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Flavor $SU(3)$ topological diagram and irreducible representation amplitudes for heavy meson charmless hadronic decays: mismatch and equivalence*

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Abstract: Flavor $SU(3)$ analysis of B meson charmless hadronic two light pseudoscalar decays can be formulated in two different ways. One is to construct the $SU(3)$ irreducible representation amplitude (IRA) according to effective Hamiltonian transformation properties, and the other is to draw the topological diagrams (TDA). We first point out that previous analyses of TDA and IRA approaches do not match in several aspects, in particular a few $SU(3)$ independent amplitudes have been overlooked in the TDA approach. This has caused confusions in the past and sometimes resulted in incorrect interpretation of data. We then demonstrate that only if these amplitudes are included, a consistent and unified picture can be obtained. With the new TDA amplitudes, all charmless hadronic decays of heavy meson must have nonzero direct CP symmetries as already predicted by the IRA approach. In addition to their notable impact on CP asymmetry, the new amplitudes are also important to extract new physics information.

Keywords: heavy flavor physics, flavor $SU(3)$ symmetry, topological diagrams

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1 Introduction

Charmless hadronic B to two light pseudoscalar decays provide an ideal platform to extract the CKM matrix elements, test the standard model description of CP violation and look for new physics effects beyond the standard model (SM). Experimentally, quite a number of physical observables like branching fractions, CP asymmetries and polarizations have been precisely measured by experiments at the electron-position colliders and hadron colliders. For a collection of these results, please see Refs. [1, 2]. On the other hand, theoretical calculations of decay amplitudes greatly rely on the factorization ansatz. Depending on explicit realizations of factorization, several QCD-based dynamic approaches have been established, such as QCDF [3, 4], PQCD [5–7], SCET [8, 9]. Apart from factorization approaches, the flavor $SU(3)$ symmetry has been also wildly used in two-body and three-body heavy meson decays [10–23]. An advantage of this method is its independence on the detailed dynamics in factorization. Since the $SU(3)$ invariant amplitudes can be determined by fitting the data, the $SU(3)$ analysis provides a bridge between experimental data and the dynamic approaches.

In the literature, the $SU(3)$ analysis has been formulated in two distinct ways. One is to derive the decay amplitudes correspond to various topological diagrams (TDA) [16–21], and another is to construct the $SU(3)$ irreducible representation amplitude (IRA) by decomposing effective Hamiltonian according to irreducible representations [11–15]. These two methods should give the same physical results in the $SU(3)$ limit when all relevant contributions are taken into account. However, as we will show we find that previous analyses in the literature using these two methods do not match consistently in several ways, in particular a few $SU(3)$ independent amplitudes have been overlooked in the TDA approach for a heavy meson decaying into two light pseudoscalar $SU(3)$

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octet (or U(3) nonet) mesons. In this work, we carry out a systematic analysis and identify possible missing amplitudes in order to establish the consistence between the RRA and TDA approaches. We find that these new amplitudes are sizable and may affect direct CP asymmetries in some channels significantly. An important consequence of the inclusion of these amplitudes is that for any charmless hadronic decay of heavy mesons, the direct CP symmetry cannot be identically zero, though in some cases it is tiny.

The rest of this paper is organized as follows. In Sec. **2**, we introduce the $SU(3)$ analysis using the TDA and IRA approaches. We summarize those amplitudes already discussed in the literature. In Sec. **3**, we first point out the mismatch problem, and then identify those missed amplitudes. The complete sets of $SU(3)$ independent amplitudes in both IRA and TDA approaches will be given to establish equivalence of these two approaches. In Sec. **4**, we include the missing amplitudes to discuss the implications for hadronic charmless decays of B and D and draw our conclusions.

2 Basics for IRA and TDA approaches

2.1 $SU(3)$ structure

We start with the electroweak effective Lagrangian for hadronic charmless B meson decays in the SM. The Hamiltonian $\mathcal{H}_{\rm eff}$ responsible for such kind of decays at one loop level in electroweak interactions is given by [24–26]:

$$\mathcal{H}_{\rm eff} = \frac{G_{\rm F}}{\sqrt{2}}\Big\{V_{ub}V_{uq}^*\big[C_1O_1+C_2O_2\big] -V_{tb}V_{tq}^*\Big[\sum_{i=3}^{10}C_iO_i\Big]\Big\}+{\rm h.c.}, \tag{1}$$

where O_i is a four-quark operator or a moment type operator. The four-quark operators O_i are given as follows:

$$\begin{aligned}
O_1 &= (\bar q^i u^j)_{\rm V-A}(\bar u^j b^i)_{\rm V-A},\\
O_2 &= (\bar q u)_{\rm V-A}(\bar u b)_{\rm V-A},\\
O_3 &= (\bar q b)_{\rm V-A}\sum_{q'}(\bar q' q')_{\rm V-A},\\
O_4 &= (\bar q^i b^j)_{\rm V-A}\sum_{q'}(\bar q'^j q'^i)_{\rm V-A},\\
O_5 &= (\bar q b)_{\rm V-A}\sum_{q'}(\bar q' q')_{V+A},\\
O_6 &= (\bar q^i b^j)_{\rm V-A}\sum_{q'}(\bar q'^j q'^i)_{V+A},\\
O_7 &= \frac{3}{2}(\bar q b)_{\rm V-A}\sum_{q'}e_{q'}(\bar q' q')_{V+A},\\
O_8 &= \frac{3}{2}(\bar q^i b^j)_{\rm V-A}\sum_{q'}e_{q'}(\bar q'^j q'^i)_{V+A},\\
O_9 &= \frac{3}{2}(\bar q b)_{\rm V-A}\sum_{q'}e_{q'}(\bar q' q')_{\rm V-A},\\
O_{10} &= \frac{3}{2}(\bar q^i b^j)_{\rm V-A}\sum_{q'}e_{q'}(\bar q'^j q'^i)_{\rm V-A}.
\end{aligned} \tag{2}$$

In the above the q denotes a d quark for the $b\to d$ transition or an s quark for the $b\to s$ transition, while $q'=u,d,s$.

At the hadron level, QCD penguin operators behave as the $\bar{\mathbf{3}}$ representation while tree and electroweak penguin operators can be decomposed in terms of a vector $H_{\bar{\mathbf 3}}$, a traceless tensor antisymmetric in upper indices, $H_{\mathbf 6}$, and a traceless tensor symmetric in upper indices, $H_{\overline{\mathbf{15}}}$. For the $\Delta S=0(b\to d)$ decays, the non-zero components of the effective Hamiltonian are [11, 14, 15]:

$$\begin{aligned}
(H_{\bar{\mathbf 3}})^2 &= 1,\\
(H_{\mathbf 6})^{12}_1 &= -(H_{\mathbf 6})^{21}_1=(H_{\mathbf 6})^{23}_3=-(H_{\mathbf 6})^{32}_3=1,\\
2(H_{\overline{\mathbf{15}}})^{12}_1 &= 2(H_{\overline{\mathbf{15}}})^{21}_1=-3(H_{\overline{\mathbf{15}}})^{22}_2=-6(H_{\overline{\mathbf{15}}})^{23}_3\\
&= -6(H_{\overline{\mathbf{15}}})^{32}_3=6,
\end{aligned} \tag{3}$$

and all other remaining entries are zero. For the $\Delta S=-1(b\to s)$ decays the nonzero entries in the $H_{\bar{\mathbf 3}}$, $H_{\mathbf 6}$, $H_{\overline{\mathbf{15}}}$ can be obtained from Eq. (3) with the exchange $2\leftrightarrow 3$ corresponding to the $d\leftrightarrow s$ exchange.

The above Hamiltonian can induce a B_i meson to decay into two light pseudoscalar nonet M^i_j mesons. There are three B mesons $(B_i)=(B(\bar bu),B(\bar bd),B(\bar bs))$ which form a flavor $SU(3)$ fundamental representation 3. The light pseudoscalar mesons M^i_j contain nine hadrons:

$$(M^i_j) = \begin{pmatrix} \frac{\pi^0}{\sqrt2}+\frac{\eta_8}{\sqrt6} & \pi^+ & K^+\\ \pi^- & -\frac{\pi^0}{\sqrt2}+\frac{\eta_8}{\sqrt6} & K^0\\ K^- & \overline{K}^0 & -2\frac{\eta_8}{\sqrt6}, \end{pmatrix} +\frac{1}{\sqrt3}\begin{pmatrix}\eta_1 & 0 & 0\\ 0 & \eta_1 & 0\\ 0 & 0 & \eta_1,\end{pmatrix}, \tag{4}$$

The first term forms an $SU(3)$ octet and the second term is a singlet. Grouping them together it is a nonet of U(3). It is similar for other light mesons, like the vector or axial-vector mesons.

2.2 Irreducible Representation Amplitudes

To obtain irreducible representation amplitudes for $B\to PP$ (P is an element in M^i_j) decays, one takes the various representations in Eq. (3) and uses one B_i and light meson M^i_j to contract all indices in the following manner

$$\begin{aligned}\mathcal{A}_t^{\rm IRA} =& A_3^T B_i(H_{\bar 3})^i(M)_k^j(M)_j^k + C_3^T B_i(M)_j^i(M)_k^j(H_{\bar 3})^k + B_3^T B_i(H_{\bar 3})^i(M)_k^k(M)_j^j + D_3^T B_i(M)_j^i(H_{\bar 3})^j(M)_k^k \\ &+ A_6^T B_i(H_6)_k^{ij}(M)_j^l(M)_l^k + C_6^T B_i(M)_j^i(H_6)_k^{jl}(M)_l^k + B_6^T B_i(H_6)_k^{ij}(M)_j^k(M)_l^l \\ &+ A_{15}^T B_i(H_{\overline{15}})_k^{ij}(M)_j^l(M)_l^k + C_{15}^T B_i(M)_j^i(H_{\overline{15}})_l^{jk}(M)_k^l + B_{15}^T B_i(H_{\overline{15}})_k^{ij}(M)_j^k(M)_l^l. \qquad (5)\end{aligned}$$

There also exist the penguin amplitudes $A_p^{\rm IRA}$ which can be obtained by the replacements $A_i^T \to A_i^P$, $B_i^T \to B_i^P$, $C_i^T \to C_i^P$ and $D_i^T \to D_i^P$ (i=3,6,15).

Expanding the above $\mathcal{A}_t^{\rm IRA}$, one obtains $B\to PP$ amplitudes in the first two columns in Tables 1 and 2. Notice that the amplitude A_6^T can be absorbed into B_6^T and C_6^T with the following redefinition:

$$C_6^{T\prime} = C_6^T - A_6^T,\, B_6^{T\prime} = B_6^T + A_6^T. \qquad (6)$$

Thus we have 18 (tree and penguin contribute 9 each) $SU(3)$ independent complex amplitudes. Since the phase of one amplitude can be freely chosen, there are 35 independent parameters to describe the two-body $B\to PP$ decays. If one also considers η–η' (or η_8–η_1) mixing, one more parameter, the mixing angle θ, is requested making total 36 independent parameters.

2.3 Topological diagram amplitudes

The topological diagram amplitudes are obtained by diagrams which connect initial and final states by quark lines as shown in Fig. 1 with vertices determined by the operators in Eq. (2). As shown in many references for instance Ref. [21], they are classified as follows:

1) T denoting the color-allowed tree amplitude with W emission;

2) C, denoting the color-suppressed tree diagram;

3) E denoting the W-exchange diagram;

4) P, corresponding to the QCD penguin contributions;

5) S, being the flavor singlet QCD penguin;

6) $P_{\rm EW}$, the electroweak penguin.

In addition, there exists annihilation diagrams, usually abbreviated as A. In Fig. 1, we have only shown the diagrams for tree operators, and those for penguin operators can be derived similarly.

The electroweak penguins contain the color-favored contribution $P_{\rm EW}$ and the color-suppressed one $P_{\rm EW}^C$. The electroweak penguin operators can be re-expressed as:

$$\bar q b\sum_{q'} e_{q'}\bar q' q' = \bar q b \bar u u - \frac{1}{3}\bar q b\sum_{q'}\bar q' q', \qquad (7)$$

where the second part can be incorporated into the penguins transforming as a $\bar 3$ of $SU(3)$. The contribution from $\bar q b\bar u u$ is similar to tree operators, and thus we will use the symbol P_T and P_C to denote this electro-weak penguin contribution. The $\bar q b\sum_{q'}\bar q' q'$ is a flavor triplet whose contribution P', as far as flavor $SU(3)$ structure is concerned, can be absorbed into penguin contribution. We can write

$$P_{\rm EW} = P_T - \frac{1}{3}P',\, P_{\rm EW}^C = P_C - \frac{1}{3}P'^C. \qquad (8)$$

The three penguin type of amplitudes P, P' and P'^C, can be grouped together. We can redefine P by $P+P'+P'^C$.

Actually these TDAs can be derived in a similar way as done for IRAs earlier by indicating $\bar q u\bar u b$ (omitting the Lorentz indices) by $\bar H_k^{ij}$. For $\Delta S=0$, the non-zero elements are $\bar H_1^{12}=1$ and for $\Delta S=-1$, $\bar H_1^{13}=1$. The penguin contribution (including P, P' and P'^C) is an $SU(3)$ triplet $\bar H^i$ with $\bar H^2=1$ for the $b\to d$ transition and H^3 for the $b\to s$ transition. Eq. (7) implies that the loop induced term proportional to $V_{tq}^* V_{tb}$ has both $\bar H_k^{ij}$ and $\bar H^i$. Note that $\bar H_k^{ij}$ is no longer traceless.

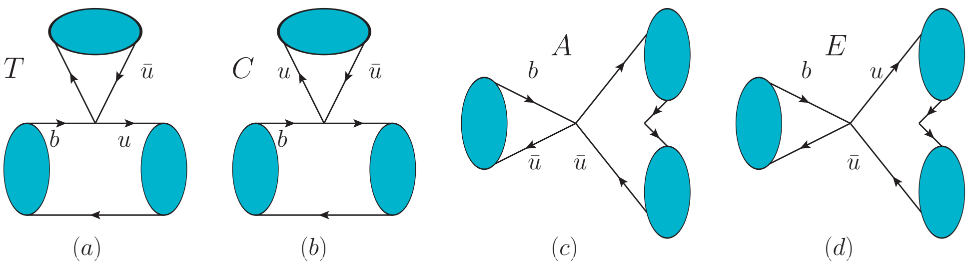

Fig. 1. (color online) Topological diagrams induced by tree amplitudes. The four panels denote: the color-allowed tree amplitude (T), color-suppressed tree amplitude (C), annihilation (A), and W-exchange (E).

Table 1. Decay amplitudes for two-body B decays induced by the $b\to d$ transition. Only tree amplitudes whose CKM matrix elements are $V_{ub}V_{ud}^*$ are shown, while penguin amplitudes can be obtained by the replacement given in Eq. (16).

channel	IRA	TDA
$B^-\to\pi^0\pi^-$	$4\sqrt{2}C_{15}^T$	$\frac{1}{\sqrt{2}}(C+T)$
$B^-\to\pi^-\eta_8$	$\sqrt{\frac{2}{3}}\left(A_6^T+3A_{15}^T+C_3^T-C_6^T+3C_{15}^T\right)$	$\frac{1}{\sqrt{6}}(2A+C+2P^u+T)$
$B^-\to\pi^-\eta_1$	$\frac{1}{\sqrt{3}}(2A_6^T+6A_{15}^T+3B_6^T+9B_{15}^T+2C_3^T+C_6^T+3C_{15}^T+3D_3^T)$	$\frac{1}{\sqrt{3}}(2A+C+3A_S^u+2P^u+3S^u+T)$
$B^-\to K^0K^-$	$A_6^T+3A_{15}^T+C_3^T-C_6^T-C_{15}^T$	$A+P^u$
$\overline{B}^0\to\pi^+\pi^-$	$2A_3^T-A_6^T+A_{15}^T+C_3^T+C_6^T+3C_{15}^T$	$E+P^u+2P_A^u+T$
$\overline{B}^0\to\pi^0\pi^0$	$2A_3^T-A_6^T+A_{15}^T+C_3^T+C_6^T-5C_{15}^T$	$-C+E+P^u+2P_A^u$
$\overline{B}^0\to\pi^0\eta_8$	$\frac{1}{\sqrt{3}}(-A_6^T+5A_{15}^T-C_3^T+C_6^T+C_{15}^T)$	$\frac{1}{\sqrt{3}}(E-P^u)$
$\overline{B}^0\to\pi^0\eta_1$	$-\frac{1}{\sqrt{6}}(2A_6^T-10A_{15}^T+3B_6^T-15B_{15}^T+2C_3^T+C_6^T-5C_{15}^T+3D_3^T)$	$\frac{1}{\sqrt{6}}(3E_S^u+2E-2P^u-3S^u)$
$\overline{B}^0\to K^+K^-$	$2\left(A_3^T+A_{15}^T\right)$	$E+2P_A^u$
$\overline{B}^0\to K^0\overline{K}^0$	$2A_3^T+A_6^T-3A_{15}^T+C_3^T-C_6^T-C_{15}^T$	$P^u+2P_A^u$
$\overline{B}^0\to\eta_8\eta_8$	$2A_3^T+A_6^T-A_{15}^T+\frac{C_3^T}{3}-C_6^T+C_{15}^T$	$\frac{1}{3}\left(C+E+P^u+6P_A^u\right)$
$\overline{B}^0\to\eta_8\eta_1$	$\frac{1}{3\sqrt{2}}(-6A_6^T+6A_{15}^T-9B_6^T+9B_{15}^T+2C_3^T-3C_6^T+3C_{15}^T+3D_3^T)$	$\frac{1}{3\sqrt{2}}(3E_S^u+2C+2E+2P^u+3S^u)$
$\overline{B}^0\to\eta_1\eta_1$	$\frac{2}{3}\left(3A_3^T+9B_3^T+C_3^T+3D_3^T\right)$	$\frac{2}{3}\left(3E_S^u+C+E+P^u+3P_A^u+3S^u+9S_S^u\right)$
$\overline{B}_s^0\to\pi^0K^0$	$\frac{1}{\sqrt{2}}(A_6^T+A_{15}^T-C_3^T-C_6^T+5C_{15}^T)$	$\frac{1}{\sqrt{2}}(C-P^u)$
$\overline{B}_s^0\to\pi^-K^+$	$-A_6^T-A_{15}^T+C_3^T+C_6^T+3C_{15}^T$	P^u+T
$\overline{B}_s^0\to K^0\eta_8$	$\frac{1}{\sqrt{6}}(A_6^T+A_{15}^T-C_3^T-C_6^T+5C_{15}^T)$	$\frac{1}{\sqrt{6}}(C-P^u)$
$\overline{B}_s^0\to K^0\eta_1$	$-\frac{1}{\sqrt{3}}(2A_6^T+2A_{15}^T+3B_6^T+3B_{15}^T-2C_3^T+C_6^T+C_{15}^T-3D_3^T)$	$\frac{1}{\sqrt{3}}(C+2P^u+3S^u)$

Table 2. Decay amplitudes for two-body B decays induced by the $b\to s$ transition. Only tree amplitudes whose CKM matrix elements are $V_{ub}V_{us}^*$ are shown, while penguin amplitudes can be obtained by the replacement given in Eq. (16).

channel	IRA	TDA
$B^-\to\pi^0K^-$	$\frac{1}{\sqrt{2}}(A_6^T+3A_{15}^T+C_3^T-C_6^T+7C_{15}^T)$	$\frac{1}{\sqrt{2}}(A+C+P^u+T)$
$B^-\to\pi^-\overline{K}^0$	$A_6^T+3A_{15}^T+C_3^T-C_6^T-C_{15}^T$	$A+P^u$
$B^-\to K^-\eta_8$	$-\frac{1}{\sqrt{6}}(A_6^T+3A_{15}^T+C_3^T-C_6^T-9C_{15}^T)$	$\frac{1}{\sqrt{6}}(-A+C-P^u+T)$
$B^-\to K^-\eta_1$	$\frac{1}{\sqrt{3}}(2A_6^T+6A_{15}^T+3B_6^T+9B_{15}^T+2C_3^T+C_6^T+3C_{15}^T+3D_3^T)$	$\frac{1}{\sqrt{3}}(2A+C+3E_S^u+2P^u+3S^u+T)$
$\overline{B}^0\to\pi^+K^-$	$-A_6^T-A_{15}^T+C_3^T+C_6^T+3C_{15}^T$	P^u+T
$\overline{B}^0\to\pi^0\overline{K}^0$	$\frac{1}{\sqrt{2}}(A_6^T+A_{15}^T-C_3^T-C_6^T+5C_{15}^T)$	$\frac{1}{\sqrt{2}}(C-P^u)$
$\overline{B}^0\to\overline{K}^0\eta_8$	$\frac{1}{\sqrt{6}}(A_6^T+A_{15}^T-C_3^T-C_6^T+5C_{15}^T)$	$\frac{1}{\sqrt{6}}(C-P^u)$
$\overline{B}^0\to\overline{K}^0\eta_1$	$-\frac{1}{\sqrt{3}}(2A_6^T+2A_{15}^T+3B_6^T+3B_{15}^T-2C_3^T+C_6^T+C_{15}^T-3D_3^T)$	$\frac{1}{\sqrt{3}}(C+2P^u+3S^u)$
$\overline{B}_s^0\to\pi^+\pi^-$	$2\left(A_3^T+A_{15}^T\right)$	$E+2P_A^u$
$\overline{B}_s^0\to\pi^0\pi^0$	$2\left(A_3^T+A_{15}^T\right)$	$E+2P_A^u$
$\overline{B}_s^0\to\pi^0\eta_1$	$-\sqrt{\frac{2}{3}}\left(2A_6^T-4A_{15}^T+3B_6^T-6B_{15}^T+C_6^T-2C_{15}^T\right)$	$\frac{1}{\sqrt{6}}(3E_S^u+C+2E)$
$\overline{B}_s^0\to K^+K^-$	$2A_3^T-A_6^T+A_{15}^T+C_3^T+C_6^T+3C_{15}^T$	$E+P^u+2P_A^u+T$
$\overline{B}_s^0\to K^0\overline{K}^0$	$2A_3^T+A_6^T-3A_{15}^T+C_3^T-C_6^T-C_{15}^T$	$P^u+2P_A^u$
$\overline{B}_s^0\to\eta_8\eta_8$	$2A_3^T-2A_{15}^T+\frac{4C_3^T}{3}-4C_{15}^T$	$\frac{1}{3}\left(-2C+E+4P^u+6P_A^u\right)$
$\overline{B}_s^0\to\eta_8\eta_1$	$\frac{1}{3}\sqrt{2}\left(6A_{15}^T+9B_{15}^T-2C_3^T+3C_{15}^T-3D_3^T\right)$	$-\frac{1}{3\sqrt{2}}(-3E_S^u+C-2E+4P^u+6S^u)$
$\overline{B}_s^0\to\eta_1\eta_1$	$\frac{2}{3}\left(3A_3^T+9B_3^T+C_3^T+3D_3^T\right)$	$\frac{2}{3}\left(3E_S^u+C+E+P^u+3P_A^u+3S^u+9S_S^u\right)$

The tree amplitude is given as

$$\mathcal{A}_t^{\rm TDA}=T\times B_i(M)^i_j\bar{H}^{jl}_k(M)^k_l+C\times B_i(M)^i_j\bar{H}^{lj}_k(M)^k_l +A\times B_i\bar{H}^{il}_j(M)^j_k(M)^k_l+E\times B_i\bar{H}^{li}_j(M)^j_k(M)^k_l, \tag{9}$$

while the penguin amplitude is given as:

$$\begin{aligned}\mathcal{A}_p^{\rm TDA} = &P\times B_i(M)^i_j(M)^j_k\bar{H}^k+S\times B_i(M)^i_j\bar{H}^j(M)^k_k \\ &+P_A\times B_i\bar{H}^i(M)^j_k(M)^k_j+P_T\times B_i(M)^i_j\bar{H}^{jl}_k(M)^k_l \\ &+P_C\times B_i(M)^i_j\bar{H}^{lj}_k(M)^k_l.\end{aligned} \tag{10}$$

Expanding Eq. (9) in the above and Eq. (12) to be given in the following, we obtain the decay amplitudes for $B\to PP$ in the third column in Tables 1 and 2. It is necessary to point out that the singlet contribution in the form M^j_j requires multi-gluon exchanges. One might naively think that its contributions are small compared with other contributions because more gluons are exchanged. However, at energy scale of B decays, the strong couplings are not necessarily very small resulting in non-negligible contributions. One should include them for a complete analysis.

3 Mismatch and equivalence

From previous discussions, one can see that the total decay amplitudes for $B\to PP$ decays for IRA and TDA can be written as

$$\begin{aligned}\mathcal{A}^{\rm IRA} &= V_{ub}V^*_{uq}\mathcal{A}^{\rm IRA}_t+V_{tb}V^*_{tq}\mathcal{A}^{\rm IRA}_p, \\ \mathcal{A}^{\rm TDA} &= V_{ub}V^*_{uq}\mathcal{A}^{\rm TDA}_t+V_{tb}V^*_{tq}\mathcal{A}^{\rm TDA}_p.\end{aligned} \tag{11}$$

For the amplitudes given in the previous section, it is clear that for both A^i_t and A^i_p, the amplitudes do not have the same number of independent parameters: there are 18 independent complex amplitudes in the IRA, while only 9 amplitudes are included in the TDA. There seems to be a mismatch between the IRA and TDA approaches. However since both approaches are rooted in the same basis, the same physical results should be obtained. It is anticipated that some amplitudes have been missed and must be added.

A close inspection shows that several topological diagrams were not included in the previous TDA analysis. For the tree amplitudes we show the relevant diagrams in Fig. 2. The missing penguin diagrams can be obtained similarly. Since there are electroweak penguin operator contributions, as far as the $SU(3)$ irreducible components are concerned, the effective Hamiltonian have the same $SU(3)$ structure as the tree contributions. Taking these contributions into account, we have the following topological amplitudes:

$$\begin{aligned}\mathcal{A}_t^{\prime\rm TDA} = &S^uB_i(M)^i_j\bar{H}^{lj}_l(M)^k_k+P^uB_i(M)^i_j(M)^j_k\bar{H}^{lk}_l \\ &+P^u_AB_i\bar{H}^{li}_l(M)^j_k(M)^k_j+S^u_SB_i\bar{H}^{li}_l(M)^j_j(M)^k_k \\ &+E^u_SB_i\bar{H}^{ji}_l(M)^l_j(M)^k_k+A^u_SB_i\bar{H}^{ij}_l(M)^l_j(M)^k_k,\end{aligned} \tag{12}$$

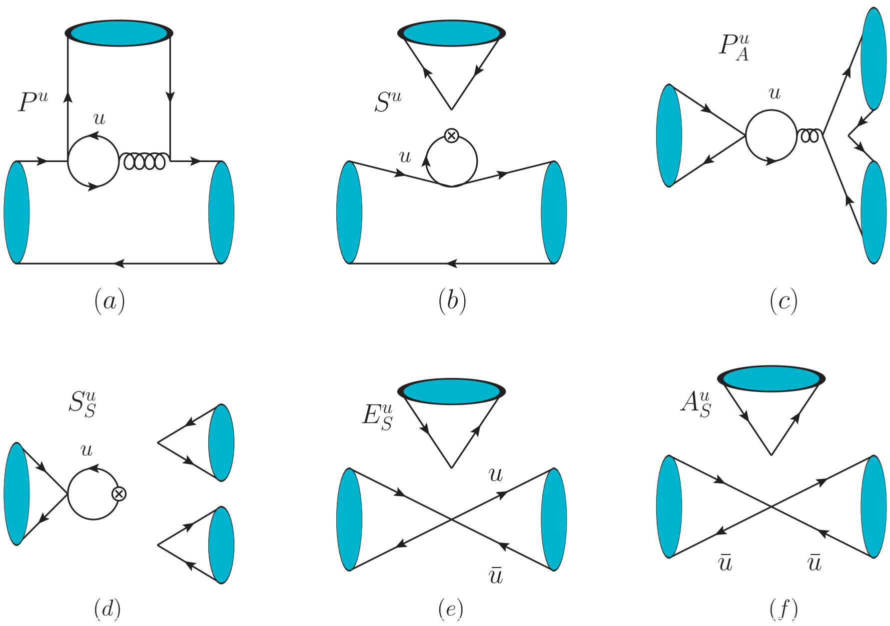

Fig. 2. (color online) Typical diagrams for the newly introduced amplitudes in Eq. (12). The crossed vertex denotes the $\bar{u}u$ annihilation and the creation of two or more gluons.

$$\begin{aligned}\mathcal{A}_p'^{\rm TDA} &= S_S^t B_i \bar{H}^i (M)_j^j (M)_k^k + A^t B_i \bar{H}_j^{il} (M)_k^j (M)_l^k \\ &+ E^t B_i \bar{H}_k^{ji} (M)_l^k (M)_j^l + E_S^t B_i \bar{H}_l^{ji} (M)_j^l (M)_k^k \\ &+ A_S^t B_i \bar{H}_l^{ij} (M)_j^l (M)_k^k. \end{aligned} \tag{13}$$

The mismatch problem can be partly traced to the fact that $\bar{H}_k^{ij}$ defined in the TDA analysis is not traceless, that is $\bar{H}_l^{lj} \neq 0$. Because of this fact, B_i and the two M_j^i can contract with $\bar{H}_l^{lj}$ to form $SU(3)$ invariant amplitudes and also the trace for M_j^i is not zero when η_1 is included in the final states. While in the previous discussions, these terms are missed.

One can expand the above new terms to obtain the results for tree amplitudes in Tables 1 and 2. With these new amplitudes at hand, one can derive the relation between the two sets of amplitudes:

$$\begin{aligned}A_3^T &= -\frac{A}{8}+\frac{3E}{8}+P_A^u, \qquad B_3^T = S_S^u + \frac{3E_S^u - A_S^u}{8}, \\ C_3^T &= \frac{1}{8}(3A-C-E+3T)+P^u, \\ D_3^T &= S^u + \frac{1}{8}(3C-E_S^u+3A_S^u-T), \\ B_6'^T &= \frac{1}{4}(A-E+A_S^u-E_S^u), \quad C_6'^T = \frac{1}{4}(-A-C+E+T), \\ A_{15}^T &= \frac{A+E}{8}, \quad B_{15}^T = \frac{A_S^u+E_S^u}{8}, \quad C_{15}^T = \frac{C+T}{8}. \end{aligned} \tag{14}$$

Here we have absorbed the A_6^T into $B_6'^T$ and $C_6'^T$. In the appendix, we give a direct derivation of relations between IRA and TDA amplitudes, in which the amplitude A_6^T is kept.

Naively there are total 10 tree amplitudes and 10 penguin amplitudes defined in Eq. (9,12). However, only 9 of the 10 tree amplitudes are independent. Choosing the option to eliminate the W-exchange E, we can express the TDA amplitudes in terms of the IRA ones:

$$\begin{aligned} T+E &= 4A_{15}^T + 2C_6'^T + 4C_{15}^T, \\ C-E &= -4A_{15}^T - 2C_6'^T + 4C_{15}^T, \\ A+E &= 8A_{15}^T, \qquad P^u - E = -5A_{15}^T + C_3^T - C_6'^T - C_{15}^T, \\ P_A^u + \frac{E}{2} &= A_3^T + A_{15}^T, \qquad E_S^u + E = 4A_{15}^T - 2B_6'^T + 4B_{15}^T, \\ A_S^u - E &= -4A_{15}^T + 2B_6'^T + 4B_{15}^T, \\ S_S^u - \frac{E}{2} &= -2A_{15}^T + B_3^T + B_6'^T - B_{15}^T, \\ S^u + E &= 4A_{15}^T - B_6'^T - B_{15}^T + C_6'^T - C_{15}^T + D_3^T. \end{aligned} \tag{15}$$

The analysis of penguin contributions is similar with the replacement for TDA amplitudes:

$$\begin{aligned} T \to P_T, C \to P_C, A \to A^t, P^u \to P, E \to E^t, \\ P_A^u \to P_A, E_S^u \to E_S^t, A_S^u \to A_S^t, S_S^u \to S_S^t, S^u \to S. \end{aligned} \tag{16}$$

From the above discussions we see that the two sets of amplitudes in IRA and TDA can be mutually expressed by each other. The IRA and TDA approaches are completely equivalent. As long as all amplitudes are taken into account in the analysis, they give the same results for $B \to PP$ decays, and we expect the equivalence for other decays[1)].

4 Discussions and conclusions

We now make a few remarks about our results obtained.

Several missing terms in the TDA analysis involve the trace M_j^j. The trace actually singles out the singlet in the nonet representation M_j^i. To have a color singlet in the diagram shown in Figs. 1 and 2, the single M_j^j need to exchange two or more gluons. As pointed out earlier that these contributions are expected to be small compared with other contributions. However, at energy scale of B decays, the strong couplings are not necessarily very small resulting in non-negligible contributions. Terms associated with the trace $\bar{H}_l^{lj}$ actually can be thought of as turning the tree operator into penguin operator with u quark exchange in the loop whose Wilson coefficient contains the large logarithms $\ln(\mu/m_u)$ which can also make non-negligible contributions. One should include them for a complete analysis.

Recently, Ref. [15] has performed a fit of $B \to PP$ decays in the IRA scheme. Depending on various options to use the data, four cases are considered in Ref. [15]. As an example, we quote the results in their case 4:

$$\begin{aligned} |C_{\bar 3}^T| &= -0.211 \pm 0.027, \\ \delta_{\bar 3}^T &= (-140 \pm 6)^\circ, \\ |B_{\overline{15}}^T| &= -0.038 \pm 0.016, \\ \delta_{B_{\overline{15}}^T} &= (78 \pm 48)^\circ, \end{aligned} \tag{17}$$

where the magnitudes and strong phases relative to $C_{\bar 3}^P$ have been given. From Eq. (14), one can see that the $C_{\bar 3}^T$ is a combination of color-allowed tree T, color-suppressed tree amplitude C and others while the $B_{\overline{15}}^T$ corresponds to $(A_S^u + T_{TS})/8$ in TDA approach. The fitted result in Eq. (17) indicates that compared to $C_{\bar 3}^T$, the $B_{\overline{15}}^T$ can reach 20% in magnitude, and more importantly, the strong phases are different significantly. The $B_{\overline{15}}^T$, equivalently A_S^u and T_{TS}, have non-negligible contributions supporting our call for a complete analysis. With more and more accurate data for $B \to PP$ from experiments,

1) In a recent study [27], TDA amplitudes have been obtained. However, the independence of amplitudes is not discussed in TDA approach.

one can now carry out a more careful analysis to obtain the amplitudes and derive implications for model calculations of the relevant amplitudes.

Without the new contributions in the TDA analysis, some of the amplitudes only have terms proportional to $V_{tq}^*V_{tb}$, such as $\overline{B}^0\to K^0\bar{K}^0$ and $\overline{B}_s^0\to K^0\bar{K}^0$. In Ref. [21], the amplitudes for $\overline{B}^0\to K^0\bar{K}^0$ are given as

$$\mathcal{A}(\overline{B}^0\to K^0\bar{K}^0)=V_{tb}V_{td}^*\left(P-\frac{1}{2}P_{\rm EW}^C+2P_A\right), \quad (18)$$

where in our work, we have observed the electro-weak penguin into the QCD penguin amplitudes. This implies that CP violation in these two decays are identically zero. However, these two decay modes receive contributions from the new terms $P^u+2P_A^u$ which is multiplied by $V_{uq}^*V_{ub}$:

$$\mathcal{A}(\overline{B}^0\to K^0\bar{K}^0)=V_{ub}V_{ud}^*(P^u+2P_A^u)+V_{tb}V_{td}^*(P+2P_A). \quad (19)$$

In principle they can have non-zero CP violation. Therefore if one takes into account the missing tree and penguin amplitudes, an important consequence is that no charmless and hadronic B decay channel has a vanishing direct CP asymmetry.

Flavor $SU(3)$ symmetry is an approximate symmetry, and symmetry breaking sources exist in QCD, mostly caused by the unequal masses for the light u,d,s quarks. How $SU(3)$ breaking effect manifest itself is not completely clear. Experimental data [2, 28] for $\overline{B}^0\to K^-\pi^+$ and $\overline{B}_s\to K^+\pi^-$ agree with relations predicted for these two modes under $SU(3)$ symmetry [13, 29]. A more conclusive analysis is inevitable in light of the large amount of data from Belle II [30] and LHCb [31] in future. One should keep in mind that for such an appropriate analysis of $SU(3)$ symmetry breaking, one must take into account all the above amplitudes, otherwise, the missing amplitudes will be disguised as symmetry breaking effects. As we have shown above, the modification due to the missing amplitudes can reach 20%, which is comparable with the generic $SU(3)$ symmetry breaking effects. Thus the additional TDAs must be treated carefully to correctly interpret the data.

Our analysis is also applicable to other decay channels of heavy mesons and baryons. In the appendix, we give a discussion on the D meson decays. For charm quark decay, penguin operators are often negligible and the $\bar{3}$ representation does not contribute either. So there are five independent tree amplitudes, while in TDA only four amplitudes, T,C,E,A, are used for the global fit.

In summary, we have carried out an analysis comparing two different approaches, the irreducible representation amplitude and topological diagram amplitude, to study $B\to PP$ decays. We find that previous analyses in the literature using these two methods do not match consistently in several ways. A few $SU(3)$ independent amplitudes have been overlooked in the TDA approach. Taking these new amplitudes into account, we find a consistent description in both approaches. These new amplitudes can affect direct CP asymmetries in some channels significantly. A consequence is that for any charmless hadronic decays of heavy mesons, the direct CP symmetry cannot be identically zero. With more data become available, we can have a a better understanding of the role of flavor $SU(3)$ symmetry in B decays.

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Appendix A

A derivation of decomposition

Using $O_1^{12}=\bar{u}b\bar{d}u$ as an example, we have the decomposition of tree operator:

$$O_1^{12} = \frac{1}{8}O_{\overline{15}}+\frac{1}{4}O_6-\frac{1}{8}O_{\bar{3}}+\frac{3}{8}O_{\bar{3}'}, \quad (A1)$$

with

$$\begin{aligned} O_{\overline{15}} &= 3\bar{u}b\bar{d}u+\bar{d}b\bar{u}u-2\bar{d}b\bar{d}d-\bar{s}b\bar{d}s-\bar{d}b\bar{s}s, \\ O_6 &= \bar{u}b\bar{d}u-\bar{d}b\bar{u}d-\bar{s}b\bar{d}s+\bar{d}b\bar{s}s, \\ O_{\bar{3}} &= \bar{d}b\bar{u}u+\bar{d}b\bar{d}d+\bar{d}b\bar{s}s, \\ O_{\bar{3}'} &= \bar{u}b\bar{d}u+\bar{d}b\bar{d}d+\bar{s}b\bar{d}s. \end{aligned} \quad (A2)$$

It implies:

$$\bar{H}_k^{ij}=\frac{1}{8}(H_{\overline{15}})_k^{ij}+\frac{1}{4}(H_6)_k^{ij}-\frac{1}{8}(H_{\bar{3}})^i\delta_k^j+\frac{3}{8}(H_{\bar{3}'})^j\delta_k^i. \quad (A3)$$

Substituting this expression into the amplitude T for instance, we have

$$\begin{aligned} &T\times B_i(M)_j^i\bar{H}_k^{jl}(M)_l^k=T\times B_i(M)_j^i(M)_l^k \\ &\times\left(\frac{1}{8}(H_{15})_k^{jl}+\frac{1}{4}(H_{\bar{6}})_k^{jl}-\frac{1}{8}(H_3)^j\delta_k^l+\frac{3}{8}(H_{3'})^l\delta_k^j\right), \quad (A4) \end{aligned}$$

contributing to

$$\begin{aligned} C_{15}^T &= \frac{1}{8}T+...,\\ C_6^T &= \frac{1}{4}T+...,\\ C_3^T &= \frac{3}{8}T+...,\\ D_3^T &= -\frac{1}{8}T+.... \end{aligned} \tag{A5}$$

Others TDA amplitudes can be analyzed similarly, and thus one has

$$\begin{aligned} &A_3^T=-\frac{A}{8}+\frac{3E}{8}+P_A^u, \quad B_3^T=S_S^u+\frac{3E_S^u-A_S^u}{8},\\ &C_3^T=\frac{1}{8}(3A-C-E+3T)+P^u, \quad D_3^T=S^u+\frac{1}{8}(3C-E_S^u+3A_S^u-T),\\ &A_6^T=\frac{1}{4}(A-E), \quad B_6^T=\frac{1}{4}(A_S^u-E_S^u),\\ &C_6^T=\frac{1}{4}(-C+T), \quad A_{15}^T=\frac{A+E}{8},\\ &B_{15}^T=\frac{A_S^u+E_S^u}{8}, \quad C_{15}^T=\frac{C+T}{8}. \end{aligned} \tag{A6}$$

The inverse relation is given as:

$$\begin{aligned} T &= 2C_6^T+4C_{15}^T, C=4C_{15}^T-2C_6^T,\\ A &= 2A_6^T+4A_{15}^T, E=4A_{15}^T-2A_6^T,\\ P^u &= -A_6^T-A_{15}^T+C_3^T-C_6^T-C_{15}^T,\\ P_A^u &= A_3^T+A_6^T-A_{15}^T, E_S^u=4B_{15}^T-2B_6^T,\\ A_S^u &= 2B_6^T+4B_{15}^T,\\ S_S^u &= B_3^T+B_6^T-B_{15}^T,\\ S^u &= -B_6^T-B_{15}^T+C_6^T-C_{15}^T+D_3^T. \end{aligned} \tag{A7}$$

From the expansion of IRA amplitudes, one can notice that the A_6^T can be absorbed into B_6^T and C_6^T.

D meson decays

The effective Hamiltonian for charm quark decay is given as

$$\begin{aligned} \mathcal{H}_{\rm eff} = \frac{G_{\rm F}}{\sqrt{2}}\big\{&V_{cs}V_{ud}^*[C_1O_1^{sd}+C_2O_2^{sd}]+V_{cd}V_{ud}^*[C_1O_1^{dd}+C_2O_2^{dd}]\\ &+V_{cs}V_{us}^*[C_1O_1^{ss}+C_2O_2^{ss}]+V_{cd}V_{us}^*[C_1O_1^{ds}+C_2O_2^{ds}]\big\}, \end{aligned} \tag{A8}$$

where

$$\begin{aligned} O_1^{sd} &= [\bar{s}^i\gamma_\mu(1-\gamma_5)c^j][\bar{u}^i\gamma^\mu(1-\gamma_5)d^j],\\ O_2^{sd} &= [\bar{s}\gamma_\mu(1-\gamma_5)c][\bar{u}\gamma^\mu(1-\gamma_5)d], \end{aligned} \tag{A9}$$

and other operators can be obtained by replacing the d,s quark fields. In the above equations, we have neglected the highly-suppressed penguin contributions. Tree operators transform under the flavor $SU(3)$ symmetry as $\bar{\mathbf{3}}\otimes\mathbf{3}\otimes\bar{\mathbf{3}}=\bar{\mathbf{3}}\oplus\bar{\mathbf{3}}\oplus\mathbf{6}\oplus\overline{\mathbf{15}}$. For the $c\to s u\bar{d}$ transition, we have

$$(H_6)_2^{31}=-(H_6)_2^{13}=1, \qquad (H_{\overline{15}})_2^{31}=(H_{\overline{15}})_2^{13}=1, \tag{A10}$$

while for the doubly Cabibbo suppressed $c\to du\bar{s}$ transition, we have

$$(H_6)_3^{21}=-(H_6)_3^{12}=-\sin^2\theta_C, (H_{\overline{15}})_3^{21}=(H_{\overline{15}})_3^{12}=-\sin^2\theta_C. \tag{A11}$$

The CKM matrix elements for $c\to u\bar{d}d$ and $c\to u\bar{s}s$ transitions are approximately equal in magnitude but different in sign: $V_{cd}V_{ud}^*=-V_{cs}V_{us}^*-V_{cb}V_{ub}^*\approx -V_{cs}V_{us}^*$ (accurate at 10^{-3}). With both contributions, the contributions from the $\bar{3}$ representation vanish, and one has the nonzero components:

$$\begin{aligned} &(H_6)_3^{31}=-(H_6)_3^{13}=(H_6)_2^{12}=-(H_6)_2^{21}=\sin(\theta_C),\\ &(H_{\overline{15}})_3^{31}=(H_{\overline{15}})_3^{13}=-(H_{\overline{15}})_2^{12}=-(H_{\overline{15}})_2^{21}=\sin(\theta_C). \end{aligned} \tag{A12}$$

A few remarks are in order.

1) The expanded amplitudes are given in Tab. A1 for Cabibbo-allowed channels, Tab. A2 for singly Cabibbo-suppressed modes, and Tab. A3 for doubly Cabibbo-suppressed decay channels.

2) One can derive the following relations between the two sets of amplitudes:

$$\begin{aligned} A_6^T &= \frac{1}{2}(A-E), A_{15}^T=\frac{1}{2}(A+E), B_6^T=\frac{1}{2}(A_S-E_S),\\ B_{15}^T &= \frac{1}{2}(A_S+E_S), C_6^T=\frac{1}{2}(T-C), C_{15}^T=\frac{1}{2}(T+C). \end{aligned} \tag{A13}$$

The superscript u has been dropped for charm quark decays.

3) The amplitudes A_6^T can be incorporated in $B_6^{T\prime}$ and $C_6^{T\prime}$, and then we have

$$\begin{aligned} A_{15}^T &= \frac{1}{2}(A+E),\\ B_6^{\prime T} &= \frac{1}{2}(A_S-E_S+A-E), B_{15}^T=\frac{1}{2}(A_S+E_S),\\ C_6^{\prime T} &= \frac{1}{2}(T-C-A+E), C_{15}^T=\frac{1}{2}(T+C), \end{aligned} \tag{A14}$$

with the inverse relation:

$$\begin{aligned} T &= A_{15}^T+C_6^{\prime T}+C_{15}^T-E,\\ C &= -A_{15}^T-C_6^{\prime T}+C_{15}^T+E,\\ A &= 2A_{15}^T-E,\\ A_S &= -A_{15}^T+B_6^{\prime T}+B_{15}^T+E,\\ E_S &= A_{15}^T-B_6^{\prime T}+B_{15}^T-E. \end{aligned} \tag{A15}$$

One of the amplitudes T,C,A,E [32, 33] is not independent, and we have eliminated E in the above equations.

Table A1. Decay amplitudes for two-body Cabibblo-Allowed D decays.

channel	IRA	TDA
$D^0\to\pi^+K^-$	$-A_6^T+A_{15}^T+C_6^T+C_{15}^T$	$E+T$
$D^0\to\pi^0\overline{K}^0$	$\frac{1}{\sqrt{2}}(A_6^T-A_{15}^T-C_6^T+C_{15}^T)$	$\frac{1}{\sqrt{2}}(C-E)$
$D^0\to\overline{K}^0\eta_8$	$\frac{1}{\sqrt{6}}(A_6^T-A_{15}^T-C_6^T+C_{15}^T)$	$\frac{1}{\sqrt{6}}(C-E)$
$D^0\to\overline{K}^0\eta_1$	$\frac{1}{\sqrt{3}}(-2A_6^T+2A_{15}^T-3B_6^T+3B_{15}^T-C_6^T+C_{15}^T)$	$\frac{1}{\sqrt{3}}(3E_S+C+2E)$
$D^+\to\pi^+\overline{K}^0$	$2C_{15}^T$	$C+T$
$D_s^+\to\pi^+\eta_8$	$\sqrt{\frac{2}{3}}(A_6^T+A_{15}^T-C_6^T-C_{15}^T)$	$\sqrt{\frac{2}{3}}(A-T)$
$D_s^+\to\pi^+\eta_1$	$\frac{1}{\sqrt{3}}(2A_6^T+2A_{15}^T+3B_6^T+3B_{15}^T+C_6^T+C_{15}^T)$	$\frac{1}{\sqrt{3}}(2A+3E_S+T)$
$D_s^+\to K^+\overline{K}^0$	$A_6^T+A_{15}^T-C_6^T+C_{15}^T$	$A+C$

Table A2. Decay amplitudes for two-body Singly Cabibblo-Suppressed D decays.

channel	IRA	TDA
$D^0\to\pi^+\pi^-$	$\sin\theta_C\left(A_6^T-A_{15}^T-C_6^T-C_{15}^T\right)$	$-\sin\theta_C(E+T)$
$D^0\to\pi^0\pi^0$	$\sin\theta_C\left(A_6^T-A_{15}^T-C_6^T+C_{15}^T\right)$	$\sin\theta_C(C-E)$
$D^0\to\pi^0\eta_8$	$-\frac{1}{\sqrt{3}}\sin\theta_C\left(A_6^T-A_{15}^T-C_6^T+C_{15}^T\right)$	$\frac{1}{\sqrt{3}}\sin\theta_C(E-C)$
$D^0\to\pi^0\eta_1$	$-\frac{1}{\sqrt{6}}\sin\theta_C\left(2A_6^T-2A_{15}^T+3B_6^T-3B_{15}^T+C_6^T-C_{15}^T\right)$	$\frac{1}{\sqrt{6}}\sin\theta_C(3E_S+C+2E)$
$D^0\to K^+K^-$	$\sin\theta_C\left(-A_6^T+A_{15}^T+C_6^T+C_{15}^T\right)$	$\sin\theta_C(E+T)$
$D^0\to\eta_8\eta_8$	$-\sin\theta_C\left(A_6^T-A_{15}^T-C_6^T+C_{15}^T\right)$	$\sin\theta_C(E-C)$
$D^0\to\eta_8\eta_1$	$\frac{1}{\sqrt{2}}\sin\theta_C\left(2A_6^T-2A_{15}^T+3B_6^T-3B_{15}^T+C_6^T-C_{15}^T\right)$	$-\frac{1}{\sqrt{2}}\sin\theta_C(3E_S+C+2E)$
$D^+\to\pi^+\pi^0$	$\sqrt{2}\sin\theta_C C_{15}^T$	$\frac{1}{\sqrt{2}}\sin\theta_C(C+T)$
$D^+\to\pi^+\eta_8$	$-\sqrt{\frac{2}{3}}\sin\theta_C\left(A_6^T+A_{15}^T-C_6^T+2C_{15}^T\right)$	$-\frac{1}{\sqrt{6}}\sin\theta_C(2A+3C+T)$
$D^+\to\pi^+\eta_1$	$-\frac{1}{\sqrt{3}}\sin\theta_C\left(2A_6^T+2A_{15}^T+3B_6^T+3B_{15}^T+C_6^T+C_{15}^T\right)$	$-\frac{1}{\sqrt{3}}\sin\theta_C(2A+3A_S+T)$
$D^+\to K^+\overline{K}^0$	$-\sin\theta_C\left(A_6^T+A_{15}^T-C_6^T-C_{15}^T\right)$	$\sin\theta_C(T-A)$
$D_s^+\to\pi^+K^0$	$\sin\theta_C\left(A_6^T+A_{15}^T-C_6^T-C_{15}^T\right)$	$\sin\theta_C(A-T)$
$D_s^+\to\pi^0K^+$	$\frac{1}{\sqrt{2}}\sin\theta_C\left(A_6^T+A_{15}^T-C_6^T+C_{15}^T\right)$	$\frac{1}{\sqrt{2}}\sin\theta_C(A+C)$
$D_s^+\to K^+\eta_8$	$-\frac{1}{\sqrt{6}}\sin\theta_C\left(A_6^T+A_{15}^T-C_6^T+5C_{15}^T\right)$	$-\frac{1}{\sqrt{6}}\sin\theta_C(A+3C+2T)$
$D_s^+\to K^+\eta_1$	$\frac{1}{\sqrt{3}}\sin\theta_C\left(2A_6^T+2A_{15}^T+3B_6^T+3B_{15}^T+C_6^T+C_{15}^T\right)$	$\frac{1}{\sqrt{3}}\sin\theta_C(2A+3A_S+T)$

Table A3. Decay amplitudes for two-body Doubly Cabibblo-Suppressed D decays.

channel	IRA	TDA
$D^0\to\pi^0K^0$	$-\frac{1}{\sqrt{2}}\sin^2\theta_C\left(A_6^T-A_{15}^T-C_6^T+C_{15}^T\right)$	$-\frac{1}{\sqrt{2}}\sin^2\theta_C(C-E)$
$D^0\to\pi^-K^+$	$-\sin^2\theta_C\left(-A_6^T+A_{15}^T+C_6^T+C_{15}^T\right)$	$-\sin^2\theta_C(E+T)$
$D^0\to K^0\eta_8$	$-\frac{1}{\sqrt{6}}\sin^2\theta_C\left(A_6^T-A_{15}^T-C_6^T+C_{15}^T\right)$	$-\frac{1}{\sqrt{6}}\sin^2\theta_C(C-E)$
$D^0\to K^0\eta_1$	$\frac{1}{\sqrt{3}}\sin^2\theta_C\left(2A_6^T-2A_{15}^T+3B_6^T-3B_{15}^T+C_6^T-C_{15}^T\right)$	$-\frac{1}{\sqrt{3}}\sin^2\theta_C(3E_S+C+2E)$
$D^+\to\pi^+K^0$	$-\sin^2\theta_C\left(A_6^T+A_{15}^T-C_6^T+C_{15}^T\right)$	$-\sin^2\theta_C(A+C)$
$D^+\to\pi^0K^+$	$-\frac{1}{\sqrt{2}}\sin^2\theta_C\left(A_6^T+A_{15}^T-C_6^T-C_{15}^T\right)$	$-\frac{1}{\sqrt{2}}\sin^2\theta_C(A-T)$
$D^+\to K^+\eta_8$	$\frac{1}{\sqrt{6}}\sin^2\theta_C\left(A_6^T+A_{15}^T-C_6^T-C_{15}^T\right)$	$-\frac{1}{\sqrt{6}}\sin^2\theta_C(T-A)$
$D^+\to K^+\eta_1$	$-\frac{1}{\sqrt{3}}\sin^2\theta_C\left(2A_6^T+2A_{15}^T+3B_6^T+3B_{15}^T+C_6^T+C_{15}^T\right)$	$-\frac{1}{\sqrt{3}}\sin^2\theta_C(2A+3A_S+T)$
$D_s^+\to K^+K^0$	$-2\sin^2\theta_C C_{15}^T$	$-\sin^2\theta_C(C+T)$

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