) | j_\mu^A | 0 \rangle = -i f_P k_\mu, \quad (21)$$

$$\langle V(k, \epsilon) | j_\mu^V | 0 \rangle = f_V m_V \epsilon_\mu, \quad (22)$$

$$\langle V(k, \epsilon) | j_\mu^A | 0 \rangle = 0, \quad (23)$$

$$\langle P(p_2) | j_\mu^V | B_c^*(\epsilon, p_1) \rangle = \varepsilon_{\mu\nu\alpha\beta} \epsilon^\nu P^\alpha q^\beta \frac{V^{B_c^* \rightarrow P}(q^2)}{m_{B_c^*} + m_P}, \quad (24)$$

$$\begin{aligned} & \langle P(p_2) | j_\mu^A | B_c^*(\epsilon, p_1) \rangle \\ &= +i 2 m_{B_c^*} \frac{\epsilon \cdot q}{q^2} q_\mu A_0^{B_c^* \rightarrow P}(q^2) + i \epsilon_\mu (m_{B_c^*} + m_P) A_1^{B_c^* \rightarrow P}(q^2) \\ &+ i \frac{\epsilon \cdot q}{m_{B_c^*} + m_P} P_\mu A_2^{B_c^* \rightarrow P}(q^2) - i 2 m_{B_c^*} \frac{\epsilon \cdot q}{q^2} q_\mu A_3^{B_c^* \rightarrow P}(q^2), \end{aligned} \quad (25)$$

$$\begin{aligned} & \langle V(\epsilon_2, p_2) | j_\mu^V | B_c^*(\epsilon_1, p_1) \rangle \\ &= -(\epsilon_1 \cdot \epsilon_2^*) \{ P_\mu V_1^{B_c^* \rightarrow V}(q^2) - q_\mu V_2^{B_c^* \rightarrow V}(q^2) \} \\ &+ \frac{(\epsilon_1 \cdot q)(\epsilon_2^* \cdot q)}{m_{B_c^*}^2 - m_V^2} \left\{ \left[P_\mu - \frac{m_{B_c^*}^2 - m_V^2}{q^2} q_\mu \right] V_3^{B_c^* \rightarrow V}(q^2) \right. \\ &+ \left. \frac{m_{B_c^*}^2 - m_V^2}{q^2} q_\mu V_4^{B_c^* \rightarrow V}(q^2) \right\} \\ &- (\epsilon_1 \cdot q) \epsilon_{2,\mu}^* V_5^{B_c^* \rightarrow V}(q^2) + (\epsilon_2^* \cdot q) \epsilon_{1,\mu} V_6^{B_c^* \rightarrow V}(q^2), \end{aligned} \quad (26)$$

$$\begin{aligned} & \langle V(\epsilon_2, p_2) | j_\mu^A | B_c^*(\epsilon_1, p_1) \rangle \\ &= -i \varepsilon_{\mu\nu\alpha\beta} \epsilon_1^\alpha \epsilon_2^{*\beta} \left\{ \left[P^\nu - \frac{m_{B_c^*}^2 - m_V^2}{q^2} q^\nu \right] A_1^{B_c^* \rightarrow V}(q^2) \right. \\ &+ \left. \frac{m_{B_c^*}^2 - m_V^2}{q^2} q^\nu A_2^{B_c^* \rightarrow V}(q^2) \right\} \\ &- \frac{i \varepsilon_{\mu\nu\alpha\beta} P^\alpha q^\beta}{m_{B_c^*}^2 - m_V^2} \{ (\epsilon_2^* \cdot q) \epsilon_1^\nu A_3^{B_c^* \rightarrow V}(q^2) - (\epsilon_1 \cdot q) \epsilon_2^{*\nu} A_4^{B_c^* \rightarrow V}(q^2) \}, \end{aligned} \quad (27)$$

where m_P (m_V) and f_P (f_V) are the mass and decay constant of final pseudoscalar (vector) mesons, respectively; ϵ_i is the polarization vector; A_i and V_i are mesonic transition form factors; the momentum $P = p_1 + p_2$ and $q = p_1 - p_2$. There are some relationships among form factors,

$$A_0^{B_c^* \rightarrow P}(0) = A_3^{B_c^* \rightarrow P}(0), \quad (28)$$

$$A_1^{B_c^* \rightarrow V}(0) = A_2^{B_c^* \rightarrow V}(0), \quad (29)$$

$$V_3^{B_c^* \rightarrow V}(0) = V_4^{B_c^* \rightarrow V}(0). \quad (30)$$

The decay constants and form factors are nonperturbative hadronic parameters. These parameters are universal, and can be obtained from data and some nonperturbative methods. The experimental data on the B_c^* meson are still unavailable. Phenomenologically, form factors at the pole $q^2 = 0$ are expressed as the overlap integrals of mesonic wave functions [45], where the mesonic wave functions are nonperturbative but process-independent physical quantities. For example, the form factors for the $B_c^* \rightarrow B_{d,s}$ transitions have been calculated with the Wirbel-Stech-Bauer model in Ref. [20]. Recently, the form factors for the $B_c^* \rightarrow J/\psi$, $B_{d,s}^*$ transitions have also been investigated with the light front quark model (LFQM) in Ref. [48]. Additionally, as noted, the latest decay constants obtained with LFQM [50, 51] are generally consistent with the data as well as those from lattice QCD simulations and QCD sum rule approaches. Following the calculation in Ref. [48], we obtain the form factors for the $B_c^* \rightarrow \psi(2S)$, $\eta_c(1S)$, $\eta_c(2S)$, $B_{d,s}$ transitions with LFQM. The numerical values of the form factors are listed in Table 1. The theoretical uncertainties are not particularly important or worthy of our attention for the moment. After all, the magnitude order estimation of branching ratios for the nonleptonic B_c^* weak decays is quite sufficient for the present purposes.

III. BRANCHING RATIO

The branching ratio of nonleptonic B_c^* decays is defined as

$$\mathcal{B}r = \frac{p_{\text{cm}}}{24\pi m_{B_c^*}^2 \Gamma_{B_c^*}} |\overline{\mathcal{A}}|^2, \quad (31)$$

where p_{cm} is the center-of-mass momentum of final states in the rest frame of the B_c^* meson. \mathcal{A} denotes the decay amplitudes, which are collected in the Appendix. With the input parameters in Table 1 and Table 2, we obtain the branching ratios, which are listed in Table 3.

Table 1. Mesonic decay constants [2, 17] and form factors at the pole $q^2 = 0$ [48].

$f_\pi = 130.2 \pm 1.2$ MeV	$f_K = 155.7 \pm 0.3$ MeV	$f_\rho = 207.7 \pm 1.6$ MeV	$f_{K^*} = 202.5^{+6.5}_{-6.7}$ MeV
$V_1^{B_c^* \rightarrow \psi(1S)} = 0.56$	$V_2^{B_c^* \rightarrow \psi(1S)} = 0.33$	$V_{3,4}^{B_c^* \rightarrow \psi(1S)} = 0.20$	$V_5^{B_c^* \rightarrow \psi(1S)} = 1.17$
$V_6^{B_c^* \rightarrow \psi(1S)} = 0.65$	$A_{1,2}^{B_c^* \rightarrow \psi(1S)} = 0.54$	$A_3^{B_c^* \rightarrow \psi(1S)} = 0.13$	$A_4^{B_c^* \rightarrow \psi(1S)} = 0.14$
$V_1^{B_c^* \rightarrow \psi(2S)} = 0.39$	$V_2^{B_c^* \rightarrow \psi(2S)} = 0.32$	$V_{3,4}^{B_c^* \rightarrow \psi(2S)} = 0.12$	$V_5^{B_c^* \rightarrow \psi(2S)} = 0.79$
$V_6^{B_c^* \rightarrow \psi(2S)} = 0.48$	$A_{1,2}^{B_c^* \rightarrow \psi(2S)} = 0.37$	$A_3^{B_c^* \rightarrow \psi(2S)} = 0.13$	$A_4^{B_c^* \rightarrow \psi(2S)} = 0.07$
$V_1^{B_c^* \rightarrow B_s^*} = 0.63$	$V_2^{B_c^* \rightarrow B_s^*} = 1.06$	$V_{3,4}^{B_c^* \rightarrow B_s^*} = 0.40$	$V_5^{B_c^* \rightarrow B_s^*} = 3.52$
$V_6^{B_c^* \rightarrow B_s^*} = 3.02$	$A_{1,2}^{B_c^* \rightarrow B_s^*} = 0.53$	$A_3^{B_c^* \rightarrow B_s^*} = 0.73$	$A_4^{B_c^* \rightarrow B_s^*} = 0.85$
$V_1^{B_c^* \rightarrow B^*} = 0.52$	$V_2^{B_c^* \rightarrow B^*} = 1.18$	$V_{3,4}^{B_c^* \rightarrow B^*} = 0.40$	$V_5^{B_c^* \rightarrow B^*} = 3.15$
$V_6^{B_c^* \rightarrow B^*} = 2.66$	$A_{1,2}^{B_c^* \rightarrow B^*} = 0.43$	$A_3^{B_c^* \rightarrow B^*} = 0.81$	$A_4^{B_c^* \rightarrow B^*} = 0.89$
$V^{B_c^* \rightarrow \eta_c(1S)} = 0.91$	$A_0^{B_c^* \rightarrow \eta_c(1S)} = 0.66$	$A_1^{B_c^* \rightarrow \eta_c(1S)} = 0.69$	$A_2^{B_c^* \rightarrow \eta_c(1S)} = 0.59$
$V^{B_c^* \rightarrow \eta_c(2S)} = 0.59$	$A_0^{B_c^* \rightarrow \eta_c(2S)} = 0.43$	$A_1^{B_c^* \rightarrow \eta_c(2S)} = 0.41$	$A_2^{B_c^* \rightarrow \eta_c(2S)} = 0.51$
$V^{B_c^* \rightarrow B_s} = 3.40$	$A_0^{B_c^* \rightarrow B_s} = 0.69$	$A_1^{B_c^* \rightarrow B_s} = 0.75$	$A_2^{B_c^* \rightarrow B_s} = 0.96$
$V^{B_c^* \rightarrow B} = 3.08$	$A_0^{B_c^* \rightarrow B} = 0.60$	$A_1^{B_c^* \rightarrow B} = 0.65$	$A_2^{B_c^* \rightarrow B} = 0.91$

Table 2. Mesonic mass (in units of MeV) [2], where their central values are regarded as the default inputs unless otherwise specified.

$m_\pi = 139.57$	$m_\rho = 775.26 \pm 0.25$	$m_{B_d} = 5279.65 \pm 0.12$	$m_{B_d^*} = 5324.70 \pm 0.21$
$m_K = 493.677 \pm 0.016$	$m_{K^*} = 891.66 \pm 0.26$	$m_{\eta_c(1S)} = 2983.9 \pm 0.5$	$m_{\psi(1S)} = 3096.9$
$m_{B_s} = 5366.88 \pm 0.14$	$m_{B_s^*} = 5415.4^{+1.8}_{-1.5}$	$m_{\eta_c(2S)} = 3637.5 \pm 1.1$	$m_{\psi(2S)} = 3686.10 \pm 0.06$

Our comments on branching ratios are as follows.

(1) For the nonleptonic B_c^* weak decays induced by the external W emission interactions, branching ratios using the NF approach are generally of the same order of magnitude as previous estimations with the QCD-inspired QCDF and pQCD approach [19–21]. The consistency of the results between different approaches indicates that our results in this paper might be reasonable. In addition, the branching ratios for the $B_c^* \rightarrow B_{s,d}^* \pi$, $B_{s,d}^* K$, $\eta_c(2S) \rho$, $\eta_c(2S) K^*$ decays are estimated for the first time. It is found that the $B_c^* \rightarrow B_s^* \pi$ decay has a relatively large branching ratio among the two-meson B_c^* weak decays. In addition, both the initial B_c^* meson and one of the final mesons of the processes in question contain one and/or two of the heavy quarks. Due to the fact that the light quarks and gluon clouds are almost blind to the spin of the heavy quark with the approximation of the heavy quark limit, the heavy quark spin symmetry should be expected to relate the initial and final mesons [52]. In the extreme non-relativistic limit, it is expected to have [52]

$$r = \frac{\mathcal{B}r(1^- \rightarrow 1^- \pi)}{\mathcal{B}r(1^- \rightarrow 0^- \pi)} \approx \frac{\left. \frac{d\mathcal{B}r(1^- \rightarrow 1^- \ell \nu)}{dq^2} \right|_{q^2=m_\pi^2}}{\left. \frac{d\mathcal{B}r(1^- \rightarrow 0^- \ell \nu)}{dq^2} \right|_{q^2=m_\pi^2}} \approx 3. \quad (32)$$

The mass difference in the phase factors furnishes the

above relations in Eq. (32). Here, some relative branching ratios are

$$r_1 = \frac{\mathcal{B}r(B_c^* \rightarrow B_s^* \pi)}{\mathcal{B}r(B_c^* \rightarrow B_s \pi)} = 3.2, \quad (33)$$

$$r_2 = \frac{\mathcal{B}r(B_c^* \rightarrow B_d^* \pi)}{\mathcal{B}r(B_c^* \rightarrow B_d \pi)} = 2.9, \quad (34)$$

$$r_3 = \frac{\mathcal{B}r(B_c^* \rightarrow B_s^* K)}{\mathcal{B}r(B_c^* \rightarrow B_s K)} = 3.8, \quad (35)$$

$$r_4 = \frac{\mathcal{B}r(B_c^* \rightarrow B_d^* K)}{\mathcal{B}r(B_c^* \rightarrow B_d K)} = 3.4, \quad (36)$$

$$r_5 = \frac{\mathcal{B}r(B_c^* \rightarrow \psi(1S) \pi)}{\mathcal{B}r(B_c^* \rightarrow \eta_c(1S) \pi)} = 3.3, \quad (37)$$

$$r_6 = \frac{\mathcal{B}r(B_c^* \rightarrow \psi(1S) K)}{\mathcal{B}r(B_c^* \rightarrow \eta_c(1S) K)} = 3.4, \quad (38)$$

$$r_7 = \frac{\mathcal{B}r(B_c^* \rightarrow \psi(2S) \pi)}{\mathcal{B}r(B_c^* \rightarrow \eta_c(2S) \pi)} = 3.8, \quad (39)$$

Table 3. Branching ratios and event numbers of nonleptonic B_c^* weak decays, assuming that about 10^8 , 10^9 and 3×10^{10} B_c^* mesons will be available in the future CEPC, FCC-ee and LHCb@HL-LHC experiments, respectively; $\lambda \approx 0.2$ is the phenomenological Wolfenstein parameter.

decay mode	CKM factor	branching ratio					event numbers		
		unit	here	previous			CEPC	FCC-ee	LHCb
$B_s \pi^+$	$V_{cs}^* V_{ud} \sim O(1)$	10^{-7}	4.4	4.0 [20]	7.3 [20]	9.8 [20]	44	442	13256
$B_s \rho^+$	$V_{cs}^* V_{ud} \sim O(1)$	10^{-6}	1.9	0.6 [20]	1.2 [20]	1.7 [20]	193	1930	57887
$B_s^* \pi^+$	$V_{cs}^* V_{ud} \sim O(1)$	10^{-6}	1.4				143	1428	42848
$B_s K^+$	$V_{cs}^* V_{us} \sim O(\lambda)$	10^{-8}	2.2	2.0 [20]	3.6 [20]	4.9 [20]	2	22	658
$B_s K^{*+}$	$V_{cs}^* V_{us} \sim O(\lambda)$	10^{-8}	7.2	2.2 [20]	4.1 [20]	6.0 [20]	7	72	2169
$B_s^* K^+$	$V_{cs}^* V_{us} \sim O(\lambda)$	10^{-8}	8.4				8	84	2533
$B_d \pi^+$	$V_{cd}^* V_{ud} \sim O(\lambda)$	10^{-8}	2.1	2.1 [20]	4.5 [20]	6.5 [20]	2	21	640
$B_d \rho^+$	$V_{cd}^* V_{ud} \sim O(\lambda)$	10^{-8}	9.7	3.5 [20]	7.5 [20]	11.5 [20]	10	97	2924
$B_d^* \pi^+$	$V_{cd}^* V_{ud} \sim O(\lambda)$	10^{-8}	6.3				6	63	1884
$B_d K^+$	$V_{cd}^* V_{us} \sim O(\lambda^2)$	10^{-9}	1.1	1.2 [20]	2.4 [20]	3.5 [20]	0	1	34
$B_d K^{*+}$	$V_{cd}^* V_{us} \sim O(\lambda^2)$	10^{-9}	4.4	1.5 [20]	3.1 [20]	4.8 [20]	0	4	133
$B_d^* K^+$	$V_{cd}^* V_{us} \sim O(\lambda^2)$	10^{-9}	3.9				0	4	117
$\eta_c(1S) \pi^+$	$V_{cb}^* V_{ud} \sim O(\lambda^2)$	10^{-8}	1.5	2.2 [19]			2	15	455
$\eta_c(1S) \rho^+$	$V_{cb}^* V_{ud} \sim O(\lambda^2)$	10^{-8}	4.3	3.0 [21]	2.5 [21]		4	43	1300
$\psi(1S) \pi^+$	$V_{cb}^* V_{ud} \sim O(\lambda^2)$	10^{-8}	5.1	9.2 [19]	2.4 [22]		5	51	1519
$\eta_c(2S) \pi^+$	$V_{cb}^* V_{ud} \sim O(\lambda^2)$	10^{-9}	4.1	2.4 [19]			0	4	123
$\eta_c(2S) \rho^+$	$V_{cb}^* V_{ud} \sim O(\lambda^2)$	10^{-8}	1.2				1	12	351
$\psi(2S) \pi^+$	$V_{cb}^* V_{ud} \sim O(\lambda^2)$	10^{-8}	1.6	3.2 [19]			2	16	471
$\eta_c(1S) K^+$	$V_{cb}^* V_{us} \sim O(\lambda^3)$	10^{-9}	1.1	1.7 [19]			0	1	33
$\eta_c(1S) K^{*+}$	$V_{cb}^* V_{us} \sim O(\lambda^3)$	10^{-9}	2.3	1.7 [21]	1.4 [21]		0	2	68
$\psi(1S) K^+$	$V_{cb}^* V_{us} \sim O(\lambda^3)$	10^{-9}	3.8	7.3 [19]			0	4	113
$\eta_c(2S) K^+$	$V_{cb}^* V_{us} \sim O(\lambda^3)$	10^{-10}	2.9	3.4 [19]			0	0	9
$\eta_c(2S) K^{*+}$	$V_{cb}^* V_{us} \sim O(\lambda^3)$	10^{-10}	6.1				0	1	18
$\psi(2S) K^+$	$V_{cb}^* V_{us} \sim O(\lambda^3)$	10^{-9}	1.2	2.4 [19]			0	1	35

$$r_8 = \frac{\mathcal{B}r(B_c^* \rightarrow \psi(2S)K)}{\mathcal{B}r(B_c^* \rightarrow \eta_c(2S)K)} = 3.9. \quad (40)$$

For the charm quark decay, the ratio r_1 of the Cabibbo-favored $B_c^* \rightarrow B_s^* \pi$, $B_s \pi$ decays is generally consistent with both the expectation Eq. (32) from the heavy quark spin symmetry and the ratio of semileptonic J/ψ weak decays $\mathcal{B}r(J/\psi \rightarrow D_s^* e^+ \nu_e) / \mathcal{B}r(J/\psi \rightarrow D_s e^+ \nu_e) \approx 3.1$ obtained with the QCD sum rules [49].

(2) For the B_c^* decays induced either by the bottom quark decay [type (1)] or by the charm quark decay [type (2)], there are some clear hierarchical relationships among branching ratios according to the CKM factors of decay amplitudes. The Cabibbo-favored $B_c^* \rightarrow B_s^* \pi$ and $B_s \rho$ decays have relatively maximum branching ratios which can reach up to $O(10^{-6})$. The CKM-suppressed $B_c^* \rightarrow \eta_c(1S, 2S)K$, $\psi(1S, 2S)K$, and $\eta_c(1S, 2S)K^*$ decays,

whose amplitudes are proportional to Wolfenstein parameter λ^3 , have relatively minimal branching ratios. In addition, there are three partial wave amplitudes for final states containing one pseudoscalar plus vector mesons, while there is only the p -wave amplitude for two final pseudoscalar mesons. Hence, there are some relationships, $\mathcal{B}r(B_c^* \rightarrow B_s^* \pi) > \mathcal{B}r(B_c^* \rightarrow B_s \pi)$ and $\mathcal{B}r(B_c^* \rightarrow B_s \rho) > \mathcal{B}r(B_c^* \rightarrow B_s \pi)$, and these hierarchical relationships are also true for other similar cases where final states have the same valence quark compositions but different orbit-spin couplings between quarks.

(3) A more careful theoretical investigation of these decays in question is necessary and helpful to explore the hadronic B_c^* weak decays experimentally. It should be pointed out that many influences can affect the final numerical results on nonleptonic B_c^* weak decays, such as the nonfactorizable contributions. It is clear from

Eq. (9)–(13) that $|V_{cb}|$ has the largest uncertainty, about 3%, among the CKM elements involved. Except for the decay constant of f_{K^*} , the uncertainties from other decay constants in Table 1 is less than 1%. For the nonleptonic decays induced by the external W boson emission interactions, the nonfactorizable contributions to the coefficient a_1 is about 15% for charm quark decays [47], *i.e.*, the $B_c^* \rightarrow B_{d,s}^{(*)}$ transitions, and about 5% for bottom quark decays [35, 36], *i.e.*, the $B_c^* \rightarrow \psi$ and η_c transitions. Large theoretical uncertainties come mainly from the hadronic transition form factors, and some of them can reach about 40% [48]. In addition, the undetermined decay width of the B_c^* meson (for example, $\Gamma(B_c^* \rightarrow \gamma B_c) = 20\text{--}135$ eV, see Table 2 of Ref. [21]) will bring very large theoretical uncertainties to branching ratios. An accurate theoretical prediction seems to be temporarily unavailable. All numbers in Table 3 are rough estimates and only indicative of the expected order of magnitude. Here, what matters to us is whether it is possible to study the unacquainted B_c^* mesons in future experiments. Hence, a rough estimate is sufficient.

(4) As it is well known that the vector ρ and K^* mesons are resonances, they will decay immediately via the strong interaction, with branching ratios $\mathcal{B}r(\rho \rightarrow \pi\pi) \sim 100\%$ and $\mathcal{B}r(K^* \rightarrow K\pi) \sim 100\%$ [2]. The vector ρ (K^*) meson is reconstructed experimentally by the final pseudoscalar mesons. An educated guess is that the branching ratios for the three-body decay modes, $B_c^* \rightarrow B_{s,d}\pi\pi$, $B_{s,d}\pi K$, $\eta_c\pi\pi$, $\eta_c\pi K$, should be of a similar order of magnitude to those for the $B_c^* \rightarrow B_{s,d}\rho$, $B_{s,d}K^*$, $\eta_c\rho$, $\eta_c K^*$, decays, respectively. If the contributions from other possible resonances reconstructed from the $\pi\pi$, πK , $B_{s,d}\pi$, $B_{s,d}K$, $\eta_c\pi$ and $\eta_c K$ states are taken into consideration, the above hadronic three-body B_c^* decays are likely to have even larger branching ratios. All in all, it is not utopian to expect that the Cabibbo-favored $B_c^* \rightarrow B_s\pi$, $B_s^*\pi$, $B_s\rho$ decays, the singly-Cabibbo-suppressed $B_c^* \rightarrow B_s K$, $B_s^* K$, $B_s K^*$, $B_d\pi$, $B_d^*\pi$, $B_d\rho$ decays, and even the CKM-suppressed $B_c^* \rightarrow \eta_c(1S)\pi$, $\eta_c(1S, 2S)\rho$, $\psi(1S, 2S)\pi$ decays might be observable in the future CEPC, FCC-ee and LHCb@HL-LHC experiments.

IV. SUMMARY

It has been established that the charged ground vector B_c^* meson carrying explicit flavor numbers should exist according to the quark model, but to date this has been merely on the theoretical calculation level rather than the experimental measurement level. The identification of the B_c^* meson in experiments is necessary and of important significance to the quark model and SM. The signal reconstruction of the B_c^* meson from the electromagnetic decay $B_c^* \rightarrow B_c\gamma$ is severely hindered by electromagnetic background pollution. Inspired by the prospects of huge

numbers of B_c^* mesons in future high-energy and high-luminosity colliders, an attractive and competitive choice might be searching for the B_c^* meson from its nonleptonic weak decays, where the charged final hadrons are comparatively easily and effectively identified in experiments. In this paper, we study two kinds of nonleptonic B_c^* meson weak decays resulting from external W boson emission interactions by using the factorization approximation and form factors from the light front quark model, one originating from bottom quark decays and the other from charm quark decays. It is found that the branching ratios for the Cabibbo-favored $B_c^* \rightarrow B_s^*\pi$, $B_s\rho$ decays can reach up to $O(10^{-6})$, and have the first priority to be studied in experiments. For the singly-Cabibbo-suppressed $B_c^* \rightarrow B_s K$, $B_s^* K$, $B_s K^*$, $B_d\pi$, $B_d^*\pi$, $B_d\rho$ decays and the CKM-suppressed $B_c^* \rightarrow \eta_c(1S, 2S)\pi$, $\eta_c(1S, 2S)\rho$, $\psi(1S, 2S)\pi$ decays, several hundred or even thousands of events might be observable in CEPC, FCC-ee and LHCb@HL-LHC experiments. This study provides a ready and helpful reference for experimental discovery and investigation of B_c^* mesons in the future.

APPENDIX A: AMPLITUDES FOR NONLEPTONIC B_c^* WEAK DECAYS

With the conventions of Ref. [46], the amplitudes for nonleptonic B_c^* weak decays can be written as follows.

Based on the conservation of angular momentum, there are only p -wave amplitudes contributing to B_c^* meson decay into two pseudoscalar mesons.

$$\mathcal{A}(B_c^{*+} \rightarrow B_s^0 \pi^+) = V_{cs}^* V_{ud} f_\pi \mathcal{M}_p^{B_s}(\epsilon_{B_c^*} \cdot p_{B_s}), \quad (\text{A1})$$

$$\mathcal{A}(B_c^{*+} \rightarrow B_s^0 K^+) = V_{cs}^* V_{us} f_K \mathcal{M}_p^{B_s}(\epsilon_{B_c^*} \cdot p_{B_s}), \quad (\text{A2})$$

$$\mathcal{A}(B_c^{*+} \rightarrow B_d^0 \pi^+) = V_{cd}^* V_{ud} f_\pi \mathcal{M}_p^{B_d}(\epsilon_{B_c^*} \cdot p_{B_d}), \quad (\text{A3})$$

$$\mathcal{A}(B_c^{*+} \rightarrow B_d^0 K^+) = V_{cd}^* V_{us} f_K \mathcal{M}_p^{B_d}(\epsilon_{B_c^*} \cdot p_{B_d}), \quad (\text{A4})$$

$$\mathcal{A}(B_c^{*+} \rightarrow \eta_c \pi^+) = V_{cb}^* V_{ud} f_\pi \mathcal{M}_p^{\eta_c}(\epsilon_{B_c^*} \cdot p_{\eta_c}), \quad (\text{A5})$$

$$\mathcal{A}(B_c^{*+} \rightarrow \eta_c K^+) = V_{cb}^* V_{us} f_K \mathcal{M}_p^{\eta_c}(\epsilon_{B_c^*} \cdot p_{\eta_c}), \quad (\text{A6})$$

with the common factor of p -wave partial amplitudes,

$$\mathcal{M}_p^{B_s} = \sqrt{2} G_F a_1 m_{B_c^*} A_0^{B_c^* \rightarrow B_s}, \quad (\text{A7})$$

$$\mathcal{M}_p^B = \sqrt{2} G_F a_1 m_{B_c} A_0^{B_c \rightarrow B}, \quad (\text{A8})$$

$$\mathcal{M}_p^{\eta_c} = \sqrt{2} G_F a_1 m_{B_c} A_0^{B_c \rightarrow \eta_c}, \quad (\text{A9})$$

where coefficient $a_1 = C_1 + C_2/N_c$ is generally influenced by nonfactorizable contributions and can be regarded as a phenomenological parameter, especially for charm quark decays. The value of $a_1 \approx 1.2$ is used in our calculation.

There are three partial wave amplitudes contributing to B_c^* meson decay into one pseudoscalar meson plus one vector meson. The general decay amplitude is written as

$$\begin{aligned} \mathcal{A}(B_c^* \rightarrow VP) = & \mathcal{M}_s(\epsilon_{B_c^*} \cdot \epsilon_V^*) \\ & + \frac{\mathcal{M}_d}{m_{V_{B_c}} m_V} (\epsilon_{B_c^*} \cdot p_V) (\epsilon_V^* \cdot p_{B_c^*}) \\ & + \frac{\mathcal{M}_p}{m_{B_c} m_V} \epsilon_{\mu\nu\alpha\beta} \epsilon_{B_c^*}^\mu \epsilon_V^{*\nu} p_{B_c^*}^\alpha p_V^\beta, \end{aligned} \quad (\text{A10})$$

where $\mathcal{M}_{s,p,d}$ correspond to the s -, p -, and d -wave partial amplitudes.

For $B_c^{*+} \rightarrow B_s^{*0} \pi^+$, $B_s^{*0} K^+$ decays, one has

$$\mathcal{M}_s^{B_c^* \pi} = -i \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} f_\pi a_1 (m_{B_c}^2 - m_{B_s}^2) V_1^{B_c^* \rightarrow B_s}, \quad (\text{A11})$$

$$\begin{aligned} \mathcal{M}_d^{B_c^* \pi} = & -i \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} f_\pi a_1 m_{B_c} m_{B_s} (V_4^{B_c^* \rightarrow B_s} \\ & - V_5^{B_c^* \rightarrow B_s} + V_6^{B_c^* \rightarrow B_s}), \end{aligned} \quad (\text{A12})$$

$$\mathcal{M}_p^{B_c^* \pi} = -2 \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} f_\pi a_1 m_{B_c} m_{B_s} A_1^{B_c^* \rightarrow B_s}, \quad (\text{A13})$$

$$\mathcal{M}_i^{B_c^* K} = \mathcal{M}_i^{B_c^* \pi} (V_{ud} \rightarrow V_{us}, f_\pi \rightarrow f_K), \quad \text{for } i = s, p, d. \quad (\text{A14})$$

For $B_c^{*+} \rightarrow B_d^{*0} \pi^+$, $B_d^{*0} K^+$ decays, one has

$$\mathcal{M}_s^{B_c^* \pi} = -i \frac{G_F}{\sqrt{2}} V_{cd}^* V_{ud} f_\pi a_1 (m_{B_c}^2 - m_{B_d}^2) V_1^{B_c^* \rightarrow B_d}, \quad (\text{A15})$$

$$\begin{aligned} \mathcal{M}_d^{B_c^* \pi} = & -i \frac{G_F}{\sqrt{2}} V_{cd}^* V_{ud} f_\pi a_1 m_{B_c} m_{B_d} (V_4^{B_c^* \rightarrow B_d} \\ & - V_5^{B_c^* \rightarrow B_d} + V_6^{B_c^* \rightarrow B_d}), \end{aligned} \quad (\text{A16})$$

$$\mathcal{M}_p^{B_c^* \pi} = -2 \frac{G_F}{\sqrt{2}} V_{cd}^* V_{ud} f_\pi a_1 m_{B_c} m_{B_d} A_1^{B_c^* \rightarrow B_d}, \quad (\text{A17})$$

$$\mathcal{M}_i^{B_c^* K} = \mathcal{M}_i^{B_c^* \pi} (V_{ud} \rightarrow V_{us}, f_\pi \rightarrow f_K), \quad \text{for } i = s, p, d. \quad (\text{A18})$$

For $B_c^{*+} \rightarrow \psi \pi^+$, ψK^+ decays, one has

$$\mathcal{M}_s^{\psi \pi} = -i \frac{G_F}{\sqrt{2}} V_{cb}^* V_{ud} f_\pi a_1 (m_{B_c}^2 - m_\psi^2) V_1^{B_c^* \rightarrow \psi}, \quad (\text{A19})$$

$$\begin{aligned} \mathcal{M}_d^{\psi \pi} = & -i \frac{G_F}{\sqrt{2}} V_{cb}^* V_{ud} f_\pi a_1 m_{B_c} m_\psi (V_4^{B_c^* \rightarrow \psi} \\ & - V_5^{B_c^* \rightarrow \psi} + V_6^{B_c^* \rightarrow \psi}), \end{aligned} \quad (\text{A20})$$

$$\mathcal{M}_p^{\psi \pi} = -2 \frac{G_F}{\sqrt{2}} V_{cb}^* V_{ud} f_\pi a_1 m_{B_c} m_\psi A_1^{B_c^* \rightarrow \psi}, \quad (\text{A21})$$

$$\mathcal{M}_i^{\psi K} = \mathcal{M}_i^{\psi \pi} (V_{ud} \rightarrow V_{us}, f_\pi \rightarrow f_K), \quad \text{for } i = s, p, d. \quad (\text{A22})$$

For $B_c^{*+} \rightarrow B_s^0 \rho^+$, $B_s^0 K^{*+}$ decays, one has

$$\mathcal{M}_s^{B_c^* \rho} = -i \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} f_\rho m_\rho a_1 (m_{B_c} + m_{B_s}) A_1^{B_c^* \rightarrow B_s}, \quad (\text{A23})$$

$$\mathcal{M}_d^{B_c^* \rho} = -i \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} f_\rho m_\rho a_1 \frac{2m_{B_c} m_\rho}{m_{B_c} + m_{B_s}} A_2^{B_c^* \rightarrow B_s}, \quad (\text{A24})$$

$$\mathcal{M}_p^{B_c^* \rho} = - \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} f_\rho m_\rho a_1 \frac{2m_{B_c} m_\rho}{m_{B_c} + m_{B_s}} V^{B_c^* \rightarrow B_s}, \quad (\text{A25})$$

$$\mathcal{M}_i^{B_c^* K^*} = \mathcal{M}_i^{B_c^* \rho} (V_{ud} \rightarrow V_{us}, f_\rho \rightarrow f_{K^*}, m_\rho \rightarrow m_{K^*}), \quad \text{for } i = s, p, d. \quad (\text{A26})$$

For $B_c^{*+} \rightarrow B_d^0 \rho^+$, $B_d^0 K^{*+}$ decays, one has

$$\mathcal{M}_s^{B_c^* \rho} = -i \frac{G_F}{\sqrt{2}} V_{cd}^* V_{ud} f_\rho m_\rho a_1 (m_{B_c} + m_{B_d}) A_1^{B_c^* \rightarrow B_d}, \quad (\text{A27})$$

$$\mathcal{M}_d^{B_c^* \rho} = -i \frac{G_F}{\sqrt{2}} V_{cd}^* V_{ud} f_\rho m_\rho a_1 \frac{2m_{B_c} m_\rho}{m_{B_c} + m_{B_d}} A_2^{B_c^* \rightarrow B_d}, \quad (\text{A28})$$

$$\mathcal{M}_p^{B_d\rho} = -\frac{G_F}{\sqrt{2}} V_{cd}^* V_{ud} f_\rho m_\rho a_1 \frac{2m_{B_c} m_\rho}{m_{B_c} + m_{B_d}} V^{B_c \rightarrow B_d}, \quad (\text{A29})$$

$$\mathcal{M}_d^{\eta_c\rho} = -i\frac{G_F}{\sqrt{2}} V_{cb}^* V_{ud} f_\rho m_\rho a_1 \frac{2m_{B_c} m_\rho}{m_{B_c} + m_{\eta_c}} A_2^{B_c \rightarrow \eta_c}, \quad (\text{A32})$$

$$\mathcal{M}_i^{B_d K^*} = \mathcal{M}_i^{B_d\rho}(V_{ud} \rightarrow V_{us}, f_\rho \rightarrow f_{K^*}, m_\rho \rightarrow m_{K^*}), \quad \text{for } i = s, p, d. \quad (\text{A30})$$

$$\mathcal{M}_p^{\eta_c\rho} = -\frac{G_F}{\sqrt{2}} V_{cb}^* V_{ud} f_\rho m_\rho a_1 \frac{2m_{B_c} m_\rho}{m_{B_c} + m_{\eta_c}} V^{B_c \rightarrow \eta_c}, \quad (\text{A33})$$

For $B_c^{*+} \rightarrow \eta_c \rho^+, \eta_c K^{*+}$ decays, one has

$$\mathcal{M}_s^{\eta_c\rho} = -i\frac{G_F}{\sqrt{2}} V_{cb}^* V_{ud} f_\rho m_\rho a_1 (m_{B_c} + m_{\eta_c}) A_1^{B_c \rightarrow \eta_c}, \quad (\text{A31})$$

$$\mathcal{M}_i^{\eta_c K^*} = \mathcal{M}_i^{\eta_c\rho}(V_{ud} \rightarrow V_{us}, f_\rho \rightarrow f_{K^*}, m_\rho \rightarrow m_{K^*}), \quad \text{for } i = s, p, d. \quad (\text{A34})$$

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