Experimental determination of one- and two-neutron separation energies for neutron-rich copper isotopes *

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Abstract: A method is proposed to determine the one-neutron $S_{\rm n}$ or two-neutron $S_{\rm 2n}$ separation energy of neutron-rich isotopes. Relationships between $S_{\rm n}$ ($S_{\rm 2n}$) and isotopic cross sections have been deduced from an empirical formula, i.e., the cross section of an isotope exponentially depends on the average binding energy per nucleon B/A. The proposed relationships have been verified using the neutron-rich copper isotopes measured in the 64A MeV ^{86}Kr + ^{9}Be reaction. $S_{\rm n}$, $S_{\rm 2n}$, and B/A for the very neutron-rich ^{77,78,79}Cu isotopes are determined from the proposed correlations. It is also proposed that the correlations between $S_{\rm n}$, $S_{\rm 2n}$ and isotopic cross sections can be used to find the location of neutron drip line isotopes.

Keywords: neutron separation energy, neutron-drip line, neutron-rich isotope

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1 Introduction

The rare isotopes, including the very neutron-rich isotopes and the very proton-rich ones, which are near the neutron and proton drip lines, consistently attract the interest of both theoretical and experimental scientists. Indicated by the easy separation of neutrons, isotopes near the neutron drip line have very small one-neutron separation energy ($S_{\rm n}$) or two-neutron separation energy ($S_{\rm 2n}$), which means that the one or two neutrons can be removed from the nucleus quite smoothly. The same happens in the one-proton separation energy ($S_{\rm p}$) or two-proton separation energy ($S_{\rm 2p}$) for isotopes near the proton drip line. New facilities for radioactive ion beams make it possible to search for the location of the neutron drip line. Besides the ∼3000 isotopes which have been found experimentally [1], most isotopes near the drip lines are only theoretically predicted to exist [2]. These rare isotopes near the drip lines test the limits of both radioactive ion beam facilities and detector systems, since they have very low production probabilities in experiments. Thus it is always important to estimate the production of rare isotopes in experiments. Empirical parameterizations including EPAX3 [3], FRACS [4] (which is an improved version of EPAX3), and SPACS [5], etc., have been developed to estimate the cross sections of rare isotopes in projectile fragmentation and spallation reactions. Besides the evaluation of binding energy and separation energy for rare isotopes, many theoretical methods have been developed to predict the binding energy of rare isotopes by extending the basic mass formula, for example, the macroscopic-microscopic approach [6], the Weizsäcker-Skyrme formula [7–10], and the improved Jänecke mass formula [11]. Some empirical formulas have also been found via the correlation between the yield of fragments and their free energy or binding energy [12–14]. In this paper, a method is proposed to determine the binding energies $S_{\rm n}$ and $S_{\rm 2n}$ from isotopic yield.

2 Formulism

Tsang et al have proposed a method to determine B for the very neutron-rich copper isotopes, for which the isotopic distribution depends exponentially on the average binding energy per nucleon [12, 15],

$$\sigma = C\exp[(B'-8)/\tau], \tag{1}$$

where $B'=(B-\varepsilon_{\rm p})/A$, with $\varepsilon_{\rm p}$ being the pairing energy. C and τ are free parameters. $\varepsilon_{\rm p}$ is introduced to minimize the odd-even staggering in the isotopic distribution,

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which is:

$$\varepsilon_{\rm p}=0.5[(-1)^N+(-1)^Z]\varepsilon\cdot A^{-3/4}. \quad (2)$$

$\varepsilon=$ 30 MeV is adopted, as done in Ref. [12]. For an isotope with charge and mass numbers (Z,A), taking the logarithm of Eq. (1) and multiplying by A, one has:

$$A\ln\sigma_{(Z,A)}=A\ln C+[B_{(Z,A)}-\varepsilon_{{\rm p}(Z,A)}-8A]/\tau, \quad (3)$$

where (Z,A) is used as an index to indicate the isotope. For the isotope $(Z,A$–$1)$, i.e., one neutron removed from the isotope (Z,A),

$$(A-1)\ln\sigma_{(Z,A-1)}=(A-1)\ln C +[B_{(Z,A-1)}-\varepsilon_{{\rm p}(Z,A-1)}-8(A-1)]/\tau, \quad (4)$$

$$(A-2)\ln\sigma_{(Z,A-2)}=(A-2)\ln C +[B_{(Z,A-2)}-\varepsilon_{{\rm p}(Z,A-2)}-8(A-2)]/\tau. \quad (5)$$

Combining Eqs. (3) and (4), for the isotope $(Z,A-1)$, i.e., one neutron removed from the isotope (Z,A),

$$A\ln\sigma_{(Z,A)}-(A-1)\ln\sigma_{(Z,A-1)} =\ln C+[B_{(Z,A)}-B_{(Z,A-1)}-\varepsilon_{{\rm p}(Z,A)}+\varepsilon_{{\rm p}(Z,A-1)}-8]/\tau. \quad (6)$$

Similarly, combining Eqs. (3) and (5), for the isotope $(Z,A-2)$, i.e., two neutrons removed from the isotope (Z,A),

$$A\ln\sigma_{(Z,A)}-(A-2)\ln\sigma_{(Z,A-2)} =2\ln C+[B_{(Z,A)}-B_{(Z,A-2)}-\varepsilon_{{\rm p}(Z,A)}+\varepsilon_{{\rm p}(Z,A-2)}-16]/\tau. \quad (7)$$

The definitions of one-neutron separation energy and two-neutron separation energy for an isotope are:

$$S_{\rm n}^{(Z,A)}=B_{(Z,A)}-B_{(Z,A-1)} \quad (8)$$

and

$$S_{\rm 2n}^{(Z,A)}=B_{(Z,A)}-B_{(Z,A-2)}, \quad (9)$$

respectively. The definitions of $S_{\rm n}$ and $S_{\rm 2n}$ in relation to binding energy are the same as those in relation to atomic mass [2]. Inserting Eq. (8) into (6), with the definition of $\sigma^{(-{\rm n})}_{(Z,A)}\equiv A\ln\sigma_{(Z,A)}-(A-1)\ln\sigma_{(Z,A-1)}$, one has

$$\sigma^{(-{\rm n})}_{(Z,A)}=\ln C+[S_{\rm n}^{(Z,A)}-\varepsilon_{{\rm p}(Z,A)}+\varepsilon_{{\rm p}(Z,A-1)}-8]/\tau, \quad (10)$$

Similarly, inserting Eq. (9) into (7), with the definition of $\sigma^{(-2{\rm n})}_{(Z,A)}\equiv A\ln\sigma_{(Z,A)}-(A-2)\ln\sigma_{(Z,A-2)}$, one has

$$\sigma^{(-2{\rm n})}_{(Z,A)}=2\ln C+[S_{\rm 2n}^{(Z,A)}-\varepsilon_{{\rm p}(Z,A)}+\varepsilon_{{\rm p}(Z,A-2)}-16]/\tau, \quad (11)$$

For simplification, $S'_{\rm n}\equiv S_{\rm n}^{(Z,A)}-\varepsilon_{{\rm p}(Z,A)}+\varepsilon_{{\rm p}(Z,A-1)}$ and $S'_{\rm 2n}\equiv S_{\rm 2n}^{(Z,A)}-\varepsilon_{{\rm p}(Z,A)}+\varepsilon_{{\rm p}(Z,A-2)}$ are defined. By Eqs. (6), (7), (10), and (11), σ, B, or $S_{\rm n}$ ($S_{\rm 2n}$) can be predicted from each other based on known parameters. The parameters C and τ in this work are determined using the least squares method. The cross sections, as well as the uncertainties, predicted by experimental $S_{\rm n}$ and $S_{\rm 2n}$, adopt the prediction technique; $S_{\rm n}$ or $S_{\rm 2n}$ and the uncertainties are determined from the cross section using the reverse prediction technique based on the least squares method [16].

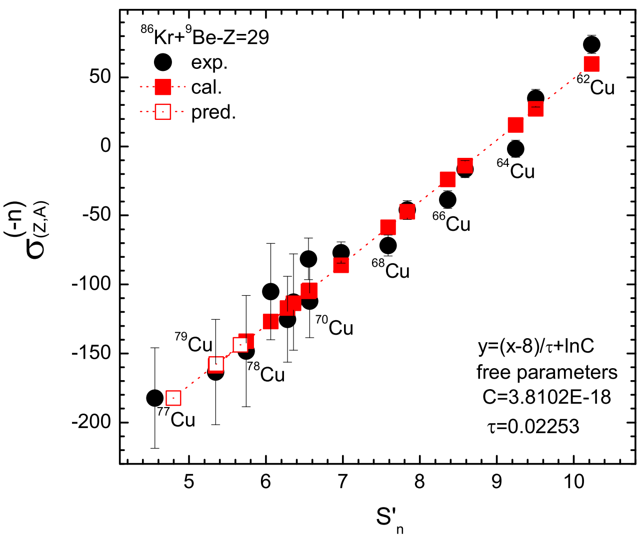

Fig. 1. (color online) The correlation between $\sigma^{(-{\rm n})}_{(Z,A)}$ and $S'_{\rm n}$ for the measured isotopes in the 64A MeV ^{86}Kr + ^{9}Be reaction. The circles denote $S'_{\rm n}$ and are calculated from the measured $S_{\rm n}$ in AME16 [2]. The line denotes the fitting to the $\sigma^{(-{\rm n})}_{(Z,A)}$ and $S'_{\rm n}$ correlation for the measured results according to Eq. (10). The full squares denote the calculated $\sigma^{(-{\rm n})}_{(Z,A)}$ using the fitting function and the experimental $S_{\rm n}$. The open squares denote the $S'_{\rm n}$ predicted from the fitting function by $\sigma^{(-{\rm n})}_{(Z,A)}$ with no measured $S_{\rm n}$ in AME16.

The cross sections for the neutron-rich copper isotopes in the 64 MeV/u ^{86}Kr + ^{9}Be reaction [12], which were measured at RIKEN by Tsang et al, are adopted to perform the analysis. The values for C and τ have been determined to be $C=2.17\times10^{-15}$ mb and $\tau=0.0213$ MeV [12]. The results of $S_{\rm n}$ and $S_{\rm 2n}$ for the copper isotopes in the new version of the Atomic Mass Evaluation (AME16) [2] are adopted. The correlation between $\sigma^{(-{\rm n})}_{(Z,A)}$ and $S'_{\rm n}$ for the neutron rich copper isotopes is plotted in Fig. 1, from which a quite a good linear correlation can be found. The fitting to the $\sigma^{(-{\rm n})}_{(Z,A)}$ and $S'_{\rm n}$ correlation yields $C=3.81\times10^{-18}$ mb and $\tau=0.0225$ MeV. The fitted τ is similar to that in Ref. [12], while C is much smaller than that in Ref. [12]. The correlation between $\sigma^{(-2{\rm n})}_{(Z,A)}$ and $S'_{\rm 2n}$ for the copper isotopes is plotted in Fig. 2, showing that it obeys the theoretical prediction much better than $\sigma^{(-{\rm n})}_{(Z,A)}$ and $S'_{\rm n}$. Meanwhile, the odd-even staggering is less obvious compared to $\sigma^{(-{\rm n})}_{(Z,A)}$.

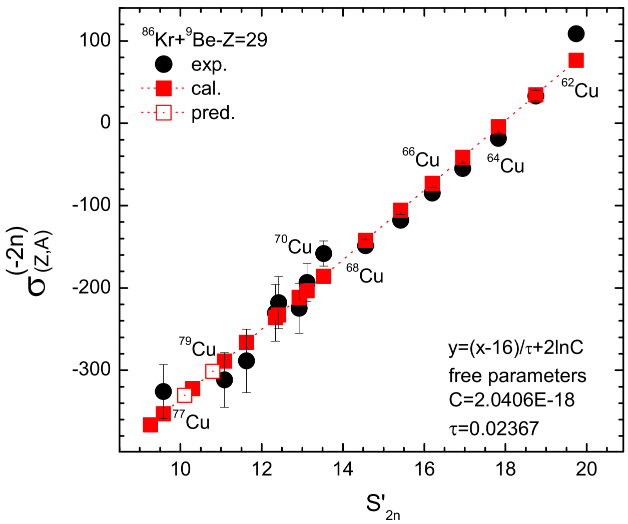

Fig. 2. (color online) The correlation between $\sigma^{(-2\mathrm{n})}_{(Z,A)}$ and $S'_{2\mathrm{n}}$ for the measured fragments in the 64A MeV ^{86}Kr + ^{9}Be reaction. The circles denote $S'_{2\mathrm{n}}$ and are calculated from the measured $S_{2\mathrm{n}}$ in AME16. The line denotes the fitting to the $\sigma^{(-2\mathrm{n})}_{(Z,A)}$ and $S'_{2\mathrm{n}}$ correlation for the measured results according to Eq. (11). The full squares denote the calculated $\sigma^{(-2\mathrm{n})}_{(Z,A)}$ using the fitting function and the experimental $S_{2\mathrm{n}}$. The open squares denote the $S'_{2\mathrm{n}}$ predicted from the fitting function by $\sigma^{(-2\mathrm{n})}_{(Z,A)}$.

In Table 1, the results for S_n and $S_{2\mathrm{n}}$ of $^{65-76}$Cu isotopes obtained by Eqs. (8) to (11) are compared to the experimental data in AME16 [2]. It can be seen that S_n obtained by Eq. (8) is very similar to the AME16 data. S_n obtained by Eq. (10) is very close to the AME16 data except for some isotopes which deviate from the fitting line. The largest difference between S_n from Eq. (10) and AME16 data is no more than $\pm$0.5 MeV, and most are within $\pm$0.3 MeV. A similar result is found for $S_{2\mathrm{n}}$ determined by Eqs. (9) and the AME16 data. The results suggest that S_n and $S_{2\mathrm{n}}$ determined by the combined isotopic cross sections are reasonable and have a high quality.

3 Results and discussion

For ^{77,78,79}Cu, the values of S_n in AME16 are estimated, as there are no experimental results. The values of S_n determined for these isotopes are listed in Table 2, and are a little larger than the evaluated results in AME16. The results for $S_{2\mathrm{n}}$ for ^{77,79}Cu are determined from $\sigma^{(-2\mathrm{n})}_{(Z,A)}$. $S_{2\mathrm{n}}$ for ^{77}Cu is close to the evaluated result in AME16, but for ^{79}Cu it is 1.54 MeV larger than that in AME16. Combining the determined S_n and $S_{2\mathrm{n}}$, it is predicted that copper should have more neutron-rich isotopes than ^{79}Cu.

The binding energy per nucleon (B/A) for these isotopes is also calculated from the values of S_n and $S_{2\mathrm{n}}$ determined in this work using Eqs. (8) and (9). In Table 2 it can be seen that the values of B/A determined from S_n and $S_{2\mathrm{n}}$ are very close to the AME16 evaluation.

Table 1. The results for S_n, $S_{2\mathrm{n}}$ determined by Eqs. (8), (9), (10) and (11) compared with AME16 [2] data (in MeV) for $^{65-76}$Cu.

nuclei	S_n			$S_{2\mathrm{n}}$		
	Eq. (8)	Eq. (10)	AME16	Eq. (9)	Eq. (11)	AME16
^{65}Cu	9.1904	9.8583	9.9104	17.8266	17.4970	17.8265
^{66}Cu	7.0659	6.7399	7.0659	16.9764	16.6566	16.9764
^{67}Cu	9.1325	9.1617	9.1326	16.1985	15.9262	16.1985
^{68}Cu	6.3188	6.0183	6.3188	15.4514	15.1666	15.4514
^{69}Cu	8.2406	8.4376	8.2405	14.5594	14.4073	14.5593
^{70}Cu	5.3115	5.8291	5.3115	13.5520	14.2071	13.552
^{71}Cu	7.8060	7.6192	7.8061	13.1175	13.3487	13.1175
^{72}Cu	5.1432	5.1495	5.1432	12.9493	12.6334	12.9493
^{73}Cu	7.2757	7.7499	7.2758	12.4190	12.7721	12.4189
^{74}Cu	5.0899	4.8923	5.09	12.3656	12.5006	12.366
^{75}Cu	6.5363	6.4106	6.536	11.6262	11.0947	11.627
^{76}Cu	4.5765	4.3968	4.576	11.1128	10.5730	11.112

Table 2. The results for S_n, $S_{2\mathrm{n}}$ and B/A (in MeV) for the neutron-rich copper isotopes determined (Prd.) in this work, in Ref. [12], and evaluated in AME16 [2].

	S_n		$S_{2\mathrm{n}}$		B/A			
	Prd.	[2]	Prd.	[2]	Prd.*	Prd.†	[12]	[2]
^{77}Cu	5.961	5.72	10.10	10.3	8.411	8.406	8.404	8.408
^{78}Cu	4.523	3.95	10.24	9.6§	8.362	8.358	8.354	8.351§
^{79}Cu	6.497	5.31	10.80	9.26	8.328	8.330	8.327	8.312

* and † denote B/A is determined by S_n and $S_{2\mathrm{n}}$, respectively. § denotes an experimental result in AME16.

4 Summary

Equations (8) and (10) both provide methods to determine S_n, and Eqs. (9) and (11) both provide the methods to determine S_{2n} for neutron-rich isotopes. For the very neutron-rich isotopes, it is not easy to measure the binding energy due to the low probability in experiments, which makes it difficult to determine S_n or S_{2n} from binding energy using Eqs. (8) and (9). Using Eqs. (10) and (11), S_n or S_{2n} can be determined from the yields or cross sections of isotopes, which are usually measured in projectile fragmentation reactions with high quality. Moreover, the neutron separation energy is linked to the neutron skin of neutron-rich isotopes [17]. The predicted S_n, S_{2n} or σ can also be used to guide the adjustment of theoretical calculations related to neutron-skin thickness [18–21].

The method proposed in this paper indicates that S_n and S_{2n} of different isotopes can be determined by the combined isotopic cross sections. Furthermore, this method can also be used to find the location of neutron-drip isotopes from known cross sections, and vice versa to predict the production cross section of an isotope. This can help to design experiments to search for the location of the neutron-drip line.

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