Exclusive production of B_c mesons in e^+e^- colliders*

Yan-Bing Wei(魏焰冰)[1,2] Run-Hui Li(李润辉)[3] Dan Luo(罗丹)[4]
Cai-Dian Lü(吕才典)[1,2] Yue-Long Shen(沈月龙)[4]

[1] Institute of High Energy Physics, CAS, P.O. Box 918(4), Beijing 100049, China
[2] School of Physics, University of Chinese Academy of Sciences, Beijing 100049, China
[3] School of Physical Science and Technology, Inner Mongolia University, Huhhot 010021, China
[4] College of Information Science and Engineering, Ocean University of China, Qingdao 266100, China

Abstract: Within the framework of the perturbative QCD approach, we calculate the time-like $B_cB_c(B_c^*)$ form factors $F(Q^2)$ and $A_2(Q^2)$. We include relativistic corrections and QCD corrections, either of which can give about 20% correction to the leading-order contribution, but there are cancellation effects between them. We calculate the cross sections of the $e^+e^- \to B_c^-B_c^+(B_c^{*+})$ processes. The cross sections are enhanced at the Z pole to be $\sigma^{PP}(Q{=}m_Z){\sim}1.3{\times}10^{-5}$pb and $\sigma^{PV}(Q{=}m_Z){\sim}2.5{\times}10^{-5}$pb, which are still too small to be detected by proposed e^+e^- colliders such as the Circular Electron Positron Collider.

Keywords: PQCD, B physics, form factor

PACS: 12.39.Hg **DOI:** 10.1088/1674-1137/42/8/083107

1 Introduction

The study of the B_c meson is of special interest, since it is the ground state of the doubly heavy-flavored ($\bar{b}c$) system, which is unique in the Standard Model. Both production and decay processes of the B_c meson contain rich heavy quark dynamics, so they are worthy of systematic study. Experimentally, observations of B_c mesons are so far available only at hadron colliders [1]. Hadron colliders only provide limited knowledge of the production of B_c mesons because the total collision energy cannot be well controlled. For B_c production in e^+e^- colliders, the total collision energy can be well controlled and many meaningful observables, such as the angular distributions of final states, could be measured. Despite these advantages, the B_c meson has not yet been observed in e^+e^- colliders [2]. The production rate is too small to have been observed at LEP. Nowadays, several high luminosity e^+e^- facilities are proposed, such as the International Linear Collider (ILC), Circular Electron Positron Collider (CEPC), and Future Circular Collider (FCC-ee). These e^+e^- colliders could provide us with the opportunity to produce B_c mesons, especially at the Z-pole.

There are many studies of B_c meson decays [3–18]. The semi-inclusive production of the B_c meson has also been investigated extensively [19–23]. However, the knowledge of B_c production in exclusive processes is very limited. To deal with hard exclusive processes involving heavy quark-anti-quark systems, two kinds of factorization approaches have been proposed in the literature. One is the non-relativistic QCD (NRQCD) factorization approach [24–27], in which the amplitude of the process can be factorized into the product of the short-distance coefficient and NRQCD matrix-element. In the other approach (called collinear factorization) the amplitude of the production process can be expressed as a convolution of the hard-kernel and the universal light-cone distribution amplitude (LCDA) [28, 29]. The LCDA of a large boosted B_c meson is defined by sandwiching the gauge invariant non-local quark bilinear operators between the vacuum and the B_c state. For the leading-twist LCDA, we have:

$$\langle B_c(^1S_0,P)|(\bar{b}W_c)(\omega n_+)\not{n}_+\gamma_5(W_c^\dagger c)(0)(\omega)|0\rangle$$
$$= -\mathrm{i}f_P n_+P\int_0^1 \mathrm{d}x\mathrm{e}^{\mathrm{i}\omega n_+Px}\hat{\phi}_P(x;\mu), \tag{1}$$
$$\langle B_c(^3S_1,P,\varepsilon^*)|(\bar{b}W_c)(\omega n_+)\not{n}_+(W_c^\dagger c)(0)(\omega)|0\rangle$$
$$= f_V m_V n_+\varepsilon^*\int_0^1 \mathrm{d}x\mathrm{e}^{\mathrm{i}\omega n_+Px}\hat{\phi}_V^{\parallel}(x;\mu), \tag{2}$$
$$\langle B_c(^3S_1,P,\varepsilon^*)|(\bar{b}W_c)(\omega n_+)\not{n}_+\gamma_\perp^\alpha(W_c^\dagger c)(0)(\omega)|0\rangle$$
$$= f_V^\perp n_+P\varepsilon_\perp^{*\alpha}\int_0^1 \mathrm{d}x\mathrm{d}x\mathrm{e}^{\mathrm{i}\omega n_+Px}\hat{\phi}_V^\perp(x;\mu), \tag{3}$$

Received 11 May 2018, Published online 10 July 2018

∗ Supported by Natural Science Foundation of Shandong Province, China (ZR2015AQ006), National Natural Science Foundation of China (11521505, 11621131001, 11447009, 11505098, 11765012), R. H. Li is partly supported by the plan of Young Creative Talents under the Talent of the Prairie project of the Inner Mongolia

where the Wilson-line

$$W_c(x)=P\exp\Big[\mathrm{i}g_s\int_{-\infty}^{0}\mathrm{d}s\ n_+A(x+sn_+)\Big], \qquad (4)$$

and $\hat{\phi}_P(x)$, $\hat{\phi}_V(x)$ and $\hat{\phi}_V^{\perp}(x)$ are LCDAs of pseudoscalar, longitudinally polarized and transversely polarized vector $\mathrm{B_c}$ mesons, respectively. Using the "refactorization" scheme [30], the $\mathrm{B_c}$-meson LCDA has been calculated to next-to-leading order with respect both to the coupling constant and the velocity of the heavy quark [31–34].

Collinear factorization breaks down if there exists an endpoint singularity. A solution is to introduce a transverse momentum to the parton, which is called k_T factorization. The PQCD approach based on k_T factorization has been employed to study a large number of exclusive processes [35–39], including pair production of light mesons [40–43]. In the present paper we will employ the PQCD approach to study the exclusive $\mathrm{B_c}$ meson pair production processes, i.e. $\mathrm{e^+e^-\to B_c^-B_c^+(B_c^{*+})}$. The amplitude can be divided into the leptonic part and the hadronic part. The hadronic part is actually the time-like $\mathrm{B_cB_c}$ or $\mathrm{B_cB_c^*}$ form factors and evaluating these form factors is the main object of this work. The form factors will be calculated at the leading power in $1/Q$ (Q being the energy of the $\mathrm{e^+e^-}$ pair in the center-of-mass frame), while power corrections from higher-twist LCDAs and quark mass effects will be considered in the future.

This paper is organized as follows. In the next section we calculate time-like $\mathrm{B_cB_c(B_c^*)}$ form factors and cross sections of $\mathrm{e^+e^-\to B_c^-B_c^+(B_c^{*+})}$ processes using PQCD approach. Numerical analysis is presented in the third section. The last section gives a summary.

2 Time-like form factors in the PQCD approach

The differential cross section of $\mathrm{e^+(k')e^-(k)}\to\gamma^*,Z^0\to\mathrm{B_c^-}(P)\mathrm{B_c^+}(P')(\mathrm{B_c^{*+}})$ is

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}=\frac{|\boldsymbol{k}|}{32\pi^2Q^3}\frac{1}{4}\sum_{s,s'}|A_\gamma+A_Z|^2, \qquad (5)$$

where s and s' are the spins of the ingoing electron and positron respectively. The momentum transfer $q=k+k'=P+P'$ and the energy scale $Q=\sqrt{q^2}$. The amplitudes are

$$A_\gamma^{PP} = \sum_{q=b,c}\frac{-\mathrm{i}e^2e_q}{Q^2}\bar{v}(k')\gamma_\mu u(k)(P'-P)^\mu F^q(Q^2),$$

$$\begin{aligned}A_Z^{PP} &= \frac{\mathrm{i}\sqrt{2}G_Fm_W^2}{\cos^2\theta_W}\sum_{q=b,c}\frac{T_3^q-2e_q\sin^2\theta_W}{Q^2-m_Z^2+\mathrm{i}\varGamma_Zm_Z}\bar{v}(k')\gamma_\mu\\ &\quad\times\left(-\frac{1}{2}+2\sin^2\theta_W+\frac{1}{2}\gamma_5\right)u(k)\\ &\quad\times\left(g^{\mu\nu}-\frac{q^\mu q^\nu}{m_Z^2-\mathrm{i}\varGamma_Zm_Z}\right)(P'-P)_\nu F^q(Q^2), \qquad (6)\end{aligned}$$

$$A_\gamma^{PV} = \sum_{q=b,c}\frac{-\mathrm{i}e^2e_q}{Q^2}\bar{v}(k')\gamma^\mu u(k)\epsilon_{\mu\nu\rho\sigma}\varepsilon^{*\nu}_{\mathrm{B_c^{*+}}}P^\rho P'^\sigma V(Q^2),$$

$$\begin{aligned}A_Z^{PV} &= \frac{\mathrm{i}\sqrt{2}G_Fm_W^2}{\cos^2\theta_W}\sum_{q=b,c}\frac{1}{Q^2-m_Z^2+\mathrm{i}\varGamma_Zm_Z}\bar{v}(k')\gamma_\mu\\ &\quad\times\left(-\frac{1}{2}+2\sin^2\theta_W+\frac{1}{2}\gamma_5\right)u(k)\left(g^{\mu\lambda}-\frac{q^\mu q^\lambda}{m_Z^2-\mathrm{i}\varGamma_Zm_Z}\right)\\ &\quad\times\Big\{[T_3^q-2e_q\sin^2\theta_W]\epsilon_{\lambda\nu\rho\sigma}\varepsilon^{*\nu}_{\mathrm{B_c^{*+}}}P^\rho P'^\sigma V^q(Q^2)\\ &\quad-T_3^q[A_1^q(Q^2)Q^2\varepsilon^{*\lambda}+A_2^q(Q^2)\varepsilon^*\cdot q(P'-P)^\lambda]\Big\}, \qquad (7)\end{aligned}$$

where $T_3^b=-1/2$ and $T_3^c=1/2$. The time-like form factors which appear in the above amplitudes are defined by:

$$\begin{aligned}\langle\mathrm{B_c^+}(P')\mathrm{B_c^-}(P)|\bar{q}\gamma_\mu q|0\rangle &= (P'-P)_\mu F^q(Q^2),\\ \langle\mathrm{B_c^{*+}}(P')\mathrm{B_c^-}(P)|\bar{q}\gamma_\mu q|0\rangle &= \epsilon_{\mu\nu\rho\sigma}\varepsilon^{*\nu}_{\mathrm{B_c^{*+}}}P^\rho P'^\sigma V^q(Q^2),\\ \langle\mathrm{B_c^{*+}}(P')\mathrm{B_c^-}(P)|\bar{q}\gamma_\mu\gamma_5 q|0\rangle &= A_1^q(Q^2)(Q^2\varepsilon^*_\mu-\varepsilon^*\cdot q q_\mu)\\ &\quad+A_2^q(Q^2)\varepsilon^*\cdot q(P'-P)_\mu, \qquad (8)\end{aligned}$$

where the equation of motion has been employed. Furthermore, form factors $V^q(Q^2)$ and $A_1^q(Q^2)$ vanish at leading power at tree level, and will not be considered in this work. Substituting the above amplitudes into Eq. (5), we have

$$\begin{aligned}\frac{\mathrm{d}\sigma^{PP}}{\mathrm{d}\Omega} &= \frac{1}{64\pi^2Q^2}\Bigg\{\Bigg\{\sum_{q=b,c}\Bigg[\left(\frac{e^2e_q}{Q^2}+Z_1^q(Q^2-m_Z^2)\right)\mathrm{Im}F_q+Z_1^qm_Z\varGamma_Z\mathrm{Re}F_q\Bigg]\Bigg\}^2\\ &\quad+\Bigg\{\sum_{q=b,c}\Bigg[\left(\frac{-e^2e_q}{Q^2}-Z_1^q(Q^2-m_Z^2)\right)\mathrm{Re}F_q+Z_1^qm_Z\varGamma_Z\mathrm{Im}F_q\Bigg]\Bigg\}^2+\Bigg\{\sum_{q=b,c}[Z_2^q(Q^2-m_Z^2)\mathrm{Im}F_q-Z_2^qm_Z\varGamma_Z\mathrm{Re}F_q]\Bigg\}^2\\ &\quad+\Bigg\{\sum_{q=b,c}[Z_2^q(Q^2-m_Z^2)\mathrm{Re}F_q+Z_2^qm_Z\varGamma_Z\mathrm{Im}F_q]\Bigg\}^2\Bigg\}\frac{1}{2}Q^4\sin^2\theta, \qquad (9)\end{aligned}$$

where

$$Z_1^q=\frac{\sqrt{2}G_F m_W^2}{\cos^2\theta_W}\frac{T_3^q-2e_q\sin^2\theta_W}{(Q^2-m_Z^2)^2+m_Z^2\Gamma_Z^2}(-\frac{1}{2}+2\sin^2\theta_W)$$

and

$$Z_2^q=\frac{\sqrt{2}G_F m_W^2}{\cos^2\theta_W}\frac{T_3^q-2e_q\sin^2\theta_W}{(Q^2-m_Z^2)^2+m_Z^2\Gamma_Z^2}\frac{1}{2}.$$

For the $B_cB_c^*$ channel, we have

$$\begin{aligned}\frac{d\sigma^{PV}}{d\Omega} &= \frac{1}{32\pi^2}\left[\frac{G_F m_W^2}{\cos^2\theta_W}\frac{1}{(Q^2-m_Z^2)^2+m_Z^2\Gamma_Z^2}\right]^2\\ &\times\left[\left\{\sum_{q=b,c}[(Q^2-m_Z^2)\mathrm{Im}A_2^q-m_Z\Gamma_Z\mathrm{Re}A_2^q]T_3^q\right\}^2\right.\\ &\left.+\left\{\sum_{q=b,c}[(Q^2-m_Z^2)\mathrm{Re}A_2^q+m_Z\Gamma_Z\mathrm{Im}A_2^q]T_3^q\right\}^2\right]\\ &\times\left(\frac{1}{2}+4\sin^4\theta_W-2\sin^2\theta_W\right)\\ &\times\frac{Q^4\sin^2\theta}{2}\left(\frac{Q^2}{4m_{B_c^*}^2}-1\right).\end{aligned} \tag{10}$$

In the proposed forthcoming accelerators, the collision energy in the center-of-mass frame is much larger than the masses of the b quark and c quark. In this paper, we concentrate on the leading power result of the relevant time-like form factors. Power suppressed contributions, such as quark mass effects and corrections from higher-twist LCDA, which may be important when Q^2 is not large enough, will be neglected in our calculations. In the PQCD approach, form factors are factorized into a convolution of the transverse momentum dependent (TMD) wave function and the hard scattering kernel. The tree-level hard kernel can be obtained by computing the Feynman diagrams plotted in Fig. 1. In the calculation, we choose the momentum fractions of charm quarks in the B_c^- meson and $B_c^{+(*)}$ meson to be x and y respectively. For form factors F^q, we have

$$\begin{aligned}F^b(Q^2) &= \frac{2\pi f_{B_c}^2 C_F}{N_c}Q^2\int_0^1 dxdy\int_0^\infty b_1db_1b_2db_2\alpha_s(t)\\ [1mm] &\times[-x\mathcal{P}_{PI}(x,b_1)\mathcal{P}_{PII}(y,b_2)]\\ [1mm] &\times\mathcal{H}(x,y,Q,b_1,b_2)\exp[-S(x,y,b_1,b_2,Q)],\\ F^c(Q^2) &= \frac{2\pi f_{B_c}^2 C_F}{N_c}Q^2\int_0^1 dxdy\int_0^\infty b_1db_1b_2db_2\alpha_s(t)\\ &\times[-\bar{x}\mathcal{P}_{PI}(x,b_1)\mathcal{P}_{PII}(y,b_2)]\\ [1mm] &\times\mathcal{H}(\bar{x},\bar{y},Q,b_1,b_2)\exp[-S(x,y,b_1,b_2,Q)],\end{aligned} \tag{11}$$

where $\bar{x}=1-x$. The hard function is:

$$\begin{aligned}\mathcal{H}(x,y,Q,b_1,b_2) &= \left(\frac{i\pi}{2}\right)^2 H_0^{(1)}(\sqrt{xy}Qb_2)\\ &\times\Big[\theta(b_1-b_2)H_0^{(1)}(\sqrt{x}Qb_1)J_0(\sqrt{x}Qb_2)\\ &+\theta(b_2-b_1)H_0^{(1)}(\sqrt{x}Qb_2)J_0(\sqrt{x}Qb_1)\Big].\end{aligned} \tag{12}$$

The TMD wave function is:

$$\mathcal{P}_P(x,\mathbf{b}) = \hat{\phi}_P(x)\Sigma(x,\boldsymbol{b}), \tag{13}$$

where $\Sigma(x,b)$ is the transverse momentum dependent part. The LCDAs can be expressed in the following expansions,

$$\hat{\phi}_P(x) = \hat{\phi}_P^{(0,0)}(x)+\hat{\phi}_P^{(1,0)}(x)+\hat{\phi}_P^{(0,1)}(x), \tag{14a}$$
$$\hat{\phi}_V^{||}(x) = \hat{\phi}_V^{||(0,0)}(x)+\hat{\phi}_V^{||(1,0)}(x)+\hat{\phi}_V^{||(0,1)}(x), \tag{14b}$$

where the superscript (i,j) denotes the order of α_s and v^2-expansion. In order to express the form factors in terms of α_s expansion, we write

$$\mathcal{P}(x,\boldsymbol{b}) = \mathcal{P}^{(0)}(x,\boldsymbol{b})+\mathcal{P}^{(1)}(x,\boldsymbol{b})+..., \tag{15a}$$
$$\begin{aligned}\mathcal{P}^{(0)}(x,\boldsymbol{b}) &= [\hat{\phi}^{(0,0)}(x)+\hat{\phi}^{(0,1)}(x)]\Sigma(x,b),\\ \mathcal{P}^{(1)}(x,\boldsymbol{b}) &= \hat{\phi}^{(1,0)}(x)\Sigma(x,b).\end{aligned} \tag{15b}$$

A typical value of x is $x\sim x_0=\frac{m_c}{m_b+m_c}$. We have $xQ^2\gg k_T^2$ when the energy scale Q is large, thus we can drop the transverse momentum in the quark propagator. Then form factors can then be simplified as

$$\begin{aligned}F^b_{(LO)}(Q^2) &= i\frac{\pi^2 f_{B_c}^2 C_F\alpha_s(\mu)}{N_c}\\ &\times\int_0^1 dxdy\int_0^\infty dbb\mathcal{P}_{PI}(x)\mathcal{P}_{PII}(y)\\ &\times\exp[-S_I(x,y,b,Q,\mu)]H_0^{(1)}(\sqrt{xy}Qb),\end{aligned} \tag{16}$$

$$\begin{aligned}F^c_{(LO)}(Q^2) &= i\frac{\pi^2 f_{B_c}^2 C_F\alpha_s(\mu)}{N_c}\int_0^1 dxdy\\ &\times\int_0^\infty dbb\mathcal{P}_{PI}^{(0)}(x)\mathcal{P}_{PII}^{(0)}(y)\\ &\times\exp[-S_I(x,y,b,Q,\mu)]H_0^{(1)}\left(\sqrt{\bar{x}\bar{y}}Qb\right),\end{aligned} \tag{17}$$

where $S_I(x,y,b,Q,\mu)\equiv S(x,b,y,b,Q,\mu)$ and the Hankel function $H_0^{(1)}(x)=J_0(x)+iY_0(x)$. Form factors $A_2^{(b,c)}$ are related to $F^{(b,c)}$ at tree level, as shown in Eqs.

(A18,A19):

$$A^b_{2,(LO)}(Q^2) = -\frac{\pi^2 f_{\rm B_c} f_V^{\parallel} m_V C_F \alpha_s(\mu)}{Q^2 N_c} \times \int_0^1 {\rm d}x{\rm d}y \int_0^\infty {\rm d}bb \mathcal{P}_{PI}(x)\mathcal{P}_{V^{\parallel}II}(y) \times \exp[-S_{\rm I}(x,y,b,Q,\mu)] H_0^{(1)}(\sqrt{xy}Qb),$$

$$A^c_{2,(LO)}(Q^2) = \frac{\pi^2 f_{\rm B_c} f_V^{\parallel} m_V C_F \alpha_s(\mu)}{Q^2 N_c} \times \int_0^1 {\rm d}x{\rm d}y \int_0^\infty {\rm d}bb \mathcal{P}_{PI}(x)\mathcal{P}_{V^{\parallel}II}(y) \times \exp[-S_{\rm I}(x,y,b,Q,\mu)] H_0^{(1)}\left(\sqrt{\bar{x}\bar{y}}Qb\right). \tag{18}$$

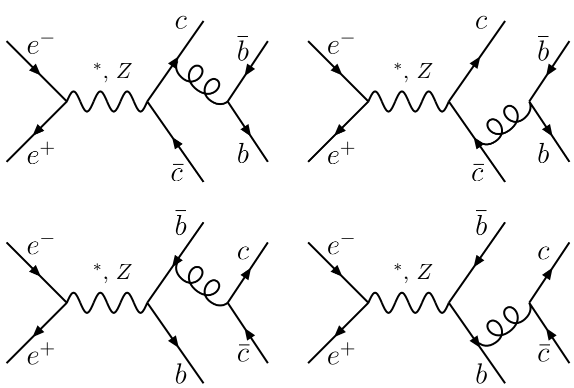

Fig. 1. Feynman diagrams for $e^+e^-\to B_c^- B_c^{+(*)}$.

The QCD corrections to form factors are of great importance in theoretical analysis. In the PQCD approach, NLO corrections to the time-like pion electromagnetic form factors have been calculated by Li et al [42]. In this paper, we consider NLO corrections to the form factors F^q. The calculation of NLO corrections in the PQCD approach is tediously complicated. To simplify the PQCD calculation, the hierarchy $xQ^2, yQ^2 \gg xyQ^2 \sim k_T^2$ is postulated. As the quark masses have been neglected, the NLO hard kernel can be borrowed directly from [42]:

$$H_{\rm F}^{(\rm NLO)}(x,k_{1T},y,k_{2T},Q^2,\mu) = h(x,y,\delta_{12},Q,\mu) H_{\rm F}^{(\rm LO)}(x,k_{1T},y,k_{2T},Q^2), \tag{19}$$

where

$$h(x,y,\delta_{12},Q,\mu) = \frac{\alpha_s(\mu)C_F}{4\pi}\left[-\frac{3}{4}\ln\frac{\mu^2}{Q^2}-\frac{17}{4}\ln^2 x +\frac{27}{8}\ln x \ln y-\frac{13}{8}\ln x+\frac{31}{16}\ln y-\ln^2\delta_{12} +\left(\frac{17}{4}\ln x+\frac{23}{8}+{\rm i}2\pi\right)\ln\delta_{12}+\frac{\pi^2}{12} +\frac{1}{2}\ln 2+\frac{53}{4}-{\rm i}\frac{3\pi}{4}\right], \tag{20}$$

and

$$\ln\delta_{12} \equiv \ln\frac{\left||\boldsymbol{k}_{1T}+\boldsymbol{k}_{2T}|^2-xyQ^2\right|}{Q^2}+i\pi\Theta\left(|\boldsymbol{k}_{1T}+\boldsymbol{k}_{2T}|^2-xyQ^2\right). \tag{21}$$

Including the NLO hard kernel and performing a Fourier transform of Eq. (19), we derive the TMD factorization formula of form factors F^q at leading power,

$$F^b_{NLO}(Q^2) = {\rm i}\frac{\pi f^2_{\rm B_c} C_F^2}{4N_c}\int_0^1 {\rm d}x{\rm d}y\int_0^\infty {\rm d}bb\alpha_s^2(\mu)\mathcal{P}^{(0)}_{PI}(x)\mathcal{P}^{(0)}_{PII}(y)\exp[-S_{\rm I}(x,y,b,Q,\mu)] \times\left[\widetilde{h}(x,y,b,Q,\mu)H_0^{(1)}(\sqrt{xy}Qb)+H_0^{(1)\prime\prime}(\sqrt{xy}Qb)\right],$$

$$F^c_{NLO}(Q^2) = {\rm i}\frac{\pi f^2_{\rm B_c} C_F^2}{4N_c}\int_0^1 {\rm d}x{\rm d}y\int_0^\infty {\rm d}bb\alpha_s^2(\mu)\mathcal{P}^{(0)}_{PI}(x)\mathcal{P}^{(0)}_{PII}(y)\exp[-S_{\rm I}(x,y,b,Q,\mu)] \times\left[\widetilde{h}(\bar{x},y,b,Q,\mu)H_0^{(1)}(\sqrt{\bar{x}\bar{y}}Qb)+H_0^{(1)\prime\prime}\left(\sqrt{\bar{x}\bar{y}}Qb\right)\right], \tag{22}$$

where

$$\widetilde{h}(x,y,b,Q,\mu) = -\frac{3}{4}\ln\frac{\mu^2}{Q^2}-\frac{1}{4}\ln^2\frac{4xy}{Q^2b^2}+\left(\frac{17}{8}\ln x+\frac{23}{16}+\gamma_E+{\rm i}\frac{\pi}{2}\right)\ln\frac{4xy}{Q^2b^2}-\frac{17}{4}\ln^2 x+\frac{27}{8}\ln x\ln y -\left(\frac{13}{8}+\frac{17\gamma_E}{4}-{\rm i}\frac{17\pi}{8}\right)\ln x+\frac{31}{16}\ln y-\frac{\pi^2}{2}+(1-2\gamma_E)\pi+\frac{1}{2}\ln 2+\frac{53}{4}-\frac{23}{8}\gamma_E -\gamma_E^2+{\rm i}\left(\frac{171}{16}+\gamma_E\right)\pi. \tag{23}$$

3 Numerical analysis

The most important nonperturbative input is the LCDA of the B_c meson, which can be studied with non-relativistic QCD effective theory as both internal quarks are heavy. At tree level and at leading order in the expansion over the relative velocity, the quark and the antiquark simply share the momentum of the meson according to their masses,

$$\hat{\phi}_P^{(0,0)}(x) = \hat{\phi}_V^{\parallel(0,0)}(x)=\delta(x-x_0). \quad (24)$$

If we include the gluon exchange effect and the power correction relevant to the quark velocity, the parton configuration will be changed. Explicit expressions of $\hat{\phi}^{(0,1)}(x)$ are given by [33]

$$\hat{\phi}_P^{(0,1)}(x)=-\frac{\langle \boldsymbol{q}^2\rangle_P}{M^2}\Big[\frac{2(1-2x_0)}{3x_0\bar{x}_0}\delta'(x-x_0)-\frac{1}{6}\delta''(x-x_0)\Big], \quad (25)$$

$$\begin{aligned}\hat{\phi}_V^{\parallel(0,1)}(x) &= \hat{\phi}_V^{\perp(0,1)}(x)\\ &= -\frac{\langle \mathbf{q}^2\rangle_V}{M^2}\Big[\frac{2(1-2x_0)}{3x_0\bar{x}_0}\delta'(x-x_0)-\frac{1}{6}\delta''(x-x_0)\Big],\end{aligned} \quad (26)$$

where $M=m_b+m_c$ and $\langle \mathbf{q}^2\rangle_{P,V}$ are the mean values of $\mathbf{q}^2$ in scalar and vector B_c mesons respectively. For QCD corrections to the LCDAs, we have

$$\hat{\phi}_P^{(1,0)}(x;\mu) = \frac{\alpha_s}{4\pi}C_F\Big\{\Phi_1(x,x_0)\Big\}, \quad (27)$$

$$\begin{aligned}\hat{\phi}_V^{\parallel(1,0)}(x;\mu) = \frac{\alpha_s}{4\pi}C_F\Big\{&\Phi_1(x,x_0)-4\Big[\frac{x}{x_0}\theta(x_0-x)\\ &+(x\leftrightarrow\bar{x},x_0\leftrightarrow\bar{x}_0)\Big]_+\Big\},\end{aligned} \quad (28)$$

with

$$\begin{aligned}\Phi_1(x,x_0) = 2\Big[&\Big(\ln\frac{\mu^2}{M^2(x_0-x)^2}-1\Big)\Big(\frac{x_0+\bar{x}}{x_0-x}\frac{x}{x_0}\theta(x_0-x)\\ &+(x\leftrightarrow\bar{x},x_0\leftrightarrow\bar{x}_0)\Big)\Big]_+ +\Big[\frac{4x\bar{x}}{(x_0-x)^2}\Big]_{++}\\ &+\Big[4x_0\bar{x}_0\ln\frac{x_0}{\bar{x}_0}+2(2x_0-1)\Big]\delta'(x-x_0).\end{aligned} \quad (29)$$

Here we adopt the NDR scheme of γ_5, as the NLO hard kernel is also obtained in this scheme. The ++- and +-distributions are defined as

$$\int_0^1 \mathrm{d}x\Big[f(x)\Big]_{++}g(x) = \int_0^1 \mathrm{d}xf(x)(g(x)-g(x_0) -g'(x_0)(x-x_0)), \quad (30a)$$

$$\int_0^1 \mathrm{d}x\Big[f(x)\Big]_{+}g(x) = \int_0^1 \mathrm{d}xf(x)(g(x)-g(x_0)), \quad (30b)$$

where $g(x)$ is a smooth test function. Because LCDA with v and α_s corrections contains a Dirac-δ function or plus distribution, we need to integrate over the momentum fraction x first. The results are given in the Appendix A.

For the TMD wave function, we use

$$\Sigma(x,\boldsymbol{b}) = \exp\Big(-\frac{b^2}{4\beta^2}\Big). \quad (31)$$

The input parameters are listed in Table 1.

Table 1. Parameters used for numerical analysis.

parameter	value	parameter	value
m_b	4.8 GeV	m_c	1.6 GeV
m_W	80.425 GeV	m_Z	91.1876 GeV
m_{B_c}	6.2749	$\langle \mathbf{q}^2\rangle$	1.59 GeV^2
Γ_Z	2.4952 GeV	$\sin^2\theta_W$	0.23129
n_f	5	Λ	0.217 GeV
γ_E	0.57721566	G_F	1.166391×10^{-5} GeV^{-2}
β	2 GeV^{-1}	f_{B_c}	0.489±0.005 GeV

We are now ready to evaluate the numerical results of the form factors and cross sections. We first concentrate on the Q dependence of the time-like form factors. In the PQCD approach the time-like form factors are complex, as the internal quark line may be on-shell. The absolute values of form factors F^c and F^b are plotted in Fig. 2. We do not show form factors A_2^c and A_2^b, since they are related to F^c and F^b. From the left-hand plot, we can see that the absolute value of form factor F^c (the dot-dashed blue curve) is about an order or magnitude smaller than that of F^b (the solid black curve) because the wave function of B_c meson is not symmetrical between charm quark and bottom quark, so the invariant mass of the internal gluon propagator is much larger in F^c. To illustrate the effects of relativity and the QCD corrections we plot the form factor F^b with these contributions. As we can see from the right-hand plot, contributions from both relativistic corrections (the dashed blue curve) and NLO corrections (the dot-dashed red curve) are about 20% of the LO contribution (the dotted black curve). However, there is cancellation between the two kinds of corrections and the total result is very close to the leading order contribution. Power corrections from higher-twist LCDAs and quark masses are not included, but they are not very significant at large Q^2.

Taking advantage of the form factors computed with the PQCD approach, the total cross sections of $e^+e^-\to B_c^-B_c^{+(*)}$ are depicted in Fig. 3. The factorization scale μ is taken to be $\mu=Q$ for the central value and $\mu\sim[Q/2,2Q]$ for the error estimate. The solid black curves correspond to central values and the dashed blue and dot-dashed red lines represent the errors of the cross sections. Considering f_{B_c} only causes tiny errors because of its high accuracy, the cross sections at our kinematic

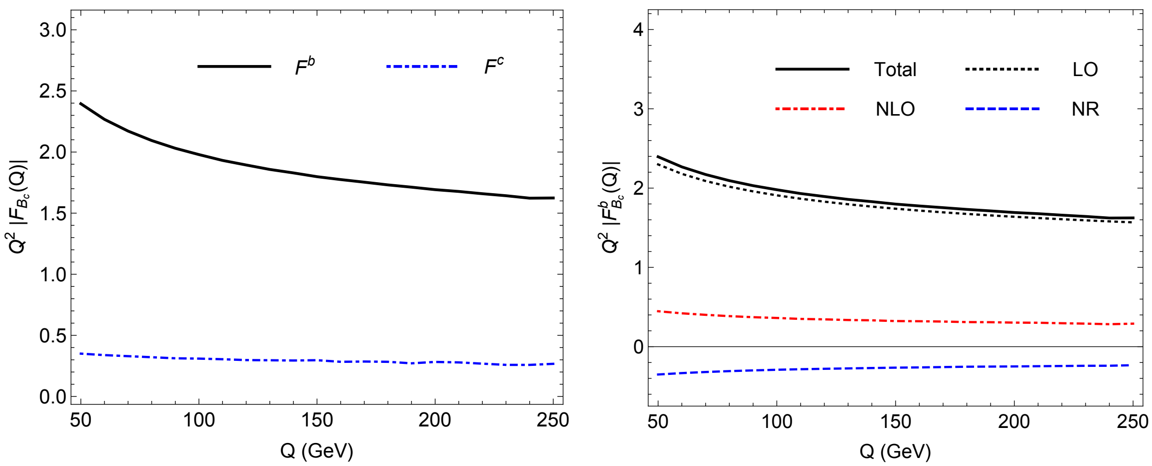

Fig. 2. (color online) Shape of $Q^2|F^c_{\mathrm{B_c}}|$ and $Q^2|F^b_{\mathrm{B_c}}|$. Left: the total results of F^b and F^c are presented by the solid black curve and the dot-dashed blue curve respectively. Right: the solid black, dotted black, dot-dashed red and dashed blue line are the total result, leading order results, NLO correction and relativistic correction of F^b, respectively.

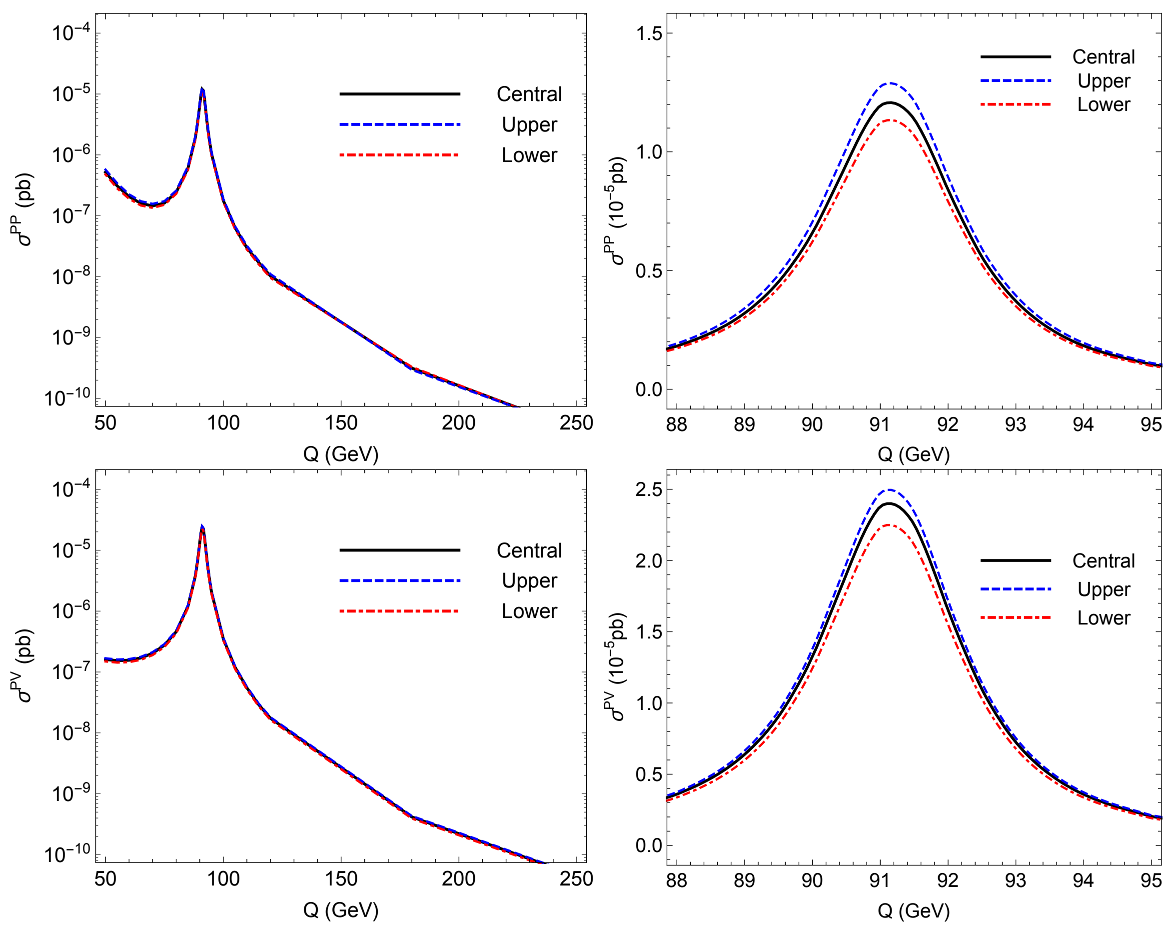

Fig. 3. (color online) Cross sections of $\mathrm{e^+e^-}\to\mathrm{B_c^-B_c^{+(*)}}$ with errors from variations of the factorization scale μ and $\mathrm{B_c}$-meson decay constant $f_{\mathrm{B_c}}$. The solid black, dashed blue and dot-dashed red curves correspond to central, upper and lower values of cross sections, respectively. The kinematic region of the left two diagrams is 50−250 GeV and that of the right two diagrams is around the Z pole.

region are not sensitive to the variation of factorization scale. As expected, the cross section has a significant peak at the Z pole ($Q=m_Z$) and the maximum values of the cross sections are $\sigma^{PP}(Q=m_Z)\sim 1.3\times10^{-5}$pb and $\sigma^{PV}(Q=m_Z)\sim 2.5\times10^{-5}$pb. They are so small that it is almost hopeless to detect a $B_c^-B_c^{+(*)}$ pair in an e^+e^- collider at the Z pole. This result is quite different from $B\bar{B}$ production, which is enhanced by a low energy resonance. For single B_c production in semi-inclusive processes, the cross section is much larger [23], and can be reached by a high luminosity e^+e^- collider.

4 Summary

B_c production processes are important because the B_c meson has unique properties and is worthy of a thorough study. Within the framework of the PQCD approach, we calculated the time-like form factors $F^q(Q^2)$ and $A_2^q(Q^2)$ at leading power in $1/Q$. The form factors are factorized into a convolution of the transverse momentum dependent wave function and the hard kernel. The wave functions employed in this work, including QCD corrections and relativistic corrections, have recently been studied using NRQCD. We evaluate the effects of relativistic corrections and QCD corrections of the form factors numerically. Both relativistic and QCD corrections can give about 20% correction to the LO contribution, but there are cancellation effects between them. We further worked out the cross sections of $e^+e^-\to B_c^-B_c^+(B_c^{*+})$ processes. The cross sections are enhanced at the Z pole, but they are still too small to be detected in future accelerators such as the CEPC.

We are grateful to Q. A. Zhang for useful discussions and comments.

Appendix A

Results after momentum fraction integration

Tree level

Leading power in v

$$F^b_{(LO)}(Q^2) = \mathrm{i}f_P P_c\int_0^\infty \mathrm{d}bb\exp[-S_{\mathrm{I}}(x_0,x_0,b,Q,\mu)]H_0^{(1)}(x_0Qb)\phi_T^2(b), \tag{A1}$$

$$F^c_{(LO)}(Q^2) = \mathrm{i}f_P P_c\int_0^\infty \mathrm{d}bb\exp[-S_{\mathrm{I}}(x_0,x_0,b,Q,\mu)]H_0^{(1)}(\bar{x}_0Qb)\phi_T^2(b), \tag{A2}$$

$$A^b_{2,(LO)}(Q^2) = -\frac{f_V^{\parallel}m_VP_c}{Q^2}\int_0^\infty \mathrm{d}bb\exp[-S_{\mathrm{I}}(x_0,x_0,b,Q,\mu)]H_0^{(1)}(x_0Qb)\phi_T^2(b), \tag{A3}$$

$$A^c_{2,(LO)}(Q^2) = \frac{f_V^{\parallel}m_VP_c}{Q^2}\int_0^\infty \mathrm{d}bb\exp[-S_{\mathrm{I}}(x_0,x_0,b,Q,\mu)]H_0^{(1)}(\bar{x}_0Qb)\phi_T^2(b), \tag{A4}$$

where $P_c=\pi^2f_PC_F\alpha_s(\mu)/N_c$, $\bar{x}_0=1-x_0$, $S_{\mathrm{I}}(x,y,b,Q,\mu)\equiv S(x,b,y,b,Q,\mu)$ and $H_n^{(1)}(x)=J_n(x)+iY_n(x)$. We use the LQCD decay constant [45] $f_B=0.489$ GeV. The Sudakov factor is

$$S(x,b_1,y,b_2,Q,\mu)=s(x,b_1,Q)+s(\bar{x},b_1,Q)+2se(\mu,b_1)+s(y,b_2,Q)+s(\bar{y},b_2,Q)+2se(\mu,b_2),$$

$$se(\mu,b)=-\frac{1}{2\beta_1}\ln\frac{\ln(\mu^2/\Lambda^2)}{\ln[1/(b^2\Lambda^2)]}, \tag{A5}$$

$$s(\xi,b)=\frac{A^{(1)}}{2\beta_1}\hat{q}\ln\left(\frac{\hat{q}}{\hat{b}}\right)-\frac{A^{(1)}}{2\beta_1}\left(\hat{q}-\hat{b}\right)+\frac{A^{(2)}}{4\beta_1^2}\left(\frac{\hat{q}}{\hat{b}}-1\right)-\frac{A^{(1)}\beta_2}{4\beta_1^3}\hat{q}\left[\frac{\ln(2\hat{b})+1}{\hat{b}}-\frac{\ln(2\hat{q})+1}{\hat{q}}\right]$$
$$-\left[\frac{A^{(2)}}{4\beta_1^2}-\frac{A^{(1)}}{4\beta_1}\ln\left(\frac{e^{2\gamma_E-1}}{2}\right)\right]\ln\left(\frac{\hat{q}}{\hat{b}}\right)+\frac{A^{(1)}\beta_2}{8\beta_1^3}\left[\ln^2(2\hat{q})-\ln^2(2\hat{b})\right], \tag{A6}$$

where

$$\hat{q}\equiv\ln\Big[\frac{\xi Q}{\sqrt{2}\Lambda_{\rm QCD}}\Big],\quad \hat{b}\equiv\ln\Big[\frac{1}{b\Lambda_{\rm QCD}}\Big], \tag{A7}$$

$$A^{(1)}=\frac{4}{3},\quad A^{(2)}=\frac{67}{9}-\frac{\pi^2}{3}-\frac{10}{27}n_f+\frac{8}{3}\beta_1\ln\Big(\frac{e^{\gamma_E}}{2}\Big). \tag{A8}$$

NRQCD corrections

The δ' term $-\frac{\langle\mathbf{q}^2\rangle}{(m_b+m_c)^2}\frac{2(1-2x_0)}{3x_0\bar{x}_0}\delta'(x-x_0)$

$$F^{b,NR,I}_{(LO)}(Q^2)=2{\rm i}f_P P_c\frac{\langle\boldsymbol{q}^2\rangle}{(m_b+m_c)^2}\frac{2(1-2x_0)}{3x_0\bar{x}_0}\int_0^\infty {\rm d}bb\exp[-S_I(x_0,x_0,b,Q,\mu)]$$
$$\times\Big\{\big[s_p(\bar{x}_0,b,Q)-s_p(x_0,b,Q)\big]H_0^{(1)}(x_0Qb)-\frac{H_1^{(1)}(x_0Qb)}{2}Qb\Big\}\phi_T^2(b), \tag{A9}$$

$$F^{c,NR,I}_{(LO)}(Q^2)=2{\rm i}f_P P_c\frac{\langle\boldsymbol{q}^2\rangle}{(m_b+m_c)^2}\frac{2(1-2x_0)}{3x_0\bar{x}_0}\int_0^\infty {\rm d}bb\exp[-S_I(x_0,x_0,b,Q,\mu)]$$
$$\times\Big\{\big[s_p(\bar{x}_0,b,Q)-s_p(x_0,b,Q)\big]H_0^{(1)}(\bar{x}_0Qb)+\frac{H_1^{(1)}(\bar{x}_0Qb)}{2}Qb\Big\}\phi_T^2(b), \tag{A10}$$

$$A^{b,NR,I}_{2,(LO)}(Q^2)={\rm i}\frac{f_V^{\parallel}m_V}{f_PQ^2}F^{b,NR,I}_{(LO)}(Q^2), \tag{A11}$$

$$A^{c,NR,I}_{2,(LO)}(Q^2)=-{\rm i}\frac{f_V^{\parallel}m_V}{f_PQ^2}F^{c,NR,I}_{(LO)}(Q^2). \tag{A12}$$

$$s_p(\xi,b,Q)=\frac{{\rm d}s(\xi,b,Q)}{{\rm d}\xi}=\frac{1}{4\xi\beta_1}\Big\{A^{(1)}\Big(2\ln\frac{\hat{q}}{\hat{b}}+\frac{1}{\hat{q}}\ln\frac{e^{2\gamma_E-1}}{2}\Big)+\frac{A^{(1)}\beta_2}{\beta_1^2}\Big[\frac{\ln(2\hat{q})+1}{\hat{q}}-\frac{\ln(2\hat{b})+1}{\hat{b}}\Big]+\frac{A^{(2)}}{\beta_1}\Big(\frac{1}{\hat{b}}-\frac{1}{\hat{q}}\Big)\Big\}, \tag{A13}$$

$$\frac{{\rm d}s(\bar{\xi},b,Q)}{{\rm d}\xi}=-s_p(\bar{\xi},b,Q), \tag{A14}$$

$$\frac{{\rm d}H_n^{(1)}(x)}{{\rm d}x}=\frac{nH_n^{(1)}(x)}{x}-H_{n+1}^{(1)}(x). \tag{A15}$$

The δ'' term $\frac{\langle\mathbf{q}^2\rangle}{6(m_b+m_c)^2}\delta''(x-x_0)$

$$F^{b,NR,II}_{(LO)}(Q^2)\ =\ 2{\rm i}f_PP_c\frac{\langle\boldsymbol{q}^2\rangle}{6(m_b+m_c)^2}\int_0^\infty{\rm d}bb\exp[-S_I(x_0,x_0,b,Q,\mu)]\Big\{[s_p(\bar{x}_0)-s_p(x_0)]$$
$$\times\Big\{[s_p(\bar{x}_0)-s_p(x_0)]H_0^{(1)}(x_0Qb)-\frac{Qb}{2}H_1^{(1)}(x_0Qb)\Big\}-[s_{pp}(\bar{x}_0)+s_{pp}(x_0)]H_0^{(1)}(x_0Qb)$$
$$-[s_p(\bar{x}_0)-s_p(x_0)]\frac{Qb}{2}H_1^{(1)}(x_0Qb)+\frac{Q^2b^2}{4}H_2^{(1)}(x_0Qb)\Big\}\phi_T^2(b), \tag{A16}$$

$$F^{c,NR,II}_{(LO)}(Q^2)\ =\ 2{\rm i}f_PP_c\frac{\langle\mathbf{q}^2\rangle}{6(m_b+m_c)^2}\int_0^\infty{\rm d}bb\exp[-S_I(x_0,x_0,b,Q,\mu)]$$
$$\times\Big\{[s_p(\bar{x}_0)-s_p(x_0)]\Big\{[s_p(\bar{x}_0)-s_p(x_0)]H_0^{(1)}(\bar{x}_0Qb)+\frac{Qb}{2}H_1^{(1)}(\bar{x}_0Qb)\Big\}$$
$$-[s_{pp}(\bar{x}_0)+s_{pp}(x_0)]H_0^{(1)}(\bar{x}_0Qb)+[s_p(\bar{x}_0)-s_p(x_0)]\frac{Qb}{2}H_1^{(1)}(\bar{x}_0Qb)+\frac{Q^2b^2}{4}H_2^{(1)}(\bar{x}_0Qb)\Big\}\phi_T^2(b), \tag{A17}$$

$$A^{b,NR,II}_{2,(LO)}(Q^2)=\mathrm{i}\frac{f_V^{\parallel}m_V}{f_PQ^2}F^{b,NR,II}_{(LO)}(Q^2), \tag{A18}$$

$$A^{c,NR,II}_{2,(LO)}(Q^2)=-\mathrm{i}\frac{f_V^{\parallel}m_V}{f_PQ^2}F^{c,NR,II}_{(LO)}(Q^2), \tag{A19}$$

$$s_{pp}(\xi,b,Q)=\frac{\mathrm{d}s_p(\xi,b,Q)}{\mathrm{d}\xi}=\frac{1}{4\xi^2\beta_1}\left\{\frac{A^{(1)}}{\hat{q}}\left[2-\frac{\ln(e^{2\gamma_E-1}/2)}{\hat{q}}\right]-\frac{A^{(1)}\beta_2}{\beta_1^2}\frac{\ln(2\hat{q})}{\hat{q}^2}+\frac{A^{(2)}}{\beta_1\hat{q}^2}\right.$$
$$\left.-\left[A^{(1)}\left(2\ln\frac{\hat{q}}{\hat{b}}+\frac{1}{\hat{q}}\ln\frac{e^{2\gamma_E-1}}{2}\right)+\frac{A^{(1)}\beta_2}{\beta_1^2}\left[\frac{\ln(2\hat{q})+1}{\hat{q}}-\frac{\ln(2\hat{b})+1}{\hat{b}}\right]+\frac{A^{(2)}}{\beta_1}\left(\frac{1}{\hat{b}}-\frac{1}{\hat{q}}\right)\right]\right\}. \tag{A20}$$

One-loop level

The NLO correction from the hard kernel is:

$$F^{b,I}_{(NLO)}(Q^2)=\mathrm{i}f_PP_c\frac{\alpha_s(\mu)C_F}{4\pi}\int_0^\infty\mathrm{d}bb\exp[-S_{\mathrm{I}}(x_0,x_0,b,Q,\mu)]\phi_T^2(b)\left[\widetilde{h}(x_0,x_0,b,Q,\mu)H_0^{(1)}(x_0Qb)+\frac{\ln^2(x_0Qb)}{3}H_0^{(1)}(x_0Qb)\right], \tag{A21}$$

$$F^{c,I}_{(NLO)}(Q^2)=\mathrm{i}f_PP_c\frac{\alpha_s(\mu)C_F}{4\pi}\int_0^\infty\mathrm{d}bb\exp[-S_{\mathrm{I}}(x_0,x_0,b,Q,\mu)]\phi_T^2(b)\left[\widetilde{h}(\bar{x}_0,\bar{x}_0,b,Q,\mu)H_0^{(1)}(\bar{x}_0Qb)+\frac{\ln^2(\bar{x}_0Qb)}{3}H_0^{(1)}(\bar{x}_0Qb)\right]. \tag{A22}$$

We add the one-loop corrections from the two wave functions together. We rewrite the wave function as:

$$\hat{\phi}_P^{(1,0)}(x)=\frac{\alpha_sC_F}{4\pi}[\hat{\phi}^I(x)+\hat{\phi}^{II}(x)], \tag{A23}$$

$$\hat{\phi}^I(x)=2\left[\left(\ln\frac{\mu^2}{(m_b+m_c)^2(x_0-x)^2}-1\right)\frac{x_0+\bar{x}}{x_0-x}\frac{x}{x_0}\theta(x_0-x)\right]_+$$
$$+2\left[\left(\ln\frac{\mu^2}{(m_b+m_c)^2(x_0-x)^2}-1\right)\frac{\bar{x}_0+x}{\bar{x}_0-\bar{x}}\frac{\bar{x}}{\bar{x}_0}\theta(x-x_0)\right]_++\left[\frac{4x\bar{x}}{(x_0-x)^2}\right]_{++}, \tag{A24}$$

$$\hat{\phi}^{II}(x)=\left[4x_0\bar{x}_0\ln\frac{x_0}{\bar{x}_0}+2(2x_0-1)\right]\delta'(x-x_0). \tag{A25}$$

The NLO corrections from $\hat{\phi}^I$ are:

$$F^{b,II}_{(NLO)}(Q^2) = 2\mathrm{i}f_PP_c\frac{\alpha_sC_F}{4\pi}\int_0^1\mathrm{d}x\int_0^\infty\mathrm{d}bb\Big\{\exp[-S_{\mathrm{I}}(x,x_0,b,Q,\mu)]H_0^{(1)}(\sqrt{xx_0}Qb)-\exp[-S_{\mathrm{I}}(x_0,x_0,b,Q,\mu)]H_0^{(1)}(x_0Qb)\Big\}$$
$$[1mm]\quad\times\Psi(x)\phi_T^2(x,b)-2\mathrm{i}f_PP_c\frac{\alpha_sC_F}{4\pi}\int_0^1\mathrm{d}x\int_0^\infty\mathrm{d}bb\frac{4x\bar{x}}{(x-x_0)}\exp[-S_I(x_0,x_0,b,Q,\mu)]$$
$$\times\left\{\left[s_p(\bar{x}_0,b,Q)-s_p(x_0,b,Q)\right]H_0^{(1)}(x_0Qb)-\frac{H_1^{(1)}(x_0Qb)}{2}Qb\right\}\phi_T^2(b), \tag{A26}$$

$$F^{c,II}_{(NLO)}(Q^2) = 2\mathrm{i}f_PP_c\frac{\alpha_sC_F}{4\pi}\int_0^1\mathrm{d}x\int_0^\infty\mathrm{d}bb\Big\{\exp[-S_{\mathrm{I}}(x,x_0,b,Q,\mu)]H_0^{(1)}(\sqrt{\bar{x}\bar{x}_0}Qb)-\exp[-S_{\mathrm{I}}(x_0,x_0,b,Q,\mu)]H_0^{(1)}(\bar{x}_0Qb)\Big\}$$
$$[1mm]\quad\times\Psi(x)\phi_T^2(x,b)-2\mathrm{i}f_PP_c\frac{\alpha_sC_F}{4\pi}\int_0^1\mathrm{d}x\int_0^\infty\mathrm{d}bb\frac{4x\bar{x}}{(x-x_0)}\exp[-S_I(x_0,x_0,b,Q,\mu)]$$
$$\times\left\{\left[s_p(\bar{x}_0,b,Q)-s_p(x_0,b,Q)\right]H_0^{(1)}(\bar{x}_0Qb)+\frac{H_1^{(1)}(\bar{x}_0Qb)}{2}Qb\right\}\phi_T^2(b), \tag{A27}$$

$$\Psi(x)=2\left(\ln\frac{\mu^2}{(m_b+m_c)^2(x_0-x)^2}-1\right)\frac{x_0+\bar{x}}{x_0-x}\frac{x}{x_0}\theta(x_0-x)+2\left(\ln\frac{\mu^2}{(m_b+m_c)^2(x_0-x)^2}-1\right)\frac{\bar{x}_0+x}{x-x_0}\frac{\bar{x}}{\bar{x}_0}\theta(x-x_0)+\frac{4x\bar{x}}{(x_0-x)^2}. \tag{A28}$$

The NLO corrections from $\hat{\phi}^{II}$ are:

$$F^{b,III}_{(NLO)}(Q^2) = \frac{\alpha_s C_F}{4\pi}\left[4x_0\bar{x}_0\ln\frac{x_0}{\bar{x}_0}+2(2x_0-1)\right]\Big/\left[-\frac{\langle \boldsymbol{q}^2\rangle}{(m_b+m_c)^2}\frac{2(1-2x_0)}{3x_0\bar{x}_0}\right]F^{b,NR,I}_{(LO)}(Q^2), \tag{A29}$$

$$F^{c,III}_{(NLO)}(Q^2) = \frac{\alpha_s C_F}{4\pi}\left[4x_0\bar{x}_0\ln\frac{x_0}{\bar{x}_0}+2(2x_0-1)\right]\Big/\left[-\frac{\langle \boldsymbol{q}^2\rangle}{(m_b+m_c)^2}\frac{2(1-2x_0)}{3x_0\bar{x}_0}\right]F^{c,NR,I}_{(LO)}(Q^2). \tag{A30}$$

The NLO corrections from the decay constant and Sudakov factor are:

$$\begin{aligned}F^{b,IV}_{(NLO)}(Q^2)=&\mathrm{i}(2f_P^{NLO})f_P P_c\int_0^\infty \mathrm{d}bb\exp[-S_{\mathrm{I}}(x_0,x_0,b,Q,\mu)]H_0^{(1)}(\bar{x}_0Qb)\phi_T^2(b)\\ &-\mathrm{i}f_P P_c\int_0^\infty \mathrm{d}bb\exp[-S_{\mathrm{I}}(x_0,x_0,b,Q,\mu)]S_{\mathrm{I}}^{NLO}(x_0,x_0,b,Q,\mu)H_0^{(1)}(x_0Qb)\phi_T^2(b),\end{aligned} \tag{A31}$$

$$\begin{aligned}F^{c,IV}_{(NLO)}(Q^2)=&\mathrm{i}(2f_P^{NLO})f_P P_c\int_0^\infty \mathrm{d}bb\exp[-S_{\mathrm{I}}(x_0,x_0,b,Q,\mu)]H_0^{(1)}(\bar{x}_0Qb)\phi_T^2(b)\\ &-\mathrm{i}f_P P_c\int_0^\infty \mathrm{d}bb\exp[-S_{\mathrm{I}}(x_0,x_0,b,Q,\mu)]S_{\mathrm{I}}^{NLO}(x_0,x_0,b,Q,\mu)H_0^{(1)}(\bar{x}_0Qb)\phi_T^2(b),\end{aligned} \tag{A32}$$

where $S_{\mathrm{I}}^{NLO}(x,y,b,Q,\mu)\equiv S^{NLO}(x,b,y,b,Q,\mu)$.

$$f_P^{NLO}=\frac{\alpha_s(m_{\mathrm{B_c}})C_F}{4\pi}\left(3\ln\frac{m_{\mathrm{B_c}}}{m_c}-4\right),$$

$$\begin{aligned}S^{NLO}(x,b_1,y,b_2,Q,\mu)=&s_{NLO}(x,b_1,Q)+s_{NLO}(\bar{x},b_1,Q)+2se_{NLO}(\mu,b_1)\\ &+s_{NLO}(y,b_2,Q)+s_{NLO}(\bar{y},b_2,Q)+2se_{NLO}(\mu,b_2),\end{aligned}$$

$$se_{NLO}(\mu,b)=\frac{\beta_2}{2\beta_1^3}\left\{\frac{\ln\ln[1/(b^2\Lambda^2)]+1}{\ln[1/(b^2\Lambda^2)]}-\frac{\ln\ln(\mu^2/\Lambda^2)+1}{\ln(\mu^2/\Lambda^2)}\right\}, \tag{A33}$$

$$\begin{aligned}s_{NLO}(\xi,b)=&\frac{A^{(1)}\beta_2}{8\beta_1^3}\ln\left(\frac{e^{2\gamma-1}}{2}\right)\left[\frac{\ln(2\hat{q})+1}{\hat{q}}-\frac{\ln(2\hat{b})+1}{\hat{b}}\right]-\frac{A^{(1)}\beta_2}{16\beta_1^4}\frac{\hat{q}-\hat{b}}{\hat{b}^2}\left[2\ln(2\hat{b})+1\right]\\ &-\frac{A^{(1)}\beta_2}{16\beta_1^4}\left[\frac{2\ln(2\hat{q})+3}{\hat{q}}-\frac{2\ln(2\hat{b})+3}{\hat{b}}\right]+\frac{A^{(2)}\beta_2^2}{432\beta_1^6}\frac{\hat{q}-\hat{b}}{\hat{b}^3}\left[9\ln^2(2\hat{b})+6\ln(2\hat{b})+2\right]\\ &+\frac{A^{(2)}\beta_2^2}{1728\beta_1^6}\left[\frac{18\ln^2(2\hat{q})+30\ln(2\hat{q})+19}{\hat{q}^2}-\frac{18\ln^2(2\hat{b})+30\ln(2\hat{b})+19}{\hat{b}^2}\right],\end{aligned} \tag{A34}$$

$$F^b(Q^2)=F^b_{(LO)}+F^{b,NR,I}_{(LO)}+F^{b,NR,II}_{(LO)}+F^{b,I}_{(NLO)}+F^{b,II}_{(NLO)}+F^{b,III}_{(NLO)}+F^{b,IV}_{(NLO)}, \tag{A35}$$

$$F^c(Q^2)=F^c_{(LO)}+F^{c,NR,I}_{(LO)}+F^{c,NR,II}_{(LO)}+F^{c,I}_{(NLO)}+F^{c,II}_{(NLO)}+F^{c,III}_{(NLO)}+F^{c,IV}_{(NLO)}, \tag{A36}$$

$$A_2^b(Q^2)=A^b_{2,(LO)}+A^{b,NR,I}_{2,(LO)}+A^{b,NR,II}_{2,(LO)}, \tag{A37}$$

$$A_2^c(Q^2)=A^c_{2,(LO)}+A^{c,NR,I}_{2,(LO)}+A^{c,NR,II}_{2,(LO)}. \tag{A38}$$

References

1 F. Abe et al (CDF Collaboration), Phys. Rev. Lett., **81**: 2432 (1998); Phys. Rev. D, **58**: 112004 (1998)

2 P. Abreu et al (DELPHI Collaboration), Phys. Letts. B, **398**: 207 (1997); R. Barate et al (ALEPH Collaboration), Phys. Letts. B, **402**: 213 (1997); K. Ackerstaff, et al (OPAL Collaboration), Phys. Letts. B, **420**: 157 (1998)

3 X. Liu, Z. J. Xiao, and C. D. Lü, Phys. Rev. D, **81**: 014022 (2010) doi: 10.1103/PhysRevD.81.014022 arXiv: 0912.1163 [hep-ph]

4 X. Liu and Z. J. Xiao, J. Phys. G, **38**: 035009 (2011) doi: 10.1088/0954-3899/38/3/035009 arXiv: 1003.3929 [hep-ph]

5 X. Liu and Z. J. Xiao, Phys. Rev. D, **81**: 074017 (2010) doi: 10.1103/PhysRevD.81.074017 arXiv: 1001.2944 [hep-ph]

6 X. Liu and Z. J. Xiao, Phys. Rev. D, **82**: 054029 (2010) doi: 10.1103/PhysRevD.82.054029 arXiv: 1008.5201 [hep-ph]

7 Z. J. Xiao and X. Liu, Phys. Rev. D, **84**: 074033 (2011) doi: 10.1103/PhysRevD.84.074033 arXiv: 1111.6679 [hep-ph]

8 Z. Rui, W. F. Wang, G. X. Wang, L. h. Song, and C. D. Lü, Eur. Phys. J. C, **75**(6): 293 (2015) doi: 10.1140/epjc/s10052-015-3528-0 arXiv: 1505.02498 [hep-ph]

9 W. F. Wang, X. Yu, C. D. Lü, and Z. J. Xiao, Phys. Rev. D, **90**(9): 094018 (2014) doi: 10.1103/PhysRevD.90.094018 arXiv: 1401.0391 [hep-ph]

10 X. Liu, R. H. Li, Z. T. Zou, and Z. J. Xiao, arXiv: 1703.05982 [hep-ph]

11 Z. J. Xiao and X. Liu, Chin. Sci. Bull., **59**: 3748 (2014) doi: 10.1007/s11434-014-0418-z arXiv: 1401.0151 [hep-ph]

12 N. Wang, Adv. High Energy Phys., **2016**: 6314675 (2016) doi: 10.1155/2016/6314675 arXiv: 1602.07027 [hep-ph]

13 Y. L. Yang, J. F. Sun, and N. Wang, Phys. Rev. D, **81**: 074012 (2010) doi: 10.1103/PhysRevD.81.074012 arXiv: 1004.2825 [hep-ph]

14 C. F. Qiao and R. L. Zhu, Phys. Rev. D, **87**(1): 014009 (2013) doi: 10.1103/PhysRevD.87.014009 arXiv: 1208.5916 [hep-ph]

15 C. F. Qiao, P. Sun, D. Yang, and R. L. Zhu, Phys. Rev. D, **89**(3): 034008 (2014) doi: 10.1103/PhysRevD.89.034008 arXiv: 1209.5859 [hep-ph]

16 J. M. Shen, X. G. Wu, H. H. Ma, and S. Q. Wang, Phys. Rev. D, **90**(3): 034025 (2014) doi: 10.1103/PhysRevD.90.034025 arXiv: 1407.7309 [hep-ph]

17 W. Wang and R. L. Zhu, Eur. Phys. J. C, **75**(8): 360 (2015) doi: 10.1140/epjc/s10052-015-3583-6 arXiv: 1501.04493 [hep-ph]

18 R. Zhu, Y. Ma, X. L. Han, and Z. J. Xiao, arXiv: 1703.03875 [hep-ph]

19 C.-H Chang and Y.-Q Chen, Phys. Rev. D, **46**: 3845 (1992); Phys. Rev. D, **50**: 6013 (1994); Phys. Letts. B, **284**: 127-132 (1992)

20 X. G. Wu, C. H. Chang, Y. Q. Chen, and Z. Y. Fang, Phys. Rev. D, **67**: 094001 (2003) doi: 10.1103/PhysRevD.67.094001 [hep-ph/0209125]

21 C. H. Chang, C. F. Qiao, J. X. Wang, and X. G. Wu, Phys. Rev. D, **71**: 074012 (2005) doi: 10.1103/PhysRevD.71.074012 [hep-ph/0502155]

22 C. H. Chang, J. X. Wang, and X. G. Wu, Phys. Rev. D, **77**: 014022 (2008) doi: 10.1103/PhysRevD.77.014022 arXiv: 0711.1898 [hep-ph]

23 X. C. Zheng, C. H. Chang, T. F. Feng, and Z. Pan, Sci. China Phys. Mech. Astron., **61**(3): 031012 (2018) doi: 10.1007/s11433-017-9121-3 arXiv: 1701.04561 [hep-ph]

24 G. T. Bodwin, E. Braaten, and G. P. Lepage, Phys. Rev. D, **51**: 1125 (1995); Phys. Rev. D, **55**: 5853 (1997) doi: 10.1103/PhysRevD.55.5853, 10.1103/PhysRevD.51.1125 [hep-ph/9407339]

25 N. Brambilla, A. Pineda, J. Soto, and A. Vairo, Rev. Mod. Phys., **77**: 1423 (2005) doi: 10.1103/RevModPhys.77.1423 [hep-ph/0410047]

26 N. Brambilla et al (Quarkonium Working Group), hep-ph/0412158

27 N. Brambilla et al, Eur. Phys. J. C, **71**: 1534 (2011) doi: 10.1140/epjc/s10052-010-1534-9 arXiv: 1010.5827 [hep-ph]

28 G. P. Lepage and S. J. Brodsky, Phys. Rev. D, **22**: 2157 (1980). doi: 10.1103/PhysRevD.22.2157

29 V. L. Chernyak and A. R. Zhitnitsky, Phys. Rept., **112**: 173 (1984) doi: 10.1016/0370-1573(84)90126-1

30 J. P. Ma and Z. G. Si, Phys. Lett. B, **647**: 419 (2007) doi: 10.1016/j.physletb.2007.02.040 [hep-ph/0608221]

31 G. Bell and T. Feldmann, JHEP, **0804**: 061 (2008) doi: 10.1088/1126-6708/2008/04/061 arXiv: 0802.2221 [hep-ph]

32 X. P. Wang and D. Yang, JHEP, **1406**: 121 (2014) doi: 10.1007/JHEP06(2014)121 arXiv: 1401.0122 [hep-ph]

33 J. Xu and D. Yang, JHEP, **1607**: 098 (2016) doi: 10.1007/JHEP07(2016)098 arXiv: 1604.04441 [hep-ph]

34 W. Wang, J. Xu, D. Yang, and S. Zhao, JHEP, **1712**: 012 (2017) doi: 10.1007/JHEP12(2017)012 arXiv: 1706.06241 [hep-ph]

35 H. n. Li and G. F. Sterman, Nucl. Phys. B, **381**: 129 (1992) doi: 10.1016/0550-3213(92)90643-P

36 H. n. Li and H. L. Yu, Phys. Rev. Lett., **74**: 4388 (1995) doi: 10.1103/PhysRevLett.74.4388 [hep-ph/9409313]

37 Y. Y. Keum, H. n. Li, and A. I. Sanda, Phys. Lett. B, **504**: 6 (2001) doi: 10.1016/S0370-2693(01)00247-7 [hep-ph/0004004]

38 Y. Y. Keum, H. N. Li, and A. I. Sanda, Phys. Rev. D, **63**: 054008 (2001) doi: 10.1103/PhysRevD.63.054008 [hep-ph/0004173]

39 C. D. Lü: K. Ukai, and M. Z. Yang, Phys. Rev. D, **63**: 074009 (2001) doi: 10.1103/PhysRevD.63.074009 [hep-ph/0004213]

40 C. D. Lü: W. Wang, and Y. M. Wang, Phys. Rev. D, **75**: 094020 (2007) doi: 10.1103/PhysRevD.75.094020 [hep-ph/0702085]

41 J. W. Chen, H. Kohyama, K. Ohnishi, U. Raha, and Y. L. Shen, Phys. Lett. B, **693**: 102 (2010) doi: 10.1016/j.physletb.2010.05.082 arXiv: 0908.2973 [hep-ph]

42 H. C. Hu and H. n. Li, Phys. Lett. B, **718**: 1351 (2013) doi: 10.1016/j.physletb.2012.12.006 arXiv: 1204.6708 [hep-ph]

43 C. D. Lü: W. Wang, Y. Xing, and Q. A. Zhang, arXiv: 1802.09718 [hep-ph]

44 T. Kurimoto, Phys. Rev. D, **74**: 014027 (2006) doi: 10.1103/PhysRevD.74.014027 [hep-ph/0605112]

45 T. W. Chiu et al (TWQCD Collaboration), Phys. Lett. B, **651**: 171 (2007) doi: 10.1016/j.physletb.2007.06.017 arXiv: 0705.2797 [hep-lat]