Chinese Physics C Vol. 46, No. 7 (2022) 073105

Searching systematics for nonfactorizable contributions to B^- and $\bar{B}^0$ hadronic decays

Maninder Kaur[†] Supreet Pal Singh[‡] R. C. Verma[§]

Department of Physics, Punjabi University, Patiala – 147002, India

Abstract: The two-body weak decays $\bar{B}\to\pi D/\bar{B}\to\rho D$ and $\bar{B}\to\pi D^*$ are examined using isospin analysis to study nonfactorizable contributions. After determining strong interaction phases and obtaining factorizable contributions from spectator-quark diagrams for N_c=3, we determine nonfactorizable isospin amplitudes from the experimental data for these modes. Our results support the universality of the ratio of nonfactorizable isospin reduced amplitudes for these decays within experimental errors. To demonstrate that these systematics are not coincidental, we also plot our results w. r. t. this ratio.

Keywords: nonfactorization, weak hadronic decays, HEP-PHE

DOI: 10.1088/1674-1137/ac600b

I. INTRODUCTION

Experimental measurements for the weak decays of charm and bottom mesons have inspired several theoretical studies exploring their dynamics [1– 8]. The phenomenological analyses of the two-body hadronic decays of heavy flavor mesons have indicated the presence of significant nonfactorizable contributions. In the naïve factorization scheme, two QCD related coefficients, a_1 and a_2, are treated as parameters to be fixed from the experimental data while ignoring the nonfactorizable contribution to decay amplitudes [9– 11]. Initially, data on the branching fractions of $D\to\bar{K}\pi$ decays seemed to require $a_1\approx c_1=1.26$, $a_2\approx c_2=-0.51$, leading to destructive interference between color-favored (CF) and color-suppressed (CS) processes for $D^+\to\bar{K}^0\pi^+$, thereby implying the $N_c\to\infty$ limit [12, 13]. This limit, which was thought to be justified with the hope that the nonfactorizable part relative to the factorizable amplitude is of the order of $1/N_c$, was expected to perform even better for B- meson decays, where the final state particles carry larger momenta than those of charm meson decays.

However, the measurement of $\bar{B}\to D\pi$ meson decays did not later favor this result empirically because these decays require $a_1\approx 1.03$, and $a_2\approx 0.23$,i.e., a positive value of a_2, in sharp contrast to the expectations based on the large N_c limit. Thus, B- meson decays, revealing constructive interference between the CF and CS diagrams for $B^-\to\pi^-D^0$, seem to favor $N_c=3$ (real value). Even in the D-meson sector, the choice of the universal parameters a_1 and a_2 proved to be problematic when more accurate measurements were obtained for other decay modes of D-mesons, even after including final state interaction (FSI) effects [14, 15]. Consequently, charm meson decays have been thoroughly reinvestigated to explicitly study the nonfactorization contributions. Using the isospin analysis for the $D\to\bar{K}\pi/\bar{K}\rho/\bar{K}^*\pi$ decay modes [16–19], these contributions are expressed in terms of two reduced matrix elements $A^{nf}_{1/2}$, $A^{nf}_{3/2}$, and systematics were recognized. It was observed that in all these decays, the nonfactorizable isospin reduced amplitude $A^{nf}_{1/2}$ not only has the same sign but also bears the same ratio (-1.12) as the $A^{nf}_{3/2}$ reduced amplitude, within the experimental errors. It is worth noting that these systematics were also found to be consistent with those of the p-wave meson emitting decays of charm mesons, $D\to\bar{K}a_1/\pi\bar{K}_1/\pi\bar{K}_{-1}/\pi\bar{K}_0/\bar{K}a_2$ [18].

Extensive work has also been conducted to study nonfactorization contributions in charmed hadronic B-decays over the past two decades. Nonfactorizable terms may appear for several reasons, such as FSI rescattering effects and soft-gluon exchange around a basic weak vertex. The rescattering effects on the outgoing mesons have been

Received 23 November 2021; Accepted 24 March 2022; Published online 23 May 2022

† E-mail: maninderphy@gmail.com
‡ E-mail: spsmudahar@gmail.com
§ E-mail: rcverma@gmail.com

studied in detail for bottom meson decays [20, 21]. Moreover, flavor $SU(3)$ symmetry and the factorization assisted topological (FAT) approach have been employed for the study of such nonfactorizable contributions because they have the advantage of absorbing various types of contributions lump-sum in terms of a few parameters, which are to be fixed empirically [22, 23].

Inspired by these efforts, we investigate nonfactorizable contributions to the weak hadronic decays of bottom mesons, including the strong interaction phases possibly through FSI. It is well known that the strong phases of the decay amplitude in B-decays are significant, and many recent analyses of $\bar{B}\to\pi D/\rho D/\pi D^*$ decays have shown large strong phases. Because non-pertubative nonfactorizable contributions cannot be calculated from first principles, we employ isospin symmetry, which is reliable for hadronic interactions to gain insight into these contributions. We perform isospin analysis to study $\bar{B}\to\pi D/\rho D/\pi D^*$ decay modes. Our aim is to investigate if such systematics, which were observed in charm meson decays, is valid for these decay modes, because their decay products also involve two different isospin states $I=1/2$ and $3/2$. We introduce strong phases, which affect the interference between the isospin-1/2 and isospin-3/2 amplitudes. Using the experimental measurements for their branching fractions, we first obtain three free parameters the two isospin amplitudes $A_{1/2}$, $A_{3/2}$, and their relative strong phase. By determining the factorizable decay amplitudes for $N_c=3$, we estimate the nonfactorizable isospin reduced amplitudes corresponding to these isospin states. We finally observe that the ratio of the nonfactorizable reduced amplitude in these isospin channels also follows a universal value for both decay modes $\bar{B}\to\pi D/\bar{B}\to\rho D$ and $\bar{B}\to\pi D^*$.

II. WEAK HAMILTONIAN

The effective weak Hamiltonian for CKM enhanced B-meson decays is given by

$$H_w=\frac{G_{\rm F}}{\sqrt{2}}V_{cb}V_{ud}^*\left[c_1\left(\bar{d}u\right)(\bar{c}b)+c_2(\bar{c}u)\left(\bar{d}b\right)\right], \tag{1}$$

where $\bar{q}_1q_2=\bar{q}_1\gamma_\mu(1-\gamma_5)q_2$ denotes the color singlet $V\!-\!A$ Dirac current, and the QCD coefficients [23, 24] on the bottom mass scale are

$$c_1=1.132,\qquad c_2=-0.287. \tag{2}$$

Because the current operators in the weak Hamiltonian are expressed in terms of fundamental quark fields, it is appropriate to have the Hamiltonian in a form such that one of these currents carries the same quantum numbers as one of the mesons emitted in the final state of bottom meson decays. Consequently, the hadronic matrix elements of an operator O receives contributions from the operator itself and the Fierz transformation of O, which generates the factorizable and nonfactorizable parts through the Fierz identity,

$$(\bar{d}u)(\bar{c}b)=\frac{1}{N_c}(\bar{c}u)(\bar{d}b)+\frac{1}{2}(\bar{c}\lambda^a u)\left(\bar{d}\lambda^a b\right), \tag{3}$$

where $\bar{q}_1\lambda^a q_2\equiv\bar{q}_1\gamma_\mu(1-\gamma_5)\lambda^a q_2$ represents the color octet current. Performing a similar treatment on the other operator $(\bar{c}u)(\bar{d}b)$, the weak Hamiltonian becomes

$$H_w^{\rm CF}=\frac{G_{\rm F}}{\sqrt{2}}V_{cb}V_{ud}^*\left[a_1\left(\bar{d}u\right)_H(\bar{c}b)_H+c_2H_w^8\right], \tag{4}$$

$$H_w^{\rm CS}=\frac{G_{\rm F}}{\sqrt{2}}V_{cb}V_{ud}^*\left[a_2(\bar{c}u)_H\left(\bar{d}b\right)_H+c_1\tilde{H}_w^8\right], \tag{5}$$

$$a_{1,2}=c_{1,2}+\frac{c_{2,1}}{N_c}, \tag{6}$$

$$H_w^8=\frac{1}{2}\sum_{a=1}^{8}(\bar{c}\lambda^a u)\left(\bar{d}\lambda^a b\right),$$
$$\tilde{H}_w^8=\frac{1}{2}\sum_{a=1}^{8}\left(\bar{d}\lambda^a u\right)(\bar{c}\lambda^a b), \tag{7}$$

which describe the color-favored (CF) and color-suppressed (CS) processes, respectively. Here, the index H in (4) and (5) indicates the change from the quark current to hadron field operator [4]. The matrix elements of the first terms in (4) and (5) lead to the factorizable contributions [4], and the second terms, involving the color octet currents, generate nonfactorized contributions.

III. DECAY MODES

A. $\bar{B}\to\pi D$ Decay mode

The branching fraction for B-meson decay into two pseudoscalar mesons is related to its decay amplitude as follows:

$$B\left(\bar{B}\to P_1P_2\right)=\tau_B\left|\frac{G_{\rm F}}{\sqrt{2}}V_{cb}V_{ud}^*\right|^2\frac{p}{8\pi m_B^2}\times\left|A\left(\bar{B}\to P_1P_2\right)\right|^2, \tag{8}$$

where τ_B denotes the lifetime of B-mesons taken from [1],

$$\tau_{\bar{B}^0} = (1.519 \pm 0.004) \times 10^{-12}\ \text{s},$$
$$\tau_{B^-} = (1.638 \pm 0.004) \times 10^{-12}\ \text{s},$$

$V_{ud}V_{cb}$ is the product of the Cabibbo –Kobayashi – Maskawa (CKM) matrix elements [1],

$$V_{ud} = 0.975, \qquad V_{cb} = 0.041,$$

and p is the magnitude of the three-momentum of the final state particles in the rest frame of the parent B-meson,

$$p = |p_1| = |p_2| = \frac{1}{2m_B}\left[\left\{m_B^2 - (m_1 + m_2)^2\right\} \times \left\{m_B^2 - (m_1 - m_2)^2\right\}\right]^{1/2}. \tag{9}$$

In heavy flavor meson decays, it has been observed that long distance strong FSI rescattering [20–21] of outgoing mesons significantly affects their branching fractions. In general, such FSI phenomena can affect a decay amplitude in two ways: The decay amplitude may itself be modulated or it may acquire a phase. It has been shown by Kamal [25] that, in the weak scattering limit, the elastic FSI effect is mainly used to obtain a phase factor, i.e.,

$$A^{\text{FSI}} = A\ \mathrm{e}^{\mathrm{i}\delta}. \tag{10}$$

Consequently, mixing of final states with the same quantum numbers can take place. Initially, it was expected that bottom meson decays may not be affected by FSI because the produced particles may not have sufficient time to interact, and there are no meson resonances near the B- meson mass corresponding to the quantum numbers of the final state. However, experimental data do not fulfill this naïve expectation [26].

To demonstrate this, we employ the isospin framework, in which $\bar{B} \to \pi D$ decay amplitudes are represented in terms of isospin reduced amplitudes, including the strong interaction phases $\delta^{\pi D}_{1/2}$, $\delta^{\pi D}_{3/2}$ in the Isospin -1/2 and 3/2 final states, respectively, as

$$A(\bar{B}^0 \to \pi^- D^+) = \frac{1}{\sqrt{3}}\left[A^{\pi D}_{3/2}\mathrm{e}^{\mathrm{i}\delta^{\pi D}_{3/2}} + \sqrt{2}A^{\pi D}_{1/2}\mathrm{e}^{\mathrm{i}\delta^{\pi D}_{1/2}}\right],$$
$$A(\bar{B}^0 \to \pi^0 D^0) = \frac{1}{\sqrt{3}}\left[\sqrt{2}A^{\pi D}_{3/2}\mathrm{e}^{\mathrm{i}\delta^{\pi D}_{3/2}} - A^{\pi D}_{1/2}\mathrm{e}^{\mathrm{i}\delta^{\pi D}_{1/2}}\right],$$
$$A(B^- \to \pi^- D^0) = \sqrt{3}A^{\pi D}_{3/2}\mathrm{e}^{\mathrm{i}\delta^{\pi D}_{3/2}}. \tag{11}$$

These lead to the following relations:

$$A^{\pi D}_{1/2} = \left[\left|A(\bar{B}^0 \to \pi^- D^+)\right|^2 + \left|A(\bar{B}^0 \to \pi^0 D^0)\right|^2 - \frac{1}{3}\left|A(B^- \to \pi^- D^0)\right|^2\right]^{1/2},$$
$$A^{\pi D}_{3/2} = \sqrt{\frac{1}{3}}\left|A(B^- \to \pi^- D^0)\right|, \tag{12}$$

and the relative phase difference, $\delta^{\pi D} = \delta^{\pi D}_{1/2} - \delta^{\pi D}_{3/2}$, is given by

$$Cos\delta^{\pi D} = \frac{(3\left|A(\bar{B}^0 \to \pi^- D^+)\right|^2 - 6\left|A(\bar{B}^0 \to \pi^0 D^0)\right|^2 + \left|A(B^- \to \pi^- D^0)\right|^2)}{6\sqrt{2}\left|A^{\pi D}_{1/2}\right|\left|A^{\pi D}_{3/2}\right|}. \tag{13}$$

Thus, $A^{\pi D}_{1/2}$ and $A^{\pi D}_{3/2}$ can be treated as real quantities in the following analysis:

Using the experimental values [1]

$$B\left(\bar{B}^0 \to \pi^- D^+\right) = (2.52 \pm 0.13) \times 10^{-3},$$
$$B\left(\bar{B}^0 \to \pi^0 D^0\right) = (2.63 \pm 0.14) \times 10^{-4},$$
$$B\left(B^- \to \pi^- D^0\right) = (4.68 \pm 0.13) \times 10^{-3},$$

we obtain

$$A^{\pi D\ \exp}_{1/2} = \pm(1.273 \pm 0.065)\ \text{GeV}^3,$$
$$A^{\pi D\ \exp}_{3/2} = \pm(1.323 \pm 0.018)\ \text{GeV}^3, \tag{14}$$

and the phase difference

$$\delta^{\pi D} = (28 \pm 7)^\circ, \tag{15}$$

which agrees with the final state rescattering analysis [22]. Although this phase difference is relatively smaller than that of the $D \to \bar{K}\pi$ mode $\delta = (86 \pm 7)^\circ$, it certainly indicates the presence of non-vanishing strong phases in the B- meson sector.

We express the decay amplitude as a sum of the factorizable and nonfactorizable parts,

$$A(\bar{B} \to \pi D) = A^f(\bar{B} \to \pi D) + A^{nf}(\bar{B} \to \pi D), \tag{16}$$

arising from the respective terms of the weak Hamiltonian given in (4) and (5).

Using the factorization scheme, the spectator-quark parts of the decay amplitudes arising from W- emission[1)] diagrams are derived for the following classes of $\bar{B} \to \pi D$ decays:

(a) Class I: Color favored (CF)

$$A^f(\bar{B}^0 \to \pi^- D^+) = a_1 f_\pi \left(m_B^2 - m_D^2\right) F_0^{\bar{B}D}\left(m_\pi^2\right), \tag{17}$$

(b) Class II: Color Suppressed (CS)

$$A^f(\bar{B}^0 \to \pi^0 D^0) = -\frac{1}{\sqrt{2}} a_2 f_D \left(m_B^2 - m_\pi^2\right) F_0^{\bar{B}\pi}\left(m_D^2\right), \tag{18}$$

(c) Class III : Interference of CF and CS

$$A^f(B^- \to \pi^- D^0) = a_1 f_\pi \left(m_B^2 - m_D^2\right) F_0^{\bar{B}D}\left(m_\pi^2\right) + a_2 f_D \left(m_B^2 - m_\pi^2\right) F_0^{\bar{B}\pi}\left(m_D^2\right). \tag{19}$$

We calculate the values of the factorization contributions for $N_c = 3$ (real value) using numerical inputs for decay constants taken as

$$f_D = (0.207 \pm 0.009)\ \text{GeV}, \quad f_\pi = (0.131 \pm 0.002)\ \text{GeV}, \tag{20}$$

from the leptonic decays of D and π mesons, respectively [27].

Assuming nearest pole dominance, momentum dependence of the form-factors, appearing in the decay amplitudes given in (17–19), is taken as

$$F_0\left(q^2\right) = \frac{F_0(0)}{\left(1 - q^2/m_s^2\right)^n}, \tag{21}$$

where the pole masses are given by the scalar meson carrying the quantum number of the corresponding weak current, which are $m_s = 5.78$ GeV and $m_s = 6.80$ GeV, and $n = 1$ for the monopole formula. The form-factors $F_0(0)$ at $q^2 = 0$ are taken from [28], as given below.

$$F_0^{\bar{B}\pi}(0) = (0.27 \pm 0.05), \quad F_0^{\bar{B}D}(0) = (0.66 \pm 0.03). \tag{22}$$

We finally obtain

$$\begin{aligned} A^f(\bar{B}^0 \to \pi^- D^+) &= 2.180 \pm 0.099\ \text{GeV}^3, \\ A^f(\bar{B}^0 \to \pi^0 D^0) &= -0.111 \pm 0.021\ \text{GeV}^3, \\ A^f(B^- \to \pi^- D^0) &= 2.339 \pm 0.103\ \text{GeV}^3. \end{aligned} \tag{23}$$

Exploiting the following isospin relations:

$$\begin{aligned} A^f_{1/2}(\bar{B} \to \pi D) &= \frac{1}{\sqrt{3}}\left\{\sqrt{2}A^f(\bar{B}^0 \to \pi^- D^+) - A^f(\bar{B}^0 \to \pi^0 D^0)\right\}, \\ A^f_{3/2}(\bar{B} \to \pi D) &= \frac{1}{\sqrt{3}}\left\{A^f(\bar{B}^0 \to \pi^- D^+) + \sqrt{2}A^f(\bar{B}^0 \to \pi^0 D^0)\right\}, \end{aligned} \tag{24}$$

we obtain

$$A^f_{1/2} = (1.845 \pm 0.082)\ \text{GeV}^3, \quad A^f_{3/2} = (1.168 \pm 0.060)\ \text{GeV}^3. \tag{25}$$

Using isospin C. G. coefficients with the convention used in [17, 18], the nonfactorizable part of the decay amplitudes can be expressed in terms of the scattering amplitudes for the spurion $+\bar{B} \to \pi D$ process.

$$\begin{aligned} A^{nf}(\bar{B}^0 \to \pi^- D^+) &= \frac{1}{3} c_2 \left(\left\langle \pi D \left\| H_w^8 \right\| \bar{B} \right\rangle_{3/2} + 2\left\langle \pi D \left\| H_w^8 \right\| \bar{B} \right\rangle_{1/2}\right), \\ A^{nf}(\bar{B}^0 \to \pi^0 D^0) &= \frac{\sqrt{2}}{3} c_1 \left(\left\langle \pi D \left\| \tilde{H}_w^8 \right\| \bar{B} \right\rangle_{3/2} - \left\langle \pi D \left\| \tilde{H}_w^8 \right\| \bar{B} \right\rangle_{1/2}\right), \\ A^{nf}(B^- \to \pi^- D^0) &= c_2 \left\langle \pi D \left\| H_w^8 \right\| \bar{B} \right\rangle_{3/2} + c_1 \left\langle \pi D \left\| \tilde{H}_w^8 \right\| \bar{B} \right\rangle_{3/2}. \end{aligned} \tag{26}$$

At present, there is no available technique to exactly calculate these quantities from the theory of strong interactions. Therefore, by subtracting the factorizable part (25) from the experimental decay amplitude (14), we determine the nonfactorizable isospin reduced amplitudes,

1) In general, W- exchange, W- annihilation and W- loop diagrams may also contribute to the bottom meson decays. Note that W-annihilation and W-loop processes do not appear for any of the decays considered in this work. W- exchange is usually suppressed due to helicity and color arguments, for which the partial decay rate depends on the wave function at the origin, and in the relative ratio of its contribution to that of the spectator diagrams is given by

$$\frac{\Gamma_{W-\text{exc/anni}}}{\Gamma_{\text{spect}}} \approx \frac{|\psi(0)|^2}{m^3} \approx \alpha_s^3\left(\frac{m_q}{M_Q}\right)^3,$$

where m_q and M_Q represents masses of the light and heavy quark in the B-mesons, As the mass of heavy quark goes up, these become less and less important [4]. Particularly for $\bar{B} \to \pi D$ decays, it has been categorically shown by Kamal and Pham [11] that W- exchange terms are highly suppressed due to smallness of the relevant form-factor $F_0^{D\pi}(m_B^2)$, and of the color factor a_2. Recently, this observation has further been supported in the FAT based analysis of these decays [23], so contribution of W- exchange diagram can be neglected specially in the presence of W-emission diagram.

$$A^{nf}_{1/2} = -(0.572 \pm 0.105)\ \text{GeV}^3,$$
$$A^{nf}_{3/2} = -(2.491 \pm 0.062)\ \text{GeV}^3, \tag{27}$$

by choosing positive and negative values for $A^{nf}_{1/2}$ and $A^{nf}_{3/2}$, respectively. Their ratio is

$$\alpha = A^{nf}_{1/2}/A^{nf}_{3/2} = 0.229 \pm 0.042. \tag{28}$$

There are several calculations for form factors, obtained from different approaches in literature, which are given in Table 1.

To observe the effect of form-factor variation on our analysis, we give the ratio α in Table 2 for the maximum and minimum values of the form-factors, which are consistent with (28) within errors.

We also plot the dependence of α on form-factors $F_0^{\bar{B}D}(0)$ and $F_0^{\bar{B}\pi}(0)$ in Fig. 1, which shows that α is not quite sensitive to them.

Table 1. Form-factor of $\bar{B} \to D$ and $\bar{B} \to \pi$ transitions at maximum recoil (q^2 =0).

Form-factor	CLFQM [27]	LQCD [28]	LCSR [29]	pQCD [30]	pQCD [31]	pQCD [32]
$F_0^{\bar{B}D}(0)$	0.67±0.01	0.66±0.03	0.65±0.08	0.673±0.063	–	–
$F_0^{\bar{B}\pi}(0)$	0.25±0.01	0.27±0.05	0.21±0.07	–	0.26±0.05	0.27±0.05

Table 2. Ratio $\alpha = A^{nf}_{1/2}/A^{nf}_{3/2}$ for maximum and minimum values of form-factors.

$F_0^{\bar{B}D}(0)$	0.69	0.69	0.63	0.63
$F_0^{\bar{B}\pi}(0)$	0.32	0.22	0.32	0.22
α	0.262	0.249	0.207	0.195

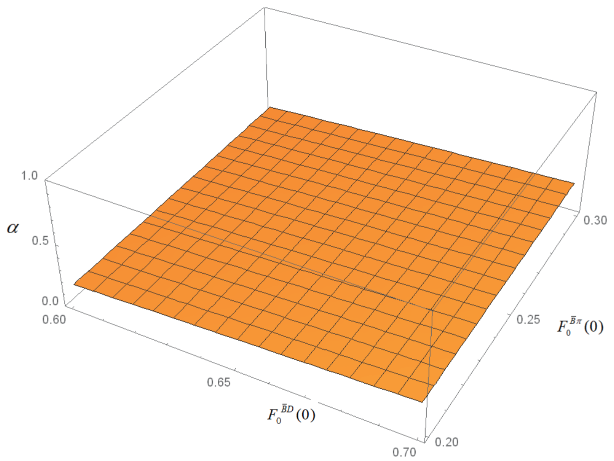

Fig. 1. (color online) Variation in α with form factors $F_0^{\bar{B}D}(0)$ and $F_0^{\bar{B}\pi}(0)$.

B. $\bar{B} \to \rho D$ Decay mode

Using the branching fraction,

$$B\left(\bar{B} \to PV\right) = \tau_B \left|\frac{G_F}{\sqrt{2}} V_{cb} V^*_{ud}\right|^2 \frac{p^3}{8\pi m_V^2} \left|A\left(\bar{B} \to PV\right)\right|^2. \tag{29}$$

Because the isospin structure of $\bar{B} \to \rho D$ decays is exactly the same as that of $\bar{B} \to \pi D$ decays,

$$A(\bar{B}^0 \to \rho^- D^+) = \frac{1}{\sqrt{3}}\left[A^{\rho D}_{3/2} e^{i\delta^{\rho D}_{3/2}} + \sqrt{2} A^{\rho D}_{1/2} e^{i\delta^{\rho D}_{1/2}}\right],$$
$$A(\bar{B}^0 \to \rho^0 D^0) = \frac{1}{\sqrt{3}}\left[\sqrt{2} A^{\rho D}_{3/2} e^{i\delta^{\rho D}_{3/2}} - A^{\rho D}_{1/2} e^{i\delta^{\rho D}_{1/2}}\right],$$
$$A(B^- \to \rho^- D^0) = \sqrt{3} A^{\rho D}_{3/2} e^{i\delta^{\rho D}_{1/2}}. \tag{30}$$

We repeat the same procedure as before. Using the experimental branching fractions

$$B\left(\bar{B}^0 \to \rho^- D^+\right) = (7.6 \pm 1.2) \times 10^{-3},$$
$$B\left(\bar{B}^0 \to \rho^0 D^0\right) = (3.21 \pm 0.21) \times 10^{-4},$$
$$B\left(B^- \to \rho^- D^0\right) = (1.34 \pm 0.18) \times 10^{-2},$$

we obtain the total isospin reduced amplitudes

$$A^{\rho D\ \exp}_{1/2} = \pm(0.143 \pm 0.025)\ \text{GeV}^2,$$
$$A^{\rho D\ \exp}_{3/2} = \pm(0.149 \pm 0.010)\ \text{GeV}^2, \tag{31}$$

and the phase difference

$$\delta^{\rho D} \equiv \delta^{\rho D}_{1/2} - \delta^{\rho D}_{3/2} = \left(8\ ^{+30}_{-8}\right)^\circ. \tag{32}$$

The factorizable decay amplitudes of the spectator-quark diagrams can be expressed as

$$A^f(\bar{B}^0 \to \rho^- D^+) = 2a_1 m_\rho f_\rho F_1^{\bar{B}D}\left(m_\rho^2\right),$$
$$A^f(\bar{B}^0 \to \rho^0 D^0) = -\sqrt{2} a_2 f_D m_\rho A_0^{\bar{B}\rho}\left(m_D^2\right),$$
$$A^f(B^- \to \rho^- D^0) = a_1 2m_\rho f_\rho F_1^{\bar{B}D}\left(m_\rho^2\right) + a_2 f_D 2m_\rho A_0^{\bar{B}\rho}\left(m_D^2\right). \tag{33}$$

It has been noted in the BSW II model [3] that consistency with heavy quark symmetry requires certain form-factors, such as $F_1(0)$ and $A_0(0)$, to have dipole q^2 dependence (n=2) in

$$F_1\left(q^2\right)=\frac{F_1(0)}{\left(1-q^2/m_V^2\right)^n},\quad A_0\left(q^2\right)=\frac{A_0(0)}{\left(1-q^2/m_P^2\right)^n},\tag{34}$$

where the vector $V(1^-)$ meson and pseudoscalar $P(0^-)$ meson pole masses are 6.34 and 5.27 GeV, respectively.

Decay constant values are taken from [27] as

$$f_D=(0.207\pm0.009)\ \text{GeV},\quad f_\rho=(0.215\pm0.005)\ \text{GeV},\tag{35}$$

and form-factors for $\bar{B}\to V$ transitions are chosen from [33],

$$A_0^{\bar{B}\rho}(0)=0.356\pm0.042,\tag{36}$$

where the $F_0^{\bar{B}D}(0)$ value is taken from Eq. (22).

$$F_1^{\bar{B}D}(0)=F_0^{\bar{B}D}(0)=0.66\pm0.03.\tag{37}$$

Thus, we calculate the factorizable contributions to the decay amplitudes,

$$\begin{aligned}A^f(\bar{B}^0\to\rho^-D^+)&=(0.235\pm0.011)\ \text{GeV}^2,\\ A^f(\bar{B}^0\to\rho^0D^0)&=-(0.010\pm0.001)\ \text{GeV}^2,\\ A^f(B^-\to\rho^-D^0)&=(0.248\pm0.011)\ \text{GeV}^2,\end{aligned}\tag{38}$$

thereby the isospin reduced amplitudes of the factorized amplitudes are calculated as

$$A^f_{1/2}=(0.197\pm0.009)\ \text{GeV}^2,\quad A^f_{3/2}=(0.127\pm0.006)\ \text{GeV}^2.\tag{39}$$

Following the procedure discussed for $\bar{B}\to\pi D$, we determine the nonfactorizable reduced isospin amplitudes

$$A^{nf}_{1/2}=-(0.054\pm0.026)\ \text{GeV}^2,\quad A^{nf}_{3/2}=-(0.277\pm0.012)\ \text{GeV}^2,\tag{40}$$

which bear the following ratio:

$$\alpha=\frac{A^{nf}_{1/2}}{A^{nf}_{3/2}}=0.200\pm0.096.\tag{41}$$

Table 3. Form-factor of $\bar{B}\to\rho$ transitions at maximum recoil (q^2=0).

Form-factor	CLFQM [27]	LCSR [33]	LCSR [34]	CLFQM [35]	PQCD [36]
$A_0^{\bar{B}\rho}(0)$	0.32±0.01	0.356±0.042	0.303	0.30±0.05	0.366±0.036

Table 4. Ratio $\alpha=A^{nf}_{1/2}/A^{nf}_{3/2}$ for maximum and minimum values of form-factors.

$F_0^{\bar{B}D}(0)$	0.69	0.69	0.63	0.63
$A_0^{\bar{B}\rho}(0)$	0.40	0.31	0.40	0.31
α	0.226	0.219	0.171	0.158

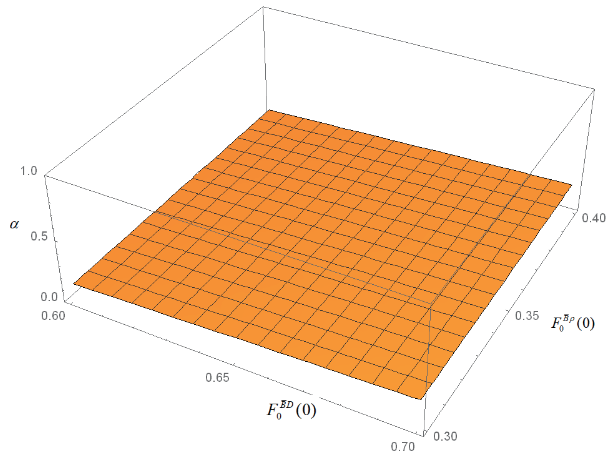

Fig. 2. (color online) Variation in α with form factors $F_0^{\bar{B}D}(0)$ and $F_0^{\bar{B}\rho}(0)$.

There are also existing calculations for $A_0^{\bar{B}\rho}(0)$, which are given in Table 3. To show the effect of form-factors on our analysis, we obtain the ratio $\alpha=A^{nf}_{1/2}/A^{nf}_{3/2}$ for the maximum and minimum value of the form factors given in Table 4, which are consistent with (41) within errors. This is also shown in Fig. 2.

C. $\bar{B}\to\pi D^*$ Decay mode

Including the strong phases between the isospin I=1/2 and 3/2 states, the decay amplitudes are given by

$$\begin{aligned}A(\bar{B}^0\to\pi^-D^{*+})&=\frac{1}{\sqrt{3}}\left[A^{\pi D^*}_{3/2}e^{i\delta^{\pi D^*}_{3/2}}+\sqrt{2}A^{\pi D^*}_{1/2}e^{i\delta^{\pi D^*}_{1/2}}\right],\\ A(\bar{B}^0\to\pi^0D^{*0})&=\frac{1}{\sqrt{3}}\left[\sqrt{2}A^{\pi D^*}_{3/2}e^{i\delta^{\pi D^*}_{3/2}}-A^{\pi D^*}_{1/2}e^{i\delta^{\pi D^*}_{1/2}}\right],\\ A(B^-\to\pi^-D^{*0})&=\sqrt{3}A^{\pi D^*}_{3/2}e^{i\delta^{\pi D^*}_{3/2}}.\end{aligned}\tag{42}$$

Using the experimental values of branching fractions [1],

$$B\left(\bar{B}^0 \to \pi^- D^{*+}\right) = (2.74 \pm 0.13) \times 10^{-3},$$
$$B\left(\bar{B}^0 \to \pi^0 D^{*0}\right) = (2.20 \pm 0.60) \times 10^{-4},$$
$$B\left(B^- \to \pi^- D^{*0}\right) = (4.90 \pm 0.17) \times 10^{-3},$$

we calculate the total isospin reduced amplitudes

$$A_{1/2}^{\pi D^*\,\mathrm{exp}} = \pm(0.226 \pm 0.042)\ \mathrm{GeV}^2,$$
$$A_{3/2}^{\pi D^*\,\mathrm{exp}} = \pm(0.231 \pm 0.040)\ \mathrm{GeV}^2, \quad (43)$$

and the phase difference

$$\delta^{\pi D^*} = (24 \pm 24)^\circ. \quad (44)$$

Therefore, the $\bar{B} \to PV$ decays also indicate the presence of strong FSI phases, as also observed in [17, 18].

The factorizable amplitudes for this mode are

$$A^f(\bar{B}^0 \to \pi^- D^{*+}) = 2a_1 m_{D^*} f_\pi A_0^{\bar{B}D^*}\left(m_\pi^2\right),$$
$$A^f(\bar{B}^0 \to \pi^0 D^{*0}) = -\sqrt{2} a_2 f_{D^*} m_{D^*} F_1^{\bar{B}\pi}\left(m_{D^*}^2\right),$$
$$A^f(B^- \to \pi^- D^{*0}) = a_1 2m_{D^*} f_\pi A_0^{\bar{B}D^*}\left(m_\pi^2\right) + a_2 f_{D^*} 2m_{D^*} F_1^{\bar{B}\pi}\left(m_{D^*}^2\right). \quad (45)$$

Using the decay constant values [27]

$$f_{D^*} = (0.245 \pm 0.034)\ \mathrm{GeV},$$
$$f_\pi = (0.131 \pm 0.002)\ \mathrm{GeV}, \quad (46)$$

and the form-factor

$$A_0^{\bar{B}D^*}(0) = (0.68 \pm 0.04), \quad (47)$$

taken from [27], with pole masses $V(1^-) = 5.32$ GeV and $P(0^-) = 6.28$ GeV for q^2-dependence (34),

$$F_1^{\bar{B}\pi}(0) = F_0^{\bar{B}\pi}(0) = (0.27 \pm 0.05),$$

we calculate the factorized amplitudes as

$$A^f(\bar{B}^0 \to \pi^- D^{*+}) = (0.371 \pm 0.022)\ \mathrm{GeV}^2,$$
$$A^f(\bar{B}^0 \to \pi^0 D^{*0}) = -(0.023 \pm 0.004)\ \mathrm{GeV}^2,$$
$$A^f(B^- \to \pi^- D^{*0}) = (0.403 \pm 0.023)\ \mathrm{GeV}^2, \quad (48)$$

which in turn yield the isospin reduced amplitudes

$$A_{1/2}^f = (0.317 \pm 0.018)\ \mathrm{GeV}^2,$$
$$A_{3/2}^f = (0.196 \pm 0.013)\ \mathrm{GeV}^2. \quad (49)$$

Subtracting the factorizable parts from the total experimental amplitudes, we calculate

$$A_{1/2}^{nf} = -(0.090 \pm 0.046)\ \mathrm{GeV}^2,$$
$$A_{3/2}^{nf} = -(0.426 \pm 0.042)\ \mathrm{GeV}^2, \quad (50)$$

with the following ratio:

$$\alpha = \frac{A_{1/2}^{nf}}{A_{3/2}^{nf}} = 0.211 \pm 0.109. \quad (51)$$

In literature, we find different values of the $A_0^{\bar{B}D^*}(0)$ form-factor, as shown in Table 5. We calculate the ratio α for the maximum and minimum values of the form-factors, as shown in Table 6, and plot the variation in α in Fig. 3. Although α remains insensitive to the $F_0^{\bar{B}\pi}(0)$ form-factor, it increases slowly for large values of $A_0^{\bar{B}D^*}(0)$. However, considering the near equality of the ratio α for $\bar{B} \to \pi D/\rho D/\pi D^*$, we expect a higher value of $A_0^{\bar{B}D^*}(0)$ is less likely.

Table 5. Form-factor of the $\bar{B} \to D^*$ transitions at maximum recoil (q^2 =0).

Form-factor	CLFQM [27]	CLFQM [35]	LQCD [37]
$A_0^{\bar{B}D^*}(0)$	0.68±0.04	0.68±0.08	0.921±0.013

Table 6. Ratio $\alpha = A_{1/2}^{nf}/A_{3/2}^{nf}$ for maximum and minimum values of form-factors.

$A_0^{\bar{B}D^*}(0)$	0.72	0.72	0.64	0.64
$F_0^{\bar{B}\pi}(0)$	0.32	0.22	0.32	0.22
α	0.264	0.245	0.192	0.173

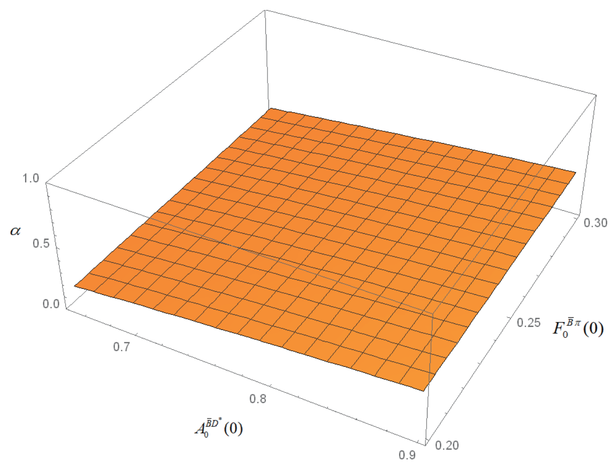

Fig. 3. (color online) Variation in α with form factors $A_0^{\bar{B}D^*}(0)$ and $F_0^{\bar{B}\pi}(0)$

IV. RESULTS AND DISCUSSIONS

The purpose of performing an isospin analysis on the $\bar{B}\to\pi D$ and $\bar{B}\to\rho D/\pi D^*$ decays is to search for systematics, which have previously been identified in the charm sector [17, 18]. By choosing a positive sign for $A^{\rm exp}_{1/2}$ and a negative sign for $A^{\rm exp}_{3/2}$ in each case, we obtain the same value of the ratio of the corresponding nonfactorizable reduced matrix elements $A^{nf}_{1/2}$ and $A^{nf}_{3/2}$, i.e.,

$$\underset{0.229\pm0.042}{\frac{A^{nf}_{1/2}(\bar{B}\to\pi D)}{A^{nf}_{3/2}(\bar{B}\to\pi D)}} = \underset{0.200\pm0.096}{\frac{A^{nf}_{1/2}(\bar{B}\to\rho D)}{A^{nf}_{3/2}(\bar{B}\to\rho D)}} = \underset{0.211\pm0.109}{\frac{A^{nf}_{1/2}(\bar{B}\to\pi D^*)}{A^{nf}_{3/2}(\bar{B}\to\pi D^*)}}, \quad (52)$$

and note that $A^{nf}_{1/2}$ has a negative sign for the cases

$$A^{nf}_{1/2}(\bar{B}\to\pi D) = -(0.572\pm0.105)\ {\rm GeV}^3, \quad (53)$$

$$A^{nf}_{1/2}(\bar{B}\to\rho D) = -(0.054\pm0.026)\ {\rm GeV}^2, \quad (54)$$

$$A^{nf}_{1/2}(\bar{B}\to\pi D^*) = -(0.090\pm0.046)\ {\rm GeV}^2. \quad (55)$$

We can generically predict the sum of the branching fractions of the $\bar{B}^0-$ meson decays in the respective modes considered here as

$$B_{-+}+B_{00} = \frac{\tau_{\bar{B}^0}}{3\tau_{B^-}}B_{0-}\left[1+\left\{\alpha+\frac{\left(\sqrt{2}-\alpha\right)A^f_{-+}-\left(1+\sqrt{2}\alpha\right)A^f_{00}}{A_{0-}}\right\}^2\right], \quad (56)$$

where α has been defined previously (27), and the experimental decay amplitude of the B^- decays is

$$A_{0-} = \sqrt{\frac{B_{0-}}{\tau_{B^-}\times(\text{kinematic factor})}},$$

where the subscripts -+, 00, and 0- denote the charge states of the non-charm and charm mesons emitted in each case. Taking the average value of α = 0.22, we predict

$$B\left(\bar{B}^0\to\pi^-D^+\right)+B\left(\bar{B}^0\to\pi^0D^0\right) = (0.28\pm0.02)\%\quad \text{Theo,}$$
$$= (0.28\pm0.01)\%\quad \text{Expt;} \quad (57)$$

$$B\left(\bar{B}^0\to\rho^-D^+\right)+B\left(\bar{B}^0\to\rho^0D^0\right) = (0.76\pm0.13)\%\quad \text{Theo,}$$
$$= (0.79\pm0.12)\%\quad \text{Expt;} \quad (58)$$

$$B\left(\bar{B}^0\to\pi^-D^{*+}\right)+B\left(\bar{B}^0\to\pi^0D^{*0}\right) = (0.29\pm0.04)\%\quad \text{Theo,}$$
$$= (0.30\pm0.01)\%\quad \text{Expt;} \quad (59)$$

which are in good agreement with the experiment. To show that this agreement is not coincidental and to study the sensitivity of the sum of the $\bar{B}^0$ branching fractions with the ratio α, we plot $\sum B\left(\bar{B}^0\to\text{decays}\right)$ against α by treating it as a free parameter for all three cases, which are shown in Figs. 4, 5, and 6. Clearly, the experiment data indicate α = 0.22 consistently. The broken curves represent the errors due to the decay constant, form factors, and branching fractions. The horizontal lines correspond to the experimental value of the sum, and its errors are indicated by broken lines.

We wish to remark that similar observations have also been made in the FAT approach [23] analysis used for B-meson decays, which separates the factorizable and nonfactorizable contributions in each topological quark level diagram. The most important result in this approach is that the non-perturbative parameters $\chi^{C,E}$ and $\varphi^{C,E}$, representing the nonfactorizable contributions, are found to be <u>universal</u> for all the $\bar{B}\to\pi D/\rho D/\pi D^*$ decay modes, which is consistent with the systematics recognized in our analysis.

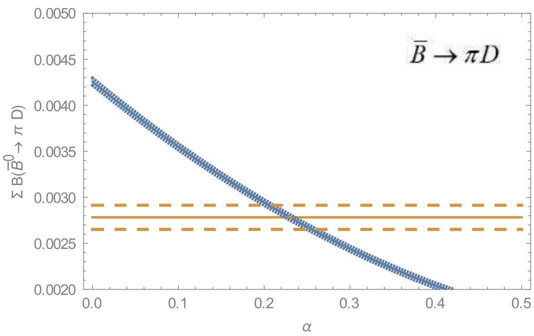

Fig. 4. (color online) Variation in the sum of $B\left(\bar{B}^0\to\pi^-D^+\right)$ and $B\left(\bar{B}^0\to\pi^0D^0\right)$ with the ratio $\alpha = A^{nf}_{1/2}/A^{nf}_{3/2}$.

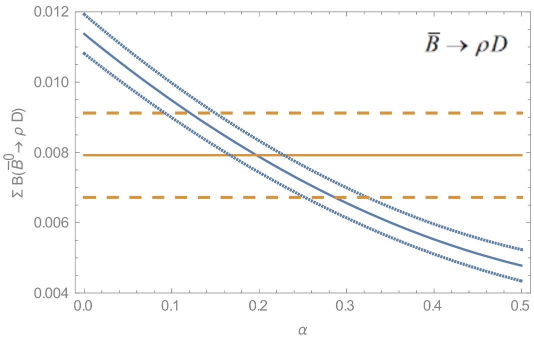

Fig. 5. (color online) Variation in the sum of $B\left(\bar{B}^0\to\rho^-D^+\right)$ and $B\left(\bar{B}^0\to\rho^0D^0\right)$ with the ratio $\alpha = A^{nf}_{1/2}/A^{nf}_{3/2}$.

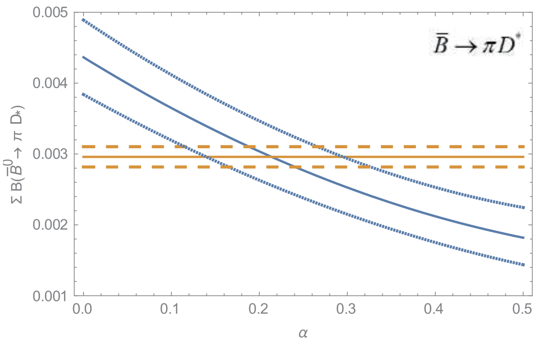

Fig. 6. (color online) Variation in the sum of $B\left(\bar{B}^0\to\pi^-D^{*+}\right)$ and $B\left(\bar{B}^0\to\pi^0D^{*0}\right)$ with the ratio $\alpha=A^{nf}_{1/2}\big/A^{nf}_{3/2}$.

V. SUMMARY AND CONCLUSIONS

We perform an analysis of CKM-favored two-body hadronic decays, $\bar{B}\to\pi D/\rho D/\pi D^*$, which involve two isospin states in the decay products, by including nonfactorizable contributions arising from the part of the weak Hamiltonian involving colored currents. Because non-perturbated nonfactorizable contributions are difficult to calculate, from the theory of strong interactions, we employ the isospin formalism and find that in all the decay modes, the nonfactorizable isospin reduced amplitude $A^{nf}_{1/2}$ consistently bears the same ratio, 0.22, as $A^{nf}_{3/2}$, within the experimental errors, and maintains the same sign. It is important to note that similar universality in nonfactorizable contributions has also been observed [23] in a recent analysis of B- decays using the FAT approach.

Because similar systematics observed for charm mesons decaying into s-wave mesons [16] have been found to be consistent with those of their p-wave meson emitting decays [18], we further expect that this universality of nonfactorizable terms in $\bar{B}\to\pi D/\rho D/\pi D^*$ may also hold true for the p-wave meson emitting decays of bottom mesons and can be used to make predictions of the branching fractions, for which experimental measurements are not yet available.

The present experimental data for B-decays clearly indicate the presence of FSI strong phase differences, and our values agree with other analyses performed for these decays [20, 21]. We also wish to mention that we are aware of a study by Sharma and Katoch [38], who assumed that the ratio of the non-factorizable amplitudes was equal to -0.828 in the absence of experimental data at that time and predicted the sum of the branching fractions of $\bar{B}^0$-decays, which was not in agreement with the latest experimental measurements. Moreover, their approach was different from ours because they did not consider the final state interaction effects.

References

[1] P. A. Zyla *et al.* (Particle Data Group), Prog. Theor. Exp. Phys **2020**, 083C01 (2020)
[2] M. Wirbel, B. Stech, and M. Bauer, Z. Phys. C **29**, 637 (1985)
[3] M. Bauer, B. Stech, and M. Wirbel, Z. Phys. C **34**, 103 (1987)
[4] M. Wirbel, Prog. Part. Nucl. Phys **21**, 33-48 (1988)
[5] N. Isgur, D. Scora, B. Grinstein *et al.*, Phys. Rev. D **39**, 799 (1989)
[6] D. Scora and N. Isgur, Phys. Rev. D **52**, 2783 (1995)
[7] T. E. Browder and K. Honscheid, Prog. Nucl. Part. Phys **35**, 81-255 (1995)
[8] H. Y. Cheng, Z. Phys. C **32**, 237 (1986)
[9] N. Deshpande, M. Gronau, and D. Sutherland, Phys. Lett. B **90**, 431 (1980)
[10] M. Shifman, Nucl. Phys. B **388**, 346 (1992)
[11] A. N. Kamal and T. N. Pham, Phys. Rev. D **50**, 395 (1994)
[12] A. J. Buras, J. M. Gerard, and R. Ruckl, Nucl. Phys. B **268**: 16-48 (1986)
[13] B. Blok and M. Shifman, Nucl. Phys. B **399**: 441 (1993).
[14] H. Y. Cheng, Int. J. Mod. Phys. A **2**, 650 (2006)
[15] M. Suzuki, *The final-state interaction in the two-body nonleptonic decay of a heavy particle*, [hep-ph/9807414]
[16] A. N. Kamal, A. B. Santra, T. Uppal *et al.*, Phys. Rev. D **53**, 2506 (1996)
[17] R. C. Verma, Z. Phys. C **69**, 253 (1996)
[18] A. C. Katoch, K. K. Sharma, and R. C. Verma, J. Phys. G **23**, 807 (1997)
[19] R. C. Verma, Phys. Lett. B **365**, 377 (1996)
[20] C. K. Chua, W. S. Hou, and K. C. Yang, Phys. Rev. D **65**, 096007 (2002)
[21] C. K. Chua and W. S. Hou, Phys. Rev. D **77**, 116001 (2008)
[22] C. W. Chiang and E. Senaha, Phys. Rev. D **75**, 074021 (2007)
[23] S. H. Zhou, Y. B. Wei, Q. Qin *et al.*, Phys. Rev. D **92**, 094016 (2015)
[24] S. H. Zhou, R. H. Li, Z. Y. Wei *et al.*, Phys. Rev. D **104**, 116012 (2021)
[25] A. N. Kamal, Int. J. Mod. Phys. A **7**: 3515 (1992)
[26] K. Watson, Phys. Rev. D **88**, 1163 (1952)
[27] R. C. Verma, J. Phys. G **39**, 025005 (2012)
[28] Neelesh sharma and Rohit Dhir, Nucl. Phys. B **946**, 114718 (2019)
[29] N. Gubernari, A. Kokulu, and D. van Dyk, JHEP **01**, 150 (2019)
[30] Y. M. Wang, Y. B. Wei, Y. L. Shen *et al.*, JHEP **06**, 062 (2017)
[31] H. n. Li, Y. L. Shen, and Y. M. Wang, Phys. Rev. D **85**, 074004 (2012)
[32] S. Cheng, Y. Y. Fan, X. Yu *et al.*, Phys. Rev. D **89**, 094004 (2014)
[33] A. Bharucha, D. M. Straub, and R. Zwicky, JHEP **08**, 098 (2016)
[34] E. Bagan, P. Ball, and V. M. Braun, Phys. Lett. B **417**, 154 (1998)
[35] Q. Chang, X. N. Lia, and L. T. Wanga, The Euro. Phys. J C **79**, 422 (2019)
[36] C. D. Lu and M. Z. Zhang, Eur. Phys. J. C **28**, 515 (2003)
[37] C. Bernard *et al.*, Phys. Rev. D **79**, 014506 (2009)
[38] K. K. Sharma and A. C. Katoch, Mod. Phys. Lett. A **14**, 1841 (1999)