Supersymmetric AdS_4 vacua in $\mathcal{N}=3$ gauged supergravity

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Abstract: We study the maximally supersymmetric AdS backgrounds of matter-coupled $\mathcal{N}=3$ gauged supergravity in four dimensions. We find that to admit supersymmetric AdS vacua, the gauge group can only be of the form $G_0\times H\subset SO(3,n)$ with $G_0=SO(3),SO(3,1)$ or $SL(3,\mathbb{R})$ and H a compact group of dimension $n+3-\dim(G_0)$. We also show that these AdS vacua have no moduli, namely they correspond to critical points in field space.

Keywords: gauge supergravity, AdS background, moduli

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1 Introduction

The AdS/CFT correspondence has been extensively studied since its original proposal. Among the various specific correspondences, the AdS_4/CFT_3 correspondence is especially interesting and important in many aspects. For example, it gives fruitful implications for the dynamics of the worldvolume theory of M2-branes [1, 2], and for condensed matter physics systems [3, 4].

Apart from the $\mathcal{N}=1,2,4,8$ cases, the AdS_4 background with $\mathcal{N}=3$ supersymmetry generally arises from a compactification of eleven-dimensional supergravity on a tri-Sasakian manifold, for example, N^{010} [5]. Its Kaluza-Klein spectrum and the properties of dual superconformal field theories (SCFT) have been studied in many articles. Also, this $AdS_4\times N^{010}$ compactification can be described by an effective four-dimensional $\mathcal{N}=3$ gauged supergravity [6, 7] which is coupled to 8 vector multiplets and has a $SO(3)\times SU(3)$ gauge group. In the AdS/CFT viewpoint, the $SO(3)$ factor corresponds to the $SO(3)$ R-symmetry, and the $SU(3)$ factor to the flavor symmetry of the dual $\mathcal{N}=3$ SCFT in three dimensions with $OSp(3|4)\times SU(3)$. Furthermore, some supersymmetric deformations and generalizations have been researched in these years (see Ref. [8] and references therein).

Besides the above compact gauge group, one can still consider other types of gauge groups. In this paper, we will consider $\mathcal{N}=3$ gauged supergravity theories with arbitrary n matter multiplets and study their AdS vacua and moduli spaces, which encode useful information for the corresponding SCFTs. We will find that, though the $\mathcal{N}=3$ supergravities can have various compact and noncompact gauge groups, to allow the maximally supersymmetric AdS vacua to exist, the constraints will reduce the group to only three forms. Furthermore, in the AdS vacua all the allowed gauge groups break spontaneously to their maximal compact subgroups. Meanwhile, the flat directions of the potential become Goldstone bosons of the symmetry breaking, which suggests that no moduli of the field space can exist.

This paper is organized in the following way. In Section 2, we briefly review the basic properties of matter-coupled four-dimensional $\mathcal{N}=3$ gauged supergravity that will be needed in the following analysis. In Section 3, we analyze the conditions for the existence of maximally supersymmetric $\mathcal{N}=3$ AdS_4 backgrounds and determine the allowed structure of the full gauge groups. In Section 4, we show that the supersymmetric AdS vacua have no moduli, namely they correspond to critical points in field space. The conclusion is given in Section 5.

2 $\mathcal{N}=3$ gauged supergravities

We begin with a brief review of the general structure of matter-coupled $\mathcal{N}=3$ gauged supergravity in $d=4$ spacetime dimensions [9]. We will take most of the conventions in Refs. [9, 10] with the metric signature $(-,+,+,+)$.

The generic spectrum consists of the supergravity multiplet and n vector multiplets. The supergravity multiplet contains the graviton $g_{\mu\nu}$, three gravitini $\psi_{\mu A}$, three vector fields $A_{\mu A}$, and one spin-1/2 fermion χ. The indices $A,B,\cdots=1,2,3$ denote the $SU(3)_{\rm R}$ R-symmetry triplets. The n vector multiplets will be labeled with the indices $i,j=1,\ldots,n$, and each multiplet contains a vector A^i_μ, four spin-1/2 gaugini λ^i_A,λ^i, which form a triplet and a singlet of $SU(3)_{\rm R}$, and three complex scalars z^i_A. The

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chirality condition of all fermionic fields are

$$\gamma_5\psi_{\mu A}=\psi_{\mu A},\quad \gamma_5\psi^A_\mu=-\psi^A_\mu,\quad \gamma_5\chi=\chi,$$
$$\gamma_5\lambda_A=\lambda_A,\quad \gamma_5\lambda^A=-\lambda^A,\quad \gamma_5\lambda=-\lambda. \tag{1}$$

The $3n$ complex scalars or $6n$ real scalars z^i_A of the vector multiplets form the coset manifold

$$\mathcal{M}=\frac{SU(3,n)}{SU(3)\times SU(n)\times U(1)}. \tag{2}$$

This coset can be parameterized by the coset representative $L_\Lambda{}^\Sigma$ with the indices $\Lambda,\Sigma,\cdots=(A,i),\cdots=1,\ldots,n+3$, which can be further split as $L_\Lambda{}^\Sigma=(L_\Lambda{}^A,L_\Lambda{}^i)$. $L_\Lambda{}^\Sigma$ is an element of $SU(3,n)$, and therefore suggests the metrics of the canonical $SU(3,n)$ and the manifold $\mathcal{M}$ respectively

$$\eta_{\Lambda\Sigma} = L^{*}_\Lambda{}^A L_\Sigma{}^A - L^{*}_\Lambda{}^i L_\Sigma{}^i, \tag{3}$$
$$\mathcal{M}_{\Lambda\Sigma} = L^{*}_\Lambda{}^A L_\Sigma{}^A + L^{*}_\Lambda{}^i L_\Sigma{}^i, \tag{4}$$

where $\eta_{\Lambda\Sigma}=(\delta_{AB},-\delta_{ij})$.

The three vector fields A_A from the gravity multiplet and the n vector fields A^i from the vector multiplets can be collectively denoted as $A_\Lambda=(A_A,A_i)$. As argued in Ref. [10], accompanied with their magnetic dual, these $n+3$ electric vector fields generally transform in the fundamental representation $(\boldsymbol{n}+3)_{\mathbb{C}}$ of $SU(3,n)$. But the Lagrangian containing only electric fields is invariant only under the subgroup $SO(3,n)$ of the global $SU(3,n)$. Then all possible gauge groups should be subgroups of $SO(3,n)$, and this leads to the result that the electric gauge fields transform only among themselves. Therefore a real fundamental representation $(3+\boldsymbol{n})_{\mathbb{R}}$ of $SO(3,n)$ will become the representation in which the adjoint representation of the gauge group is embedded.

To gauge a particular subgroup $G\subset SO(3,n)\subset SU(3,n)$, the structure constants $f_{\Lambda\Sigma}{}^\Gamma$ which appear in the gauge algebra,

$$[T_\Lambda,T_\Sigma]=f_{\Lambda\Sigma}{}^\Gamma T_\Gamma, \tag{5}$$

where T_Λ are gauge generators, should satisfy two conditions,

$$f_{\Lambda\Sigma\Gamma}=f_{\Lambda\Sigma}{}^\Pi\eta_{\Pi\Gamma}=f_{[\Lambda\Sigma\Gamma]}, \tag{6}$$
$$f_{[\Lambda\Sigma}{}^\Delta f_{\Gamma]\Delta}{}^\Pi=0, \tag{7}$$

which are the well-known linear and quadratic constraints, respectively.

The bosonic Lagrangian, which only contains nonvanishing metric and scalars, is

$$e^{-1}\mathcal{L}=\frac{1}{4}R-\frac{1}{2}P^{Ai}_\mu P^\mu_{Ai}-V, \tag{8}$$

where P^{Ai}_μ is the veilbein of the coset manifold (2) and is given by the $(A,i)-$components of the Maurer-Cartan form

$$\Omega_\Lambda{}^\Gamma=L^{-1}{}_\Lambda{}^\Sigma\left(dL_\Sigma{}^\Gamma+f_\Sigma{}^{\Delta\Pi}A_\Delta L_\Pi{}^\Gamma\right). \tag{9}$$

The scalar potential is written as

$$V=-2S_{AB}S^{AB}+\frac{2}{3}\mathcal{U}_A\mathcal{U}^A+\frac{1}{6}\mathcal{N}_{iA}\mathcal{N}^{iA}+\frac{1}{6}\mathcal{M}^{iB}{}_A\mathcal{M}_{iB}{}^A, \tag{10}$$

where after dressing the structure constants,

$$C^\Lambda{}_{\Sigma\Gamma}=L_\Pi{}^\Lambda L^{-1}{}_\Sigma{}^\Delta L^{-1}{}_\Gamma{}^\Xi f_{\Delta\Xi}{}^\Pi, \tag{11}$$

the fermion-shift matrices are defined by:

$$\begin{aligned} S_{AB} &= \frac{1}{4}\left(\epsilon_{BCD}C_A{}^{CD}+\epsilon_{ABC}C_D{}^{DC}\right)\\ \mathcal{U}^A &= -\frac{1}{4}C_B{}^{BA},\qquad \mathcal{N}_{iA}=-\frac{1}{2}\epsilon_{ABC}C_i{}^{BC},\\ \mathcal{M}_{iA}{}^B &= \frac{1}{2}\left(\delta^B_A C_{iD}{}^D-2C_{iA}{}^B\right). \end{aligned} \tag{12}$$

The potential can also be expressed as

$$V=\frac{1}{8}|C_{iA}{}^B|^2+\frac{1}{8}|C_i{}^{AB}|^2-\frac{1}{4}\left(|C_A{}^{BD}|^2-|C_A|^2\right), \tag{13}$$

where $C_A=-C_{AB}{}^B$.

We still need the supersymmetric transformations of the fermions, which will take important roles for finding supersymmetric solutions in the following section:

$$\delta\psi_{\mu A} = D_\mu\epsilon_A+S_{AB}\gamma_\mu\epsilon^B, \tag{14}$$
$$\delta\chi = \mathcal{U}^A\epsilon_A, \tag{15}$$
$$\delta\lambda_i = -P^A_{i\mu}\gamma^\mu\epsilon_A+\mathcal{N}_{iA}\epsilon^A, \tag{16}$$
$$\delta\lambda_{iA} = -P^B_{i\mu}\gamma^\mu\epsilon_{ABC}\epsilon^C+\mathcal{M}_{iA}{}^B\epsilon_B. \tag{17}$$

The covariant derivative of the parameter ϵ_A is defined as

$$D_\mu\epsilon_A=\partial_\mu\epsilon_A+\frac{1}{4}\omega^{ab}_\mu\gamma_{ab}\epsilon_A+Q_{\mu A}{}^B\epsilon_B+\frac{1}{2}nQ_\mu\epsilon_A, \tag{18}$$

where $Q_A{}^B$ and Q are the $SU(3)\times U(1)$ composite connections.

3 Maximally supersymmetric AdS backgrounds

Now we follow the strategy of Refs. [11–13] and study the conditions for the theory to admit maximally supersymmetric AdS vacua. For the supersymmetry to be unbroken, we need that in the background all the variations of the fermion fields (14)-(17) vanish,

$$\langle\delta\psi_{\mu A}\rangle=\langle\delta\chi\rangle=\langle\delta\lambda_i\rangle=\langle\delta\lambda_{iA}\rangle=0, \tag{19}$$

where $\langle\cdot\rangle$ suggests that we calculate the quantity in the AdS vacua. As γ^μ and the identity matrix are linearly independent from each other, the vanishing of $\langle\delta\lambda_i\rangle$ infers that

$$\mathcal{N}_{iA}=-\frac{1}{2}\epsilon_{ABC}C_i{}^{BC}=0, \tag{20}$$

in which Eq. (12) is used. Then we can get

$$C_{iAB}=0. \tag{21}$$

This is also the condition we can get from $\mathcal{M}_{iA}{}^{B}=0$ by $\langle\delta\lambda_{iA}\rangle=0$. The condition $\langle\delta\chi\rangle=0$ also suggests

$$\mathcal{U}^{A}=-\frac{1}{4}C_{B}{}^{BA}=0. \tag{22}$$

With these results, we conclude that the background value of the scalar potential V (13), which is equal to the cosmological constant, is reduced to

$$\Lambda=\langle V\rangle=-\frac{1}{4}|C_{ABC}|^{2}. \tag{23}$$

We see that as long as C_{ABC} is nontrivial the background is indeed AdS. Actually, as we will show below, these structure constants always form a group $SO(3)$ and take the form $C_{ABC}=g\epsilon_{ABC}$ where g is the coupling constant. Then the maximally supersymmetric AdS vacua infer that

$$\Lambda=\langle V\rangle=-\frac{3}{2}g^{2}. \tag{24}$$

For further analysis, we still need to specify the following structure constants:

$$C_{ABC},\quad C_{Aij},\quad C_{ijk}. \tag{25}$$

Different choices of these structure constants will lead to different gauged supergravities that admit the above maximally supersymmetric AdS_4 vacua. Now we will use the linear and quadratic constants (6), (7) to find the various possible gauge groups.

Firstly, for the most simple case, we can let all $C_{Aij}=C_{ijk}=0$, and only $C_{ABC}\neq 0$. Then Eq. (7) becomes the usual Jacobi identity and Eq. (6) shows that these non-vanishing structure constants should take the form $C_{ABC}=g\epsilon_{ABC}$, which form an $SO(3)$ algebra. Actually this corresponds to the pure gauged supergravity with no vector multiplets.

For the case $C_{Aij}=0$ but $C_{ABC}\neq 0$, $C_{ijk}\neq 0$, Eq. (7) becomes two separate Jacobi identities for C_{ABC} and C_{ijk}, which forms two commuting compact groups. The gauge group is therefore

$$G=SO(3)\times H\subset SO(3,n), \tag{26}$$

where $SO(3)$ is related to the unbroken R-symmetry of the theory, and H is a compact subgroup of $SO(n)$. As G is compact, clearly it satisfies Eq. (6).

Now we analyse the most general case where all C_{ABC},C_{Aij},C_{ijk} in Eq. (25) can take non-vanishing values. For this case, we can choose a specific vector multiplet basis so as to split C_{ijk} into two disjoint subsets $C_{\tilde{\imath}\tilde{\jmath}\tilde{k}}$ and $C_{\hat{\imath}\hat{\jmath}\hat{k}}$, where the former subset has no common indices with C_{Aij}. Then from Eq. (7) these $C_{\tilde{\imath}\tilde{\jmath}\tilde{k}}$ satisfy the usual Jacobi identity again, and correspond to a group $H\subset SO(q)$, $q\leq n$. The remaining C_{ABC},$C_{A\hat{\imath}\hat{\jmath}}$ and $C_{\hat{\imath}\hat{\jmath}\hat{k}}$ correspond to a non-compact group $G_0\subset SO(3,m)$, $m=n-q$. Clearly $SO(3)$ is a subgroup of G_0. Then we get the total gauge group

$$G=G_0\times H\subset SO(3,n). \tag{27}$$

Generally, as with other gauge supergravities, we can further suppose that the gauge group G should be semisimple. The Cartan-Killing form of G_0 must be embeddable in the $SO(3,n)$ canonical metric $\eta_{\Lambda\Sigma}$ (3), and this says that G_0 can have at most three compact or three non-compact generators. Still considering G_0 should contain $SO(3)$ as a subgroup, the only possibilities are

$$G_0=SO(3),\quad SO(3,1),\quad SL(3,\mathbb{R}). \tag{28}$$

This is in agreement with the result in Ref. [10]. It is worth noting that the compact group H takes no role in the AdS vacua, consistent with the fact that all scalars in the coset manifold are H singlets.

For the $\mathcal{N}=3$ gauge supergravity, besides the above forms of G_0, this group factor can still take the following forms

$$G_0=SO(2,2),\quad SO(2,1),\quad SO(2,1\times SO(2,2). \tag{29}$$

These $G_0{\times}H$ satisfy the constraints (6) and (7) and so are admissible gauge groups. But as analyzed in Ref. [10], all these gauge supergravity theories will either give no AdS vacua or suggest that the AdS vacua are neither supersymmetric nor stable.

4 Moduli spaces of AdS background

We have determined the $\mathcal{N}=3$ maximally supersymmetric AdS_4 backgrounds, and now we begin to study the moduli spaces of these backgrounds. The moduli are the flat directions of the manifold $\mathcal{M}$ (2) that are not specified by the conditions (19). To do so, we vary the supersymmetry conditions

$$\delta\mathcal{N}_{iA}=\delta\mathcal{M}_{iA}{}^{B}=\delta\mathcal{U}^{A}=0, \tag{30}$$

and look for their continuous solutions in the vicinity of the supersymmetric background.

As stated below (4) in Section 2, though we originally have $3n$ complex scalars z^{i}_{A}, when considering the gauge groups which transform the vector fields among themselves but not with their magnetic dual, the global $SU(3,n)$ is restricted to $SO(3,n)$. At the same time, its complex, fundamental representation $(3+\boldsymbol{n})_{\mathbb{C}}$ can also split into two real, fundamental representation $(3+\boldsymbol{n})_{\mathbb{R}}$ of $SO(3,n)$. In the gauged $(3+\boldsymbol{n})_{\mathbb{R}}$, the corresponding scalar fields are real and we denote them as ϕ^{i}_{A}.

Then we parameterize the variations of the coset representative,

$$\delta L_{\Lambda}{}^{A}=\langle L_{\Lambda}{}^{i}\rangle\delta\phi^{i}_{A},\quad \delta L_{\Lambda}{}^{i}=\langle L_{\Lambda}{}^{A}\rangle\delta\phi^{i}_{A}, \tag{31}$$

and the variations of their inverse,

$$\delta L^{-1}{}_{A}{}^{\Lambda}=\langle L^{-1}{}_{i}{}^{\Lambda}\rangle\delta\phi^{i}_{A},\quad \delta L^{-1}{}_{i}{}^{\Lambda}=\langle L^{-1}{}_{A}{}^{\Lambda}\rangle\delta\phi^{i}_{A}. \tag{32}$$

Then as discussed in Ref. [11], the only nontrivial condition of Eq. (30) is

$$\delta C_{ABi}=-C_{ABC}\delta\phi^i_C+2C_{ij[A}\delta\phi^j_{B]}, \tag{33}$$

and all solutions of this equation take the form

$$\delta\phi_{Ai}=C_{Aij}\lambda^j. \tag{34}$$

where λ^j are some real parameters. In these equations (also in the following) we suppress the bracket $\langle\cdot\rangle$ to simplify the notation. As we have split the index i into $\tilde{\imath}$ and $\hat{\imath}$, with $C_{A\tilde{\imath}\tilde{\jmath}}=0$, Eq. (34) actually is

$$\delta\phi_{A\hat{\imath}}=C_{A\hat{\imath}\hat{\jmath}}\lambda^{\hat{\jmath}}, \quad \delta\phi_{A\tilde{\imath}}=0. \tag{35}$$

For all the nontrivial scalars $\delta\phi_{A\hat{\imath}}$, we note that the $(A,\hat{\imath})$-components of the gauged Maurer-Cartan form (9) are

$$P_{A\hat{\imath}}=L^{-1\Lambda}_{A}dL_{\Lambda\hat{\imath}}+C_{A\hat{\imath}\hat{\jmath}}A^{\hat{\jmath}}. \tag{36}$$

This form appears quadratically in the bosonic Lagrangian (8) and therefore suggests a mass term for every $A^{\hat{\imath}}$ in the preimage of $C_{A\hat{\imath}\hat{\jmath}}$. As there is precisely one massive vector $A^{\hat{\imath}}$ for every $\lambda^{\hat{\imath}}$, we conclude that all non-trivial scalars (35) are Goldstone bosons eaten by massive vectors. Therefore there are no physical flat directions left and the moduli spaces only consist of critical points in the field space. For a further step, as $C_{A\hat{\imath}\hat{\jmath}}$ correspond to non-compact generators of G_0, the massive vectors also break the gauge group to its maximally compact subgroup

$$G=G_0\times H\rightarrow SO(3)\times H. \tag{37}$$

5 Conclusion

In this paper, we have studied the AdS backgrounds and the moduli spaces of the matter-coupled four-dimensional $\mathcal{N}=3$ gauged supergravity theories. We have shown that though these gauge supergravities can have various compact and noncompact gauge groups, to admit the maximally supersymmetric stable AdS vacua, the constraints on the structure constants will reduce the group to only three forms, namely $G_0{\times}H\subset SO(3,n)$ with $G_0=SO(3),SO(3,1),SL(3,\mathbb{R})$ and H a compact group of dimension $n{+}3{-}dim(G_0)$. We have further shown that in the maximally supersymmetric AdS vacua all the allowed gauge groups break spontaneously to their maximal compact subgroups, which suggests that the flat directions of the potential becomes Goldstone bosons and are eaten by the massive vector fields. Then no moduli of the field space can exist.

Finally we note that all gaugings studied here involve only electric vector fields, namely they are of electric type. But similar to Ref. [14], we expect searches for more general gaugings in which the dual magnetic fields are also involved and some other possible gauge groups may arise.

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