Mixing of the lowest-lying qqq configurations with $J^P=1/2^-$ in different hyperfine interaction models*

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Abstract: We investigate mixing of the lowest-lying qqq configurations with $J^P=1/2^-$ caused by the hyperfine interactions between quarks mediated by Goldstone Boson Exchange, One Gluon Exchange, and both Goldstone Boson and One Gluon exchange, respectively. The first orbitally excited nucleon, Σ, Λ and Ξ states are considered. Contributions of both the contact term and tensor term are taken into account. Our numerical results show that mixing of the studied configurations in the two employed hyperfine interaction models are very different. Therefore, the present results, which should affect the strong and electromagnetic decays of baryon resonances, may be used to examine the present employed hyperfine interaction models.

Keywords: quark model, pentaquark states, baryon resonances

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1 Introduction

The traditional constituent quark model, within which the spectrum of the low-lying baryon resonances can be reproduced very well, is one of the most successful phenomenological models in hadronic physics. However, the mechanism of quark-quark interaction in the baryons is still a controversial subject. There are two widely accepted hyperfine interaction models, namely the Isgur-Kark model [1–7] and the Glozman-Riska model [8–11]. In the former, the hyperfine interaction between quarks is suggested to be mediated by One Gluon Exchange (OGE), but it is proposed to be mediated by Goldstone Boson Exchange (GBE) in the latter.

At the beginning of this century, a critique of the GBE model was raised by Isgur: four defects of the pion exchange model for interquark forces were described in Ref. [12]. A rebuttal was written by Glozman later [13], arguing that all statements in Ref. [12] were shown to lack a basis. Consequently, it is not in fact possible for us to judge which model is more reasonable based on the previous theoretical and experimental investigations. Therefore, to investigate the properties of baryon resonances, one often employs both the OGE and GBE quark-quark interaction models, and compares results in these two models [14], or chooses one of the models which may be appropriate for the corresponding study [15, 16]. In addition, there are also some interesting works on meson-baryon bound states [17–19], the d^* resonance [20, 21], and bottomonium-like resonances Z_b [22], in which both the OGE and GBE contributions are taken into account.

It has been argued that the $N\eta$ photoproduction data should support the mixing angle of the configurations $|N_8^2P_M 1/2^-\rangle$ and $|N_8^4P_M 1/2^-\rangle$ obtained by the OGE model [23], and the same conclusion was drawn by studying the amplitudes of the first orbitally excited nucleon resonances for photoproduction and electroproduction [24]. However, in Ref. [25], based on investigations of the photoproduction of nucleon resonances, it was concluded that the GBE model is the more reasonable one, if the η', ω_0 and σ exchanges are included. On the other hand, it was recently suggested that the OGE model should reproduce the experimental data for strong decays of $S_{11}(1535)$ and $S_{11}(1650)$ [26].

In any case, it is possible for us to examine the OGE and GBE models by investigations of the configuration mixing in these two models, and applying the obtained wave functions to the electromagnetic and strong decays of the baryon resonances, or the meson-baryon photoproduction [27], since the experimental data is now abundant [28].

Accordingly, in the present work, we investigate the mixing of the first orbitally excited states with $J^P=1/2^-$ in the OGE and GBE models. The nucleon, Σ, Λ and Ξ configurations are considered, respectively. In both these

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models, the $SU(3)$ symmetry breaking effects are taken into account. In the GBE model, all the pseudoscalar meson exchanges are included. Note that the configuration mixing caused by the OGE interaction has been studied in Isgur-Karl's work [3], and mixing of the nucleon excitations caused by both OGE and GBE interactions has been studied by He and Dong in Refs. [24, 25]. Consequently, here we compare our results to the predictions in these references. As shown in Section 3, our results are consistent with these. Mixing of the Σ, Λ and Ξ excitations has not been studied before, and the numerical results obtained in the three hyperfine interaction models are very different.

This manuscript is organized as follows. The main ingredients of the constituent quark model with the Hamiltonian for a three-quark system, and the quark-quark interaction being mediated by OGE and GBE are addressed briefly in Section 2. Mixing of the studied baryon configurations are also deduced in this section. In Section 3, we show the present numerical results. A summary and conclusions are presented in Section 4.

2 The OGE and GBE models

Both the OGE and GBE models have been extensively studied in the literature. Here we just briefly address the key ingredients of these models in the following subsections. In Section 2.1, we show the theoretical frameworks for quark-quark interactions mediated by OGE and GBE, respectively. The wave functions for the studied baryon configurations are presented in Section 2.2, and the configuration mixing caused by OGE and GBE interactions are deduced in Section 2.3.

2.1 Quark-quark hyperfine interaction models

In the OGE model, the quark-quark hyperfine interaction takes a form similar to a magnetic dipole-dipole interaction in electrodynamics, as follows [7]:

$$H_{\rm hyp}^{\rm OGE}=\sum_{i<j}\frac{2\alpha_s}{3m_i m_j}\left[\frac{8\pi}{3}\vec{s}_i\cdot\vec{s}_j\delta^3(\vec{r}_{ij})+\frac{1}{r_{ij}^3}\hat{S}_{ij}\right], \quad (1)$$

where $\vec{s}_n$ and m_n are the spin and mass operators acting on the nth quark, and $\hat{S}_{ij}$ is the tensor operator:

$$\hat{S}_{ij}=3\vec{s}_i\cdot\hat{r}_{ij}\vec{s}_j\cdot\hat{r}_{ij}-\vec{s}_i\cdot\vec{s}_j. \quad (2)$$

Apart from the overall constant, Eq. (1) is just the common magnetic-dipole-magnetic-dipole component of the Breit Hamiltonian, which was first applied to the quark-quark interaction mediated by one gluon exchange in Ref. [29], and α_s represents the corresponding quark-gluon fine-structure constant. As we can see in Eq. (1), the contact interaction and the tensor term are included in the quark-quark interaction. The former only contributes to the interaction of quark pairs with relative angular momentum $L_{ij}=0$, while the latter only contributes to the interaction of $L_{ij}\neq 0$ quark pairs.

In the GBE model, the quark-quark interaction is assumed to be mediated by Goldstone bosons but not one gluon, and the corresponding Hamiltonian is of the following form [8]:

$$H_{\rm hyp}^{\rm GBE}=\sum_{i<j}\vec{\lambda}_i^F\cdot\vec{\lambda}_j^F[V_C^M(\vec{r}_{ij})\vec{s}_i\cdot\vec{s}_j+V_T^M(\vec{r}_{ij})\hat{S}_{ij}], \quad (3)$$

where $\vec{\lambda}_n^a$ denotes the ath Gell-Mann matrix in the flavor $SU(3)$ space acting on the nth quark, and the corresponding meson exchange potentials $V_{C(T)}^M(\vec{r}_{ij})$ can be written in the form of the Yukawa interactions between constituent quarks:

$$V_C^M(\vec{r}_{ij})=\frac{1}{4}\frac{g^2}{4\pi}\frac{1}{12m_i m_j}\left[\mu^2\frac{{\rm e}^{-\mu\vec{r}_{ij}}}{\vec{r}_{ij}}-4\pi\delta(\vec{r}_{ij})\right], \quad (4)$$

$$V_T^M(\vec{r}_{ij})=\frac{1}{4}\frac{g^2}{4\pi}\frac{\mu^3}{12m_i m_j}\left(1+\frac{3}{\mu\vec{r}_{ij}}+\frac{3}{\mu^2\vec{r}_{ij}^2}\right)\frac{{\rm e}^{-\mu\vec{r}_{ij}}}{\mu\vec{r}_{ij}}, \quad (5)$$

where μ denotes the mass of the exchanged meson. In addition, the flavor structure of the pseudoscalar octet exchange interaction between two quarks i and j in Eq. (3) is:

$$V_{C(T)}^M(\vec{r}_{ij})\vec{\lambda}_i^F\cdot\vec{\lambda}_j^F = \sum_{a=1}^{3}V_{C(T)}^{\pi}(\vec{r}_{ij})\hat{\lambda}_i^a\hat{\lambda}_j^a +\sum_{a=4}^{7}V_{C(T)}^{K}(\vec{r}_{ij})\hat{\lambda}_i^a\hat{\lambda}_j^a+V_{C(T)}^{\eta}(\vec{r}_{ij})\hat{\lambda}_i^8\hat{\lambda}_j^8, \quad (6)$$

where the three terms just correspond to interactions between quarks mediated by the π meson, K meson and η meson exchanges, respectively. The η-exchange, which is denoted by the third term, takes place in any quark pair state.

Finally, we denote the matrix elements of the interaction potentials $V_{C(T)}^M(\vec{r}_{ij})$ in GBE models which can be understood as the coupling strength constants of meson exchanges by P_{nl}^M and T_{nl}^M, namely

$$P_{nl}^M=\langle\varphi_{nl}(\vec{r}_{ij})|V_C^M(\vec{r}_{ij})|\varphi_{nl}(\vec{r}_{ij})\rangle, \quad (7)$$

$$T_{nl}^M=\langle\varphi_{nl}(\vec{r}_{ij})|V_T^M(\vec{r}_{ij})|\varphi_{nl}(\vec{r}_{ij})\rangle, \quad (8)$$

respectively, where $\varphi_{nl}(\vec{r}_{ij})$ denotes the wave function of the relative orbital motions between the ith and jth quarks. Tentatively, we take the empirical values for P_{nl}^{π} and T_{nl}^{π} as shown in Ref. [8], and the coupling strength for interactions mediated by K and η exchanges can be obtained by

$$P(T)_{nl}^K=\frac{m}{m_s}P(T)_{nl}^{\pi},$$
$$P(T)_{nl}^{\eta(qq)}=P(T)_{nl}^{\pi},$$
$$P(T)_{nl}^{\eta(ss)}=(\frac{m}{m_s})^2P(T)_{nl}^{\pi}. \quad (9)$$

In fact, the relation between P_{nl}^M for different meson exchanges was first suggested by Glozman and Riska in [8], and here we assume that these relation should be also applicable for T_{nl}^M.

2.2 Wave functions for the studied baryon states

As we know, different conventions for the baryon wave functions may result in different signs for the final results for configuration mixing [26], although these signs will not really affect any physical results if one takes a consistent convention in the calculations. Here we briefly show the wave functions we employ in the present work. The studied baryon configurations in the present work are those with the quantum number $J^P=1/2^-$, namely, those in the $[70,1^-]$ multiplet in the constituent quark model. As in the literature, here we denote these baryon configurations by $|B_M^{2S+1}P_\Pi J^P\rangle$, where $B=N$, Σ, Λ or Ξ, S represents the total spin of quarks, and M indicates the flavor symmetry of the three quarks. Namely, we denote $[21]_F$ by $M=8$, $[1^3]_F$ by $M=1$ and $[3]_F$ by $M=10$, respectively. The label P indicates that the total quark angular momentum is $L=1$, Π denotes the permutational symmetry of the quark flavor wave functions, and J^P is just the quantum numbers of the studied baryon states.

Accordingly, the general form for the wave functions of the $|B_8^2P_M\frac{1}{2}^-\rangle$, $|B_8^4P_M\frac{1}{2}^-\rangle$, $|B_1^2P_A\frac{1}{2}^-\rangle$, and $|B_{10}^2P_S\frac{1}{2}^-\rangle$ baryon configurations can be written as follows:

$$|B_8^2P_M\frac{1}{2}^-\rangle = C_A\frac{1}{2}\{\phi_B^\rho[\psi_{1M}^\rho\chi_{\frac{1}{2}}^\lambda+\psi_{1M}^\lambda\chi_{\frac{1}{2}}^\rho] +\phi_B^\lambda[\psi_{1M}^\rho\chi_{\frac{1}{2}}^\rho-\psi_{1M}^\lambda\chi_{\frac{1}{2}}^\lambda]\}[1^3]_C, \quad (10)$$

$$|B_8^4P_M\frac{1}{2}^-\rangle = C_A\chi_{\frac{3}{2}}^s\frac{1}{\sqrt{2}}(\phi_B^\rho\psi_{1M}^\rho+\phi_B^\lambda\psi_{1M}^\lambda)[1^3]_C, \quad (11)$$

$$|B_1^2P_A\frac{1}{2}^-\rangle = C_A\phi_B^a\frac{1}{\sqrt{2}}(\psi_{1M}^\rho\chi_{\frac{1}{2}}^\lambda-\psi_{1M}^\lambda\chi_{\frac{1}{2}}^\rho)[1^3]_C, \quad (12)$$

$$|B_{10}^2P_S\frac{1}{2}^-\rangle = C_A\phi_B^s\frac{1}{\sqrt{2}}(\psi_{1M}^\rho\chi_{\frac{1}{2}}^\rho+\psi_{1M}^\lambda\chi_{\frac{1}{2}}^\lambda)[1^3]_C, \quad (13)$$

where ϕ_B^π represents the flavor wave function, ψ_{1M}^π the orbital wave function, and χ_S^π the spin wave function. π denotes the permutational symmetry between the first two quarks (s represents symmetric, ρ and λ represent two different mixed symmetries, and a represents anti-symmetric) of the wave functions; explicit forms for all these wave functions for the studied states are given in Appendix A. $[1^3]_C$ is just the wave function for the color singlet. Finally, C_A is the Clebsch-Gordan coefficient for the L-S coupling to form the total orbital angular momentum $J=1/2$.

2.3 Mixing of the studied baryon configurations

As we know, the perturbative hyperfine interaction between quarks should not only cause splitting of the energies for baryon configurations, but also mixing between the baryon states. Once the coefficients for configuration mixing are obtained, the physical baryon states can be written as

$$|\mathcal{B}_{\rm phy}\rangle=\mathcal{C}_B|B_M^{2S+1}P_\Pi J^P\rangle. \quad (14)$$

Explicitly, physical states for the present studied N, Σ, Λ and Ξ configurations are then

$$\begin{pmatrix} N_1 \\ N_2 \end{pmatrix}=\mathcal{C}_N\begin{pmatrix} |N_8^4P_M\frac{1}{2}^-\rangle \\ |N_8^2P_M\frac{1}{2}^-\rangle \end{pmatrix}, \quad (15)$$

$$\begin{pmatrix} \Sigma_1 \\ \Sigma_2 \\ \Sigma_3 \end{pmatrix}=\mathcal{C}_\Sigma\begin{pmatrix} |\Sigma_{10}^2P_S\frac{1}{2}^-\rangle \\ |\Sigma_8^4P_M\frac{1}{2}^-\rangle \\ |\Sigma_8^2P_M\frac{1}{2}^-\rangle \end{pmatrix}, \quad (16)$$

$$\begin{pmatrix} \Lambda_1 \\ \Lambda_2 \\ \Lambda_3 \end{pmatrix}=\mathcal{C}_\Lambda\begin{pmatrix} |\Lambda_8^4P_M\frac{1}{2}^-\rangle \\ |\Lambda_8^2P_M\frac{1}{2}^-\rangle \\ |\Lambda_1^2P_A\frac{1}{2}^-\rangle \end{pmatrix}, \quad (17)$$

$$\begin{pmatrix} \Xi_1 \\ \Xi_2 \\ \Xi_3 \end{pmatrix}=\mathcal{C}_\Xi\begin{pmatrix} |\Xi_{10}^2P_S\frac{1}{2}^-\rangle \\ |\Xi_8^4P_M\frac{1}{2}^-\rangle \\ |\Xi_8^2P_M\frac{1}{2}^-\rangle \end{pmatrix}. \quad (18)$$

The coefficient matrices can be obtained by diagonalization of the following hyperfine interaction matrices:

$$\mathcal{H}_N=\begin{pmatrix} \langle N_8^4P_M\frac{1}{2}^-|H_{\rm hyp}|N_8^4P_M\frac{1}{2}^-\rangle & \langle N_8^4P_M\frac{1}{2}^-|H_{\rm hyp}|N_8^2P_M\frac{1}{2}^-\rangle \\ \langle N_8^2P_M\frac{1}{2}^-|H_{\rm hyp}|N_8^4P_M\frac{1}{2}^-\rangle & \langle N_8^2P_M\frac{1}{2}^-|H_{\rm hyp}|N_8^2P_M\frac{1}{2}^-\rangle \end{pmatrix}, \quad (19)$$

$$\mathcal{H}_\Sigma=\begin{pmatrix} \langle \Sigma_{10}^2P_S\frac{1}{2}^-|H_{\rm hyp}|\Sigma_{10}^2P_S\frac{1}{2}^-\rangle & \langle \Sigma_{10}^2P_S\frac{1}{2}^-|H_{\rm hyp}|\Sigma_8^4P_M\frac{1}{2}^-\rangle & \langle \Sigma_{10}^2P_S\frac{1}{2}^-|H_{\rm hyp}|\Sigma_8^2P_M\frac{1}{2}^-\rangle \\ \langle \Sigma_8^4P_M\frac{1}{2}^-|H_{\rm hyp}|\Sigma_{10}^2P_S\frac{1}{2}^-\rangle & \langle \Sigma_8^4P_M\frac{1}{2}^-|H_{\rm hyp}|\Sigma_8^4P_M\frac{1}{2}^-\rangle & \langle \Sigma_8^4P_M\frac{1}{2}^-|H_{\rm hyp}|\Sigma_8^2P_M\frac{1}{2}^-\rangle \\ \langle \Sigma_8^2P_M\frac{1}{2}^-|H_{\rm hyp}|\Sigma_{10}^2P_S\frac{1}{2}^-\rangle & \langle \Sigma_8^2P_M\frac{1}{2}^-|H_{\rm hyp}|\Sigma_8^4P_M\frac{1}{2}^-\rangle & \langle \Sigma_8^2P_M\frac{1}{2}^-|H_{\rm hyp}|\Sigma_8^2P_M\frac{1}{2}^-\rangle \end{pmatrix}, \quad (20)$$

$$\mathcal{H}_\Lambda=\begin{pmatrix} \langle \Lambda_8^4P_M\frac{1}{2}^-|H_{\rm hyp}|\Lambda_8^4P_M\frac{1}{2}^-\rangle & \langle \Lambda_8^4P_M\frac{1}{2}^-|H_{\rm hyp}|\Lambda_8^2P_M\frac{1}{2}^-\rangle & \langle \Lambda_8^4P_M\frac{1}{2}^-|H_{\rm hyp}|\Lambda_1^2P_A\frac{1}{2}^-\rangle \\ \langle \Lambda_8^2P_M\frac{1}{2}^-|H_{\rm hyp}|\Lambda_8^4P_M\frac{1}{2}^-\rangle & \langle \Lambda_8^2P_M\frac{1}{2}^-|H_{\rm hyp}|\Lambda_8^2P_M\frac{1}{2}^-\rangle & \langle \Lambda_8^2P_M\frac{1}{2}^-|H_{\rm hyp}|\Lambda_1^2P_A\frac{1}{2}^-\rangle \\ \langle \Lambda_1^2P_A\frac{1}{2}^-|H_{\rm hyp}|\Lambda_8^4P_M\frac{1}{2}^-\rangle & \langle \Lambda_1^2P_A\frac{1}{2}^-|H_{\rm hyp}|\Lambda_8^2P_M\frac{1}{2}^-\rangle & \langle \Lambda_1^2P_A\frac{1}{2}^-|H_{\rm hyp}|\Lambda_1^2P_A\frac{1}{2}^-\rangle \end{pmatrix}, \quad (21)$$

$$\mathcal{H}_{\Xi}=\begin{pmatrix} \langle\Xi^2_{10}P_S\frac{1}{2}^-|H_{\rm hyp}|\Xi^2_{10}P_S\frac{1}{2}^-\rangle & \langle\Xi^2_{10}P_S\frac{1}{2}^-|H_{\rm hyp}|\Xi^4_8P_M\frac{1}{2}^-\rangle & \langle\Xi^2_{10}P_S\frac{1}{2}^-|H_{\rm hyp}|\Xi^2_8P_M\frac{1}{2}^-\rangle \\ \langle\Xi^4_8P_M\frac{1}{2}^-|H_{\rm hyp}|\Xi^2_{10}P_S\frac{1}{2}^-\rangle & \langle\Xi^4_8P_M\frac{1}{2}^-|H_{\rm hyp}|\Xi^4_8P_M\frac{1}{2}^-\rangle & \langle\Xi^4_8P_M\frac{1}{2}^-|H_{\rm hyp}|\Xi^2_8P_M\frac{1}{2}^-\rangle \\ \langle\Xi^2_8P_M\frac{1}{2}^-|H_{\rm hyp}|\Xi^2_{10}P_S\frac{1}{2}^-\rangle & \langle\Xi^2_8P_M\frac{1}{2}^-|H_{\rm hyp}|\Xi^4_8P_M\frac{1}{2}^-\rangle & \langle\Xi^2_8P_M\frac{1}{2}^-|H_{\rm hyp}|\Xi^2_8P_M\frac{1}{2}^-\rangle \end{pmatrix}. \quad (22)$$

Explicit calculations lead to the matrix elements in the OGE and GBE models given in Appendices B and C, respectively.

Note that the hyperfine interactions taken in the present work are singular. Accordingly, the present calculations could only be a guide, since a perturbative calculation does not need to include any regulator that is needed when the interactions are used to obtain the orbital wave functions. While the balance between the short range and long range parts of the wave functions is very important, and could be fixed by the present employed harmonic oscillator wave functions, a full model which leads to more realistic wave functions of baryon states may change the balance between the long range and short parts.

3 Numerical results

In this section, we show the numerical results of the present work. Firstly, we have to discuss the model parameters.

As mentioned in Section 2, to obtain the numerical results for the mixing coefficients, the coupling strength constants P^{π}_{nl} and T^{π}_{nl} in the GBE model, and the ratio between constituent masses of the light (m) and strange (m_s) quarks in both the GBE and OGE models must be taken explicitly. None of the other parameters in the present investigations contribute to the final results.

The configuration mixing in different models should result in different mass splittings for the physical baryon states, which must depend on the explicit values of the constituent quark masses, the parameters in the quark confinement model, and the quark-gluon fine-structure constant α_s in the OGE interactions. Since all the baryon states considered in present work are the first orbitally excited ones, eigenvalues of the kinetic term and a harmonic oscillator confinement potential in the Hamiltonian of the three-quark system for all the studied states can be given by

$$M'_0=3m+4\omega+3V_0+n\delta_m=M_0+n\delta_m, \quad (23)$$

where ω and V_0 are the parameters in the harmonic oscillator model, n indicates the number of strange quarks in a baryon state, and $\delta_m=m_s-m$. To reduce the model parameters, here we just treat M_0 as a free parameter without giving explicit values for the harmonic oscillator parameters.

Accordingly, here we calculate the numerical results in the three hyperfine models, namely, the OGE model, the GBE model, and the mixed model in which both OGE and GBE hyperfine interactions are included. The explicit values of the parameters in these models are as follows. i) OGE model: $m=340$ MeV, $m/m_s=7/10$, $M_0=1610$ MeV, and a common factor $\delta=\frac{4\alpha_s}{3\sqrt{2}m^2}\omega^3\pi^{-\frac{1}{2}}=300$ MeV; all these empirical values are taken from Ref. [3]. ii) GBE model: $m=340$ MeV, $\delta_m=127$ MeV, $M_0=1503$ MeV, $P^{\pi}_{00}=29.05$ MeV, $P^{\pi}_{11}=45.5$ MeV and $T^{\pi}_{11}=4.2$ MeV; all these values are taken from Ref. [8]. iii) The mixed model: a tentative value $M_0=1550$ MeV is taken, the constituent mass for the light quark is taken to be 340 MeV, the ratio between the constituent masses of light and strange quarks is 7/10, and values for all the other parameters are taken to be the same as in the OGE and GBE models.

Table 1. The masses of the studied nucleon, Λ, Σ, Ξ resonances in the OGE and GBE models, without corrections from configuration mixing (in units of MeV).

state	OGE	GBE	mixed
$N^2_8P_M\frac{1}{2}^-$	1535	1527	1525
$N^4_8P_M\frac{1}{2}^-$	1610	1661	1629
$\Lambda^2_1P_A\frac{1}{2}^-$	1576	1494	1539
$\Lambda^2_8P_M\frac{1}{2}^-$	1681	1633	1658
$\Lambda^4_8P_M\frac{1}{2}^-$	1741	1784	1765
$\Sigma^2_8P_M\frac{1}{2}^-$	1711	1667	1692
$\Sigma^4_8P_M\frac{1}{2}^-$	1771	1734	1753
$\Sigma^2_{10}P_S\frac{1}{2}^-$	1816	1806	1812
$\Xi^2_8P_M\frac{1}{2}^-$	1844	1761	1813
$\Xi^4_8P_M\frac{1}{2}^-$	1891	1879	1895
$\Xi^2_{10}P_S\frac{1}{2}^-$	1949	1899	1933

With the parameters given above, and the diagonal terms of the hyperfine interaction matrices in the OGE and GBE models given in Appendices A and B, we can obtain the numerical results for the masses of the studied lowest-lying qqq configurations with $J^P=1/2^-$ in the three models without corrections of configuration mixing, as shown in Table 1. Mixing coefficients of the nucleon resonances $N^2_8P_M\frac{1}{2}^-$ and $N^4_8P_M\frac{1}{2}^-$, Λ resonances $\Lambda^2_1P_A\frac{1}{2}^-$, $\Lambda^2_8P_M\frac{1}{2}^-$ and $\Lambda^4_8P_M\frac{1}{2}^-$, Σ resonances $\Sigma^2_8P_M\frac{1}{2}^-$, $\Sigma^4_8P_M\frac{1}{2}^-$ and $\Sigma^2_{10}P_S\frac{1}{2}^-$, and Ξ resonances $\Xi^2_8P_M\frac{1}{2}^-$, $\Xi^4_8P_M\frac{1}{2}^-$ and $\Xi^2_{10}P_S\frac{1}{2}^-$ in the three hyperfine models are shown in Tables 2-5, compared to the numerical results obtained in Ref. [3], which are shown in column IK in the tables. The masses of the obtained physical baryon resonances with mass splitting caused by configuration mixing in the three models are shown in Table 6, compared to the numerical results obtained in Ref. [3], which are listed in column IK. Note that we have denoted the final physical states by the names listed in Ref. [28], according to the main properties of the physical resonances. In fact, the present assignments for the physical baryon resonances are also consistent with those in Refs. [7, 8].

Table 2. Mixing of the nucleon resonances $N^2_8P_M\frac{1}{2}^-$ and $N^4_8P_M\frac{1}{2}^-$ in the OGE and GBE models.

N^*	OGE	GBE	mixed	IK [3]	state
$N(1535)$	0.85	0.97	0.98	0.85	$N^2_8P_M\frac{1}{2}^-$
	−0.53	0.23	−0.19	0.53	$N^4_8P_M\frac{1}{2}^-$
$N(1650)$	0.53	−0.23	0.19	0.53	$N^2_8P_M\frac{1}{2}^-$
	0.85	0.97	0.98	−0.85	$N^4_8P_M\frac{1}{2}^-$

Table 3. Mixing of the Λ resonances $\Lambda^2_1P_A\frac{1}{2}^-$, $\Lambda^2_8P_M\frac{1}{2}^-$ and $\Lambda^4_8P_M\frac{1}{2}^-$ in the OGE and GBE models.

Λ^*	OGE	GBE	mixed	IK [3]	state
$\Lambda(1405)$	0.98	0.95	0.96	0.90	$\Lambda^2_1P_A\frac{1}{2}^-$
	−0.20	−0.30	−0.28	0.43	$\Lambda^2_8P_M\frac{1}{2}^-$
	−0.01	−0.01	0.01	0.06	$\Lambda^4_8P_M\frac{1}{2}^-$
$\Lambda(1670)$	0.16	0.30	0.28	−0.39	$\Lambda^2_1P_A\frac{1}{2}^-$
	0.81	0.93	0.95	0.75	$\Lambda^2_8P_M\frac{1}{2}^-$
	−0.56	0.21	−0.19	0.58	$\Lambda^4_8P_M\frac{1}{2}^-$
$\Lambda(1800)$	0.12	−0.05	0.05	−0.18	$\Lambda^2_1P_A\frac{1}{2}^-$
	0.55	−0.21	0.18	0.50	$\Lambda^2_8P_M\frac{1}{2}^-$
	0.83	0.98	0.98	−0.85	$\Lambda^4_8P_M\frac{1}{2}^-$

Table 4. Mixing of the Σ resonances $\Sigma^2_8P_M\frac{1}{2}^-$, $\Sigma^4_8P_M\frac{1}{2}^-$ and $\Sigma^2_{10}P_S\frac{1}{2}^-$ in the OGE and GBE models.

Σ^*	OGE	GBE	mixed	IK [3]	state
$\Sigma(1620?)$	0.86	0.94	0.97	0.82	$\Sigma^2_8P_M\frac{1}{2}^-$
	0.50	0.29	−0.22	0.54	$\Sigma^4_8P_M\frac{1}{2}^-$
	0.05	−0.19	−0.10	−0.17	$\Sigma^2_{10}P_S\frac{1}{2}^-$
$\Sigma(1750)$	0.48	−0.27	0.21	−0.46	$\Sigma^2_8P_M\frac{1}{2}^-$
	0.85	0.96	0.98	0.81	$\Sigma^4_8P_M\frac{1}{2}^-$
	0.23	0.12	−0.05	0.35	$\Sigma^2_{10}P_S\frac{1}{2}^-$
$\Sigma(1900?)$	−0.16	0.21	0.11	0.33	$\Sigma^2_8P_M\frac{1}{2}^-$
	−0.18	−0.07	0.03	−0.21	$\Sigma^4_8P_M\frac{1}{2}^-$
	0.97	0.97	0.99	0.92	$\Sigma^2_{10}P_S\frac{1}{2}^-$

Table 5. Mixing of Ξ resonances $\Xi^2_8P_M\frac{1}{2}^-$, $\Xi^4_8P_M\frac{1}{2}^-$ and $\Xi^2_{10}P_S\frac{1}{2}^-$ in the OGE and GBE models.

Ξ^*	OGE	GBE	mixed	IK [3]	state
$\Xi(1690)$	0.84	0.97	0.98	0.88	$\Xi^2_8P_M\frac{1}{2}^-$
	−0.54	0.19	−0.17	0.42	$\Xi^4_8P_M\frac{1}{2}^-$
	0.03	−0.14	−0.07	0.22	$\Xi^2_{10}P_S\frac{1}{2}^-$
$\Xi(1900?)$	0.54	−0.15	0.16	−0.18	$\Xi^2_8P_M\frac{1}{2}^-$
	0.84	0.97	0.99	0.73	$\Xi^4_8P_M\frac{1}{2}^-$
	0.12	0.21	−0.04	−0.66	$\Xi^2_{10}P_S\frac{1}{2}^-$
$\Xi(1930?)$	−0.09	0.18	0.08	0.43	$\Xi^2_8P_M\frac{1}{2}^-$
	−0.08	−0.18	0.03	−0.56	$\Xi^4_8P_M\frac{1}{2}^-$
	0.99	0.97	0.99	−0.73	$\Xi^2_{10}P_S\frac{1}{2}^-$

Table 6. The masses of the studied nucleon, Λ, Σ, Ξ resonances in the OGE and GBE models, with configuration mixing (in units of MeV).

state	OGE	GBE	model M	IK [3]
$N(1535)$	1489	1519	1521	1490
$N(1650)$	1656	1669	1633	1655
$\Lambda(1405)$	1571	1479	1528	1490
$\Lambda(1670)$	1639	1641	1666	1650
$\Lambda(1800)$	1788	1791	1769	1800
$\Sigma(1620?)$	1680	1654	1688	1650
$\Sigma(1750)$	1800	1741	1756	1750
$\Sigma(1900?)$	1817	1813	1813	1810
$\Xi(1690)$	1810	1753	1810	1780
$\Xi(1900?)$	1925	1883	1898	1900
$\Xi(1930?)$	1949	1903	1933	1930

Obviously, as we can see in Tables 1 and 6, the mass splittings caused by hyperfine interactions in the present three models fall in a reasonable range. The results obtained in the GBE model shown in Table 1 are slightly different from those obtained in Ref. [8], because the contributions from the tensor term of the hyperfine interaction have not been included explicitly in Ref. [8]. Also, in Table 6, our results obtained by employing the OGE model are somewhat different from those in Ref. [3], because the parameters are taken to be different for different baryon states in that work, whereas we use a unified set for all the parameters.

The mixing angle θ_s for the S_{11} states has already been calculated employing the OGE model in Ref. [23], and applied to studying the amplitudes of the first orbitally excited nucleon resonances for photoproduction and electroproduction in Ref. [24]. The mixing angle is found to be $\theta_s=-32^\circ$ in those works. However, as we can see in Table 2, the present obtained value is $\theta_s=32^\circ$. This is because we have use a different convention for the flavor wave functions. This has also been clarified in Ref. [26]. The same situation can also be found in the results for the Λ, Σ and Ξ resonances. In addition, except for the signs, some of the absolute values of the mixing coefficients obtained in the present calculations using the OGE model are also different from those in Ref. [3]. These differences are also only from the setting of model parameters, as we have claimed in last paragraph. As we can see in Appendix B, the present expressions for the matrix elements of the OGE hyperfine interaction are consistent with those in Ref. [3] if the same convention for flavor wave functions is used.

As we can see in Tables 2-5, the results obtained in the three models are very different, although the dominant components of all the obtained physical states are almost the same in the three hyperfine interaction models. Especially, as shown in the tables, the signs for the coefficients of the dominant configurations in $N(1535)$, $N(1650)$, $\Lambda(1670)$, $\Lambda(1800)$, $\Sigma(1750)$, $\Sigma(1900)$, $\Xi(1690)$,

$\Xi(1900)$ are different. Accordingly, fitting data for the electromagnetic or strong decays of these baryon resonances using the present results should be a possible way to examine the OGE and GBE models.

For instance, a recent calculation of the strong decays of the S_{11} resonances indicates that the experimental data should favor a negative sign for the coefficient of the $N^4_8P_M\frac{1}{2}^-$ component in $N(1535)$ [26], because of the large partial width for the $N(1535)\to\eta N$ decay. In other words, the OGE model may be the more reasonable one. This is consistent with the results in Refs. [23, 24]. Although it is claimed that the higher Fock components in $N(1535)$ may result in the large ηN partial decay width [13], contributions of the lowest pentaquark components were in fact already included in Ref. [26]. However, in Ref. [25], it was shown that the experimental data would favor the GBE model if the scalar meson exchanges were taken into account. Consequently, we cannot get a solid conclusion from only fitting the experimental data on the decays of the S_{11} resonances.

As we have mentioned above, the signs for the dominant components of 8 baryon resonances are different in the OGE and GBE models, and values of the mixing coefficients for all the other resonances are also somewhat different. However, no-one has compared the decay properties of the Λ^*, Σ^* and Ξ^* resonances in these two models. Therefore, we hope the present results can be used to examine the OGE and GBE models by investigations of the electromagnetic and strong decays of these resonances. In addition, it would be very interesting to predict the favored decay channels of the baryon resonances obtained in the present three models. However, because of the similar properties of baryon states with the same quantum numbers, it is very difficult for us to give such predictions without explicit calculations.

4 Summary

Here we briefly summarize the present work. In this article, we have investigated mixing of the N^*, Λ^*, Σ^* and Ξ^* configurations with quantum number $J^P=1/2^-$. The hyperfine interactions between quarks are taken to be mediated by one gluon exchange and Goldstone boson exchange, or by both kinds of exchange.

Our numerical results show that the configuration mixing in the three employed hyperfine interaction models are very different, although the dominant components of all the obtained physical states are almost the same in these models. The differences between the one gluon exchange model and Goldstone boson exchange model involve not only the absolute values, but also the signs of the probability amplitudes for the dominant components in several physical baryon resonances.

Consequently, to conclude, this is a possible way to examine the hyperfine interaction models by investigations of the electromagnetic and strong decays of the baryon resonances using the present results, which should be very sensitive to the coefficients of the configuration mixing. One might expect a prediction of the favored decay channels of the obtained baryon resonances in the three hyperfine models. However, because of the similar properties of baryon states with the same quantum numbers, it is very difficult for us to give such a prediction without explicit calculations. Investigations of the strong decays of baryon resonances using the present numerical results are in progress now.

Appendix A

Explicit flavor, spin and orbital wave functions

The explicit flavor wave functions of the studied resonances are as follows:

$$\phi_p^\rho=\frac{1}{\sqrt{2}}[|udu\rangle-|duu\rangle]. \quad (A1)$$

$$\phi_p^\lambda=\frac{1}{\sqrt{6}}[2|uud\rangle-|duu\rangle-|udu\rangle]. \quad (A2)$$

$$\phi_n^\rho=\frac{1}{\sqrt{2}}[|udd\rangle-|dud\rangle]. \quad (A3)$$

$$\phi_n^\lambda=-\frac{1}{\sqrt{6}}[2|ddu\rangle-|udd\rangle-|dud\rangle]. \quad (A4)$$

$$\phi_\Lambda^a=\frac{1}{\sqrt{6}}[|uds\rangle+|dsu\rangle+|sud\rangle-|usd\rangle-|dus\rangle-|sdu\rangle]. \quad (A5)$$

$$\phi_\Lambda^\rho=\frac{1}{2\sqrt{3}}[|usd\rangle-|dsu\rangle-|sud\rangle+|sdu\rangle+2|uds\rangle-2|dus\rangle]. \quad (A6)$$

$$\phi_\Lambda^\lambda=\frac{1}{2}[|usd\rangle+|sud\rangle-|sdu\rangle-|dsu\rangle]. \quad (A7)$$

$$\phi_{\Sigma^0}^\rho=\frac{1}{2}[|usd\rangle+|dsu\rangle-|sud\rangle-|sdu\rangle]. \quad (A8)$$

$$\phi_{\Sigma^0}^\lambda=-\frac{1}{2\sqrt{3}}[|usd\rangle+|dsu\rangle+|sud\rangle+|sdu\rangle-2|uds\rangle-2|dus\rangle]. \quad (A9)$$

$$\phi_{\Sigma^+}^\rho=\frac{1}{\sqrt{2}}[|usu\rangle-|suu\rangle]. \quad (A10)$$

$$\phi_{\Sigma^+}^\lambda=\frac{1}{\sqrt{6}}[2|uus\rangle-|suu\rangle-|usu\rangle]. \quad (A11)$$

$$\phi^{\rho}_{\Sigma^-}=-\frac{1}{\sqrt{2}}[|sdd\rangle-|dsd\rangle]. \quad (A12)$$

$$\phi^{\lambda}_{\Sigma^-}=\frac{1}{\sqrt{6}}[2|dds\rangle-|sdd\rangle-|dsd\rangle]. \quad (A13)$$

$$\phi^{s}_{\Sigma^0}=\frac{1}{\sqrt{6}}[|usd\rangle+|dsu\rangle+|sud\rangle+|sdu\rangle+|uds\rangle+|dus\rangle]. \quad (A14)$$

$$\phi^{s}_{\Sigma^+}=\frac{1}{\sqrt{3}}[|uus\rangle+|usu\rangle+|suu\rangle]. \quad (A15)$$

$$\phi^{s}_{\Sigma^-}=\frac{1}{\sqrt{3}}[|dds\rangle+|dsd\rangle+|sdd\rangle]. \quad (A16)$$

$$\phi^{\rho}_{\Xi^0}=\frac{1}{\sqrt{2}}[|uss\rangle-|sus\rangle]. \quad (A17)$$

$$\phi^{\lambda}_{\Xi^0}=-\frac{1}{\sqrt{6}}[2|ssu\rangle-|uss\rangle-|sus\rangle]. \quad (A18)$$

$$\phi^{\rho}_{\Xi^-}=-\frac{1}{\sqrt{2}}[|sds\rangle-|dss\rangle]. \quad (A19)$$

$$\phi^{\lambda}_{\Xi^-}=-\frac{1}{\sqrt{6}}[2|ssd\rangle-|dss\rangle-|sds\rangle]. \quad (A20)$$

$$\phi^{s}_{\Xi^0}=\frac{1}{\sqrt{3}}[|ssu\rangle+|uss\rangle+|sus\rangle]. \quad (A21)$$

$$\phi^{s}_{\Xi^-}=\frac{1}{\sqrt{3}}[|ssd\rangle+|dss\rangle+|sds\rangle]. \quad (A22)$$

The explicit spin wave functions for the $S_{3q}=1/2$ and $S_{3q}=3/2$ states are

$$\chi^{\rho}_{\frac{1}{2}}(S_z=1/2)=\frac{1}{\sqrt{2}}[|\uparrow\downarrow\uparrow\rangle-|\downarrow\uparrow\uparrow\rangle]. \quad (A23)$$

$$\chi^{\lambda}_{\frac{1}{2}}(S_z=1/2)=\frac{1}{\sqrt{6}}[2|\uparrow\uparrow\downarrow\rangle-|\uparrow\downarrow\uparrow\rangle-|\downarrow\uparrow\uparrow\rangle]. \quad (A24)$$

$$\chi^{\rho}_{\frac{1}{2}}(S_z=-1/2)=\frac{1}{\sqrt{2}}[|\uparrow\downarrow\downarrow\rangle-|\downarrow\uparrow\downarrow\rangle]. \quad (A25)$$

$$\chi^{\lambda}_{\frac{1}{2}}(S_z=-1/2)=-\frac{1}{\sqrt{6}}[2|\downarrow\downarrow\uparrow\rangle-|\downarrow\uparrow\downarrow\rangle-|\uparrow\downarrow\downarrow\rangle]. \quad (A26)$$

$$\chi_{\frac{3}{2}}(S_z=3/2)=|\uparrow\uparrow\uparrow\rangle. \quad (A27)$$

$$\chi_{\frac{3}{2}}(S_z=1/2)=\frac{1}{\sqrt{3}}[|\uparrow\uparrow\downarrow\rangle+\uparrow\downarrow\uparrow\rangle+|\downarrow\uparrow\uparrow\rangle]. \quad (A28)$$

$$\chi_{\frac{3}{2}}(S_z=-1/2)=\frac{1}{\sqrt{3}}[|\uparrow\downarrow\downarrow\rangle+\downarrow\uparrow\downarrow\rangle+\downarrow\downarrow\uparrow\rangle]. \quad (A29)$$

The orbital wave functions of the three-quark system in the harmonic oscillator model are:

$$\psi^{S}_{00}=\frac{\alpha_\rho^{\frac{3}{2}}\alpha_\lambda^{\frac{3}{2}}}{\pi^{\frac{3}{2}}}\mathrm{e}^{-(\alpha_\rho^2\vec{\rho}^2+\alpha_\lambda^2\vec{\lambda}^2)/2}. \quad (A30)$$

$$\psi^{\rho}_{1\pm1}=\mp\frac{\alpha_\rho^{\frac{5}{2}}\alpha_\lambda^{\frac{3}{2}}}{\pi^{\frac{3}{2}}}\rho_\pm\mathrm{e}^{-(\alpha_\rho^2\vec{\rho}^2+\alpha_\lambda^2\vec{\lambda}^2)/2}. \quad (A31)$$

$$\psi^{\lambda}_{1\pm1}=\mp\frac{\alpha_\rho^{\frac{3}{2}}\alpha_\lambda^{\frac{5}{2}}}{\pi^{\frac{3}{2}}}\lambda_\pm\mathrm{e}^{-(\alpha_\rho^2\vec{\rho}^2+\alpha_\lambda^2\vec{\lambda}^2)/2}. \quad (A32)$$

$$\psi^{\rho}_{10}=\frac{\alpha_\rho^{\frac{5}{2}}\alpha_\lambda^{\frac{3}{2}}}{\pi^{\frac{3}{2}}}\sqrt{2}\rho_0\mathrm{e}^{-(\alpha_\rho^2\vec{\rho}^2+\alpha_\lambda^2\vec{\lambda}^2)/2}. \quad (A33)$$

$$\psi^{\lambda}_{10}=\frac{\alpha_\rho^{\frac{3}{2}}\alpha_\lambda^{\frac{5}{2}}}{\pi^{\frac{3}{2}}}\sqrt{2}\lambda_0\mathrm{e}^{-(\alpha_\rho^2\vec{\rho}^2+\alpha_\lambda^2\vec{\lambda}^2)/2}, \quad (A34)$$

where $\vec{\rho}_\pm=\vec{\rho}_x\pm i\vec{\rho}_y$, $\vec{\rho}_0=\vec{\rho}_z$, and $\vec{\lambda}_\pm=\vec{\lambda}_x\pm i\vec{\lambda}_y$, $\vec{\lambda}_0=\vec{\lambda}_z$.

Appendix B

Matrix elements of the hyperfine interactions in OGE model

The matrix elements of the OGE hyperfine interaction $H^{\mathrm{OGE}}_{\mathrm{hyp}}$ in the nucleon resonances $|N^4_8P_M\frac{1}{2}^-\rangle$ and $|N^2_8P_M\frac{1}{2}^-\rangle$ are

$$\langle N^4_8P_M\frac{1}{2}^-|H^{\mathrm{OGE}}_{\mathrm{hyp}}|N^4_8P_M\frac{1}{2}^-\rangle=0, \quad (B1)$$

$$\langle N^4_8P_M\frac{1}{2}^-|H^{\mathrm{OGE}}_{\mathrm{hyp}}|N^2_8P_M\frac{1}{2}^-\rangle=\frac{\alpha_s}{3\sqrt{2}m^2}\omega^3\pi^{-\frac{1}{2}}, \quad (B2)$$

$$\langle N^2_8P_M\frac{1}{2}^-|H^{\mathrm{OGE}}_{\mathrm{hyp}}|N^4_8P_M\frac{1}{2}^-\rangle=\frac{\alpha_s}{3\sqrt{2}m^2}\omega^3\pi^{-\frac{1}{2}}, \quad (B3)$$

$$\langle N^2_8P_M\frac{1}{2}^-|H^{\mathrm{OGE}}_{\mathrm{hyp}}|N^2_8P_M\frac{1}{2}^-\rangle=-\frac{\alpha_s}{3\sqrt{2}m^2}\omega^3\pi^{-\frac{1}{2}}. \quad (B4)$$

The matrix elements of $H^{\mathrm{OGE}}_{\mathrm{hyp}}$ in the resonances $|\Lambda^4_8P_M\frac{1}{2}^-\rangle$, $|\Lambda^2_8P_M\frac{1}{2}^-\rangle$ and $|\Lambda^2_1P_A\frac{1}{2}^-\rangle$ are

$$\langle \Lambda^4_8P_M\frac{1}{2}^-|H^{\mathrm{OGE}}_{\mathrm{hyp}}|\Lambda^4_8P_M\frac{1}{2}^-\rangle=-\frac{\alpha_s}{15\sqrt{2}m^2}\omega^3\pi^{-\frac{1}{2}}, \quad (B5)$$

$$\langle \Lambda^4_8P_M\frac{1}{2}^-|H^{\mathrm{OGE}}_{\mathrm{hyp}}|\Lambda^2_8P_M\frac{1}{2}^-\rangle=\frac{3\alpha_s}{10\sqrt{2}m^2}\omega^3\pi^{-\frac{1}{2}}, \quad (B6)$$

$$\langle \Lambda^4_8P_M\frac{1}{2}^-|H^{\mathrm{OGE}}_{\mathrm{hyp}}|\Lambda^2_1P_A\frac{1}{2}^-\rangle=\frac{\alpha_s}{15\sqrt{2}m^2}\omega^3\pi^{-\frac{1}{2}}, \quad (B7)$$

$$\langle \Lambda^2_8P_M\frac{1}{2}^-|H^{\mathrm{OGE}}_{\mathrm{hyp}}|\Lambda^4_8P_M\frac{1}{2}^-\rangle=\frac{3\alpha_s}{10\sqrt{2}m^2}\omega^3\pi^{-\frac{1}{2}}, \quad (B8)$$

$$\langle \Lambda^2_8P_M\frac{1}{2}^-|H^{\mathrm{OGE}}_{\mathrm{hyp}}|\Lambda^2_8P_M\frac{1}{2}^-\rangle=-\frac{\alpha_s}{3\sqrt{2}m^2}\omega^3\pi^{-\frac{1}{2}}, \quad (B9)$$

$$\langle \Lambda^2_8P_M\frac{1}{2}^-|H^{\mathrm{OGE}}_{\mathrm{hyp}}|\Lambda^2_1P_A\frac{1}{2}^-\rangle=\frac{\alpha_s}{10\sqrt{2}m^2}\omega^3\pi^{-\frac{1}{2}}, \quad (B10)$$

$$\langle \Lambda^2_1P_A\frac{1}{2}^-|H^{\mathrm{OGE}}_{\mathrm{hyp}}|\Lambda^4_8P_M\frac{1}{2}^-\rangle=\frac{\alpha_s}{15\sqrt{2}m^2}\omega^3\pi^{-\frac{1}{2}}, \quad (B11)$$

$$\langle \Lambda_1^2 P_A \frac{1}{2}^- | H_{\rm hyp}^{\rm OGE} | \Lambda_8^2 P_M \frac{1}{2}^- \rangle = \frac{\alpha_s}{10\sqrt{2}m^2}\omega^3\pi^{-\frac{1}{2}}, \quad \text{(B12)}$$

$$\langle \Lambda_1^2 P_A \frac{1}{2}^- | H_{\rm hyp}^{\rm OGE} | \Lambda_1^2 P_A \frac{1}{2}^- \rangle = -\frac{4\alpha_s}{5\sqrt{2}m^2}\omega^3\pi^{-\frac{1}{2}}. \quad \text{(B13)}$$

The matrix elements of $H_{\rm hyp}^{\rm OGE}$ in the resonances $|\Sigma_{10}^2 P_S \frac{1}{2}^-\rangle$, $|\Sigma_8^4 P_M \frac{1}{2}^-\rangle$ and $|\Sigma_8^2 P_M \frac{1}{2}^-\rangle$ are

$$\langle \Sigma_{10}^2 P_S \frac{1}{2}^- | H_{\rm hyp}^{\rm OGE} | \Sigma_{10}^2 P_S \frac{1}{2}^- \rangle = \frac{4\alpha_s}{15\sqrt{2}m^2}\omega^3\pi^{-\frac{1}{2}}, \quad \text{(B14)}$$

$$\langle \Sigma_{10}^2 P_S \frac{1}{2}^- | H_{\rm hyp}^{\rm OGE} | \Sigma_8^4 P_M \frac{1}{2}^- \rangle = 0, \quad \text{(B15)}$$

$$\langle \Sigma_{10}^2 P_S \frac{1}{2}^- | H_{\rm hyp}^{\rm OGE} | \Sigma_8^2 P_M \frac{1}{2}^- \rangle = -\frac{\alpha_s}{30\sqrt{2}m^2}\omega^3\pi^{-\frac{1}{2}}, \quad \text{(B16)}$$

$$\langle \Sigma_8^4 P_M \frac{1}{2}^- | H_{\rm hyp}^{\rm OGE} | \Sigma_{10}^2 P_S \frac{1}{2}^- \rangle = 0, \quad \text{(B17)}$$

$$\langle \Sigma_8^4 P_M \frac{1}{2}^- | H_{\rm hyp}^{\rm OGE} | \Sigma_8^4 P_M \frac{1}{2}^- \rangle = \frac{\alpha_s}{15\sqrt{2}m^2}\omega^3\pi^{-\frac{1}{2}}, \quad \text{(B18)}$$

$$\langle \Sigma_8^4 P_M \frac{1}{2}^- | H_{\rm hyp}^{\rm OGE} | \Sigma_8^2 P_M \frac{1}{2}^- \rangle = \frac{7\alpha_s}{30\sqrt{2}m^2}\omega^3\pi^{-\frac{1}{2}}, \quad \text{(B19)}$$

$$\langle \Sigma_8^2 P_M \frac{1}{2}^- | H_{\rm hyp}^{\rm OGE} | \Sigma_{10}^2 P_S \frac{1}{2}^- \rangle = -\frac{\alpha_s}{30\sqrt{2}m^2}\omega^3\pi^{-\frac{1}{2}}, \quad \text{(B20)}$$

$$\langle \Sigma_8^2 P_M \frac{1}{2}^- | H_{\rm hyp}^{\rm OGE} | \Sigma_8^4 P_M \frac{1}{2}^- \rangle = \frac{7\alpha_s}{30\sqrt{2}m^2}\omega^3\pi^{-\frac{1}{2}}, \quad \text{(B21)}$$

$$\langle \Sigma_8^2 P_M \frac{1}{2}^- | H_{\rm hyp}^{\rm OGE} | \Sigma_8^2 P_M \frac{1}{2}^- \rangle = -\frac{\alpha_s}{5\sqrt{2}m^2}\omega^3\pi^{-\frac{1}{2}}. \quad \text{(B22)}$$

The matrix elements of $H_{\rm hyp}^{\rm OGE}$ in the resonances $|\Xi_{10}^2 P_S \frac{1}{2}^-\rangle$, $|\Xi_8^4 P_M \frac{1}{2}^-\rangle$ and $|\Xi_8^2 P_M \frac{1}{2}^-\rangle$ are

$$\langle \Xi_{10}^2 P_S \frac{1}{2}^- | H_{\rm hyp}^{\rm OGE} | \Xi_{10}^2 P_S \frac{1}{2}^- \rangle = \frac{189\alpha_s}{900\sqrt{2}m^2}\omega^3\pi^{-\frac{1}{2}}, \quad \text{(B23)}$$

$$\langle \Xi_{10}^2 P_S \frac{1}{2}^- | H_{\rm hyp}^{\rm OGE} | \Xi_8^4 P_M \frac{1}{2}^- \rangle = 0, \quad \text{(B24)}$$

$$\langle \Xi_{10}^2 P_S \frac{1}{2}^- | H_{\rm hyp}^{\rm OGE} | \Xi_8^2 P_M \frac{1}{2}^- \rangle = -\frac{7\alpha_s}{300\sqrt{2}m^2}\omega^3\pi^{-\frac{1}{2}}, \quad \text{(B25)}$$

$$\langle \Xi_8^4 P_M \frac{1}{2}^- | H_{\rm hyp}^{\rm OGE} | \Xi_{10}^2 P_S \frac{1}{2}^- \rangle = 0, \quad \text{(B26)}$$

$$\langle \Xi_8^4 P_M \frac{1}{2}^- | H_{\rm hyp}^{\rm OGE} | \Xi_8^4 P_M \frac{1}{2}^- \rangle = -\frac{7\alpha_s}{150\sqrt{2}m^2}\omega^3\pi^{-\frac{1}{2}}, \quad \text{(B27)}$$

$$\langle \Xi_8^4 P_M \frac{1}{2}^- | H_{\rm hyp}^{\rm OGE} | \Xi_8^2 P_M \frac{1}{2}^- \rangle = \frac{7\alpha_s}{30\sqrt{2}m^2}\omega^3\pi^{-\frac{1}{2}}, \quad \text{(B28)}$$

$$\langle \Xi_8^2 P_M \frac{1}{2}^- | H_{\rm hyp}^{\rm OGE} | \Xi_{10}^2 P_S \frac{1}{2}^- \rangle = -\frac{7\alpha_s}{300\sqrt{2}m^2}\omega^3\pi^{-\frac{1}{2}}, \quad \text{(B29)}$$

$$\langle \Xi_8^2 P_M \frac{1}{2}^- | H_{\rm hyp}^{\rm OGE} | \Xi_8^4 P_M \frac{1}{2}^- \rangle = \frac{7\alpha_s}{30\sqrt{2}m^2}\omega^3\pi^{-\frac{1}{2}}, \quad \text{(B30)}$$

$$\langle \Xi_8^2 P_M \frac{1}{2}^- | H_{\rm hyp}^{\rm OGE} | \Xi_8^2 P_M \frac{1}{2}^- \rangle = -\frac{231\alpha_s}{900\sqrt{2}m^2}\omega^3\pi^{-\frac{1}{2}}. \quad \text{(B31)}$$

The relation between the constituent masses of light and strange quarks $m/m_s = 7/10$ has been applied, which is why many strange numbers appear in the above equations. We have treated the harmonic oscillator parameters α_ρ and α_λ as the same, $\alpha_\rho = \alpha_\lambda = \omega$, and it has been shown that this approximation cannot affect the final results too much.

Appendix C

Matrix elements of the hyperfine interactions in GBE model

The matrix elements of the GBE hyperfine interaction $H_{\rm hyp}^{\rm GBE}$ in the nucleon resonances $|N_8^4 P_M \frac{1}{2}^-\rangle$ and $|N_8^2 P_M \frac{1}{2}^-\rangle$ are

$$\langle N_8^4 P_M \frac{1}{2}^- | H_{\rm hyp}^{\rm GBE} | N_8^4 P_M \frac{1}{2}^- \rangle = -\frac{3}{2}P_{00}^{\pi} - \frac{1}{2}P_{00}^{uu} + \frac{9}{2}P_{11}^{\pi} - \frac{1}{2}P_{11}^{uu} + 9T_{11}^{\pi} - T_{11}^{uu}, \quad \text{(C1)}$$

$$\langle N_8^4 P_M \frac{1}{2}^- | H_{\rm hyp}^{\rm GBE} | N_8^2 P_M \frac{1}{2}^- \rangle = -9T_{11}^{\pi} + T_{11}^{uu}, \quad \text{(C2)}$$

$$\langle N_8^2 P_M \frac{1}{2}^- | H_{\rm hyp}^{\rm GBE} | N_8^4 P_M \frac{1}{2}^- \rangle = -9T_{11}^{\pi} + T_{11}^{uu}, \quad \text{(C3)}$$

$$\langle N_8^2 P_M \frac{1}{2}^- | H_{\rm hyp}^{\rm GBE} | N_8^2 P_M \frac{1}{2}^- \rangle = -\frac{15}{2}P_{00}^{\pi} + \frac{1}{2}P_{00}^{uu} + \frac{9}{2}P_{11}^{\pi} + \frac{1}{2}P_{11}^{uu}. \quad \text{(C4)}$$

The matrix elements of $H_{\rm hyp}^{\rm GBE}$ in the resonances $|\Lambda_8^4 P_M \frac{1}{2}^-\rangle$, $|\Lambda_8^2 P_M \frac{1}{2}^-\rangle$ and $|\Lambda_1^2 P_A \frac{1}{2}^-\rangle$ are

$$\langle \Lambda_8^4 P_M \frac{1}{2}^- | H_{\rm hyp}^{\rm GBE} | \Lambda_8^4 P_M \frac{1}{2}^- \rangle = P_{00}^{us} - 3P_{00}^{K} + 3P_{11}^{\pi} - \frac{1}{3}P_{11}^{uu} + \frac{1}{3}P_{11}^{us} + P_{11}^{K} + 6T_{11}^{\pi} + 2T_{11}^{K} - \frac{2}{3}T_{11}^{uu} + \frac{2}{3}T_{11}^{us}, \quad \text{(C5)}$$

$$\langle \Lambda_8^4 P_M \frac{1}{2}^- | H_{\rm hyp}^{\rm GBE} | \Lambda_8^2 P_M \frac{1}{2}^- \rangle = -6T_{11}^{\pi} - 2T_{11}^{K} + \frac{2}{3}T_{11}^{uu} - \frac{2}{3}T_{11}^{us}, \quad \text{(C6)}$$

$$\langle \Lambda_8^4 P_M \frac{1}{2}^- | H_{\rm hyp}^{\rm GBE} | \Lambda_1^2 P_A \frac{1}{2}^- \rangle = -6T_{11}^{\pi} + 4T_{11}^{K} + \frac{2}{3}T_{11}^{uu} + \frac{4}{3}T_{11}^{us}, \quad \text{(C7)}$$

$$\langle \Lambda_8^2 P_M \frac{1}{2}^- | H_{\text{hyp}}^{\text{GBE}} | \Lambda_8^4 P_M \frac{1}{2}^- \rangle = -6T_{11}^{\pi} - 2T_{11}^{K} + \frac{2}{3}T_{11}^{uu} - \frac{2}{3}T_{11}^{us}, \quad \text{(C8)}$$

$$\langle \Lambda_8^2 P_M \frac{1}{2}^- | H_{\text{hyp}}^{\text{GBE}} | \Lambda_8^2 P_M \frac{1}{2}^- \rangle = -\frac{9}{2}P_{00}^{\pi} + \frac{1}{2}P_{00}^{uu} - 3P_{00}^{K} + \frac{3}{2}P_{11}^{\pi} - \frac{1}{6}P_{11}^{uu} - \frac{4}{3}P_{11}^{us} + 5P_{11}^{K}, \quad \text{(C9)}$$

$$\langle \Lambda_8^2 P_M \frac{1}{2}^- | H_{\text{hyp}}^{\text{GBE}} | \Lambda_1^2 P_A \frac{1}{2}^- \rangle = \frac{9}{2}P_{00}^{\pi} - 3P_{00}^{K} - P_{00}^{us} - \frac{1}{2}P_{00}^{uu} + \frac{3}{2}P_{11}^{\pi} - P_{11}^{K} - \frac{1}{6}P_{11}^{uu} - \frac{1}{3}P_{11}^{us}, \quad \text{(C10)}$$

$$\langle \Lambda_1^2 P_A \frac{1}{2}^- | H_{\text{hyp}}^{\text{GBE}} | \Lambda_8^4 P_M \frac{1}{2}^- \rangle = -6T_{11}^{\pi} + 4T_{11}^{K} + \frac{2}{3}T_{11}^{uu} + \frac{4}{3}T_{11}^{us}, \quad \text{(C11)}$$

$$\langle \Lambda_1^2 P_A \frac{1}{2}^- | H_{\text{hyp}}^{\text{GBE}} | \Lambda_8^2 P_M \frac{1}{2}^- \rangle = \frac{9}{2}P_{00}^{\pi} - 3P_{00}^{K} - P_{00}^{us} - \frac{1}{2}P_{00}^{uu} + \frac{3}{2}P_{11}^{\pi} - P_{11}^{K} - \frac{1}{6}P_{11}^{uu} - \frac{1}{3}P_{11}^{us}, \quad \text{(C12)}$$

$$\langle \Lambda_1^2 P_A \frac{1}{2}^- | H_{\text{hyp}}^{\text{GBE}} | \Lambda_1^2 P_A \frac{1}{2}^- \rangle = -\frac{9}{2}P_{00}^{\pi} + \frac{1}{2}P_{00}^{uu} - 2P_{00}^{us} - 6P_{00}^{K} + \frac{3}{2}P_{11}^{\pi} - \frac{1}{6}P_{11}^{uu} + \frac{2}{3}P_{11}^{us} + 2P_{11}^{K}. \quad \text{(C13)}$$

The matrix elements of $H_{\text{hyp}}^{\text{GBE}}$ in the resonances $|\Sigma_{10}^2 P_S \frac{1}{2}^-\rangle$, $|\Sigma_8^4 P_M \frac{1}{2}^-\rangle$ and $|\Sigma_8^2 P_M \frac{1}{2}^-\rangle$ are

$$\langle \Sigma_{10}^2 P_S \frac{1}{2}^- | H_{\text{hyp}}^{\text{GBE}} | \Sigma_{10}^2 P_S \frac{1}{2}^- \rangle = -\frac{1}{2}P_{00}^{\pi} - \frac{1}{6}P_{00}^{uu} + \frac{2}{3}P_{00}^{us} - 2P_{00}^{K} + \frac{3}{2}P_{11}^{\pi} + \frac{1}{2}P_{11}^{uu} - 2P_{11}^{us} + 6P_{11}^{K}, \quad \text{(C14)}$$

$$\langle \Sigma_{10}^2 P_S \frac{1}{2}^- | H_{\text{hyp}}^{\text{GBE}} | \Sigma_8^4 P_M \frac{1}{2}^- \rangle = 0, \quad \text{(C15)}$$

$$\langle \Sigma_{10}^2 P_S \frac{1}{2}^- | H_{\text{hyp}}^{\text{GBE}} | \Sigma_8^2 P_M \frac{1}{2}^- \rangle = \frac{1}{2}P_{00}^{\pi} + \frac{1}{6}P_{00}^{uu} + \frac{1}{3}P_{00}^{us} - P_{00}^{K} + \frac{3}{2}P_{11}^{\pi} + \frac{1}{2}P_{11}^{uu} + P_{11}^{us} - 3P_{11}^{K}, \quad \text{(C16)}$$

$$\langle \Sigma_8^4 P_M \frac{1}{2}^- | H_{\text{hyp}}^{\text{GBE}} | \Sigma_{10}^2 P_S \frac{1}{2}^- \rangle = 0, \quad \text{(C17)}$$

$$\langle \Sigma_8^4 P_M \frac{1}{2}^- | H_{\text{hyp}}^{\text{GBE}} | \Sigma_8^4 P_M \frac{1}{2}^- \rangle = -P_{00}^{\pi} - \frac{1}{3}P_{00}^{uu} + \frac{1}{3}P_{00}^{us} - P_{00}^{K} + P_{11}^{us} + 3P_{11}^{K} + 6T_{11}^{K} + 2T_{11}^{us}, \quad \text{(C18)}$$

$$\langle \Sigma_8^4 P_M \frac{1}{2}^- | H_{\text{hyp}}^{\text{GBE}} | \Sigma_8^2 P_M \frac{1}{2}^- \rangle = -6T_{11}^{K} - 2T_{11}^{us}, \quad \text{(C19)}$$

$$\langle \Sigma_8^2 P_M \frac{1}{2}^- | H_{\text{hyp}}^{\text{GBE}} | \Sigma_{10}^2 P_S \frac{1}{2}^- \rangle = \frac{1}{2}P_{00}^{\pi} + \frac{1}{6}P_{00}^{uu} + \frac{1}{3}P_{00}^{us} - P_{00}^{K} + \frac{3}{2}P_{11}^{\pi} + \frac{1}{2}P_{11}^{uu} + P_{11}^{us} - 3P_{11}^{K}, \quad \text{(C20)}$$

$$\langle \Sigma_8^2 P_M \frac{1}{2}^- | H_{\text{hyp}}^{\text{GBE}} | \Sigma_8^4 P_M \frac{1}{2}^- \rangle = -6T_{11}^{K} - 2T_{11}^{us}, \quad \text{(C21)}$$

$$\langle \Sigma_8^2 P_M \frac{1}{2}^- | H_{\text{hyp}}^{\text{GBE}} | \Sigma_8^2 P_M \frac{1}{2}^- \rangle = -\frac{1}{2}P_{00}^{\pi} - \frac{1}{6}P_{00}^{uu} - \frac{4}{3}P_{00}^{us} - 5P_{00}^{K} + \frac{3}{2}P_{11}^{\pi} + \frac{1}{2}P_{11}^{uu} + 3P_{11}^{K}. \quad \text{(C22)}$$

The matrix elements of $H_{\text{hyp}}^{\text{GBE}}$ in the resonances $|\Xi_{10}^2 P_S \frac{1}{2}^-\rangle$, $|\Xi_8^4 P_M \frac{1}{2}^-\rangle$ and $|\Xi_8^2 P_M \frac{1}{2}^-\rangle$ are

$$\langle \Xi_{10}^2 P_S \frac{1}{2}^- | H_{\text{hyp}}^{\text{GBE}} | \Xi_{10}^2 P_S \frac{1}{2}^- \rangle = \frac{2}{3}P_{00}^{us} - \frac{2}{3}P_{00}^{ss} - 2P_{00}^{K} - 2P_{11}^{us} + 2P_{11}^{ss} + 6P_{11}^{K}, \quad \text{(C23)}$$

$$\langle \Xi_{10}^2 P_S \frac{1}{2}^- | H_{\text{hyp}}^{\text{GBE}} | \Xi_8^4 P_M \frac{1}{2}^- \rangle = 0, \quad \text{(C24)}$$

$$\langle \Xi_{10}^2 P_S \frac{1}{2}^- | H_{\text{hyp}}^{\text{GBE}} | \Xi_8^2 P_M \frac{1}{2}^- \rangle = -\frac{1}{3}P_{00}^{us} - \frac{2}{3}P_{00}^{ss} + P_{00}^{K} - P_{11}^{us} - 2P_{11}^{ss} + 3P_{11}^{K}, \quad \text{(C25)}$$

$$\langle \Xi_8^4 P_M \frac{1}{2}^- | H_{\text{hyp}}^{\text{GBE}} | \Xi_{10}^2 P_S \frac{1}{2}^- \rangle = 0, \quad \text{(C26)}$$

$$\langle \Xi_8^4 P_M \frac{1}{2}^- | H_{\text{hyp}}^{\text{GBE}} | \Xi_8^4 P_M \frac{1}{2}^- \rangle = \frac{1}{3}P_{00}^{us} - \frac{4}{3}P_{00}^{ss} - P_{00}^{K} + P_{11}^{us} + 3P_{11}^{K} + 6T_{11}^{K} + 2T_{11}^{us}, \quad \text{(C27)}$$

$$\langle \Xi_8^4 P_M \frac{1}{2}^- | H_{\text{hyp}}^{\text{GBE}} | \Xi_8^2 P_M \frac{1}{2}^- \rangle = -6T_{11}^{K} - 2T_{11}^{us}, \quad \text{(C28)}$$

$$\langle \Xi_8^2 P_M \frac{1}{2}^- | H_{\text{hyp}}^{\text{GBE}} | \Xi_{10}^2 P_S \frac{1}{2}^- \rangle = -\frac{1}{3}P_{00}^{us} - \frac{2}{3}P_{00}^{ss} + P_{00}^{K} - P_{11}^{us} - 2P_{11}^{ss} + 3P_{11}^{K}, \quad \text{(C29)}$$

$$\langle \Xi_8^2 P_M \frac{1}{2}^- | H_{\text{hyp}}^{\text{GBE}} | \Xi_8^4 P_M \frac{1}{2}^- \rangle = -6T_{11}^{K} - 2T_{11}^{us}, \quad \text{(C30)}$$

$$\langle \Xi_8^2 P_M \frac{1}{2}^- | H_{\text{hyp}}^{\text{GBE}} | \Xi_8^2 P_M \frac{1}{2}^- \rangle = -\frac{4}{3}P_{00}^{us} - \frac{2}{3}P_{00}^{ss} - 5P_{00}^{K} + 2P_{11}^{ss} + 3P_{11}^{K}. \quad \text{(C31)}$$

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