Magnetic moments and g-factors in odd-A Ho isotopes *

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Abstract: The ground-state magnetic moment, g_K factor and quenching spin gyromagnetic ratio have been calculated using the microscopic method based on the Quasiparticle Phonon Nuclear Model (QPNM) for $^{155-169}$Ho nuclei for the first time. It is shown that the residual spin-spin interactions are responsible for the core polarization, and because of the core polarization the spin gyromagnetic factors are quenched. By considering the core polarization effects, a satisfactory agreement is obtained for the computed ground state g_K factor, which gives an intrinsic contribution to the magnetic moments. In order to assess the collective contribution to the magnetic moments, the rotational gyromagnetic factors g_R have been also calculated within the cranking approximation using the single particle wave function of the axially symmetric Woods-Saxon potential. For the ground-state magnetic moments of odd-proton $^{155-165}$Ho nuclei, a good description of the experimental data is obtained with an accuracy of 0.01–0.1 μ_N. From systematic trends, the quenching spin gyromagnetic factor, g_K factor and magnetic moment have also been theoretically predicted for ^{167,169}Ho where there is no existing experimental data.

Keywords: deformed odd-A, Ho isotopes, magnetic moment, magnetic g-factors, core-polarization, QRPA

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1 Introduction

Rare-earth nuclei have attracted strong interest in the nuclear physics community in recent years because of their complex structure. As a result, they have become some of the most frequently studied nuclei and a considerable amount of effort has been spent on understanding their complex structure.

One of the basic tools to gain information on the complex structure of rare-earth nuclei is to measure the nuclear magnetic moments [1]. During the last four decades, the experimental results of ground-state nuclear spins and magnetic moments for a great number of rare earth odd-mass nuclei have been reported [2].

The limits of the magnetic moments of odd-mass nuclei are described by the Schmidt lines theoretically derived from the single-particle (shell) model [3]. However, the magnetic moments of odd-mass deformed nuclei which are far from the closed shells deviate systematically from the Schmidt predictions [4]. It is well known that these deviations are mainly due to the core polarization and meson exchange current (MEC) [5–7]. Core polarization arises as a result of the interaction between M1 excitations of the core and odd-particle. It is responsible for quenching of the spin gyromagnetic factor [8–15]. It is important to state that the meson exchange current is beyond the scope of the present paper.

In an earlier paper, we proposed a microscopic method in the QPNM framework [16] in which core polarization effects are assumed to be due to the scattering of an odd particle by 1^+ excitations of the doubly even core [12]. The influence of these effects can be taken into account by introducing an effective spin gyromagnetic factor calculated in theory. The method allows proper description of core polarization effects. Such a calculation also makes it possible to analyse underlying the microscopic structure of the ground states of nuclei investigated. The numerical calculations have been performed in the rare earth elements $^{155-165}$Dy [13], $^{157-167}$Er [12], $^{165-175}$Hf [14] for the ground states with $K>1/2$. It was demonstrated that such a quenching of the spin gyromagnetic factors is due to the residual spin-spin interactions and explains the deviation between theoretical and experimental magnetic moments.

The current work focuses on applying the QPNM method to investigate the magnetic moments of the ground state for odd-mass holmium isotopes. Hyperfine structure, nuclear spins as well as magnetic mo-

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ments of the rare earth element $^{155-165}$Ho have been investigated experimentally using resonance ionization spectroscopy [17]. However, no systematic theoretical analysis of experimental data on magnetic moments of odd-mass $^{155-165}$Ho nuclei has been performed. Therefore, holmium with long isotopic chains provides us a strong test area for the method. This is the first study in which the QPNM method has been used to explain the magnetic properties of an odd-proton nucleus.

The paper is organised as follows. In Section 2 a brief introduction of the theory is given. In Section 3 the numerical results of the magnetic moments of the ground states in $^{155-165}$Ho nuclei are presented. In Section 4 a short summary is given.

2 Theory

2.1 QPNM description of ground-state of an odd-nucleus

The renormalization of g_s is associated with the spin dependent part of the spin-spin residual interaction [4]. The spin-polarization effect can be described in terms of the coupling to excitations of the even-even core produced by spin-dependent fields [4]. This was confirmed by early calculations [4, 10, 11]. Therefore, the starting point of the method is that the nucleons in the axially symmetric average field interact via pairing and spin-spin residual forces. The QPNM Hamiltonian of such a nucleus consists of a single-quasiparticle term, a collective part and an interaction term [12–14],

$$H \approx H_{\rm sqp}+H_{\rm coll.}+H_{\rm int.}, \quad (1)$$

where

$$H_{\rm sqp}=\sum_{s,\tau}\varepsilon_s(\tau)\alpha^+_{s\rho}\alpha_{s\rho}, \quad (2)$$

$$\begin{aligned} H_{\rm coll.} &= \frac{1}{2}\sum_{\tau,\tau'}\chi_{\tau\tau'}\sum_{ss'}\sigma^{(\mu)}_{ss'}L_{ss'}g^i_{ss'}[Q^+_i(\tau)+Q_i(\tau)] \\ &\quad\times\sum_{mm'}\sigma^{(\mu)}_{mm'}L_{mm'}g^i_{mm'}[Q^+_i(\tau')+Q_i(\tau')], \end{aligned} \quad (3)$$

$$\begin{aligned} H_{\rm int.} &= \sum_{\tau,\tau'}\chi_{\tau\tau'}\sum_{mm'}\sum_{ss'}\{\sigma^{(\mu)}_{ss'}M_{ss'}\sigma^{(\mu)}_{mm'}L_{mm'}g^i_{mm'}D_{ss'}(\tau) \\ &\quad\times[Q^+_i(\tau')+Q_i(\tau')]+\sigma^{(\mu)}_{ss'}L_{ss'}\sigma^{(\mu)}_{mm'}M_{mm'}g^i_{ss'} \\ &\quad\times[Q^+_i(\tau)+Q_i(\tau)]D_{mm'}(\tau')\}. \end{aligned} \quad (4)$$

Here the first two terms ($H_{\rm sqp}$ and $H_{\rm coll.}$) describe quasiparticle motion and phonon excitations, respectively, and the last term ($H_{\rm int.}$) represents the interaction between quasiparticles and phonons [12–14]

In Eqs. (2)–(4), $\varepsilon_s(\tau)$is the quasiparticle energy of the nucleons, the quantities $M_{ss'}=u_su'_s+v_sv'_s$ and $L_{ss'}=u_sv'_s-u'_sv_s$ are expressed in terms of Bogoliubov canonical transformation parameters, and u_s and v_s. $\sigma^{(\mu)}_{ss'}=\langle s|\sigma_\mu|s'\rangle$ are single-particle matrix elements of the Pauli spin operator. $D_{ss'}=\sum_\rho\rho\alpha^+_{s-\rho}\alpha_{s'-\rho}$, where $\alpha^+_{s\rho}(\alpha_{s\rho})$ are quasi-particle creation (annihilation) operators.$g^i_{ss'}=\psi^i_{ss'}+\varphi^i_{ss'}$ is the sum of the two-quasiparticle amplitudes. The summation over $ss'(mm')$ means that the sum is taken over the average field single particle levels of the neutron (proton) system. $\chi_{\tau\tau'}$ is the spin-spin interaction parameter with τ,τ', which denotes the corresponding proton-proton, neutron-neutron ($\chi=\chi_{\rm nn}=\chi_{\rm pp},$) and proton-neutron ($\chi_{\rm np}=q\chi, q$ expresses the isovector and isoscalar characteristics of the neutron-proton spin-spin interactions) spin interactions. Both spin interaction parameters (χ) are expressed in terms of spin interaction strengths (κ), $\chi=\frac{\kappa}{A}$ MeV [12–14].

In the case of odd-mass nuclei the QPNM Hamiltonian is constructed as a superposition of one-quasiparticle and single-quasiparticles $\otimes$ phonon components

$$\Psi^j_{\rm K}(\tau)=\left\{N^j_{\rm K}\alpha^+_{\rm K}(\tau)+\sum_{i\nu}G^{K\nu}_{ij}\alpha^+_\nu(\tau)Q^+_i\right\}|\psi_0\rangle, \quad (5)$$

where the index j describes the states with a given spin and parity K^π value in an odd-mass nucleus. The quantities $N^j_{\rm K}$ and $G^{K\nu}_{ij}$ determine the contribution of the one-quasiparticle and the quasiparticle $\otimes$ phonon component in the wave function, respectively [12–14].

By using the variation principle the secular equation is determined as following:

$$-P(\eta)\equiv\varepsilon_{\rm K}-\eta_{\rm K}-\sum_{iv}\frac{1}{Z(\omega_i)}\frac{q^2\sigma^2_{Kv}M^2_{Kv}}{\left(1+\chi F^i_p\right)^2(\omega_i+\varepsilon_\nu-\eta_{\rm K},)}=0 \quad (6)$$

where

$$\begin{aligned} Z(\omega_i) &= \frac{1}{(-\chi F_{\rm n})^2}Y_{\rm n}(\omega_i)+\frac{q^2}{(1+\chi F_{\rm p})^2}Y_{\rm p}(\omega_i), \\ Y_\tau &= 4\omega_i\sum_{ss'}\frac{\varepsilon_{ss'}\sigma^2_{ss'}L^2_{ss'}}{(\varepsilon^2_{ss'}-\omega^2_i)^2}, \\ F_\tau &= 2\sum_{ss'}\frac{\varepsilon_{ss'}\sigma^2_{ss'}L^2_{ss'}}{\varepsilon^2_{ss'}-\omega^2_i}. \end{aligned} \quad (7)$$

Here, ω_i is the energy of collective 1^+ states in the even-even core. The energies $\eta^j_{\rm K}(\tau)$ of the ground and excited states of odd-mass nuclei are the roots of the secular equation. The functions G^{Kv}_{ij} and $N^j_{\rm K}$ can be easily derived by using the secular Eq. (6) and the normalization condition of the wave function [12–14].

$$N^{-2}_{\rm K}=1+\frac{1}{Z(\omega_i)}\left[\frac{q\sigma_{K\nu}M_{K\nu}}{(1+\chi F_{\rm p})(\varepsilon_\nu+\omega_i-\eta_{\rm K})}\right]^2, \quad (8)$$

$$R^{K\nu}_i=\frac{N_{\rm K}(\tau)}{Z(\omega_i)}\left[\frac{q\sigma_{K\nu}M_{K\nu}}{\chi(1+\chi F_{\rm p})(\varepsilon_\nu+\omega_i-\eta_{\rm K})}\right]. \quad (9)$$

2.2 Core polarization and ground-state magnetic momen

According to the Unified Model of Bohr and Mottelson, the ground-state (with $K>1/2$) magnetic moment of an odd-mass deformed nuclei reads

$$\mu=\frac{K}{I+1}(g_{\mathrm{K}}K+g_{\mathrm{R}}). \tag{10}$$

As can be seen in Eq. (10), the ground-state magnetic moment of an odd-A nucleus contains contributions from both intrinsic and rotational motion. Therefore, it is dependent on two parameters, namely intrinsic (g_{K}) and rotational (g_{R}) gyromagnetic factor [4].

For a $K>1/2$ state of an odd-mass nucleus the intrinsic magnetic moment $\mu_{\mathrm{K}}=g_{\mathrm{K}}K$ is the expectation value of μ_z, which is the projection of the magnetic dipole moment operator on the symmetry axis. Using the QPNM wave function, i.e Eq. (5) one can see that g_{K} is a sum of quasiparticle-quasiparticle (qp-qp), phonon-phonon (ph-ph) and quasiparticle-phonon (qp-ph) terms:

$$\mu_{\mathrm{K}} = \langle \Psi_{\mathrm{K}}^j(\tau)|\hat{\mu}_z|\Psi_{\mathrm{K}}^j(\tau)\rangle=\underbrace{\left(N_{\mathrm{K}}^j\right)^2\langle\psi_0|\alpha_{\mathrm{K}}|\hat{\mu}_z|\alpha_{\mathrm{K}}^+|\psi_0\rangle}_{(\mathrm{qp-qp})\mathrm{term}}$$
$$+\underbrace{N_{\mathrm{K}}^j\sum_{i\nu}G_{ij}^{K\nu}\left[\langle\psi_0|\alpha_{\mathrm{K}}|\hat{\mu}_z|\alpha_\nu^+Q_i^+|\psi_0\rangle+\langle\psi_0|\alpha_\nu Q_i|\hat{\mu}_z|\alpha_{\mathrm{K}}^+|\psi_0\rangle\right]}_{(\mathrm{qp-ph})\mathrm{term}}+\underbrace{\left(\sum_{i\nu}G_{ij}^{K\nu}\right)^2\langle\psi_0|\alpha_\nu Q_i|\hat{\mu}_z|\alpha_\nu^+Q_i^+|\psi_0\rangle}_{(\mathrm{ph-ph})\mathrm{term}}. \tag{11}$$

After the calculation of expectation values in Eq. (11) the analytical formula is then given by

$$\mu_{\mathrm{K}}=\left\{(g_{\mathrm{s}}^{\mathrm{p}}-g_{\mathrm{l}}^{\mathrm{p}})\left(1+N_{\mathrm{K}}^2\frac{1}{Z(\omega)}\frac{2M_{KK}(1+\chi F_{\mathrm{n}})}{(\chi^2F_{\mathrm{n}})(1+\chi F_{\mathrm{p}})(\varepsilon_{\mathrm{K}}+\omega_i-\eta_{\mathrm{K}})}\right)-g_{\mathrm{s}}^{\mathrm{n}}N_{\mathrm{K}}^2\frac{1}{Z(\omega)}\frac{2qM_{KK}}{\chi(1+\chi F_{\mathrm{p}})(\varepsilon_{\mathrm{K}}+\omega_i-\eta_{\mathrm{K}})}\right\}\langle K|\hat{s}_z|K\rangle+g_{\mathrm{l}}^{\mathrm{p}}K. \tag{12}$$

From the comparison of Eq. (12) with conventional Nilsson formula, i.e.

$$\mu=g_{\mathrm{K}}K=(g_{\mathrm{s}}^{\mathrm{p}}-g_{\mathrm{l}}^{\mathrm{p}})\langle K|\hat{s}_z|K\rangle+g_{\mathrm{l}}^{\mathrm{p}}K, \tag{13}$$

it can easily be seen that

$$g_{\mathrm{s}}^{\mathrm{eff}}-g_{\mathrm{l}}^{\mathrm{p}}=(g_{\mathrm{s}}^{\mathrm{p}}-g_{\mathrm{l}}^{\mathrm{p}})\left(1+2N_{\mathrm{K}}^2\sum_i\frac{(1+\chi F_{\mathrm{n}})}{\chi^2Z(\omega_i)F_{\mathrm{n}}(1+\chi F_{\mathrm{p}})}\frac{1}{(\varepsilon_{\mathrm{K}}+\omega_i-\eta_{\mathrm{K}})}\right)-g_{\mathrm{s}}^{n}N_{\mathrm{K}}^2\sum_i\frac{2q}{\chi Z(\omega)(1+\chi F_{\mathrm{p}})(\varepsilon_{\mathrm{K}}+\omega_i-\eta_{\mathrm{K}})}, \tag{14}$$

where $g_{\mathrm{s}}^{\mathrm{eff}}$ is the effective spin gyromagnetic factor for odd-proton nuclei. The second term in the bracket and last term on the right side of Eq. (14) express the coherent contribution coming from the quasi particle-phonon interactions in the polarized core. This contribution leads to a significant reduction of the spin g_{s} factor [12–14].

Calculations of the collective gyromagnetic ratio (g_{R}) for odd-mass nuclei have been carried out in the Inglis-Belyaev cranking model [18]. As an improvement to the early calculations in which the Nilsson Model wave functions were used, the single-particle wave function of the axially symmetric Wood-Saxon potential is used in the present calculations. According to the Inglis-Belyaev cranking model, g_{R} is given for an axially symmetric odd-mass nucleus as follows:

$$g_{\mathrm{R}}^{(p)}=\frac{1}{J_0}[J_{\mathrm{p}}+(g_{\mathrm{s}}^{\mathrm{n}}-g_{\mathrm{l}}^{\mathrm{n}})X_{\mathrm{n}}+(g_{\mathrm{s}}^{\mathrm{p}}-g_{\mathrm{l}}^{\mathrm{p}})X_{\mathrm{p}}], \tag{15}$$

where

$$X_{\mathrm{n}} = 2\sum_{ss'}\frac{(j_x)_{ss'}(s_x)_{ss'}L_{ss'}^2}{\varepsilon_{\mathrm{s}}+\varepsilon_{s'}}$$

$$X_{\mathrm{p}} = 2\sum_{t\neq K}\frac{(j_x)_{tK}(s_x)_{tK}M_{tK}^2}{\varepsilon_t-\varepsilon_{\mathrm{K}}}+2\sum_{t,t'\neq K}\frac{(j_x)_{tt'}(s_x)_{tt'}L_{tt'}^2}{\varepsilon_t+\varepsilon_{t'}}$$

$$J_{\mathrm{p}} = 2\sum_{t\neq K}\frac{(j_x)_{tK}^2M_{tK}^2}{\varepsilon_t-\varepsilon_{\mathrm{K}}}+2\sum_{t,t'\neq K}\frac{(j_x)_{tt'}^2L_{tt'}^2}{\varepsilon_t+\varepsilon_{t'}}. \tag{16}$$

Here, $(j_x)_\rho$ and $(s_x)_\rho$ are the single particle matrix elements of the x-component of angular momentum and spin operator, respectively. $J_0=J_{\mathrm{n}}+J_{\mathrm{p}}$ is the moment of inertia of odd-proton nuclei [18].

The empirical values of $g_{\mathrm{s}}^{\mathrm{eff}}/g_{\mathrm{s}}^{\mathrm{free}}$ and g_{K} can be computed by using experimental magnetic moments [2] and empirical rotational gyromagnetic ratios in following equations

$$g_{\mathrm{K}}^{\mathrm{emp.}}=\frac{K+1}{K^2}(\mu_{\exp}-g_{\mathrm{R}}K)+g_{\mathrm{R}}, \tag{17}$$

$$g_{\mathrm{s}}^{\mathrm{eff}}(\mathrm{emp})=\frac{2(g_{\mathrm{K}}^{\mathrm{emp}}-g_{\mathrm{l}}^{\mathrm{p}})K+g_{\mathrm{l}}^{\mathrm{p}}\sigma_{KK}}{g_{\mathrm{s}}^{\mathrm{p}}\sigma_{KK}}. \tag{18}$$

3 Results and discussion

The calculations of single particle basis for both neutron and protons are performed using a Woods-Saxon potential with β_2 quadrupole deformation parameters derived from the experimental quadrupole moments [19]. The parameters of the Woods –Saxon potential are the same as in Ref, [20]. The proton and neutron pairing gaps were taken from Ref [21]. λ_p and λ_n chemical potentials were calculated according to Ref. [22], based on the single-particle states of the nuclei under investigation. For the ground-state calculations of odd-mass holmium isotopes the phonon basis was constructed using one-phonon QRPA states of the corresponding even-even core with $I^\pi K=1^+0$.

Table 1. The ground-state configurations, pairing correlation quantities Δ and λ (in MeV) and the mean-field deformation parameters.

isotope	$[Nn_z\Lambda]\Sigma$	Δ_n/MeV	Δ_p/MeV	λ_n/MeV	λ_p/MeV	δ_2
^{155}Ho	[402]↑	1.011	1.046	−8.631	−5.015	0.205
^{157}Ho	[523]↑	1.017	1.021	−8.340	−5.625	0.254
^{159}Ho	[523]↑	1.006	1.010	−8.072	−6.219	0.283
^{161}Ho	[523]↑	0.972	1.023	−7.748	−6.810	0.295
^{163}Ho	[523]↑	0.952	1.021	−7.379	−7.398	0.299
^{165}Ho	[523]↑	0.919	1.023	−6.957	−7.974	0.304
^{167}Ho	[523]↑	0.886	1.024	−6.437	−8.594	0.256*
^{169}Ho	[523]↑	0.848	1.030	−6.029	−9.149	0.265*

*Calculated using β_2 quadrupole deformation parameters taken from Ref [21].

The interaction constant (κ) of the spin-spin force is determined by comparison of the theoretical and empirical values of ground-state g_K factors of odd-mass ^{165}Ho nuclei. However, this is done for only ^{165}Ho since the empirical g_R factor is only available for this nucleus. Then, the determined spin-spin interaction strength was fixed and it was used in magnetic moment calculation of other Ho isotopes. The empirical g_K for ^{165}Ho is found to be g_K=1.410 (11) using empirical g_R ($g_R^{\text{emp.}}$=0.429) [18] and the experimental magnetic moment value [2] in Eq. (17). Figure 1 shows the computed g_K as a function of q and κ for ^{165}Ho. As can be seen from Fig. 1, a satisfactory description of empirical g_K factor for ^{165}Ho is obtained with q =-1 and κ=35 MeV. Therefore, the strength of the spin-spin interaction is fixed to be κ=35 MeV for $^{155-169}$Ho isotopes. This value of κ is close to that used earlier in the scissors mode calculations [23, 24].

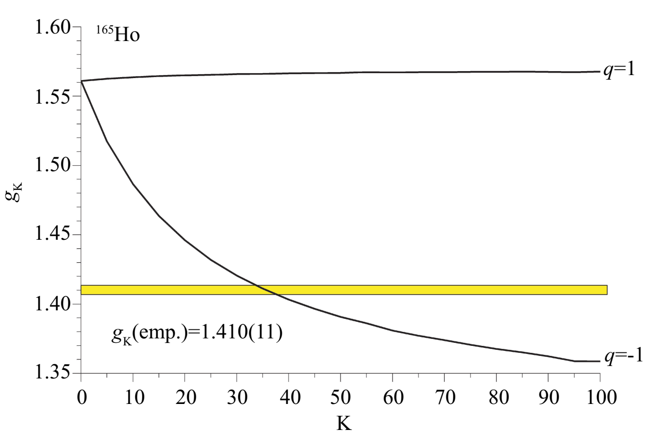

Fig. 1. Computed g_K as a function of q and κ for ^{165}Ho.

According to the secular equation there should be an energy shift of one-quasiparticle spectra due to the quasiparticle phonon admixtures. The theoretical results show that the energy shift is always small (∼80 keV) and almost homogeneous for all single-quasiparticle states lying below the collective vibrational states of odd-mass nuclei under investigation.

The microscopic structure of odd-mass $^{155-169}$Ho nuclei calculated in the QPNM is presented in Table 2. The ground-states of odd-mass $^{157-169}$Ho nuclei originate from the single-quasiparticle state with [523]↑ and the couplings of the single quasiparticle state to the one-phonon core excitations, i.e. [523]↑$\otimes Q_i$. In the case of ^{155}Ho, the ground state has a [523]↑ single-quasiparticle state with [402]↑ and contains a component [402]↑$\otimes Q_i$. Because of having the same Nilsson configuration, the structures of the ground states of odd-mass $^{157-169}$Ho are very similar. Calculations show that as a rule the contribution of the single-quasiparticle state to the wave functions is predominant (∼99.9%) so that N_K^{j2} is very close to unity. The quasiparticle⊗phonon admixtures are less important in ground states of odd-mass Ho isotopes and do not exceed 0.1%. This means that the ground state of these nuclei can be approximated as a pure one-quasiparticle state. This approximation may not affect the calculations of reduced M1 transitions between ground and excited states in a significant way owing to small admixture of quasiparticles⊗phonon states [23-25]. However, as discussed in the next paragraph, these small admixtures in the ground state are enough to cause a significant renormalization of spin matrix elements. Therefore, in the calculation of the ground state of odd-mass nuclei, the quasiparticles⊗phonon interaction has to be taken into account.

Table 2. Ground-state structures of odd-mass $^{155-169}$Ho nuclei.

nucleus	K^{π}	structure		phonon structure			
		single quasiparticle	quasiparticles⊗phonon	ω_i	$\psi_{ss'}$	$\varphi_{ss'}$	$[Nn_z\Lambda]\Sigma$
^{155}Ho	$5/2^+$	99.83%[402]↑	0.05%[402]↑⊗Q_{47}	9.630	0.298	0.001	nn420↑-640↑
					0.363	0.006	pp523↑-514↓
			0.03%[402]↑⊗Q_{53}	10.344	−0.408	−0.002	nn530↑-510↑
					−0.243	0.005	pp523↑-514↓
			0.08%[402]↑⊗Q_{56}	10.612	−0.306	0.003	nn541↑-512↓
					−0.255	0.008	pp523↑-514↓
			0.02%[402]↑⊗Q_{60}	11.073	−0.149	0.001	pp413↓-402↓
					−0.658	0.002	pp523↓-503↓
^{57}Ho	$7/2^-$	99.88%[523]↑	0.02%[523]↑⊗Q_{70}	10.060	0.380	0.001	nn422↓-651↑
					−0.164	0.004	pp532↑-512↑
			0.04%[523]↑⊗Q_{79}	10.855	0.173	0.001	nn512↓-501↑
					−0.383	0.001	pp550↑-521↓
			0.02%[523]↑⊗Q_{81}	10.925	−0.375	0.001	nn512↓-501↑
					0.408	−0.001	pp550↑-521↓
^{159}Ho	$7/2^-$	99.76%[523]↑	0.04%[523]↑⊗Q_{80}	11.113	−0.379	−0.001	nn550↑-550↑
					−0.196	0.002	pp440↑-411↓
			0.02%[523]↑⊗Q_{82}	11.250	0.458	−0.001	nn550↑-550↑
					0.158	−0.005	pp523↑-514↓
^{161}Ho	$7/2^-$	99.88%[523]↑	0.03%[523]↑⊗Q_{73}	10.354	0.186	−0.006	nn514↑-505↓
					−0.361	0.004	pp532↑-512↑
			0.05%[523]↑⊗Q_{84}	11.271	0.203	0.001	nn420↑-400↑
					0.330	−0.005	pp550↑-521↓
^{163}Ho	$7/2^-$	99.75%[523]↑	0.02%[523]↑⊗Q_{69}	10.385	0.163	−0.005	nn514↑-505↓
					0.415	−0.004	pp532↓-512↓
			0.02%[523]↑⊗Q_{74}	10.923	−0.137	−0.001	nn651↑-642↓
					0.463	0.003	pp550↑-521↓
			0.03%[523]↑⊗Q_{81}	11.360	0.485	−0.001	nn550↑-530↑
					−0.243	0.003	pp550↑-521↓
^{165}Ho	$7/2^-$	99.75%[523]↑	0.02%[523]↑⊗Q_{71}	10.123	0.279	−0.005	nn514↑-505↓
					0.463	0.004	pp532↓-512↓
			0.02%[523]↑⊗Q_{72}	10.437	0.146	−0.005	nn514↑-505↓
					−0.487	0.004	pp532↓-512↓
			0.02%[523]↑⊗Q_{74}	10.971	-0.396	-0.003	pp550↑-521↓
			0.02%[523]↑⊗Q_{77}	11.299	−0.347	−0.001	nn541↑-532↓
					0.371	−0.003	pp550↑-521↓
^{167}Ho	$7/2^-$	99.76%[523]↑	0.02%[523]↑⊗Q_{74}	10.691	0.508	0.003	pp530↑-521↓
					0.289	−0.001	pp440↑-411↓
			0.02%[523]↑⊗Q_{75}	10.966	0.467	0.001	nn541↑-512↓
					−0.365	0.003	pp530↑-521↓
			0.04%[523]↑⊗Q_{78}	11.196	0.344	-0.002	nn550↑-510↑
					−0.245	0.004	pp530↑-521↓
^{169}Ho	$7/2^-$	99.76%[523]↑	0.02%[523]↑⊗Q_{61}	9.777	−0.178	0.001	nn550↑-750↑
					−0.468	−0.004	pp532↓-512↓
			0.02%[523]↑⊗Q_{70}	10.780	0.154	0.001	nn541↑-521↑
					0.410	−0.001	pp440↑-411↓
			0.02%[523]↑⊗Q_{74}	11.069	−0.443	−0.001	nn541↑-512↓
					0.370	−0.003	pp530↑-521↓
			0.04%[523]↑⊗Q_{77}	11.280	0.445	−0.002	nn550↑-510↑
					0.249	−0.004	pp530↑-521↓

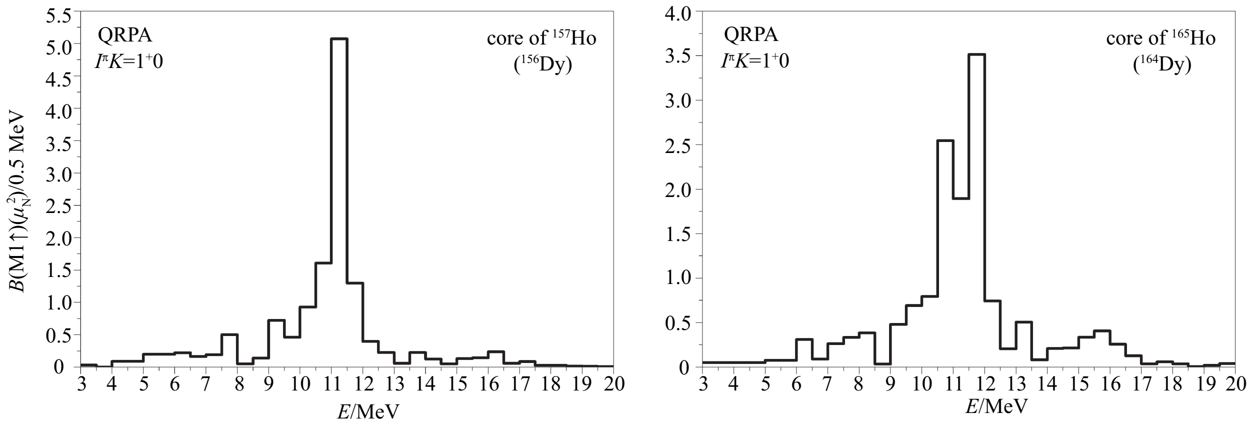

Fig. 2. The $B(\mathrm{M1}\uparrow)$ distributions with $I^{\pi}K=1^{+}0$ computed within QRPA, summed in bins of 0.2 MeV, for the core of ^{157}Ho and ^{165}Ho in 3–20 MeV energy range.

Table 3. Comparison of QPNM results for $g_s^{\text{eff}}/g_s^{\text{free}}$ and g_K with those obtained in other theories and empirically.

	$g_s^{\text{eff}}/g_s^{\text{free}}$				g_K					
isotope	KPM [8]	TDA	QPNM	Semi-Emp.	SM	MSM [28]	KPM [8]	TDA	QPNM	Semi-Emp.
^{155}Ho	0.630	0.635	0.786	0.866(15)	1.886	-	1.486	1.491	1.655	1.741(16)
^{157}Ho	0.632	0.639	0.787	0.782(17)	1.548		1.302	1.307	1.405	1.402(11)
^{159}Ho	0.623	0.633	0.785	0.784(16)	1.556		1.301	1.307	1.411	1.410(11)
^{161}Ho	0.617	0.623	0.784	0.789(16)	1.559	1.508	1.298	1.303	1.412	1.416(11)
^{163}Ho	0.612	0.619	0.782	0.790(22)	1.560	1.512	1.295	1.300	1.412	1.416(15)
^{165}Ho	0.607	0.614	0.781	0.778(16)	1.561	1.515	1.292	1.297	1.411	1.410(11)
^{167}Ho	0.596	0.631	0.778		1.546	1.518	1.277	1.301	1.398	
^{169}Ho	0.602	0.610	0.779		1.549	-	1.282	1.288	1.401	

According to our calculation, the phonon states lying around 9–12 MeV give the largest contribution to the structure of the ground states of odd-mass Ho isotopes. This is because these core states have strong M1 transitions known as spin-flip resonances. As can be seen from Table 2, the backward amplitudes ($\phi_{ss'}$) of these high-lying 1^+ phonons are very small. For illustrative purposes, the QRPA results for the $B(\mathrm{M1}\uparrow)$ distributions with $I^{\pi}K=1^{+}0$, summed in bins of 0.5 MeV, are shown in Fig. 2 for only the core of ^{157}Ho and ^{165}Ho. The QRPA calculations for $I^{\pi}K=l^{+}0$ states presented in Fig. 2 are performed as in Ref. [26].

In general, $B(\mathrm{M1}\uparrow)$ strength below 5 MeV is due to the orbital motion. In contrast, the contribution of the spin part to the M1 strength increases above 5 MeV and the states in the 9-12 MeV energy range are purely spin excitations. Although the contributions to ground-state wave function of odd-mass nuclei coming from coupling of these phonons with single-quasiparticles are small, they significantly affect the magnetic properties of odd-mass Ho nuclei.

Let us now consider the correction to the ground-state g_s and g_K factors of $^{155-169}$Ho isotopes from the small admixture of quasiparticles⊗phonon interaction. A direct comparison between QPNM results (for g_K and g_s^{eff}) and the semi-empirical data is presented in Table 3. The semi-empirical values of $g_s^{\text{eff}}/g_s^{\text{free}}$and g_K were computed by using experimental magnetic moments [2] and empirical rotational gyromagnetic factors in Eqs. (17, 18) as pointed out by Y.F. Bow [27]. Empirical rotational gyromagnetic ratio is only available for ^{165}Ho. Therefore, in the determination of semi-empirical g_K for the $^{155-169}$Ho isotopic chain, except for ^{165}Ho , the estimations of the cranking model presented in Table 4 (column 6) were used. In this table also the results of the early theoretical attempts such as the Kuliev-Pyatov Method (KPM), Tamm-Damcof Approximation (TDA), Shell-Model (SM) and Modified-Shell Model (MSM) were included.

As seen in Table 3, a satisfactory overall description of the ground-state g_K and g_s^{eff} factors of odd-Z $^{155-169}$Ho isotopes was obtained by the QPNM calculations. The TDA and KPM provide very similar results and all of them deviate noticeably from the empirical data. This is because the ground-state of the core corresponds to an independent quasiparticle vacuum in both methods. For all nuclei, the QPNM results are closer to the empirical values than TDA and KPM. This indicates that the inclusion of ground-state correlations with QPNM in the core improves the description of not only the one-phonon vibrational states in an even-even core, but also the ground-state of the corresponding odd-mass nuclei. The basic premise of the TDA is to accept the independent quasiparticle vacuum as the ground state whereas QPNM admits the possibility that the ground state is not of purely independent quasiparticle character and contain correlations. In other word, the QPNM ground state is different from the TDA ground state because of the ground-state correlations. The results of TDA for $g_s^{\rm eff}$ and $g_{\rm K}$ are therefore much smaller than the empirical values.

As expected, in the case of $g_{\rm K}$, SM and MSM show very similar behaviours and the calculations are in poorer agreement with empirical data than QPNM due to the neglect of the core. In these models it is assumed that protons and neutrons do not interact. The key difference between them is only the pairing correlations introduced in the MSM [28]. From the above comparisons it can be concluded that the magnetic properties are sensitive to small admixtures of the spin-dependent interaction, which are neglected in SM and MSM. Although TDA and KPM include the residual spin-spin interaction, the interactions in the ground-state of core are neglected. This causes the asymmetric behaviours of the ground- and excited- states in TDA and KPM. However, the QPNM method considers quasiparticle interactions in both excited- and ground-states so that it allows proper description of core polarization effects in odd-mass nuclei.

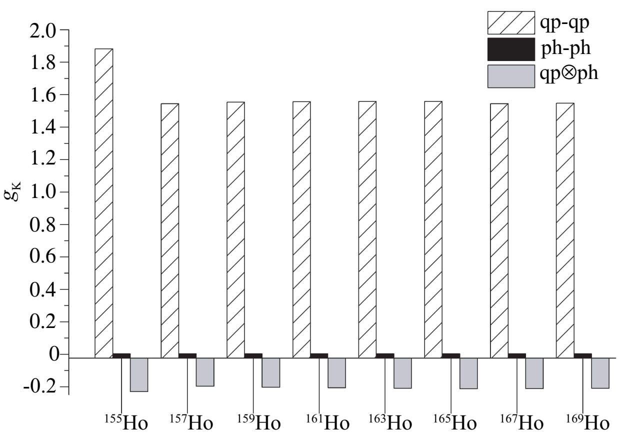

Fig. 3. Contribution of (qp-qp), (ph-ph) and (qp-ph) terms to the ground state $g_{\rm K}$ of odd-mass $^{155-169}$Ho nuclei.

As can be seen in Eq. (11), $g_{\rm K}$ is indeed a sum of three terms, namely quasiparticle-quasiparticle (qp-qp), phonon-phonon (ph-ph) and quasiparticle-phonon (qp-ph). The contribution of these terms to the $g_{\rm K}$ factors of odd-A Ho isotopes are illustrated in Fig. 3. As expected, the main contribution to $g_{\rm K}$ comes from qp-qp terms. The ph-ph terms give a positive but rather weak contribution. The qp-ph terms turn out to be always of negative sign and are also quite weak compared to the qp-qp terms. It must be emphasized once more that the small admixtures induced by quasiparticle⊗phonon interactions, i.e. qp-ph terms, lead to a decrease in the magnitude of the $g_{\rm K}$ factor, which improves the agreement with empirical data. The quasiparticle⊗phonon admixtures are usually less than 1% of the norm of the wave function. However, because of the large density ($\rho\sim30$ MeV^{-1}) and coherent contribution, small quasiparticle⊗phonon admixtures in the wave function strongly affect the nuclear magnetic moments. That the magnetic moments are sensitive to such small admixtures were first pointed out by Arima and Horie [8, 9].

Table 4 shows the calculated moment of inertia and $g_{\rm R}$ values for odd-mass nuclei.The same calculations were also performed for core nuclei and presented in Table 4, since it may be of interest to compare the results for odd-A nuclei with those for the corresponding cores. From these comparisons it can be concluded that adding an extra proton to the core induces a considerable contribution to the $g_{\rm R}$ factor and moment of inertia in some cases. The quantum state of the odd-proton determines whether the contribution is additive or destructive. For all cases the contribution of the odd-proton to the $g_{\rm R}$ factors is additive, so that the ground state $g_{\rm R}$ factor for odd mass Ho nuclei is higher than the one for doubly even cores, except for ^{155}Ho. This is because the ground-state quantum numbers, i.e Nilsson state ([402]), of ^{155}Ho are different from other odd-mass Ho isotopes. In the case of ^{155}Ho, when an odd proton is added to the core nucleus, i.e ^{154}Dy, the proton contribution ($J_{\rm p}$) to the total moment of inertia (J_0)decreases because of the negative sign of $J_{\rm p}$. Besides, the negative sign of the $(j_x)_{tK}(s_x)_{tK}$ term in Eq. (16) reduces to 0.257 from 0.099. In view of Eq. (15), the decrease in $J_{\rm p}$ and X_p reduces $g_{\rm R}$ from 0.580 to 0.562.

In Table 4 the calculated values of the gyromagnetic ratio are also compared to the simple estimate Z/A and to empirical data [18]. In general the calculated $g_{\rm R}$ factors significantly deviate from Z/A. Only in ^{168}Dy are the calculated $g_{\rm R}$ factors nearly equal to Z/A. In all cases, the calculated $g_{\rm R}$ factors are in excellent agreement with the available empirical data. The core polarization also affects the rotational gyromagnetic ratio. However, its impact on $g_{\rm R}$ is weak compared to its effect on the spin gyromagnetic ratios. To show this, the rotational gyromagnetic ratio has been calculated with core polarization and is also presented in Table 4.

In order to calculate $g_{\rm R}$ with core polarization, i.e $g_{\rm R}^{\rm c}({\rm eff})$, the QPNM results for $g_s^{\rm eff}/g_s^{\rm free}$ presented in Table 3 were used in Eq. (15).

Figure 4 shows the dependence of the results obtained for ground-state magnetic moment of odd-proton

Table 4. Ground-state moments of inertia and gyromagnetic ratios (g_R). Here, the quantities J_n and J_p are proton and neutron contributions to the total moment of inertia (J_0), respectively.

nucleus	J_n	J_p	J_0	$g_R=Z/A$	g_R^c	$g_R^c(eff)$	$g_R^{emp.}$ [18]
^{154}Dy	14.877	15.434	30.311	0.429	0.580	-	-
^{155}Ho	14.877	13.405	28.282	0.432	0.562	0.536	-
^{156}Dy	20.621	16.212	36.833	0.423	0.518	-	-
^{157}Ho	20.621	29.236	49.857	0.427	0.686	0.655	-
^{158}Dy	26.543	16.744	43.287	0.418	0.433	-	-
^{159}Ho	26.543	27.937	54.480	0.421	0.569	0.544	-
^{160}Dy	30.086	16.492	46.578	0.412	0.382	-	0.370(40)
^{161}Ho	30.086	27.190	57.276	0.416	0.510	0.486	-
^{162}Dy	32.140	16.356	48.496	0.407	0.359	-	0.362(24)
^{163}Ho	32.140	26.938	59.078	0.411	0.481	0.459	-
^{164}Dy	34.342	16.075	50.417	0.402	0.338	-	0.331(16)
^{165}Ho	34.342	26.566	60.908	0.406	0.456	0.435	0.429(58)
^{166}Dy	34.256	15.102	49.358	0.398	0.326	-	-
^{167}Ho	34.256	28.977	63.234	0.401	0.500	0.475	-
^{168}Dy	36.170	14.989	51.159	0.393	0.317	-	-
^{169}Ho	36.170	28.387	64.557	0.396	0.479	0.455	-

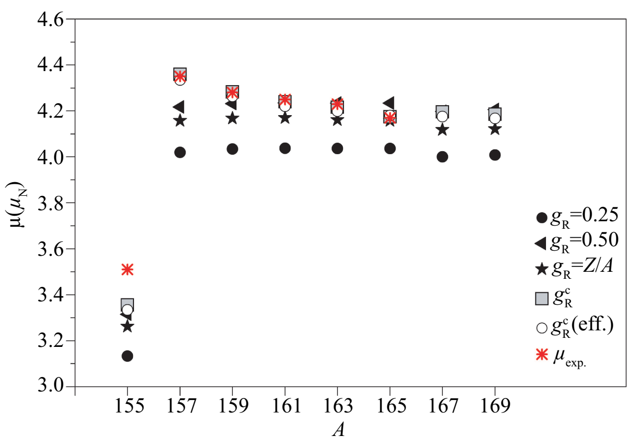

Fig. 4. Theoretical and experimental magnetic moments as a function of mass number A. The experimental data are shown by red stars.

$^{155-169}$Ho on the mass number A. In the majority of cases for g_R, the theoretical results are close to each other, but the values calculated using cranking g_R provides a better description of experimental data.

The differences between the theoretical results of magnetic dipole moment calculated using both g_R^c and g_R^c(eff) and the experimental results are somewhat smaller in $^{155-169}$Ho. For illustrative purposes, the quantitative difference between theory and experiment may be expressed by $\Delta\mu$. One can see that, except for ^{155}Ho, the characteristic value of this quantity is $\Delta\mu \approx 0.1\mu_N$, indicating a reasonably good agreement between the theoretical and experimental results. In the case of ^{155}Ho, the discrepancy between theoretical and experimental ground-state magnetic moments may be due to the fact that the Nilsson state ([402]) has been not exactly confirmed by experiments. Although quantitative agreement is rather poor, the sharp drop of magnetic moment of ^{155}Ho is well reproduced.

The analyses of the results indicate that the ground-state magnetic moments are much more sensitive to the value of g_s than to that of g_R. As seen in Fig. 4, a variation of g_R from 0.25 to 0.50 corresponds to a variation of the magnetic moment $\approx 0.2\mu_N$. On the other hand, a variation in g_s from 0.7 to 1 leads to $\approx 0.6\mu_N$. This stresses the importance of accurate determination of the effective gyromagnetic factor of a nucleus for the calculation of its ground-state magnetic moment. Therefore, determination of effective spin gyromagnetic ratios is an important problem for comparing theoretical and experimental magnetic moments.

The quadrupole deformation dependence of the ground-state magnetic properties of odd-mass Ho isotopes has been also investigated. As an example, the results for ^{163}Ho are illustrated in Fig. 5. In all the odd-mass Ho nuclei considered, the g_s^{eff} factors exhibit a weak dependence on the quadrupole deformation. On the other hand, the calculations show that there is pronounced sensitivity of the magnetic moments and g_K to the quadrupole deformation. This situation reflects the important role of the single-particle matrix elements in the calculation of the magnetic moments and g_K.

In the present paper it has been shown that the quenching of spin gyromagnetic ratio can be well explained by microscopic QPNM calculations including residual spin-spin interactions. It should be noted that the effective spin gyromagnetic ratios is important not only for calculation of ground-state magnetic moments but also for the calculation of reduced M1 transitions between ground and excited states.

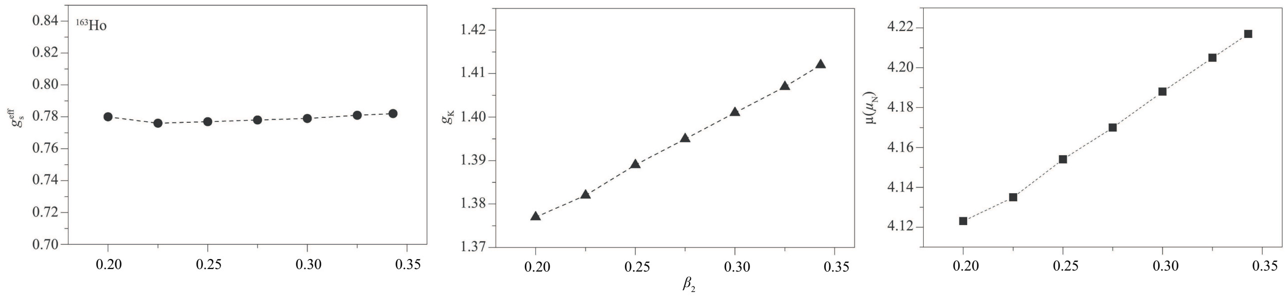

Fig. 5. Deformation dependence of ground-state magnetic properties of ^{163}Ho.

The reasonably good agreement with experiment has encouraged us to give predictions of the ground-state magnetic moments of ^{167,169}Ho isotopes for which there is not yet any experimental data. Using effective gyromagnetic factors and $g_{\mathrm{R}}^{\mathrm{c}}$ values, the ground-state magnetic moments of ^{167}Ho and ^{169}Ho nuclei have been predicted to be μ=+4.195μ_{N} and μ=+4.18μ_{N}, respectively.

4 Conclusion

The present study has provided the first theoretical systematic investigation of the ground-state magnetic moments in odd-mass $^{155-169}$Ho nuclei. Core polarization corrections to ground-state magnetic moments of odd-mass $^{155-169}$Ho nuclei have been calculated in QPNM theory. It has been shown that the effective g_{s} factors in the calculation of magnetic moments can be explained in terms of the polarized core.

In all cases, the results for the ground-state magnetic moments calculated using cranking rotational gyromagnetic factors ($g_{\mathrm{R}}^{\mathrm{c}}$) are closer to the experimental values than those calculated using different g_{R} values.

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