Fixed point and anomaly mediation in partial $N=2$ supersymmetric standard models

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Abstract: Motivated by the simple toroidal compactification of extra-dimensional SUSY theories, we investigate a partial $N=2$ supersymmetric (SUSY) extension of the standard model which has an $N=2$ SUSY sector and an $N=1$ SUSY sector. We point out that below the scale of the partial breaking of $N=2$ to $N=1$, the ratio of Yukawa to gauge couplings embedded in the original $N=2$ gauge interaction in the $N=2$ sector becomes greater due to a fixed point. Since at the partial breaking scale the sfermion masses in the $N=2$ sector are suppressed due to the $N=2$ non-renormalization theorem, the anomaly mediation effect becomes important. If dominant, the anomaly-induced masses for the sfermions in the $N=2$ sector are almost UV-insensitive due to the fixed point. Interestingly, these masses are always positive, i.e. there is no tachyonic slepton problem. From an example model, we show interesting phenomena differing from ordinary MSSM. In particular, the dark matter particle can be a sbino, i.e. the scalar component of the $N=2$ vector multiplet of $U(1)_Y$. To obtain the correct dark matter abundance, the mass of the sbino, as well as the MSSM sparticles in the $N=2$ sector which have a typical mass pattern of anomaly mediation, is required to be small. Therefore, this scenario can be tested and confirmed in the LHC and may be further confirmed by the measurement of the $N=2$ Yukawa couplings in future colliders. This model can explain dark matter, the muon $g-2$ anomaly, and gauge coupling unification, and relaxes some ordinary problems within the MSSM. It is also compatible with thermal leptogenesis.

Keywords: extended supersymmetry, MSSM, extra-dimension, muon $g-2$, dark matter, gravitino problem, LHC

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1   Introduction

The supersymmetric (SUSY) extension of the Standard Model (SM) is a leading candidate for new physics above the TeV scale, and can explain the discrepancy between soft SUSY breaking and the fundamental scales. Theoretically, SUSY is needed to construct a self-consistent quantum gravity [1].

However, from a theoretical point of view, there is no reason that the minimal SUSY extension of the SM (MSSM) is the physics next to the electroweak (EW) scale. The SUSY SM may include more particle content than the MSSM. In particular, $N=1$ SUSY in higher dimensional space-time, such as the effective theory of superstrings [1], with a simple toroidal compactification, is represented as $N=2$ SUSY [2] in four-dimensional space-time [3][2)]. Phenomenologically, $N=2$ SUSY should be partially broken down to $N=1$ and chirality appears at

a scale, say $M_\mathrm{p} \simeq 2\times10^{16}$GeV. If the partial breaking takes place only in one sector (the $N=1$ sector), with the other sector (the $N=2$ sector) sequestered, some of the $N=2$ SUSY partners ($N=2$ partners) of the SM in the $N=2$ sector may have masses around the soft SUSY mass scale, which is supposed to be around the TeV scale.

This possibility gives several alternative experimental features to the ordinary MSSM case [4]. In this paper, we investigate the possibility that the $N=2$ vector and hyper-partners all remain until the TeV scale. In particular, we will show that in this setup at a sufficiently low energy scale the gauge and Yukawa ($N=2$ Yukawa) couplings, originating from the $N=2$ gauge interaction for an $N=2$ hypermultiplet (hypermultiplet), are related by the fixed point of the renormalization group (RG). The ratio of the Yukawa to gauge couplings is raised by the RG effect towards this fixed point from the ratio at the

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2) The Lagrangian with $N>2$ SUSY can be represented by $N=2$ SUSY.

$N = 2$ SUSY limit. Namely, we predict typical Yukawa couplings for these hypermultiplets, which depend less on the partial breaking mechanism as well as the threshold corrections at $M_{\rm p}$.

Suppose that the soft breaking terms are generated above the partial breaking scale. The soft breaking mass squares of the hypermultiplets then vanish due to the $N = 2$ non-renormalization theorem [5]. Then the soft masses for the sfermions in the $N = 2$ sector below the partial breaking scale are generated via radiative corrections. One of the corrections is from the anomaly mediation effect [6, 7]. Since is a function of the particle's couplings and the gravitino mass, the anomaly-induced masses in the $N=2$ sector are almost UV-insensitive due to the fixed point. Interestingly, due to the large ratio of the Yukawa and gauge couplings at the fixed point, the anomaly-induced mass squares for the hypermultiplets are positive. Namely, the tachyonic slepton problem[1] in the $N = 2$ sector is automatically solved near the fixed point.

In a concrete partial $N = 2$ SSM, we confirm that the $N = 2$ Yukawa couplings do converge to the fixed point at 2-loop level, and also the corresponding positive anomaly-induced slepton mass squares. When the typical soft mass scale in the $N = 1$ sector is around $\mathcal{O}(10)$ TeV, the radiative corrections from the top loops can be large enough to explain the Higgs boson mass [11]. Due to the light smuons and bino in the $N = 2$ sector, we also find that the muon $g-2$ anomaly [12–14] can be explained within its 1σ level error for the gravitino mass of $\mathcal{O}(100)$ TeV[2].

The aspect which is quite different from the ordinary MSSM is dark matter. Ordinary neutralinos cannot be candidates for dark matter because they are all heavier than the smuon, due to the mass relation of anomaly mediation with additional multiplets. The interesting candidates are the $N = 2$ vector partners, which are stabilized by a new Z_2-parity introduced to solve the tadpole problem. The scalar component of the $N = 2$ U(1)$_Y$ vector multiplet, the sbino, can explain the correct dark matter abundance when its mass is up to 700 GeV, and the annihilation products, some sleptons, are even lighter. Due to these mass constraints, if these light sleptons are long-lived enough, they can be fully tested in the LHC. The other $N=2$ sector sparticles of the MSSM have a typical mass pattern, which is related by anomaly mediation, with a scale smaller than ~ 2 TeV. This typical spectrum could be measured in the LHC. Furthermore, depending on their mass range, the $N=2$ partners could be tested in future colliders, such as SPPC, FCC, CLIC and a Muon Collider Higgs Factory, which could further

confirm our scenario [16].

From the cosmological viewpoint, this scenario is favored because of the heavy gravitino, which relaxes the gravitino problem [17]. In particular, the reheating temperature can be large enough, while avoiding the overproduction of the produced dark matter. Thus, our scenario is compatible with thermal leptogenesis [18] and can produce the correct baryon asymmetry. CP and FCNC problems in the ordinary MSSM are also relaxed due to the heavy $N=1$ sector sfemions.

This paper is organized as follows. In Section 2, we introduce the partial $N = 2$ SUSY models, and derive the fixed point for the $N = 2$ Yukawa couplings. In Section 3, we explain the anomaly mediation effect on the fixed point and show the absence of the tachyonic slepton problem. The $N = 2$ non-renormalization theorem will also be explained. In Section 4, we discuss a concrete example of a partially SSM and its several phenomenological and cosmological aspects. Section 5 gives some discussion and conclusions.

2   Fixed point in partial $N=2$ supersymmetric model

We will focus on the possibility that the $N=2$ SUSY is broken down to $N=1$ at a high energy scale $M_{\rm p}$ in the $N=1$ sector, while the $N=2$ sector remains approximately $N=2$ SUSY at this scale. This possibility is not peculiar because the following theoretical backgrounds exist.

$N=2$ to $N=1$ partial SUSY breaking can take place spontaneously as an $N=1$ SUSY gauge theory can be described by non-linear realized $N=2$ SUSY theories with chiral matter of any representation [19]. In particular, $N=2$ SUSY non-linear Abelian gauge models with electric and magnetic $N=2$ Fayet-Iliopoulos terms are proved to have such breaking [20]. Thus, a sector which does not directly couple to the $N=2$ SUSY gauge fields inducing the partial breaking, has the partial breaking only at the higher order and can be identified as the $N=2$ sector.

The extra-dimensional theory with branes is also a candidate to realize this possibility. For example, $N=1$ SUSY on $R_{1,3} \times S_1$ spacetime can be compactified into $d = 4$, $N = 2$ SUSY at low energy [3]. If "our world" is localized on one of the four-dimensional branes perpendicular to the extra dimension, the brane fields have $d = 4$, $N = 1$ SUSY while the compactified bulk fields have $d=4$, $N=2$ SUSY. Ordinarily, these $N=2$ partners of the MSSM particles are projected out by assuming an orbifold parity, but this is not necessary. Therefore, at

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1) There are typically two ways to solve this problem: (a) the anomaly mediation effect is canceled or negligible [5, 6, 8, 9], and (b) the sfermions have large Yukawa interactions [10]. The solution here is of the latter type.

2) For other explanations of the muon $g-2$ anomaly with heavy stops and light smuons, see Refs. [9, 15].

the compactification scale, we have two sectors, an $N=2$ sector composed of the bulk fields and an $N=1$ sector composed of the brane fields.

In this section, we will introduce the partially $N=2$ SUSY model as the effective theory of the previous backgrounds (or others) and study the RG behavior for the dimensionless couplings in its $N=2$ sector.

2.1   Introduction to partial $N=2$ supersymmetric model

To simplify the discussion, let us consider an $U(N_c)=$ $SU(N_c) \times U(1)$ Yang-Mills theory with partial $N=2$ SUSY defined by the following Lagrangian as a toy model.

$$\mathcal{L} = \int d^4\theta (K_{N=2}+K_{N=1})+\int d^2\theta W_{N=2}$$
$$+\int d^2\theta W_{\text{mass}}+\int d^2\theta W_H$$
$$+\frac{1}{2}\left(\int d^2\theta \text{tr}[\mathcal{W}^c\mathcal{W}^c]+\frac{1}{2}\int d^2\theta \mathcal{W}\mathcal{W}\right)+\text{h.c.} , \quad (1)$$

$$K_{N=2} = 2\text{tr}[W^\dagger e^{-2g_cV_c}We^{2g_cV_c}]+|\phi_Y|^2$$
$$+\sum_i^{N_F}\left(L_i^\dagger e^{2g_cV_c+2Y_igV}L^i+\overline{L}_i e^{-2g_cV_c-2Y_igV}\overline{L}^{i\dagger}\right),$$
$$(2)$$

$$W_{N=2}=-\sum_i^{N_F}\left(\sqrt{2}\omega_i g_c\overline{L}_iWL_i+\sqrt{2}\tilde{Y}_ig\overline{L}_i\phi_YL_i\right), \quad (3)$$

$$K_{N=1}=\sum_i^{N_F}S_i^\dagger e^{2g_cV_c+2Y_igV}S^i+\sum_a^{N_f}H_a^\dagger e^{2g_cV_c+2Y_a^HgV}H^a,$$
$$(4)$$

$$W_{\text{mass}}=-M_W\text{tr}[W^2]-M\phi_Y^2-\sum_i^{N_F}M_i\overline{L}_iS^i. \quad (5)$$

Here, $\mathcal{W}^c$ ($\mathcal{W}$) and V_c (V) are the field strength and the corresponding gauge multiplet of the $SU(N_c)$ ($U(1)$) with gauge coupling, g_c (g), respectively; W (ϕ_Y) is an $SU(N_c)$ adjoint (singlet) chiral multiplet which stands for the $N=2$ vector partner (see Section 2.2) of the $SU(N_c)$ ($U(1)$) gauge particle. L_i, $\overline{L}_j$, S_k and H_a are chiral multiplets with representations of $(r_i,Y_i),(\overline{r}_j,-Y_j)$, (r_k,Y_k) and (r_a^H,Y_a^H) under $(SU(N_c),U(1))$, respectively, where L_i stand for the MSSM matter multiplets or new matter multiplets embedded in hypermultiplets (see Section 2.2), $\overline{X}$ are the $N=2$ hyperpartners of chiral multiplet X, S_X are the spectators needed to cancel the chiral anomaly, and H_a stand for the matter multiplets in the $N=1$ sector; N_F is the number of L_i, and also is that of $\overline{L}_i$ or S_j, i.e. i runs from 1 to N_F; N_f is the number of H_a; M_W and M are the Majorana SUSY mass for W and ϕ_Y, respectively; M_i is the Dirac SUSY masses for $\overline{L}_i$ and S_i; and $\tilde{Y}_i$ and $\tilde{w}_i$ are the $N=2$ Yukawa couplings

(see Section 2.2) in units of the corresponding gauge couplings.

$K_{N=2}$ and $W_{N=2}$ represent the Kähler potential and super-potential respectively, of the same forms as those in $N=2$ SUSY QCD [2], while $K_{N=1}$ and W_H are the ordinary $N=1$ Kähler potential and super- potential, respectively. Here, $W_H=W_H(H_a,S_j;L_i,\overline{L}_i,W,\phi_Y)\simeq$ $W_H(H_a,S_j)$ is a function of H_a and S_j, and is assumed to depend weakly on L_i, $\overline{L}_i$, W, and ϕ_Y. Thus, we will neglect the dependence on $L_i,\overline{L}_i$, W and ϕ_Y of W_H. In particular, we will neglect the Yukawa interactions for L_i, which can stand for the SM Yukawa couplings and breaks the $N=2$ SUSY explicitly. The definitions are summarized in Table 1

2.2 $N=2$ SUSY limit and two sectors

With the decoupling of the multiplets, H_a and S_j,

$$\omega_i\to 1, \quad \tilde{Y}_i\to Y_i, \quad (6)$$

is the $N=2$ SUSY limit [2]. To see the property at this limit, let us focus on some of the Yukawa couplings obtained from $W_{N=2}$ and $K_{N=2}$:

$$\mathcal{L} \supset \sum_{i=1}^{N_F}\left(-i\sqrt{2}\tilde{L}_i^\dagger(g_c\lambda_c+Y_ig\lambda)\cdot\psi_{Li}\right.$$
$$\left.-\sqrt{2}\tilde{\overline{L}}_i(\omega_ig_c\psi_W+\tilde{Y}_ig\psi)\cdot\psi_{Li}\right). \quad (7)$$

Here, ψ_W (ψ) and λ_c (λ) are the gaugini, the fermionic component of W (ϕ_Y), and the gaugino in the adjoint representation of the $SU(N_c)$ ($U(1)$) gauge groups, respectively, and $\tilde{X}(\psi_X)$ are the scalar (fermion) components of X. An $SU(2)_R$ symmetry manifests itself in Eq. (7) with Eq. (6), under which $\{i\lambda_c,\psi_W\}$, $\{i\lambda,\psi_{\phi_Y}\}$ and $\{\tilde{L}_i,\tilde{\overline{L}}_i^\dagger\}$ are doublets while the other fields participating in this limit are singlets. In the light of $SU(2)_R$, we have two kinds of enlarged multiplets in $N=2$ SUSY, e.g. $N=2$ vector multiplets $\{V,\phi_Y\}$ and $\{V_c,W\}$, and hypermultiplets $\{L_i,\overline{L}_i\}$.

Since the first and second terms of Eq. (7) arise from the Kähler and super-potentials, respectively, the $SU(2)_R$ transformation mixes the terms in these two potentials. Moreover, the Kähler potential and the SUSY gauge kinetic term are mixed due to the rotation of the components of $\{V,\phi_Y\}$ or $\{V_c,W\}$. This fact will be essential to derive the $N=2$ non-renormalization theorem in Section 3.1.

The particles present in the $N=2$ SUSY limit compose the $N=2$ sector, namely V_c,V,W,ϕ_Y, L_i, and $\overline{L}_i$ are the components. The multiplets decoupling at this limit compose the $N=1$ sector, where S_i and H_a are the components. The gauge couplings of S_i and H_a, and Yukawa couplings in W_H are hard breakings of the $SU(2)_R$ symmetry.

Table 1.   Particle contents of a partially $N=2$ SUSY model, Eq. (1). The representation r_i is allowed to be unity. "Vector" and "hyper-" denote that the corresponding particles are embedded in the vector and hyper- multiplets, respectively. $i_{\max}$ and $a_{\max}$ denotes the maximum value of i and a, respectively.

sector	$N=2$ sector				$N=1$ sector	
Chiral multiplets	W	ϕ_Y	L_i	$\overline{L}_i$	S_i	H_a
$N=2$ partner	V_c	V	$\overline{L}_i$	L_i	/	/
$N=2$ multiplet	vector	vector	hyper-	hyper-	/	/
SU(N_c) representation	adjoint	1	r_i	$\overline{r_i}$	r_i	r_a^H
U(1) charge	0	0	Y_i	$-Y_i$	Y_i	Y_a^H
$i_{\max}/a_{\max}$	/	/	N_F	N_F	N_F	N_f

We will use the definition made here to explain the phenomena in the $N=2$ sector, even with an explicit breaking of $N=2$ SUSY. The definitions made here are summarized in Table 1.

2.3   Radiative corrections

The 1-loop RG equations for the dimensionless couplings in the $N=2$ sector are given as follows [24, 25]:

$$\frac{\mathrm{d}}{\mathrm{d}t}g_c \equiv \beta_c = \frac{1}{16\pi^2}g_c^3(F_2+F_1-2N_c), \quad \frac{\mathrm{d}}{\mathrm{d}t}g \equiv \beta = \frac{1}{16\pi^2}g^3(f_2+f_1), \tag{8}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}(g_c w_i) = g_c\omega_i(\gamma_W + \gamma_{Li} + \gamma_{\overline{L}i}), \tag{9}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}(g\tilde{Y}_i) = g\tilde{Y}_i(\gamma_{\phi_Y} + \gamma_{Li} + \gamma_{\overline{L}i}), \tag{10}$$

where the anomalous dimensions for W, ϕ_Y, and $L_i, \overline{L}_i$ are given as

$$\gamma_W = \frac{1}{16\pi^2}\left(\sum_i^{N_F} 2T(r_i)\omega_i^2 - 2N_c\right)g_c^2, \tag{11}$$

$$\gamma_{\phi_Y} = \frac{1}{16\pi^2}\sum_i^{N_F} 2d(r_i)\tilde{Y}_i^2 g^2, \tag{12}$$

and $\gamma_{Li} = \gamma_{\overline{L}i} = \dfrac{1}{16\pi^2}\left(2(\omega_i^2-1)C(r_i)g_c^2 + 2(\tilde{Y}_i^2-Y_i^2)g^2\right),$ (13)

respectively. Here $t = \log(\frac{\mu_{\mathrm{RG}}}{\mathrm{GeV}})$; $T(r)$, $C(r)$ and $d(r)$ denote the Dynkin index, the quadratic Casimir invariant and the dimension of the representation r, respectively; β_c and β are the 1-loop $\beta-$functions for SU(N_c) and $U(1)$, respectively; and F_2 (F_1) and f_2 (f_1) are the sums of Dynkin indices of SU(N_c) and U(1) in the $N=2$ ($N=1$) sector, respectively:

$$F_2 \equiv \sum_i^{N_F} 2T(r_i), \quad F_1 \equiv \sum_i^{N_F} T(r_i) + \sum_i^{N_f} T(r_a^H), \tag{14}$$

and $f_2 \equiv \sum_i^{N_F} 2d(r_i)Y_i^2, \quad f_1 \equiv \sum_i^{N_F} d(r_i)Y_i^2 + \sum_i^{N_f} d(r_a^H)(Y_i^H)^2.$ (15)

At the $N=2$ SUSY limit, where $f_1=0$, $F_1=0$ with Eq. (6), we evaluate $\frac{\mathrm{d}}{\mathrm{d}t}\omega_i = \frac{\mathrm{d}}{\mathrm{d}t}\tilde{Y}_j = \gamma_{Li,\overline{L}i} = 0$ so that Eq. (6)

and the vanishing of Eq. (13) are satisfied perturbatively at any scale.

2.4   A fixed point of the $N=2$ Yukawa couplings

Let us investigate the dimensionless couplings at low energy analytically.

As shown in **Sec.2.3**, at the limit of $N=2$ SUSY, Eqs. (6) are satisfied at any scale. This implies that the limit (6) represents a fixed point in the parameter space characterized by $\{\omega_i, \tilde{Y}_j\}$. We will show that in the presence of the degrees of H_a and S_i, the IR fixed point still exists and moves to a different position, $\{\overline{\omega}_i, \overline{\tilde{Y}}_j\}$.

First, for convenience, we divide N_F into N_F^s and $N_F-N_F^s$, where N_F^s is the total number of SU(N_c) singlets in L_i. Without loss of generality, we can rearrange the indices of the $N=2$ sector, such that the superfields labeled by $i=1\sim N_F^s$ are SU(N_c) singlets, while those of $i=N_F^s+1\sim N_F$ are not. We also divide f_2 into

$$f_2^s \equiv \sum_i^{N_F^s} 2Y_i^2, \quad f_2^{ns} \equiv \sum_{i=N_F^s+1}^{N_F} 2d(r_i)Y_i^2. \tag{16}$$

In the calculation we assume

$$C(r_i)g_c^2 \sim C(r_i)g_c^2\omega_i^2 \gg Y_i^2 g^2 \sim \tilde{Y}_i^2 g^2. \tag{17}$$

This condition stands for $(4/3)g_3^2$ or $(3/4)g_2^2 \gg Y_i^2 g_Y^2$, where g_Y, g_2, and g_3 are the SM gauge couplings of U(1)$_Y$, SU(2)$_L$, and SU(3), respectively.

A solution of the vanishing condition for $\frac{\mathrm{d}}{\mathrm{d}t}\omega_i, \frac{\mathrm{d}}{\mathrm{d}t}(g\tilde{Y}_i)$ is,

$$\overline{\omega}_i^2 - 1 \simeq \frac{d(r_i)}{2T(r_i)}\frac{F_1}{\sum_{i=N_F^s+1}^{N_F} d(r_i) + 2(N_c^2-1)} + \mathcal{O}\left(\frac{g^2 Y_i^2}{g_c^2 C(r_i)}\right), \tag{18}$$

$$g^2\overline{\tilde{Y}}_i^2 \simeq 0 \quad (i > N_F^s), \tag{19}$$

The second equation is obtained by neglecting terms of $\mathcal{O}(g^2)$ while considering $(f_1+f_2)g^2$. This approximation stands for $(f_1+f_2)g_Y^2 > g_2^2, g_3^2$ due to the large coefficient $f_1+f_2 \geqslant 11$ in the realistic case.

Hence we find a general relation,

$$\overline{\omega}_i^2 - 1 > 0. \tag{20}$$

Now let us check whether $\{\overline{\omega}_i, \overline{\tilde{Y}}_i\}(i \geqslant N_F^s)$ with approximation Eq. (17) represents an IR fixed point. Rewriting Eqs. (9) and (10) in terms of $\delta\omega_i^2 \equiv \omega_i^2 - \overline{\omega}_i^2$ and $\delta\tilde{Y}_i^2 \equiv \tilde{Y}_i^2 - \overline{\tilde{Y}}_i^2$, we obtain the RG equations for the differences

$$\frac{\mathrm{d}}{\mathrm{d}t}\delta\omega_i^2 \simeq \sum_{j=N_F^s+1}^{N_F} \delta\omega_j^2 \frac{g_c^2}{8\pi^2} A_{ij},$$

$$\frac{\mathrm{d}}{\mathrm{d}t}(g^2\delta\tilde{Y}_i^2) \simeq \sum_{j=N_F^s+1}^{N_F} g^2\delta\tilde{Y}_j^2 \frac{g_c^2}{8\pi^2} B_{ij} \quad (i > N_F^s) \qquad (21)$$

at the leading order. Here A_{ij} and B_{ij} are positive-definite matrices:

$$A_{ij} = 2\overline{\omega}_i^2(T(r_j) + 2C(r_j)\delta_{ij}), \quad B_{ij} = 4(\overline{\omega}_i^2 - 1)C(r_i)\delta_{ij}. \quad (22)$$

Employing the analytic solutions of Eqs. (8), we can solve Eqs. (21) and obtain,

$$\delta\omega_i^2(\mu_{\mathrm{RG}}) \simeq \sum_{j=N_F^s+1}^{N_F} \left[\left(\frac{\frac{1}{\alpha_{\mathrm{p}}}}{\frac{4\pi}{g_c^2(\mu_{\mathrm{RG}})}}\right)^{\frac{A}{F_1+F_2-2N_c}}\right]_{ij} \delta\omega_j^2(M_{\mathrm{p}}), \qquad (23)$$

$$\delta(g^2\tilde{Y}_i^2)|_{\mu_{\mathrm{RG}}} \simeq \sum_{j=N_F^s+1}^{N_F} \left[\left(\frac{\frac{1}{\alpha_{\mathrm{p}}}}{\frac{4\pi}{g_c^2(\mu_{\mathrm{RG}})}}\right)^{\frac{B}{F_1+F_2-2N_c}}\right]_{ij} \delta(g^2\tilde{Y}_j^2)|_{M_{\mathrm{p}}} \qquad (i > N_F^s), \qquad (24)$$

where $X|_\mu$ denotes the variable X at the renormalization scale μ and $\alpha_{\mathrm{p}} \equiv g_c(M_{\mathrm{p}})^2/4\pi$. Thus, for $F_1+F_2-2N_c \neq 0$ Eqs. (18) and (19) represent an IR fixed-point.

Following the same procedure, the $N=2$ Yukawa couplings for the $SU(N_c)$ singlets have a fixed point,

$$\overline{Y}_i^2 - Y_i^2 \simeq \frac{1}{2}\left(\frac{f_1 + f_2^{ns}}{N_F^s + 2}\right) \quad (i \leqslant N_F^s), \qquad (25)$$

where we have set $Y_i = \overline{Y}_i, w_i = \overline{w}_i$ for $i > N_F^s$. Hence,

$$\overline{Y}_i^2 - Y_i^2 > 0. \qquad (26)$$

The difference, $\delta\tilde{Y}_i^2 \equiv \tilde{Y}_i^2 - \overline{\tilde{Y}}_i^2$, at the scale μ_{RG} is given by

$$\delta\tilde{Y}_i^2(\mu_{\mathrm{RG}}) \simeq \sum_{j}^{N_F^s} \left[\left(\frac{\frac{1}{\tilde{\alpha}_{\mathrm{p}}}}{\frac{4\pi}{g^2(\mu)}}\right)^{\frac{C}{f_1+f_2}}\right]_{ij} \delta\tilde{Y}_j^2(M_{\mathrm{p}}) \quad (i \leqslant N_F^s), \qquad (27)$$

where

$$C_{ij} = \overline{\tilde{Y}}_i^2(1+\delta_{ij}) \qquad (28)$$

is a positive-definite matrix, and $\tilde{\alpha}_{\mathrm{p}} \equiv g(M_{\mathrm{p}})^2/4\pi$.

In summary, the position $\{\overline{\omega}_i, \overline{\tilde{Y}}_j\}$ given by Eqs. (18), (19), and (25) in the parameter space represents an IR fixed-point at 1-loop order. Namely, at low energy the $N=2$ Yukawa couplings approach to $\{\overline{\omega}_i, \overline{\tilde{Y}}_j\}$ from the value around $\{1, Y_j\}$ at the partially breaking scale.

Therefore, a partially $N=2$ SUSY model has a striking feature, the typical pattern of Yukawa couplings controlled by IR physics and matter contents, which is insensitive to the partial breaking mechanism or the threshold corrections at M_{p}. The IR fixed point is easily reached in the realistic case because the additional matter contents enhance the gauge couplings at M_{p} through the RG running and thus Eqs. (23), (24), and (27) are suppressed.

3 $N=2$ to $N=0$ SUSY breaking and anomaly mediation

The $N=2$ SUSY breaking to $N=0$ is turned on in this section, and we will show that if $SO(2)_R$ remains after the SUSY breaking, the sfermion masses are forbidden by the $N=2$ non-renormalization theorem. The anomaly mediation effect at the previously discussed fixed point will be investigated. In particular, the tachyonic slepton problem is resolved automatically near the fixed point. We will also discuss the condition that suppresses the RG running effect so that the spectrum in the $N=2$ sector is mostly induced by anomaly mediation.

3.1 $N = 2$ Non-renormalization theorem and splitting mass spectra

Before a general discussion, let us consider a concrete model for $N=2$ SUSY breaking to $N=0$: an $N=2$ gauge mediation model [5, 21]. Suppose that the SUSY breaking is mediated by $N=2$ messengers, $\{\phi_m, \overline{\phi}_m\}$, which are introduced as hypermultiplets charged under $U(N_c)$. The $N=2$ messengers are characterized by the superpotential,

$$W_{SB} = \sqrt{2}\overline{\phi}_m(\tilde{Y}_m g\phi_Y + \omega_m g_c W + Z)\phi_m, \qquad (29)$$

where $Z \equiv M + \theta^2 F_Z$ is a SUSY breaking field with $M \gg \sqrt{F_Z}$, where M (F_Z) represents the messenger scale (SUSY breaking F-term), and $\tilde{Y}_m, \omega_m$ are the $N=2$ Yukawa couplings for the messengers. Z can be identified as the vacuum expectation value for the chiral com-

ponent of an abelian $N=2$ vector multiplet[1]. The $N=2$ SUSY limit of the Yukawa couplings is given as

$$\tilde{Y}_m \to Y_m, \quad \omega_m \to 1, \tag{30}$$

where Y_m is the U(1) charge of ϕ_m.

The soft mass squares for L_i at the $N=2$ SUSY limit, induced by radiative corrections from the messengers, are given by [21]

$$m_i^2 = \frac{1}{2}\left|\frac{F_Z}{Z}\right|^2\left(\frac{\mathrm{d}}{\mathrm{d}t}\gamma_i^- - \frac{\mathrm{d}}{\mathrm{d}t}\gamma_i^+\right), \tag{31}$$

where i denotes L_i, H_a; $+(-)$ denotes the value evaluated above (below) M. Since Eqs. (6) and (30) are satisfied, from Eqs. (8)-(13), we find

$$m_{Li}^2 = m_{\overline{L}i}^2 = 0. \tag{32}$$

In fact, these vanishing masses are the consequence of symmetry and holomorphy. From the non-vanishing expectation value, F_Z, in Eq. (29), the potential acquires

$$
\begin{aligned}
\delta V &= F_Z \tilde{\phi}_m \tilde{\overline{\phi}}_m + \text{h.c.} \\
&= (\tilde{\phi}_m^*, \tilde{\overline{\phi}}_m)^* \cdot (\Re[F_Z]\sigma_1 - \Im[F_Z]\sigma_2) \cdot (\tilde{\phi}_m^*, \tilde{\overline{\phi}}_m)^T,
\end{aligned} \tag{33}
$$

where σ_1 and σ_2 are the Pauli matrices while T denotes the transpose. Since Eq. (33) represents an isovector in $SU(2)_R \sim SO(3)_R$ space, non-vanishing F_Z breaks SUSY but preserves a $U(1)_R \sim SO(2)_R$ symmetry, which is a subgroup of $SU(2)_R$ with rotating axis $\{\Re F_Z, -\Im F_Z, 0\}$ in isovector space. Thus, this symmetry must mix $\tilde{\phi}_m^*$ and $\tilde{\overline{\phi}}_m$, which are anti-chiral and chiral scalar fields, respectively. The effective theory at low energy has this $SO(2)_R$ symmetry.

In fact, the soft breaking mass squares from the Kähler potential, like

$$\delta K \sim \frac{|Z|^2}{M^2}\left(L_i^\dagger \mathrm{e}^{2g_c V_c + 2Y_i gV}L^i + \overline{L}_i \mathrm{e}^{-2g_c V_c - 2Y_i gV}\overline{L}^{i\dagger}\right), \tag{34}$$

are forbidden by the $SO(2)_R$ symmetry and the holomorphy. This is because with the $SO(2)_R$ symmetry the potential should include the term Eq. (7) multiplied by $\frac{|Z|^2}{M^2}$ while the second term $\frac{|Z|^2}{M^2}\tilde{\overline{L}}_i(\omega_i g_c \psi_W + \tilde{Y}_i g\psi)\cdot\psi_{Li}$ is never generated from the superpotential due to holomorphy. Thus, $SO(2)_R$ symmetry and holomorphy forbid soft breaking mass squares for hypermultiplets. This is nothing but the consequence of the $N=2$ non-renormalization

theorem for the wave function renormalization and is independent of the mediation mechanism.

Now we come back to the setup of Section 2. Suppose that the soft SUSY breaking terms are generated preserving the $SO(2)_R$ symmetry above the partial breaking scale. Since is sequestered from the partially breaking sector at M_p[2], the $N=2$ sector has an approximate $SO(2)_R$ symmetry. Therefore, the soft mass squares from the SUSY breaking of the sfermions are suppressed in the $N=2$ sector at M_p.

As a consequence of the approximate $SO(2)_R$ symmetry in the $N=2$ sector at M_p, the following relations among the parameters are obtained,

$$\omega_i(M_p) \simeq 1, \quad \tilde{Y}_i(M_p) \simeq Y_i, \tag{35}$$

and

$$m_{L_i,\overline{L}_i}^2(M_p) \simeq 0. \tag{36}$$

Throughout this paper, we do not specify the parameters in the $N=1$ sector.

3.2 Anomaly mediation and Tachyonic Slepton problem in the $N=2$ sector

Since the SM is chiral, such an $SO(2)_R$ symmetry[3] should be broken by radiative corrections, and the sfermion masses in the $N=2$ sector are generated. One of the radiative corrections, which must be considered from supergravity, is anomaly mediation [6, 7]. The scalar and gaugino masses induced by the anomaly mediation effect are given as

$$m_{L_i,\overline{L}_i,W,\phi_Y}^2 = \frac{1}{2}m_{3/2}^2\frac{\mathrm{d}}{\mathrm{d}t}\gamma_{L_i,\overline{L}_i,W,\phi_Y}, \tag{37}$$

$$M_c = m_{3/2}\frac{\beta_c}{g_c}, \quad M = m_{3/2}\frac{\beta}{g}. \tag{38}$$

where $m_{3/2}$ is the gravitino mass. Interestingly, this relation is a renormalization invariant of $N=1$ SUSY, and hence the anomaly induced mass is UV-insensitive. Notice that the anomaly induced mass squares of the sleptons in the MSSM, which have asymptotically non-free gauge interactions with small Yukawa couplings, are negative, i.e. the tachyonic slepton problem [6].

Substituting Eqs. (8) and (13) with the $N=2$ Yukawa couplings at the fixed point into Eq. (37), the anomaly-induced mass for the partially $N=2$ SUSY model is obtained as

---

1) For example, in a simple model where the $N=2$ vector multiplet has an $N=2$ Fayet-Iliopoulos term, $W=\xi Z$ can induce an $F-$term to Z spontaneously while the messenger scale is the scalar component of Z. (See Ref. [20] for reference.) If the messengers carry the charge of this abelian gauge group, then Eq. (29) is obtained.

2) We are assuming that the $SO(2)_R$ breaking Yukawa couplings for the fields in the $N=2$ sector are small enough to be neglected, e.g. L_i are likely to be the first two generation sfermions, which have small SM Yukawa couplings.

3) It is difficult to introduce the chiral partners of the SM fermions to have an exact $SO(2)_R$ symmetry. Since a fermion of the $N=2$ partner is charged under the SM gauge group, it should be heavier than $\mathcal{O}(100)$GeV due to the LEP and LHC constraints [22]. A Dirac mass term between fermions of the SM and the $N=2$ partner is forbidden, otherwise the SM fermion would become too massive. A mass term between two chiral fermions in the $N=2$ partners is likely to be forbidden because the mass in this case should be generated via the EW symmetry breakdown, and cannot be too large due to the constraint from the precision measurement of the S parameter [22, 23].

$$m^2_{L_i,\overline{L}_i}|_{\mathrm{fp}} \simeq \frac{1}{16\pi^2}m^2_{3/2}(\overline{\omega}^2_i-1)C(r_i)\frac{\mathrm{d}}{\mathrm{d}t}g^2_c + \frac{1}{16\pi^2}m^2_{3/2}(\overline{\tilde{Y}}^2_i-Y^2_i)\frac{\mathrm{d}}{\mathrm{d}t}g^2$$

$$= \frac{1}{16\pi^2}m^2_{3/2}\left(\frac{(N^2_c-1)F_1}{\sum^{N_F}_{i=N^s_F+1}d(r_i)+2(N^2_c-1)}g_c\beta_c - 2Y^2_i g\beta\right) \simeq \frac{1}{16\pi^2}m^2_{3/2}\left(\frac{(N^2_c-1)F_1}{\sum^{N_F}_{i=N^s_F+1}d(r_i)+2(N^2_c-1)}g_c\beta_c\right), \tag{39}$$

for non-singlets of $\mathrm{SU}(N_c)$, while

$$m^2_{L_i,\overline{L}_i}\Big|_{\mathrm{fp}} \simeq \frac{1}{16\pi^2}m^2_{3/2}(\overline{\tilde{Y}}^2_i-Y^2_i)\frac{\mathrm{d}}{\mathrm{d}t}g^2$$

$$= \frac{1}{16\pi^2}m^2_{3/2}\left(\frac{f_1+f^{ns}_2}{N^s_F+2}\right)g\beta, \tag{40}$$

for singlets of $\mathrm{SU}(N_c)$. These masses are functions of $g,g_c,m_{3/2}$ and the model constants, independent of the UV physics.

We can see that these mass squares are always positive for positive β,β_c. Therefore, the sleptons in the $N=2$ sector do not have the tachyonic slepton problem near the fixed point.

We note that instead of the tachyonic slepton problem, the negative anomaly induced mass squared for the scalar component of W may be generated. However, this is not so problematic as the case of the sleptons since we are allowed to have a tree-level SUSY mass as in Eq. (5)[1].

3.3   Anomaly induced $N=2$ sector

An interesting possibility is that the masses of sfermions in the $N=2$ sector are dominantly induced by anomaly mediation and their spectrum becomes almost UV-insensitive. Since the anomaly mediation effect for the sfermion mass is at 2-loop order, we would like to find the condition suppressing the 1-loop and 2-loop RGE effects [24, 25]. The 1-loop RG effect can be suppressed if the following is satisfied,

$$m^2_W(M_{\mathrm{p}}) \simeq m^2_{\phi_Y}(M_{\mathrm{p}}) \simeq 0, \tag{41}$$

$$A-\mathrm{terms}|_{M_{\mathrm{p}}} \simeq 0, \; M_c(M_{\mathrm{p}}) \simeq M(M_{\mathrm{p}}) \simeq 0, \tag{42}$$

$$S \equiv \sum^{N_f}_i d(r^H_a)m^2_{H_a}Y^H_a + \sum^{N_F}_i d(r_i)(m^2_{L_i}-m^2_{\overline{L}_i}+m^2_{S_i})Y_i \simeq 0, \tag{43}$$

where m^2_X is the soft mass of scalar $\tilde{X}$; M_c and M are Majorana masses for the gaugino of $\mathrm{SU}(N_c)$ and $\mathrm{U}(1)$, respectively; and S is the $D-$term of the $\mathrm{U}(1)$ gauge interaction.

Eqs. (41) and (42) can be obtained by simply assuming that the SUSY breaking spurion field $Z = M+\theta^2 F_Z$ is charged under some hidden symmetry. Obviously, the gaugino mass terms, e.g.

$$\frac{Z}{M_{\mathrm{p}}}\mathrm{tr}[\mathcal{W}_c\mathcal{W}_c], \tag{44}$$

are forbidden. Furthermore, the soft scalar masses of W and ϕ_Y are also suppressed due to the approximate $\mathrm{SO}(2)_R$ symmetry. This is because the soft scalar mass square for a vector partner, restricted by the $\mathrm{SO}(2)_R$ symmetry (similarly to the discussion in **Sec.3.1**), can only originate from a kinetic term, like

$$\delta K = \frac{Z}{M_{\mathrm{p}}}\mathrm{tr}[W^\dagger \mathrm{e}^{-2g_c V_c}W\mathrm{e}^{2g_c V_c}], \tag{45}$$

which is forbidden by the hidden symmetry.

A vanishing $D-$term of $\mathrm{U}(1)$ is quite general in several mediation mechanisms, e.g. the gauge mediation model, mSUGRA, etc. We do not discuss this further.

To suppress the 2-loop RG effect characterized by $(\frac{1}{16\pi^2})^2 m^2_{H_a,S_i}$,

$$m^2_{H_a,S_i} \ll m^2_{3/2}, \tag{46}$$

should be satisfied. If $m^2_{H_a,S_i}$ are generated via partial breaking of $N=2$ to $N=1$ at M_{p}, $m^2_{H_a,S_i}$ may be suppressed to the gravitino mass, $m^2_{H_a,S_i} \sim \epsilon m^2_{3/2}$, by assuming the order parameter of the partial breaking to be $\epsilon = (\frac{M_{\mathrm{p}}}{M_{\mathrm{pl}}})$ or assuming further sequestering between the visible and the $N=2$ to $N=0$ SUSY breaking sectors.

In summary, we have explained a possible setup in which the sfermion and gaugino masses are dominantly induced by anomaly mediation in the $N=2$ sector. Since the $N=2$ Yukawa couplings are controlled by the fixed point at low energy, light sfermions and gauginos in the $N=2$ sector have a typical spectrum that is almost UV-insensitive.

4   A partial $N=2$ SSM

We will calculate the 2-loop RG running of the couplings and the corresponding anomaly induced masses in a partial $N=2$ SSM numerically to confirm the previous fixed point phenomena at higher loop level. In particular, we take the condition discussed in **Sec.3.3**, and suppose that the soft masses in the $N=2$ sector are dominantly induced by anomaly mediation.

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1) In fact, Eq. (5) includes the Majorana mass term for the gaugini, which breaks the $\mathrm{SO}(2)_R$ symmetry. This term may arise from supergravity without spoiling the approximate $\mathrm{SO}(2)_R$ symmetry at the global limit. This is because a holomorphic Käher term, like $K = c\mathrm{tr}[WW]$, vanishes and does not break $\mathrm{SO}(2)_R$ in global SUSY, while in supergravity a gaugini Majorana mass term, like $W = cm_{3/2}\mathrm{tr}[WW]$, is generated.

We consider an example model where the additional particle content satisfies gauge coupling unification at 1-loop level and the perturbativity of dimensionless couplings. We will take this restriction, but we do not impose that the additional particles or the $N=2$ multiplets in the $N=2$ sector should be embedded into complete GUT multiplets. This is because, as proposed by Ref. [26] as a solution to the doublet-triplet and proton decay problems, GUT breaking may be due to an orbifold projection in extra dimensions which leads to missing GUT partners. We take this possibility because an extra dimension scenario is one of the leading candidates for the partial breaking. This is also the reason why we set the partial breaking scale to the GUT scale,

$$M_{\rm p}=2\times10^{16}{\rm GeV}. \qquad (47)$$

4.1   An example model

Now let us introduce a partial SSM model. The two sectors are composed by

$N=2$ **sector** $\{e_1,\overline{e}_1\},\{e_2,\overline{e}_2\},\{L_2,\overline{L}_2\},\{V_2,W\},\{V_Y,\phi_Y\}.$

$N=1$ **sector** $S_{L2},S_{e1},S_{e2},G,$
$\qquad\qquad Q_{1,2,3},u_{1,2,3},d_{1,2,3},L_{1,3},e_3,H_u,H_d,V_3.$

Here, X_i are the ith generation chiral multiplets including the SM fermion ψ_{X_i}; $V_Y,V_2,$ and V_3 are the MSSM gauge multiplets of U(1)$_Y$,SU(2)$_L$, and SU(3), respectively; H_u and H_d are the MSSM Higgs multiplets; $\overline{X}$ denotes the hyperpartner of the chiral multiplet X; S_X are the spectators; ϕ_Y and W are the $N=2$ vector partners of the gauge multiplets, V_Y and V_2, respectively; and G, which is an octet of the SU(3) and a singlet under SU(2)$_L\times$U(1)$_Y$, is introduced to satisfy the gauge coupling unification at $M_{\rm p}$. The $N=2$ sector fields are chosen to be the first two generation sfermions because they have small SM Yukawa couplings, which may be due to the SO(2)$_R$ symmetry. L_2,e_2 are assigned to be within the $N=2$ sector because there is a hint from the discrepancy of the muon $g-2$ [12–14] suggesting that $\tilde{L}_2,\tilde{e}_2$ are light, while the absence of the electron electric dipole moment implies that one of the selectrons, $\tilde{L}_1$, may be heavy. To summarize, other than the MSSM particle contents, there are chiral multiplets of $\overline{L}_2,\overline{e}_2,\overline{e}_1$ as the hyperpartners of some of the leptons, the vector partners of ϕ_Y,W to guarantee the partial $N=2$ SUSY, the spectators of S_{L2},S_{e1},S_{e2} for cancelling the chiral anomaly, and an SU(3) octet of G for the gauge coupling unification. The superpotential is given as follows.

$$W=W_{N=2}+W_{\rm mass}+W_{\rm MSSM}, \qquad (48)$$

$$W_{N=2}=\sqrt{2}\overline{L}_2(\omega_Lg_2W+\tilde{Y}_Lg_Y\phi_Y)L_2+\sqrt{2}\sum_i^2\overline{e}_ig_Y\tilde{Y}_{e_i}\phi_Ye_i, \qquad (49)$$

$$W_{\rm mass} = -M_G{\rm tr}[G^2]-M_W{\rm tr}[W^2]$$
$$-M\phi_Y^2-M_L\overline{L}_2S_{L_2}-\sum_i^2M_{e_i}\overline{e}_iS_{e_i}, \qquad (50)$$

$$W_{\rm MSSM}\simeq y_tH_uQ_3u_3+y_bH_dQ_3d_3+y_\tau H_dL_3e_3+\mu H_uH_d. \qquad (51)$$

Here y_t,y_b, and y_τ are the ordinary MSSM Yukawa couplings for top, bottom, and tau, respectively; μ is the Higgs mixing parameter; and the other parameters are defined in analogy to those in Eq. (1). We have neglected the Yukawa couplings including the first two generation fermions.

Following **Sec.3.2**, the SO(2)$_R$ symmetry and a hidden charge for the SUSY breaking field imply the following boundary conditions for the parameters at $M_{\rm p}$,

$$\omega_L=1, \quad \tilde{Y}_L=1/2, \quad \tilde{Y}_{e_1}=\tilde{Y}_{e_2}=1, \qquad (52)$$

$$m_{e_1}^2 = \frac{1}{2}m_{3/2}^2\frac{\rm d}{{\rm d}t}\gamma_{e1},$$
$$m_{e_2}^2 = \frac{1}{2}m_{3/2}^2\frac{\rm d}{{\rm d}t}\gamma_{e2},$$
$$m_{L_2}^2 = \frac{1}{2}m_{3/2}^2\frac{\rm d}{{\rm d}t}\gamma_{L2}, \qquad (53)$$

$$m_W^2=\frac{1}{2}m_{3/2}^2\frac{\rm d}{{\rm d}t}\gamma_W, \quad m_{\phi_Y}^2=\frac{1}{2}m_{3/2}^2\frac{\rm d}{{\rm d}t}\gamma_{\phi_Y}, \qquad (54)$$

$$M_1=m_{3/2}\frac{\beta_Y}{g_Y}, \quad M_2=m_{3/2}\frac{\beta_2}{g_2}. \qquad (55)$$

and the $A-$terms in the $N=2$ sector are also assumed to be induced by anomaly mediation. β_Y and β_2 are the $\beta-$functions of g_Y and g_2, respectively; M_1 and M_2 are the bino and wino masses. We do not specify the parameters in the $N=1$ sector except for the assumption of the vanishing $D-$term as discussed in Section 3.2. We also do not specify the SUSY Dirac and Majorana masses in Eq. (50).

We assume the unification condition at the scale $M_{\rm p}$ with

$$\frac{3}{5}4\pi/g_Y^2=4\pi/g_2^2=4\pi/g_3^2=1/\alpha_{\rm p}\simeq11.5. \qquad (56)$$

4.2   Low energy mass parameters and Yukawa couplings

We calculate the 2-loop RG equations with the boundary conditions of Eq. (52) where the 2-loop anomalous dimensions and the $\beta-$functions are derived following Ref. [24]. The RG runnings of the relevant dimensionless couplings are illustrated in Fig. 1.

The gray solid lines represent the scale dependence of the SM gauge couplings $\{\sqrt{\frac{5}{3}}g_Y,g_2,g_3\}$, and the {red dotted (black dotted), green dashed (zero-axis line), blue dot-dashed (black dot-dashed)} lines represent that of $\{g_2\omega_L,2\sqrt{\frac{5}{3}}g_Y\tilde{Y}_L,g_Y\sqrt{\frac{5}{3}}\tilde{Y}_{e1,e2}\}$ (at the fixed point), respectively. We can see that the $N=2$ Yukawa couplings approach their fixed point values.
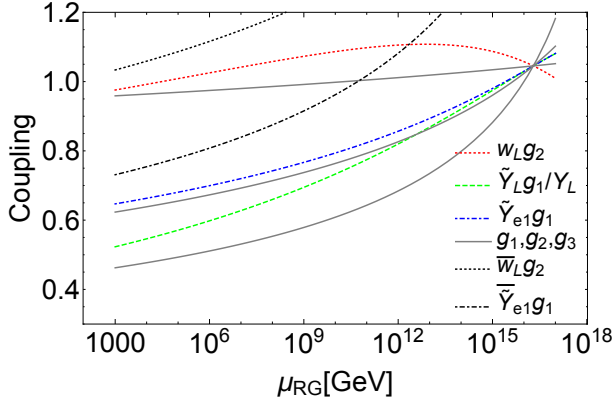
Fig. 1. (color online) The 2-loop RG running of the gauge and Yukawa couplings. See the text for details.

The convergence of the $N=2$ Yukawa couplings toward a fixed point is shown in Fig. 2.
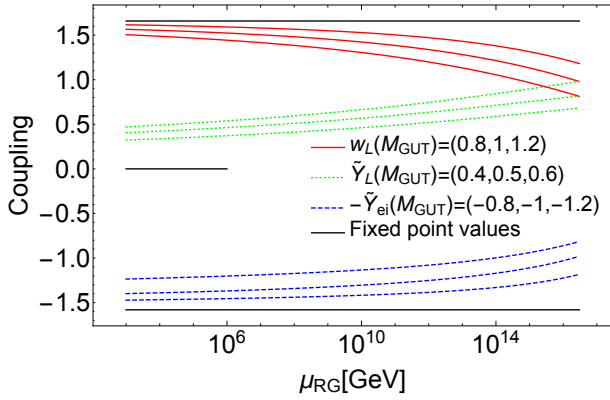

Fig. 2. (color online) The UV-insensitivity of the $N=2$ Yukawa couplings. For illustrative purposes, we flip the signs for some parameters shown in the figure.

The red solid, green dotted and blue dashed lines are obtained with the boundary conditions: $\{\omega_L,\tilde{Y}_L,\tilde{Y}_{e1},\tilde{Y}_{e2}\} = \{0.8,\frac{1}{2},1,1\},\{1,\frac{1}{2},1,1\},\{1.2,\frac{1}{2},1,1\}$, $\{\omega_L,\tilde{Y}_L,\tilde{Y}_{e1},\tilde{Y}_{e2}\} = \{1,0.4,1,1\},\{1,0.5,1,1\},\{1,0.6,1,1\}$, and $\{\omega_L,\tilde{Y}_L,\tilde{Y}_{e1},\tilde{Y}_{e2}\}=\{1,\frac{1}{2},1,0.8\},\{1,\frac{1}{2},1,1\},\{1,\frac{1}{2},1,1.2\}$, respectively. The black solid line denotes the fixed point values of the $N=2$ Yukawa couplings evaluated by Eqs. (18), (19), and (25). Therefore, we find that the $N=2$ Yukawa couplings, especially ω_L, are almost UV-insensitive. Even if Eqs. (52) in the $N=2$ sector are more or less violated at $M_{\rm p}$, the $N=2$ Yukawa couplings at low energy are approximated well by these fixed point values, Eqs. (18), (19), and (25).

The numerically evaluated values (fixed point values at 1-loop order) of the couplings at the renormalization scale, $\mu_{\rm RG}=10$ TeV, are

$$\omega_L^2 \simeq 2 \ (\frac{11}{4}),\ g_Y^2\tilde{Y}_L^2 \simeq 0.05\ (0),\ \tilde{Y}_{e1}^2 \simeq \tilde{Y}_{e2}^2 \sim 2\ (\frac{5}{2}), \quad (57)$$

with

$$g_Y^2 \simeq 0.1, \quad g_2^2 \simeq 0.4, \quad g_3^2 \simeq 0.9. \quad (58)$$

Since the third generation and Higgs fields are in the $N=1$ sector, these values depend less on y_t, y_b and y_τ and hence we do not specify $\tan\beta$.

In Fig. 3, the scale dependence of some relevant anomaly-induced masses is shown with $m_{3/2}=100$ TeV at the 3-loop level (gaugino masses are evaluated at 2-loop level). The gray solid and dotted lines represent the scale dependence of bino and wino masses, respectively. The red solid (black solid) and green dashed (black dashed) lines represent $\text{sign}(m_{L2}^2)|m_{L2}|$, and $-\text{sign}(m_{e1}^2)|m_{e1}| \simeq -\text{sign}(m_{e2}^2)|m_{e2}|$ (at the fixed point), respectively. Here, the sign of the vertical axis denotes the sign of the mass squared. We can see that below $\mu_{\rm RG}=10^9$ GeV all the slepton mass squares become positive.
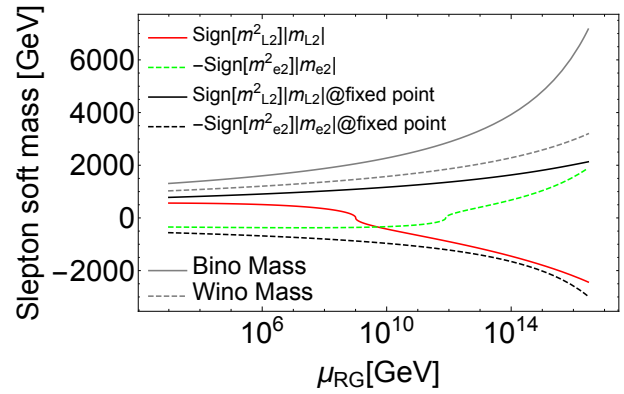

Fig. 3. (color online) The 2-loop RG running of the anomaly-induced masses for the MSSM particles in the $N=2$ sector. The sign of the vertical axis represents the sign of the mass squared. See the text for details.

In Fig. 4, we express the UV-insensitivity of these anomaly induced masses. The red solid and green dashed lines represent the runnings of the anomaly induced masses with the boundary conditions:

$$\{\omega_L,\tilde{Y}_L,\tilde{Y}_{e1},\tilde{Y}_{e2}\}=\{0.8,\frac{1}{2},1,1\},\{1,\frac{1}{2},1,1\},\{1.2,\frac{1}{2},1,1\},$$

and $\{\omega_L,\tilde{Y}_L,\tilde{Y}_{e1},\tilde{Y}_{e2}\}=\{1,\frac{1}{2},1,0.8\},\{1,\frac{1}{2},1,1\},\{1,\frac{1}{2},1,1.2\}$,

respectively. The black solid (dashed) line represents the anomaly induced mass at the fixed point, Eqs. (39) and (40).

Therefore, we have shown that by using a concrete model the tachyonic slepton problem in the $N=2$ sector is solved in an UV-insensitive manner.

In particular, the numerical values of the relevant anomaly-induced masses in the $N=2$ sector are evaluated at $\mu_{\rm RG}=10$ TeV,

$$m_{e1}=m_{e2}\simeq 0.004 m_{3/2},\ m_{L_2}\simeq 0.006 m_{3/2}, \quad (59)$$

$$M_1 \simeq 0.01 m_{3/2}, \quad M_2 \simeq 0.01 m_{3/2}. \tag{60}$$

Notice that the bino and wino are both heavier than the sleptons, and they cannot be the dark matter particles as in the ordinary MSSM.
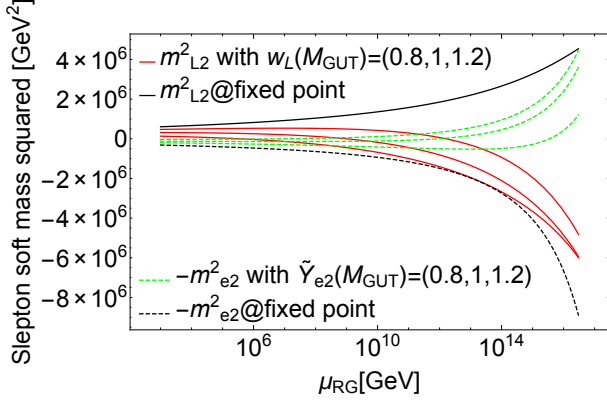

Fig. 4.　(color online) The UV-insensitivity of the anomaly-induced mass squares in the $N=2$ sector. For illustrative purposes, we flip the signs for some parameters shown in the figure.

4.3　Phenomenological aspects

Tadpole problem and dark matter candidates

Before discussing the phenomenology, let us focus on a tadpole problem due to the existence of the gauge singlet ϕ_Y [27]. Since cubic radiative divergences are not forbidden in the softly broken SUSY theory, a fundamental singlet scalar has a large tadpole term $\sim m_{3/2}^2 M_{\rm p} \phi_Y$, which would lead to a large vacuum expectation value of $\langle \phi_Y \rangle \sim M_{\rm p}$. This is quite problematic for our scenario because the SM fermions embedded in the hypermultiplets become fairly massive through the $N=2$ Yukawa couplings. A simple way to solve this problem is to impose an exact Z_2 symmetry[1], under which ϕ_Y is odd, and this term is forbidden. Then, we can find that W should also be Z_2-odd. Thus, if a component of ϕ_Y or W is the lightest Z_2-odd particle, it can be the dark matter particle. Since this Z_2 is not necessarily an R−parity, the light sfermions can be even lighter than the dark matter, while with R-parity violation the lightest sfermion decays. This possibility provides a significant feature of our scenario differing from the MSSM. Therefore, we will consider that R-parity is violated, while Z_2 symmetry is exact and a component of ϕ_Y or W, the lightest Z_2-odd particle, is stable. From the superpotential (48), the additional particles to the MSSM except for G are all odd, to guarantee the Z_2 symmetry.

The dark matter physics will be discussed later in detail.

Muon $g-2$

The sleptons in this scenario might be excluded up to 250　GeV (350 GeV) for right-handed (left-handed) ones [28][2]. Therefore, we may have a constraint,

$$m_{3/2} > 70 \text{ TeV}. \tag{61}$$

If the $\mu-$term, and the ratio of the vacuum expectation value of H_u^0 to H_d^0, i.e. $\tan\beta$, are large enough, the muon $g-2$ contribution can be evaluated as [30, 31] at the 1-loop level[3],

$$\delta\alpha_\mu \simeq \left(\frac{1}{1+\Delta_\mu} \right) \frac{g_Y^2}{16\pi^2} \frac{m_\mu^2 \mu \tan\beta M_1}{m_{L_2}^2 m_{e_2}^2} f_N \left(\frac{m_{L2}^2}{M_1^2}, \frac{m_{e2}^2}{M_1^2} \right), \tag{62}$$

$$= 25 \times 10^{-10} \left(\frac{1.4}{1+\Delta_\mu} \right) \left(\frac{\mu \tan\beta}{300 \text{ TeV}} \right) \left(\frac{80 \text{ TeV}}{m_{3/2}} \right)^2, \tag{63}$$

where,

$$\Delta_\mu \simeq \mu \tan\beta \frac{g_Y^2 M_1}{16\pi^2} I(M_1^2, m_{L_2}^2, m_{e_2}^2)$$

$$= 0.4 \left(\frac{\mu \tan\beta}{300 \text{ TeV}} \right) \left(\frac{80 \text{ TeV}}{m_{3/2}} \right), \tag{64}$$

and we have substituted the anomaly-induced masses for the sfermions and the gauginos. Here, $I(x,y,z)$ and $f_N(x,y)$ are loop functions which can be found in the references. If we quote [12, 13]

$$\delta\alpha_{\rm exp} = (26.1 \pm 8.0) \times 10^{-10}. \tag{65}$$

as a reference value of the experimental deviation from the SM prediction, the muon $g-2$ anomaly can be explained within the error at the 1σ level (2σ level) with $m_{3/2} \lesssim 100$ TeV (120 TeV) for $\mu \tan\beta = 300$ TeV, for instance.

Higgs boson mass

The mass scale, $m_{N=1}$, of the $N=1$ sector can be much larger than that of the $N=2$ sector, depending on the detail of the partial SUSY breaking. Such a heavy scalar mass may raise the sfermion masses in the $N=2$ sector via 2-loop RG running, the contribution of which can be approximated by [24, 25]

$$\delta m_{L_1, e_{1,2}}^2 \sim \frac{(g_2^2 \text{ or } g_Y^2)}{(16\pi^2)^2} \ln \left(\frac{m_{N=1}}{M_{\rm p}} \right) (g_3^2 \text{ or } y_t^2) m_{N=1}^2$$

$$\sim (400 \text{ GeV})^2 \left(\frac{m_{N=1}}{40 \text{ TeV}} \right)^2. \tag{66}$$

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1) This symmetry could also be approximate but precise enough.

2) The process we consider can be either a slepton decaying to lepton and neutrino (R−parity violation) or slepton decaying to lepton and lightest singlet (it might be bini or wini). We have assumed the decoupling of the other particles absent in the process. For the left-handed smuon, the bound should be over-estimated, as we have only one light flavor. For the case of long-lived slepton see Ref. [29].

3) The additional particles contribute to the muon $g-2$ effective vertex at more than 2-loop level due to Z_2 parity conservation. This is the reason we have used the formula for the MSSM.

Here we have used the fact that the $N=2$ sector fields are color singlets of SU(3), and hence the RG effect must contain EW gauge couplings or the $N=2$ Yukawa couplings which are of the same order.

Therefore, the parameter region, where the $N=2$ sector sfermion masses are dominantly induced by anomaly mediation, can be roughly characterized as

$$m_{N=1} \lesssim 40 \text{ TeV} \left(\frac{m_{3/2}}{100 \text{ TeV}} \right). \qquad (67)$$

Since there is a small threshold correction due to the suppressed mixing term of the stops, the Higgs mass is mainly raised by the RGE effect through the top-loop, which is cut off by the stop mass [11], and thus a large stop mass scale is predicted. By using FeynHiggs [32], we found that $m_{\text{stop}} \gtrsim 6$ TeV is obtained for $\tan\beta \gg 10$ for the typical spectra where squarks, higgsino, MSSM Higgs are heavy while gauginos are light[1]. Thus, the stops in the $N=1$ sector are allowed to give a large quantum correction to explain the correct Higgs boson mass [11]. Unfortunately, a large stop mass implies we have several amount of fine tuning to obtain the correct EW vacuum.

Constraints from particle physics and cosmology

Paying the cost of the tuning, we can obtain several relaxations for the ordinary problems of the MSSM simply due to the heavy $N=1$ sfermions and the gravitino. In the light of the heavy sfermions in the $N=1$ sector, SUSY contributions to the FCNC and CP-violating processes are suppressed[2]. Furthermore, the heavy gravitino, $m_{3/2} \sim 100$ TeV, decays much earlier than the BBN era and the cosmological gravitino problem is alleviated [17]. Also, the SUSY breaking field is not a singlet, and we do not have the cosmological moduli/Polonyi problem [33].

One may worry about the vacuum decay problem because we have large trilinear terms proportional to $\mu\tan\beta$, which implies the existing of charge breaking deeper minima than the EW vacuum of the potential. Since the fields in the $N=1$ sector are heavy and can be neglected in the discussion, the EW vacuum dominantly decays into the smuon number violating one. This was studied in Ref. [34], from which we get $\mu\tan\beta < 1140$ TeV(1400 TeV) with $m_{3/2} = 100$ TeV(120 TeV) in our case.

Let me comment on a problem due to the additional color octet and its possible solutions. The anomaly-induced mass for the gluino vanishes at 1-loop level, and is generated at 2-loop level as

$$M_3 \sim 0.002 m_{3/2}, \qquad (68)$$

at $\mu_{\text{RG}} = 10$ TeV. The gluino mass, if given by this formula, is too small to survive the experimental constraints with $m_{3/2} = \mathcal{O}(100)$ TeV [35].

There are two ways to tackle this problem. One is to introduce a Dirac gluino mass term, as $W = \frac{D^a}{M_{\text{p}}} \text{tr}[GW_a^{(3)}]$, where $D^a = D_Z \theta^a$ and $W_a^{(3)}$ are a spurion SUSY breaking field[3] and the field strength of the SU(3) gauge interaction, respectively. Notice that such a Dirac mass term is not allowed for the bino or wino due to the Z_2 parity, and our prediction would not change unless D_Z is extremely large. The other way is to have a large M_G with a supergravity induced "b−term", $V = M_G m_{3/2} \text{tr}[GG]$. Then the decoupling of G induces a gauge mediation effect Eq. (29) to raise the gluino mass to be the MSSM anomaly-induced one, $\gtrsim 2$ TeV, with $m_{3/2} \gtrsim 100$ TeV. The $N=2$ sector spectrum does not change at leading order. In the Higgs mass calculation, we have taken the latter possibility.

Dark matter and leptogenesis

For simplicity, suppose that the lightest Z_2-odd particle is much lighter than the other Z_2-odd particles, so we can discuss the dark matter physics in a generic manner. If the dark matter particle is a wini (bini), i.e. the fermionic component of ϕ_Y (W), it does not have any Yukawa interactions, due to this assumption. The physics of wini dark matter is similar to the pure wino case [36], the difference of which will be discussed in Section 5, while the bini is decoupled from the SM sector.

The interesting candidates for dark matter are the sbino and swino, i.e. the scalar components of the ϕ_Y and W, respectively. For instance, let us consider the imaginary part of the sbino, $\phi \equiv \frac{1}{\sqrt{2}} \Im \tilde{\phi}_Y$. It has a quartic potential only with the $N=2$ sector sfermions given by

$$V_\phi \sim \sum_{i=1}^{2} \left(\frac{m_{S_{ei}}^2}{m_{S_{ei}^2 + M_{S_{ei}}^2}} \right) g_Y^2 \left| \tilde{Y}_{e_i} \tilde{e}_i \phi \right|^2$$

$$+ \left(\frac{m_{S_L}^2}{m_{S_L}^2 + M_{S_L}^2} \right) g_Y^2 \left| \tilde{Y}_L \tilde{L}_2 \phi \right|^2 \qquad (69)$$

The ratio of the mass terms denotes the effect of the non-SUSY decoupling of the spectators (we have neglected the "b−terms" for illustrative purposes), and at the SUSY limit, this vanishes. However, this becomes an $\mathcal{O}(1)$ coefficient in general due to the SUSY breaking terms.

Now let us discuss the abundance of dark matter. At the fixed point $\tilde{Y}_L = 0$, the first term of Eq. (69) denotes the dominant interaction for the sbino and thus controls

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1) In the estimation, we have not included the effect of the additional particles, which is at 2-loop order.

2) We have assumed flavor symmetry conservation for the $N=2$ sector field for this example model.

3) This is a natural introduction when the SUSY breaking $N=2$ multiplet is a vector multiplet, as D^a can be recognized as field strength for the vector partner of the chiral SUSY breaking field, Z.

the annihilation process represented by

$$\phi+\phi\rightarrow\tilde{e}_i+\tilde{e}_i^\dagger. \tag{70}$$

Since in the early universe the annihilation of dark matter occurs only when $\tilde{e}_i$ is lighter than the dark matter particle,

$$m_{e_i}<m_\phi \tag{71}$$

is required. The thermal averaged total cross section is approximated by

$$\langle\sigma_{\phi\phi}|v|\rangle\sim\frac{1}{8\pi}\frac{\sum_i^2\left|g_Y^2\tilde{Y}_{e_i}^2\right|^2}{m_\phi^2}\simeq(0.1/\text{TeV})^2\left(\frac{700\ \text{GeV}}{m_\phi}\right)^2, \tag{72}$$

where we have used the fixed point value of $\tilde{Y}_{e_i}^2=5/2$ in the second approximation. Thus, the thermal abundance given is approximated by

$$\Omega_{\text{th}}h^2\simeq0.1\left(\frac{0.01\ \text{TeV}^{-2}}{\langle\sigma_{\phi\phi}|v|\rangle}\right)\sim0.1\left(\frac{m_\phi}{700\ \text{GeV}}\right)^2, \tag{73}$$

compared with the observed dark matter abundance $\Omega_{\text{DM}}h^2\simeq0.12$ [37]. Notice that the over-abundance problem, which needs to be addressed in the ordinary case of bino-like neutralino dark matter, is avoided in the sbino case due to the light annihilation products of sleptons and the large quartic couplings.

A heavy gravitino can decay into dark matter, which contributes to the dark matter abundance nonthermally [36]. The contribution in our scenario is

$$\Omega_{\text{nt}}h^2\simeq2\text{Br}_{Z_2}\Omega_{3/2}h^2\frac{m_\phi}{m_{3/2}} \tag{74}$$

$$\sim0.16\left(\frac{2n_\chi^{\text{odd}}/12}{n_V+n_\chi/12}\right)\left(\frac{m_\phi}{300\text{GeV}}\right)\left(\frac{T_R}{10^{10}\text{GeV}}\right). \tag{75}$$

Here T_R is the reheating temperature; Br_{Z_2} is the branching ratio of the gravitino decay to the Z_2 odd particles; and $\Omega_{3/2}$ is the energy density of the gravitino before its decay. n_V $(n_\chi,n_\chi^{\text{odd}})$ is the effective number of the vector (chiral, Z_2-odd chiral) multiplet, and is $1+3+8$, $(49+12+8,\ 12)$. There is a suppression factor of Br_{Z_2} because the gravitino is Z_2-even and it rarely eventually decays into dark matter when the direct decay products are Z_2-even particles. Thus, the abundance should be multiplied by 2Br_{Z_2}[1].

In summary, we find that for

$$m_\phi\lesssim700\text{GeV} \tag{76}$$

the correct dark matter abundance $\Omega_{DM}h^2=\Omega_{\text{th}}h^2+\Omega_{\text{nt}}h^2$ can be obtained with a certain $T_R\gtrsim2\times10^{10}$ GeV.

In particular, thermal leptogenesis [18] requires $T_R\gtrsim10^{9.5}$ GeV [36]. We conclude that our scenario is compatible with thermal leptogenesis.

Predictions

The direct detection constraints should not be stringent. This is because the sbino couples to a nucleon with a spin-dependent suppressed interaction through a Z-boson coupling induced by a slepton loop. The direct and indirect detections will be discussed in detail elsewhere.

Since the annihilation of the sbino dark matter is viable only when an $N=2$ slepton is lighter than its mass, the dark matter mass range turns to be a robust prediction of our scenario, that is

$$m_{e_1,e_2}\lesssim700\ \text{GeV}. \tag{77}$$

Notice that the muon $g-2$ anomaly can be explained at the 1σ level with a certain $\mu\tan\beta$ in this mass range satisfying all the constraints discussed above. If the R-parity violating decay of the light slepton occurs out of the detector, this mass range can be fully tested in the LHC with Drell-Yan production in a spectrum-independent manner [39]. If they decay within the detector they could be also tested as the $R-$parity violating scenario [28].

From the relation of the anomaly mediation, we predict the upper bound of

$$m_{L_2}\lesssim1.1\ \text{TeV and}\ M_{1,2}\lesssim1.8\ \text{TeV}. \tag{78}$$

The chargino with this mass range could also be produced in the LHC and would be followed by a typical decay to smuon and muon neutrino ($\chi_1^-\rightarrow\tilde{\mu}_2^-+\nu_2$), or smuon neutrino and muon ($\chi_1^-\rightarrow\tilde{\nu}_2+\mu_2^-$). In this case, our scenario could be confirmed by measuring the typical mass pattern, especially for that for wino and light sleptons.

Furthermore, the $N=1$ sector sfermions as well as the $N=2$ partners, if are light enough, could be produced in high energy future colliders, such as FCC, SPPC, CLIC, or a Muon Collider Higgs Factory [16]. In particular, if the mass of a fermionic hyperpartner of the electron (muon) is within reach in the electron (muon) collider, the hyperpartners are pair-produced via a sbino-propagating t-channel process, $e^-+e^+\rightarrow\overline{e}^++\overline{e}^-$ ($\mu^-+\mu^+\rightarrow\overline{\mu}^++\overline{\mu}^-$). In this case, the production rate is proportional to the fourth power of the $N=2$ Yukawa coupling. Thus, the typical Yukawa coupling controlled by the fixed point can be obtained if the production rate is carefully measured, which could be striking evidence for our scenario.

5   Discussion and conclusions

Since we are essentially relying on the behavior around an IR fixed point, several discussions can apply to models without partial $N=2$ SUSY but with adjoint chiral multiplets. We may also consider the possibility

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1) The additional chiral multiplets do not change the thermally produced gravitino abundance at leading order, because the production is dominated by the dimension-5 gravitino-gauge interaction [38] which does not differ from the MSSM interaction in our scenario.

that some of the quarks are within the $N = 2$ sector, which can lead to light squarks and gluinos.

I would like to mention the difference between pure wini and pure wino dark matter in the light of leptogenesis. The thermal abundance estimation of pure wini dark matter is quite similar to the pure wino case, the mass of which is bounded up to 2.7 TeV from the correct abundance. It is known that ordinary pure wino dark matter should have $M_2 \lesssim 1$ TeV to be compatible with thermal leptogenesis [36]. This is bounded from the nonthermal production of dark matter abundance. In our case, thanks to the suppression factor in Eq. (74), the pure wini dark matter has a larger mass range compatible with thermal leptogenesis, which is $M_W \lesssim 2.7$ TeV for the example model.

We have investigated partial $N = 2$ supersymmetric (SUSY) extensions of the standard model, composed of two sectors with almost $N = 2$ SUSY and $N = 1$ SUSY at the partial breaking scale, respectively. Since the global $N = 2$ SUSY is expected from the simple toroidal compactification of extra-dimensional SUSY theory, including the effective theory of superstrings, the SM may originate from $N = 2$ SUSY. If the partial breaking of $N = 2$ to $N = 1$ takes place in a sequestered sector, the $N = 2$ partners in the $N = 2$ sector may be light enough to give interesting low energy phenomena. In particular, we have shown that the light $N = 2$ partners in the $N = 2$ sector have almost UV-insensitive significant Yukawa interactions due to an IR fixed point.

From these interactions, the typical anomaly induced masses for the sfermions and the gauginos in the $N = 2$ sector are almost UV-insensitive. In fact, we have clari-

fied that the anomaly mediation goes well with the partial $N = 2$ SSMs in two aspects: (a) partial $N = 2$ SUSY can provide a good condition for the anomaly mediation to be effective in the $N = 2$ sector due to the $N = 2$ non-renormalization theorem, and (b) the tachyonic slepton problem is automatically solved due to the large $N = 2$ Yukawa couplings around the IR fixed point.

In a concrete model of a partial $N = 2$ SSM with gauge coupling unification, we have shown that the muon $g-2$ anomaly can be explained within its 1σ level error by the light smuons and gauginos. The masses are anomaly-induced and are 1-loop suppressed to the gravitino mass of $\mathcal{O}(100)$ TeV. We have discussed the phenomenological and cosmological aspects. In particular, we have considered the dark matter candidate as a sbino, the scalar component of the singlet $N = 2$ vector multiplet. To explain the dark matter abundance, the sbino and right-handed smuon are required to be lighter than 700 GeV. Then from the anomaly mediation relation we predict that all the sparticles of the MSSM in the $N = 2$ sector have masses below ~ 2 TeV, with a pattern. These are robust predictions that can be tested and could be confirmed in the LHC or in future colliders. If the $N = 2$ partners are also reachable in future colliders, the predicted $N = 2$ Yukawa couplings might be measured as striking evidence for the scenario.

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