b → ss$\bar{\rm d}$ decay in Randall-Sundrum models *

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Abstract: The extremely small branching ratio of the b → ss$\bar{\rm d}$ decay in the Standard Model makes it a suitable channel to explore new-physics signals. We study this $\Delta S = 2$ process in Randall-Sundrum models, including the custodially protected and the bulk-Higgs Randall-Sundrum models. Exploring the experimentally favored parameter spaces of these models suggests a possible enhancement of the decay rate, compared to the Standard Model result, by at most two orders of magnitude.

Keywords: rare B decays, new physics, flavor-changing neutral-current

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1 Introduction

In studying flavor-changing neutral-current (FCNC) transitions in rare B decays for exploring new physics (NP), one major difficulty is how to reliably subtract the Standard Model (SM) background. Theoretical uncertainties in FCNC transitions make it hard to draw conclusions about definite new physics signals against SM predictions. For this reason, an alternative approach suggested in Refs. [1, 2] is to consider processes which have tiny strengths in the SM so that mere detection of such processes will indicate NP. One such process is the rare b → ss$\bar{\rm d}$ decay, as reported in Refs. [1, 2], which can serve the purpose of exposing NP.

The $\Delta S = 2$ b → ss$\bar{\rm d}$ process is box-mediated in the SM and is found to occur with a branching ratio of the order of 10^{-12}. The authors of Ref. [1] suggested $\rm B^- \to K^-K^-\pi^+$ as the most appropriate mode for experimental searches and many other studies of the b → ss$\bar{\rm d}$ decay have been conducted in various beyond-SM scenarios [3–5]. The first search was reported in Ref. [6] and upper limits were given by both B factories [7–9], with the current upper limit reported by the LHCb collaboration to be $\mathcal{B}(\rm B^+ \to K^+K^+\pi^-) < 1.1 \times 10^{-8}$ [10]. Moreover, two-body exclusive decays of B^- [11] and $B_{\rm c}$ [12], which are driven by the b → ss$\bar{\rm d}$ transition, have also been studied in the SM and in various extensions.

In this paper, we consider the inclusive b → ss$\bar{\rm d}$ decay in Randall-Sundrum (RS) models [13, 14]. We shall study two models known as the RS model with custodial protection ($\rm RS_c$) [15–19] and the bulk-Higgs RS model [20], in both of which FCNC transitions occur at tree level.

2 RS model with custodial protection

The $\rm RS_c$ model is based on a single warped extra dimension with the bulk gauge group $SU(3)_{\rm c} \times SU(2)_{\rm L} \times SU(2)_{\rm R} \times U(1)_{\rm X} \times P_{\rm LR}$. In the $\rm RS_c$ model, the $\Delta S = 2$ b → ss$\bar{\rm d}$ decay receives tree level contributions from the Kaluza-Klein (KK) gluons, the heavy KK photons, new heavy electroweak (EW) gauge bosons $\rm Z_H$ and $\rm Z'$, and in principle the $\rm Z^0$ boson. Custodial protection of the $\rm Zb_L\bar{b}_L$ coupling through the discrete $P_{\rm LR}$ symmetry in order to satisfy EW precision constraints renders tree-level $\rm Z^0$ contributions negligible. It was pointed out in [21] that for the $\rm RS_c$ model the $\Delta F = 2$ contributions from Higgs boson exchange are of $\mathcal{O}(v^4/{M_{\rm KK}}^4)$ ($v \approx 246$ GeV is the Higgs vacuum expectation value and $M_{\rm KK}$ is the KK scale, which is always larger than 1 TeV) and the importance of Higgs FCNCs is limited, with the most pronounced effects occurring in the case of the CP-violating parameter $\epsilon_{\rm K}$, but even there they are typically smaller than the corrections due to KK-gluon exchanges

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[22]. Therefore, in view of the possible Higgs-boson effects being insignificant in $\Delta F=2$ processes, we simply neglect them in our study of the $\mathrm{b}\to\mathrm{ss\bar{d}}$ decay in the $\mathrm{RS_c}$ model.

For the $\mathrm{RS_c}$ model, we consider only first KK excitations of gauge bosons, with M_{KK} setting the mass scale for the low-lying KK excitations of the SM particles such that the mass of the first KK bosons are given by $M_{g^{(1)}}\approx 2.45\ M_{\mathrm{KK}}$. Here it is important to mention that we have used a different notation for the mass of the first KK gluon than in Ref. [23]; our M_{KK} corresponds to their f. The dominant contribution comes from the KK gluon, while the new heavy EW gauge bosons $(\mathrm{Z_H},\mathrm{Z}')$ can compete with it. The tree-level Z^0 and KK photon contributions are very small. The effective Hamiltonian for the $\Delta S=2$ $\mathrm{b}\to\mathrm{ss\bar{d}}$ decay mediated by exchanges of the lightest KK gluon, the lightest KK photon and $(\mathrm{Z_H},\mathrm{Z}')$ with the Wilson coefficients corresponding to $\mu=\mathcal{O}(M_{g^{(1)}})$ is given by

$$\begin{aligned}[\mathcal{H}_{\mathrm{eff}}^{\Delta S=2}]_{\mathrm{KK}}=&\frac{1}{2(M_{g^{(1)}})^2}[C_1^{\mathrm{VLL}}\mathcal{Q}_1^{\mathrm{VLL}}+C_1^{\mathrm{VRR}}\mathcal{Q}_1^{\mathrm{VRR}}\\&+C_1^{\mathrm{LR}}\mathcal{Q}_1^{\mathrm{LR}}+C_2^{\mathrm{LR}}\mathcal{Q}_2^{\mathrm{LR}}\\&+C_1^{\mathrm{RL}}\mathcal{Q}_1^{\mathrm{RL}}+C_2^{\mathrm{RL}}\mathcal{Q}_2^{\mathrm{RL}}],\end{aligned}\tag{1}$$

where

$$\begin{aligned}\mathcal{Q}_1^{V\mathrm{LL}}&=(\bar{s}\gamma_\mu P_{\mathrm{L}}b)(\bar{s}\gamma^\mu P_{\mathrm{L}}d),\\\mathcal{Q}_1^{V\mathrm{RR}}&=(\bar{s}\gamma_\mu P_{\mathrm{R}}b)(\bar{s}\gamma^\mu P_{\mathrm{R}}d),\\\mathcal{Q}_1^{\mathrm{LR}}&=(\bar{s}\gamma_\mu P_{\mathrm{L}}b)(\bar{s}\gamma^\mu P_{\mathrm{R}}d),\\\mathcal{Q}_2^{\mathrm{LR}}&=(\bar{s}P_{\mathrm{L}}b)(\bar{s}P_{\mathrm{R}}d),\\\mathcal{Q}_1^{\mathrm{RL}}&=(\bar{s}\gamma_\mu P_{\mathrm{R}}b)(\bar{s}\gamma^\mu P_{\mathrm{L}}d),\\\mathcal{Q}_2^{\mathrm{RL}}&=(\bar{s}P_{\mathrm{R}}b)(\bar{s}P_{\mathrm{L}}d),\end{aligned}\tag{2}$$

and

$$\begin{aligned}C_i^j(M_{g^{(1)}})=&[C_i^j(M_{g^{(1)}})]^G+[\Delta C_i^j(M_{g^{(1)}})]^{\mathrm{QED}}\\&+[\Delta C_i^j(M_{g^{(1)}})]^{\mathrm{EW}},\end{aligned}\tag{3}$$

with $i=1,2$ and $j=$ VLL, VRR, LR, RL. Note that in the $\mathrm{RS_c}$ model, compared to the analogous processes K^0-$\bar{\mathrm{K}}^0$ and B_s^0-$\bar{\mathrm{B}}_s^0$ mixings [23], the $\mathrm{b}\to\mathrm{ss\bar{d}}$ decay receives additional contributions from the RL operators. $[C_i^j(M_{g^{(1)}})]^G$ in Eq. (3) denote the contributions from the KK gluon to the Wilson coefficients, and are calculated to be

$$\begin{aligned}[C_1^{\mathrm{VLL}}(M_{g^{(1)}})]^G&=\frac{2}{3}{p_{\mathrm{UV}}}^2\Delta_{\mathrm{L}}^{sb}\Delta_{\mathrm{L}}^{sd},\\[C_1^{\mathrm{VRR}}(M_{g^{(1)}})]^G&=\frac{2}{3}{p_{\mathrm{UV}}}^2\Delta_{\mathrm{R}}^{sb}\Delta_{\mathrm{R}}^{sd},\\[C_1^{\mathrm{LR}}(M_{g^{(1)}})]^G&=-\frac{1}{3}{p_{\mathrm{UV}}}^2\Delta_{\mathrm{L}}^{sb}\Delta_{\mathrm{R}}^{sd},\\[C_2^{\mathrm{LR}}(M_{g^{(1)}})]^G&=-2{p_{\mathrm{UV}}}^2\Delta_{\mathrm{L}}^{sb}\Delta_{\mathrm{R}}^{sd},\\[C_1^{\mathrm{RL}}(M_{g^{(1)}})]^G&=-\frac{1}{3}{p_{\mathrm{UV}}}^2\Delta_{\mathrm{R}}^{sb}\Delta_{\mathrm{L}}^{sd},\\[C_2^{\mathrm{RL}}(M_{g^{(1)}})]^G&=-2{p_{\mathrm{UV}}}^2\Delta_{\mathrm{R}}^{sb}\Delta_{\mathrm{L}}^{sd},\end{aligned}\tag{4}$$

where p_{UV} parameterizes the influence of brane kinetic terms on the $SU(3)_{\mathrm{c}}$ coupling. In our analysis we set $p_{\mathrm{UV}}\equiv 1$. Similarly, for the KK photon and $(\mathrm{Z_H},\mathrm{Z}')$ contributions, we find the following corrections to the Wilson coefficients $C_i^j(M_{g^{(1)}})$,

$$\begin{aligned}[\Delta C_1^{\mathrm{VLL}}(M_{g^{(1)}})]^{\mathrm{QED}}&=2[\Delta_{\mathrm{L}}^{sb}(A^{(1)})][\Delta_{\mathrm{L}}^{sd}(A^{(1)})],\\[\Delta C_1^{\mathrm{VRR}}(M_{g^{(1)}})]^{\mathrm{QED}}&=2[\Delta_{\mathrm{R}}^{sb}(A^{(1)})][\Delta_{\mathrm{R}}^{sd}(A^{(1)})],\\[\Delta C_1^{\mathrm{LR}}(M_{g^{(1)}})]^{\mathrm{QED}}&=2[\Delta_{\mathrm{L}}^{sb}(A^{(1)})][\Delta_{\mathrm{R}}^{sd}(A^{(1)})],\\[\Delta C_2^{\mathrm{LR}}(M_{g^{(1)}})]^{\mathrm{QED}}&=0,\\[\Delta C_1^{\mathrm{RL}}(M_{g^{(1)}})]^{\mathrm{QED}}&=2[\Delta_{\mathrm{R}}^{sb}(A^{(1)})][\Delta_{\mathrm{L}}^{sd}(A^{(1)})],\\[\Delta C_2^{\mathrm{RL}}(M_{g^{(1)}})]^{\mathrm{QED}}&=0,\end{aligned}\tag{5}$$

$$\begin{aligned}[\Delta C_1^{\mathrm{VLL}}(M_{g^{(1)}})]^{\mathrm{EW}}&=2[\Delta_{\mathrm{L}}^{sb}(Z^{(1)})\Delta_{\mathrm{L}}^{sd}(Z^{(1)})\\&\quad+\Delta_{\mathrm{L}}^{sb}(Z_X^{(1)})\Delta_{\mathrm{L}}^{sd}(Z_X^{(1)})],\\[\Delta C_1^{\mathrm{VRR}}(M_{g^{(1)}})]^{\mathrm{EW}}&=2[\Delta_{\mathrm{R}}^{sb}(Z^{(1)})\Delta_{\mathrm{R}}^{sd}(Z^{(1)})\\&\quad+\Delta_{\mathrm{R}}^{sb}(Z_X^{(1)})\Delta_{\mathrm{R}}^{sd}(Z_X^{(1)})],\\[\Delta C_1^{\mathrm{LR}}(M_{g^{(1)}})]^{\mathrm{EW}}&=2[\Delta_{\mathrm{L}}^{sb}(Z^{(1)})\Delta_{\mathrm{R}}^{sd}(Z^{(1)})\\&\quad+\Delta_{\mathrm{L}}^{sb}(Z_X^{(1)})\Delta_{\mathrm{R}}^{sd}(Z_X^{(1)})],\\[\Delta C_2^{\mathrm{LR}}(M_{g^{(1)}})]^{\mathrm{EW}}&=0,\\[\Delta C_1^{\mathrm{RL}}(M_{g^{(1)}})]^{\mathrm{EW}}&=2[\Delta_{\mathrm{R}}^{sb}(Z^{(1)})\Delta_{\mathrm{L}}^{sd}(Z^{(1)})\\&\quad+\Delta_{\mathrm{R}}^{sb}(Z_X^{(1)})\Delta_{\mathrm{L}}^{sd}(Z_X^{(1)})],\\[\Delta C_2^{\mathrm{RL}}(M_{g^{(1)}})]^{\mathrm{EW}}&=0,\end{aligned}\tag{6}$$

where the overlap integrals $\Delta_{\mathrm{L,R}}^{sb}(Z^{(1)})$, $\Delta_{\mathrm{L,R}}^{sb}(Z_X^{(1)})$, $\Delta_{\mathrm{L,R}}^{sd}(Z^{(1)})$ and $\Delta_{\mathrm{L,R}}^{sd}(Z_X^{(1)})$ are given in Appendix B of [23]. These overlap integrals contain the profiles of the zero mode fermions and shape functions of the KK gauge bosons. We estimate the size of EW contributions compared to the KK gluon contributions in the $\mathrm{b}\to\mathrm{ss\bar{d}}$ decay by factoring out all the couplings and charge factors from $\Delta_{\mathrm{L,R}}^{sb}$ and $\Delta_{\mathrm{L,R}}^{sd}$. The remaining $\tilde{\Delta}_{\mathrm{L,R}}^{sb}$ and $\tilde{\Delta}_{\mathrm{L,R}}^{sd}$ are then universal for all the gauge bosons considered up to the different boundary conditions. Combining contributions in Eqs. (4), (5) and (6) and evaluating the various couplings, we have

$$\begin{aligned}C_1^{\mathrm{VLL}}(M_{g^{(1)}})&=[0.67+0.02+0.56]\tilde{\Delta}_{\mathrm{L}}^{sb}\tilde{\Delta}_{\mathrm{L}}^{sd}=1.25\tilde{\Delta}_{\mathrm{L}}^{sb}\tilde{\Delta}_{\mathrm{L}}^{sd},\\C_1^{\mathrm{VRR}}(M_{g^{(1)}})&=[0.67+0.02+0.98]\tilde{\Delta}_{\mathrm{R}}^{sb}\tilde{\Delta}_{\mathrm{R}}^{sd}=1.67\tilde{\Delta}_{\mathrm{R}}^{sb}\tilde{\Delta}_{\mathrm{R}}^{sd},\\C_1^{\mathrm{LR}}(M_{g^{(1)}})&=[-0.333+0.02+0.56]\tilde{\Delta}_{\mathrm{L}}^{sb}\tilde{\Delta}_{\mathrm{R}}^{sd}=0.25\tilde{\Delta}_{\mathrm{L}}^{sb}\tilde{\Delta}_{\mathrm{R}}^{sd},\\C_1^{\mathrm{RL}}(M_{g^{(1)}})&=[-0.333+0.02+0.56]\tilde{\Delta}_{\mathrm{R}}^{sb}\tilde{\Delta}_{\mathrm{L}}^{sd}=0.25\tilde{\Delta}_{\mathrm{R}}^{sb}\tilde{\Delta}_{\mathrm{L}}^{sd},\end{aligned}\tag{7}$$

where the three contributions in the bracket correspond to the KK gluon, the KK photon and combined

(Z_H, Z') exchange, respectively. The Wilson coefficients $C_2^{LR}(M_{g^{(1)}})$ and $C_2^{RL}(M_{g^{(1)}})$ receive only the KK-gluon contributions. We see that the EW contributions, dominated by (Z_H, Z') exchanges, give +87% and +150% corrections in the case of $C_1^{VLL}(M_{g^{(1)}})$ and $C_1^{VRR}(M_{g^{(1)}})$, respectively, while corrections of −174% are observed for $C_1^{LR}(M_{g^{(1)}})$ and $C_1^{RL}(M_{g^{(1)}})$. The Hamiltonian in Eq. (1) is valid at scales of $\mathcal{O}(M_{g^{(1)}})$ and has to be evolved to a low energy scale $\mu_b = 4.6$ GeV. For that, the anomalous dimension matrices for $\Delta F=2$ four-quark dimension-six operators have already been calculated at two loop level in Refs. [24, 25]. As gluons are flavor blind and QCD preserves chirality, the anomalous dimension matrices of the operators in $b\to ss\bar{d}$ are the same as for the case of $B^0_{d,s}$-$\bar{B}^0_{d,s}$ mixing operators. Therefore, the renormalization group running of the Wilson coefficients for the $b\to ss\bar{d}$ decay is performed by using analytic formulae for the relevant QCD factors given in Section 3.1 and Appendix C of [26]. Finally, the decay width for the $b\to ss\bar{d}$ decay in the RS_c model is given by

$$
\begin{aligned}
\Gamma = {} & \frac{m_b^5}{3072(2\pi)^3(M_{g^{(1)}})^4}[16(|C_1^{VLL}(\mu_b)|^2+|C_1^{VRR}(\mu_b)|^2) \\
& +12(|C_1^{LR}(\mu_b)|^2+|C_1^{RL}(\mu_b)|^2) \\
& +3(|C_2^{LR}(\mu_b)|^2+|C_2^{RL}(\mu_b)|^2) \\
& -2\mathcal{R}e(C_1^{LR}(\mu_b)C_2^{*LR}(\mu_b)+C_2^{LR}(\mu_b)C_1^{*LR}(\mu_b) \\
& +C_1^{RL}(\mu_b)C_2^{*RL}(\mu_b)+C_2^{RL}(\mu_b)C_1^{*RL}(\mu_b))]. \qquad (8)
\end{aligned}
$$

3 Bulk-Higgs RS model

The bulk-Higgs RS model is based on the 5D gauge group $SU(3)_c\times SU(2)_V\times U(1)_Y$, where all the fields are allowed to propagate in the 5D space-time [20]. The $b\to ss\bar{d}$ decay in the bulk-Higgs RS model results from tree-level exchanges of Kaluza-Klein gluons and photons, the Z^0 boson and the Higgs boson as well as their KK excitations and the extended scalar fields $\phi^{Z(n)}$. For the bulk-Higgs RS model we consider the summation over the contributions from the entire KK towers, with the lightest KK states having mass $M_{g^{(1)}}\approx 2.45\ M_{KK}$. We start with the effective NP Hamiltonian

$$
[\mathcal{H}_{\rm eff}^{\Delta S=2}]_{KK}=\sum_{n=1}^{5}[C_n\mathcal{O}_n+\tilde{C}_n\tilde{\mathcal{O}}_n], \qquad (9)
$$

where

$$
\begin{aligned}
\mathcal{O}_1 &= (\bar{s}_L\gamma_\mu b_L)(\bar{s}_L\gamma^\mu d_L), \\
\mathcal{O}_2 &= (\bar{s}_R b_L)(\bar{s}_R d_L), \\
\mathcal{O}_3 &= (\bar{s}_R^\alpha b_L^\beta)(\bar{s}_R^\beta d_L^\alpha), \\
\mathcal{O}_4 &= (\bar{s}_R b_L)(\bar{s}_L d_R), \\
\mathcal{O}_5 &= (\bar{s}_R^\alpha b_L^\beta)(\bar{s}_L^\beta d_R^\alpha). \qquad (10)
\end{aligned}
$$

A summation over color indices α and β is understood. The $\tilde{O}_n$ operators are obtained from O_n by L↔R exchange. Wilson coefficients at $\mathcal{O}(M_{KK})$ are given by

$$
\begin{aligned}
C_1 &= \frac{4\pi L}{M_{KK}^2}(\tilde{\Delta}_D)_{23}\otimes(\tilde{\Delta}_D)_{21}\Big[\frac{\alpha_s}{2}\Big(1-\frac{1}{N_c}\Big) \\
&\quad +\alpha Q_d^2+\frac{\alpha}{s_w^2c_w^2}(T_3^d-Q_ds_w^2)^2\Big], \\
\tilde{C}_1 &= \frac{4\pi L}{M_{KK}^2}(\tilde{\Delta}_d)_{23}\otimes(\tilde{\Delta}_d)_{21}\Big[\frac{\alpha_s}{2}\Big(1-\frac{1}{N_c}\Big) \\
&\quad +\alpha Q_d^2+\frac{\alpha}{s_w^2c_w^2}(-Q_ds_w^2)^2\Big], \\
C_4 &= -\frac{4\pi L\alpha_s}{M_{KK}^2}(\tilde{\Delta}_D)_{23}\otimes(\tilde{\Delta}_d)_{21} \\
&\quad -\frac{L}{\pi\beta M_{KK}^2}(\tilde{\Omega}_d)_{23}\otimes(\tilde{\Omega}_D)_{21}, \\
\tilde{C}_4 &= -\frac{4\pi L\alpha_s}{M_{KK}^2}(\tilde{\Delta}_d)_{23}\otimes(\tilde{\Delta}_D)_{21} \\
&\quad -\frac{L}{\pi\beta M_{KK}^2}(\tilde{\Omega}_D)_{23}\otimes(\tilde{\Omega}_d)_{21}, \\
C_5 &= \frac{4\pi L}{M_{KK}^2}(\tilde{\Delta}_D)_{23}\otimes(\tilde{\Delta}_d)_{21}\Big[\frac{\alpha_s}{N_c} \\
&\quad -2\alpha Q_d^2+\frac{2\alpha}{s_w^2c_w^2}(T_3^d-Q_ds_w^2)(Q_ds_w^2)\Big], \\
\tilde{C}_5 &= \frac{4\pi L}{M_{KK}^2}(\tilde{\Delta}_d)_{23}\otimes(\tilde{\Delta}_D)_{21}\Big[\frac{\alpha_s}{N_c} \\
&\quad -2\alpha Q_d^2+\frac{2\alpha}{s_w^2c_w^2}(T_3^d-Q_ds_w^2)(Q_ds_w^2)\Big], \qquad (11)
\end{aligned}
$$

where $Q_d=-1/3$, $T_3^d=-1/2$, and $N_c=3$. Higgs and scalar field ϕ^Z give opposite contributions to the Wilson coefficient C_2, thus they cancel each other, giving $C_2=0$. Similarly, $\tilde{C}_2=0$. The expressions of the mixing matrices $(\tilde{\Delta}_{F(f)})_{mn}\otimes(\tilde{\Delta}_{F(f)})_{m'n'}$ and $(\tilde{\Delta}_{F(f)})_{mn}\otimes(\tilde{\Delta}_{F(f)})_{m'n'}$ (with $F=U,D$ and $f=u,d$, and similarly in the lepton sector) in terms of the overlap integrals of boson and fermion profiles in the bulk-Higgs RS model, will be reported in future.[1)] For the present study, we restrict ourselves to the 3×3 submatrices governing the couplings of the SM fermion fields. In the zero mode approximation (ZMA), the required expressions are simplified considerably with (see also Ref. [27])

$$
\begin{aligned}
(\tilde{\Delta}_D)_{23}\otimes(\tilde{\Delta}_d)_{21} &\to (U_d^\dagger)_{2i}(U_d)_{i3}(\tilde{\Delta}_{Dd})_{ij}(W_d^\dagger)_{2j}(W_d)_{j1}, \\
(\tilde{\Delta}_{Dd})_{ij} &= \frac{F^2(c_{Q_i})}{3+2c_{Q_i}}\frac{3+c_{Q_i}+c_{d_j}}{2(2+c_{Q_i}+c_{d_j})}\frac{F^2(c_{d_j})}{3+2c_{d_j}}, \\
(\tilde{\Omega}_D)_{23}\otimes(\tilde{\Omega}_d)_{21} &\to (U_d^\dagger)_{2i}(W_d)_{j3}(\tilde{\Omega}_{Dd})_{ijkl}(W_d^\dagger)_{2k}(U_d)_{l1}, \\
(\tilde{\Omega}_{Dd})_{ijkl} &= \frac{\pi(1+\beta)}{4L}\frac{F(c_{Q_i})F(c_{d_j})}{2+\beta+c_{Q_i}+c_{d_j}}\frac{(Y_d)_{ij}(Y_d^\dagger)_{kl}}{1}
\end{aligned}
$$

1) A. Acosta, C. D. Lü, M. Neubert and Q. Qin, Flavor phenomenology in the bulk-Higgs Randall-Sundrum model, In preparation.

$$\times\frac{(4+2\beta+c_{Q_i}+c_{d_j}+c_{d_k}+c_{Q_l})}{4+c_{Q_i}+c_{d_j}+c_{d_k}+c_{Q_l}}$$
$$\times\frac{F(c_{d_k})F(c_{Q_l})}{2+\beta+c_{d_k}+c_{Q_l}},$$

where U_d and W_d are flavor matrices diagonalising the SM down-type Yukawa matrix. β is a parameter of the model related to the Higgs profile and the $c's$ are bulk-mass parameters of fermions, which control the localization of fermions in the warped extra dimension. The 5D Yukawa matrix Y_d has anarchic $\mathcal{O}(1)$ complex elements, which together with other flavor parameters generate the right quark masses. Summation over indices i,j,k and l is understood. Analogous expressions hold for remaining combinations of D and d. The effective Hamiltonian given in Eq. (9) is valid at $\mathcal{O}(M_{\mathrm{KK}})$, which must be evolved to a low-energy scale μ_b. Hence for the evolution of the Wilson coefficients we use the formulae of the NLO QCD factors given in Ref. [28]. After that, the decay width in the bulk-Higgs RS model is given by

$$\begin{aligned}\Gamma=\frac{m_{\mathrm{b}}^5}{3072(2\pi)^3}&[64(|C_1(\mu_b)|^2+|\tilde{C}_1(\mu_b)|^2)\\ &+12(|C_4(\mu_b)|^2+|\tilde{C}_4(\mu_b)|^2+|C_5(\mu_b)|^2+|\tilde{C}_5(\mu_b)|^2)\\ &+4\mathcal{R}e(C_4(\mu_b)C_5^*(\mu_b)+C_4^*(\mu_b)C_5(\mu_b)\\ &+\tilde{C}_4(\mu_b)\tilde{C}_5^*(\mu_b)+\tilde{C}_4^*(\mu_b)\tilde{C}_5(\mu_b))].\end{aligned}\tag{12}$$

4 Phenomenological bounds on RS models

In this section we discuss the relevant constraints on the parameter spaces of the RS models coming from the EW precision tests and the latest measurements of the Higgs signal strengths at the LHC. In addition, we will also consider the constraints coming from K^0-$\bar{\mathrm{K}}^0$ and B_s^0-$\bar{\mathrm{B}}_s^0$ mixing in Section 5.

First, considering the $\mathrm{RS_c}$ model, the bounds induced from EW precision tests allow for KK masses in the few TeV range. A recent tree-level analysis of the S and T parameters yields $M_{g^{(1)}}>4.8$ TeV at 95% confidence level (CL) for the mass of the lightest KK gluon and photon resonances [29]. While comparing the predictions of the signal rates for the various Higgs-boson decays with the latest data from the LHC, it is suggested in [30] that the most stringent bounds emerge from the signal rates for $\mathrm{pp}\to\mathrm{h}\to\mathrm{ZZ}^*,\mathrm{WW}^*$. In the $\mathrm{RS_c}$ model, KK gluon masses lighter than 22.7 TeV$\times(y_\star/3)$ in the brane-Higgs case and 13.2 TeV$\times(y_\star/3)$ in the narrow bulk-Higgs scenario are excluded at 95% CL, where the $y_\star=\mathcal{O}(1)$ is a free parameter and is defined as the upper bound on the various entries of the Yukawa matrices that are taken to be complex random numbers such that $|(Y_f)_{ij}|\leqslant y_\star$. Thus, for $y_\star=3$ the bounds derived from Higgs physics are much stronger than those stemming from EW precision measurements. In order to lower these bounds, smaller values of $y_\star$ can be considered. For that, it was also presented in Ref. [30] that for the lowest value of the lightest KK gluon mass $M_{g^{(1)}}=4.8$ TeV implied by EW precision constraints, in the $\mathrm{RS_c}$ model, the constraints at 95% CL on the values of the $y_\star$ are given by $y_\star<0.3$ for the brane-Higgs scenario, and $y_\star<1.1$ for the narrow bulk-Higgs case. However, realizing the fact that too-small Yukawa couplings would give rise to enhanced corrections to ϵ_{K} and hence would reinforce the RS flavor problem, relatively loose bounds on the values of the $y_\star$ can be obtained for the lightest KK gluon mass of $M_{g^{(1)}}=10$ TeV. For instance, in the $\mathrm{RS_c}$ model, the constraints on the value of $y_\star$ at 95% CL valid for $M_{g^{(1)}}=10$ TeV are given by $y_\star<1.1$ and $y_\star<2.25$ for the brane-Higgs case and the narrow bulk-Higgs case, respectively [30].

Next, we consider the bulk-Higgs RS model. The constraints on the KK mass scale in the bulk-Higgs RS model implied by the analyses of EW precision data are given in Ref. [20]. Under a constrained fit (i.e. $U=0$), the obtained lower bounds on the KK mass scale at 95% CL vary between $M_{\mathrm{KK}}>3.0$ TeV for $\beta=0$ to $M_{\mathrm{KK}}>5.1$ TeV for $\beta=10$. With an unconstrained fit, these bounds relax to $M_{\mathrm{KK}}>2.5$ TeV and $M_{\mathrm{KK}}>4.3$ TeV, respectively. For significantly larger values of β, the lower bounds increase towards the brane localized Higgs limit.

Table 1. Default values of the input parameters used in the SM calculation [31].

$G_F=1.16637\times10^{-5}$ GeV^{-2}, $m_{\mathrm{b}}=4.66^{+0.04}_{-0.03}$ GeV,
$m_{\mathrm{c}}=1.27\pm0.03$ GeV, $m_{\mathrm{t}}=173.21\pm0.51$ GeV,
$m_{\mathrm{W}}=80.385\pm0.015$ GeV, $\sin2\beta=0.691\pm0.017$,
$\tau_B=(1.566\pm0.003)\times10^{-12}$ sec, $\lvert V_{tb}V_{ts}^*\rvert=(40\pm2)\times10^{-3}$,
$\lvert V_{td}V_{ts}^*\rvert=(32\pm3)\times10^{-5}$, $\lvert V_{cd}V_{cs}^*\rvert=(21.8\pm0.6)\times10^{-2}$.

5 Numerical analysis

In this section we present the results of the $\mathrm{b}\to\mathrm{ss\bar{d}}$ decay rate in RS models. Before proceeding to analyze the NP, we first estimate the size of the leading order SM result. The numerical values of the parameters that are involved in the SM calculation are listed in Table 1. Employing the formula of the SM $\mathrm{b}\to\mathrm{ss\bar{d}}$ decay rate [2], we get

$$\mathcal{B}(\mathrm{b}\to\mathrm{ss\bar{d}})_{\mathrm{SM}}=(2.19\pm0.38)\times10^{-12}.\tag{13}$$

Next, we explore the parameter space of the $\mathrm{RS_c}$ model using the strategy outlined in Ref. [23]. It was pointed out in Ref. [23] that there exist regions in parameter space, without much fine-tuning in the 5D Yukawa couplings, which satisfy all existing $\Delta F=2$ and EW precision constraints for the scales of the masses of the lightest KK gauge bosons $M_{\mathrm{KK}}\simeq3$ TeV. However, as mentioned above, for the anarchic Yukawa couplings with

$y_\star = 3$ in the RS$_\mathrm{c}$ model with a brane Higgs, the constraints on $M_{g^{(1)}}$ emerging from Higgs physics are much stronger than the EW precision constraints, so in our study of the RS$_\mathrm{c}$ model, we generate two sets of fundamental 5D Yukawa matrices with $y_\star = 1.5$ and 3. For the first set the 28 parameters contained in the fundamental 5D Yukawa matrices are randomly chosen in their respective ranges, $[0,\pi/2]$, $[0,2\pi]$ and $[0.1,1.5]$ for angles, phases and $|(Y_f)_{ij}|$, respectively. In the second set, $|(Y_f)_{ij}|$ are chosen randomly in the range $[0.1,3]$, keeping the ranges for angles and phases the same as for the first set. In order to determine the nine quark bulk-mass parameters $c^i_{Q,u,d}$, we take $0.4 \leqslant c^3_Q \leqslant 0.45$ in our scan, allowing for consistency with EW precision data, so that the remaining bulk mass parameters are determined making use of the analytic formulae presented in Section 3 of [23]. Finally, by diagonalising numerically the obtained effective 4D Yukawa coupling matrices, we keep only those parameter sets that in addition to the quark masses and CKM mixing angles also reproduce the proper value of the Jarlskog determinant, all within their respective 2σ ranges. The flavor transitions that would be involved in the $\mathrm{b}\to\mathrm{ss\bar{d}}$ mode will commonly also give contributions to K^0-$\bar{\mathrm{K}}^0$ and B^0_s-$\bar{\mathrm{B}}^0_s$ mixings, so we consider ΔM_K, ϵ_K and ΔM_{B_s} constraints on the parameter space in addition to EW precision constraints and the Higgs constraints mentioned above. Expressions of $(M^K_{12})_\mathrm{KK}$ and $(M^s_{12})_\mathrm{KK}$ relevant to K^0-$\bar{\mathrm{K}}^0$ and B^0_s-$\bar{\mathrm{B}}^0_s$ mixing constraints, calculated in the RS$_\mathrm{c}$ model, are contained in Eqs. (4.32) and (4.33) of [23], respectively.

Figure 1 shows the branching ratio of the RS$_\mathrm{c}$ predictions for the $\mathrm{b}\to\mathrm{ss\bar{d}}$ decay as a function of $M_{g^{(1)}}$ with two different values of $y_\star$. Note that we have excluded the SM contribution to display the decoupling behavior of the NP contribution as $M_{g^{(1)}}$ increases. The red and blue scatter points represent the cases of $y_\star = 1.5$ and 3, respectively. While imposing the experimental constraints for ΔM_K, ΔM_{B_s} and ϵ_K in both cases, we set the input parameters in Table 2 to their central values and allow the resulting observables to deviate by ±50%, ±30% and ±30%, respectively.

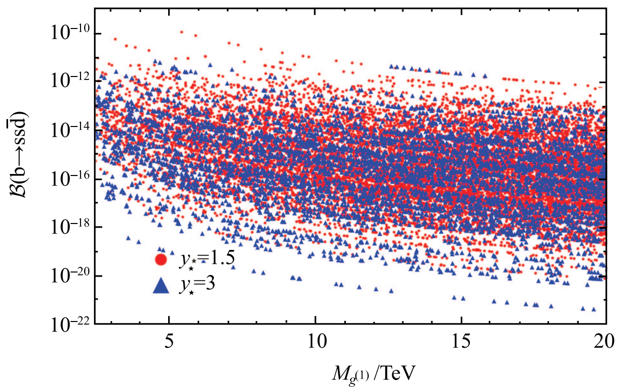
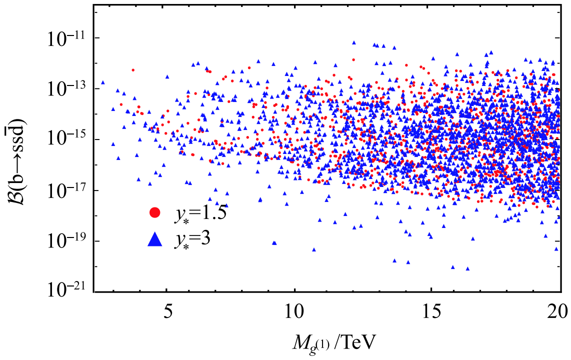


Fig. 1. (color online) The branching ratio of $\mathrm{b}\to\mathrm{ss\bar{d}}$ as a function of the KK gluon mass $M_{g^{(1)}}$ in the RS$_\mathrm{c}$ model. The red and blue points correspond to $y_\star = 1.5$ and 3, respectively.

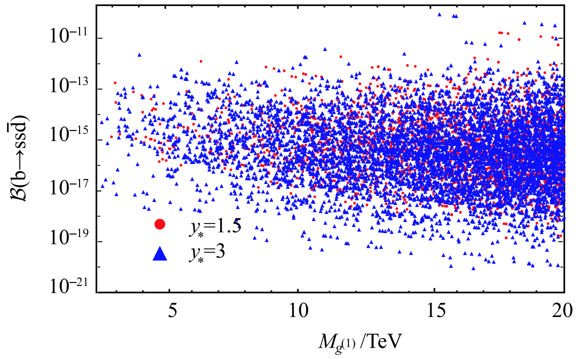
The predictions of the $\mathrm{b}\to\mathrm{ss\bar{d}}$ decay rates for the parameter points with $y_\star = 1.5$ are generally larger than those with $y_\star = 3$, but it can be seen in Fig. 1 that after applying the ΔM_K, ϵ_K and ΔM_{B_s} constraints simultaneously, the maximum possible $y_\star = 1.5$ prediction is reduced to relatively close to that for the case of $y_\star = 3$. However, after imposing the K^0-$\bar{\mathrm{K}}^0$ and B^0_s-$\bar{\mathrm{B}}^0_s$ mixings constraints, still for some parameter points with $y_\star = 1.5$ in the low $M_{g^{(1)}}$ range, the branching ratio of $\mathrm{b}\to\mathrm{ss\bar{d}}$ decay in the RS$_\mathrm{c}$ model can be close to the order of 10^{-10}, which is approximately two orders of magnitude larger than the SM result. Considering the effects

Table 2. Values of experimental and theoretical quantities used as input parameters while scanning the parameter spaces of the RS models and in calculation of ΔM_K, ΔM_{B_s} and ϵ_K. Values of the parameters B^K_i at $\mu_\mathrm{L} = 2$ GeV and B^s_i at $\mu_b = 4.6$ GeV are given in the $\overline{\mathrm{MS}}$-NDR scheme obtained for K^0-$\bar{\mathrm{K}}^0$ and B^0_s-$\bar{\mathrm{B}}^0_s$ mixings, respectively.

$\lvert V_{us}\rvert = 0.226(2)$		$s^2_w = 0.2312$		
$\lvert V_{ub}\rvert = 3.8(4)\times10^{-3}$		$\alpha(m_\mathrm{Z}) = 1/127.9$		
$\lvert V_{cb}\rvert = 4.1(1)\times10^{-2}$	[32]	$\alpha_s(m_\mathrm{Z}) = 0.1182\pm0.0012$		[31]
$\lambda = 0.2250\pm0.0005$		$m_\mathrm{K} = 497.611$ MeV		
$A = 0.811\pm0.026$		$m_{\mathrm{B}_s} = 5366.82$ MeV		[31]
$\bar{\rho} = 0.124^{+0.019}_{-0.018}$		$\eta_{tt} = 0.57\pm0.01$		[35]
$\bar{\eta} = 0.356\pm0.011$	[31]	$\eta_{cc} = 1.50\pm0.37$		[36]
$\Delta M_\mathrm{K} = (3.484\pm0.006)\times10^{-15}$ GeV		$\eta_{ct} = 0.47\pm0.05$		[37, 38]
$\Delta M_{\mathrm{B}_s} = (1.1688\pm0.0014)\times10^{-11}$ GeV		$\eta_\mathrm{B} = 0.55\pm0.01$		[35]
$\lvert\epsilon_\mathrm{K}\rvert = (2.228\pm0.011)\times10^{-3}$	[31]	$F_\mathrm{K} = 156$ MeV		
$\phi_\epsilon = (43.52\pm0.05)^\circ$	[31]	$F_{\mathrm{B}_s} = 245\pm25$ MeV		[34]
$\kappa_\epsilon = 0.92\pm0.02$	[33]			
$\hat{B}_\mathrm{K} = 0.75$		$\mu_\mathrm{L} = 2$ GeV	$B^K_1 = 0.57, B^K_4 = 0.81, B^K_5 = 0.56$	[39]
$\hat{B}_{\mathrm{B}_s} = 1.22$	[34]	$\mu_b = 4.6$ GeV	$B^s_1 = 0.87, B^s_4 = 1.16, B^s_5 = 1.75$	[40]



(a) $\beta=1$;



(b) $\beta=10$.

Fig. 2. (color online) The branching ratio of $\mathrm{b}\to\mathrm{ss}\bar{\mathrm{d}}$ as a function of the KK gluon mass $M_{g^{(1)}}$ in the bulk-Higgs RS model with $\beta=1$ and $\beta=10$. The red and blue scatter points correspond to $y_\star=1.5$ and 3, respectively.

of the new heavy EW gauge bosons $\mathrm{Z_H}$ and $\mathrm{Z'}$ in the $\mathrm{RS_c}$ model, we found in agreement with [23] that imposing the ΔM_{K} and ϵ_{K} constraints $\mathrm{Z_H}$ and $\mathrm{Z'}$ gives subleading contributions because the strong QCD renormalization group enhancement of the C_2^{LR} coefficient and the chiral enhancement of the $\mathcal{Q}_2^{\mathrm{LR}}$ hadronic matrix element in $(M_{12}^{\mathrm{K}})_{\mathrm{KK}}$ assure that the first KK gluon contributions still dominate over EW contributions. However, for the prediction of the branching ratio in the $\mathrm{b}\to\mathrm{ss}\bar{\mathrm{d}}$ decay, the QCD renormalization group enhancement in the C_2^{LR} and C_2^{RL} coefficients is smaller and the chiral enhancement is absent. Therefore, for a parameter point that satisfies the ΔM_{K}, ΔM_{B_s} and ϵ_{K} constraints simultaneously, $\mathrm{Z_H}$ and $\mathrm{Z'}$ increase the prediction of the branching ratio with comparable contributions to that of the first KK gluon.

For the bulk-Higgs RS model, following the directions given in Refs. [20, 21], for a given value of β and M_{KK}, we generate two sets of random and anarchic 5D Yukawa matrices, whose entries satisfy $|(Y_{u,d})_{ij}| \leqslant y_\star$ with $y_\star=1.5$ and 3. These values of $y_\star$ lie below the perturbativity bound, which is given by $y_\star < y_{\max}$ with $y_{\max}\sim 8.3/\sqrt{1+\beta}$ [20]. Moreover, for values of $y_\star<1$ it becomes increasingly difficult to fit the top-quark mass. Next, we require that the 5D Yukawa matrices with proper bulk-mass parameters $c_{Q_i}<1.5$ and $c_{q_i}<1.5$ reproduce the correct values for the SM quark masses evaluated at the scale $\mu=1$ TeV. In our analysis, we consider the two representative values $\beta=1$ and $\beta=10$ corresponding to broad Higgs profile and narrow Higgs profile, respectively.

In Fig. 2, we show the NP predictions with $\beta=1$ and 10, respectively, for the $\mathrm{b}\to\mathrm{ss}\bar{\mathrm{d}}$ decay rate as a function of $M_{g^{(1)}}$, after simultaneously imposing the ΔM_{K}, ϵ_{K} and ΔM_{B_s} constraints. The red and blue scatter points again correspond to model points obtained using $y_\star=1.5$ and 3, respectively. For the case of $y_\star=1.5$, the branching ratios are generally larger because of less-suppressed FCNCs than the $y_\star=3$ case, but as mentioned earlier the lower values of $y_\star$ are subject to more stringent constraints from flavour physics, so after imposing the ΔM_{K}, ϵ_{K} and ΔM_{B_s} constraints, the maximum possible branching ratio of the parameter points with $y_\star=1.5$ in the bulk-Higgs RS model lies close to the SM result, as shown in Fig. 2(a). For the case of $y_\star=3$ in Fig. 2(a), subject to relatively less severe constraints from the K^0-$\bar{\mathrm{K}}^0$ and B_s^0-$\bar{\mathrm{B}}_s^0$ mixings compared with $y_\star=1.5$ case, the maximum possible branching ratio for some of the parameter points, even with suppressed FCNCs, lies close to the order 10^{-11}. The situation is similar in the $\beta=10$ case, except that compared to the $\beta=1$ scenario, an order of magnitude enhancement for the maximum possible branching ratio is observed for both cases of $y_\star$, as displayed in Fig. 2(b).

6 Conclusions

We studied the $\mathrm{b}\to\mathrm{ss}\bar{\mathrm{d}}$ decay in the $\mathrm{RS_c}$ and the bulk-Higgs RS model. In both models, the main contribution to the $\mathrm{b}\to\mathrm{ss}\bar{\mathrm{d}}$ decay comes from tree level exchanges of KK gluons, while in the $\mathrm{RS_c}$ model the contributions from the new heavy EW gauge bosons $\mathrm{Z_H}$ and $\mathrm{Z'}$ can compete with the KK-gluon contributions. We employed renormalization group runnings of the Wilson coefficients with NLO QCD factors in both models. Although this decay receives tree level contributions, the parameter space is severely constrained by K^0-$\bar{\mathrm{K}}^0$ mixing and B_s^0-$\bar{\mathrm{B}}_s^0$ mixing experiments such that for a broad Higgs profile corresponding to the $\beta=1$ case no significant increase in the branching ratio is observed in the bulk-Higgs RS model compared to the SM result. For the value $\beta=10$, however, it is possible to achieve an order of magnitude enhancement of the branching ratio for some of the parameter points. The $\mathrm{RS_c}$ model with additional contributions from the new heavy EW gauge bosons $\mathrm{Z_H}$ and $\mathrm{Z'}$ enhances the branching ratio, compared to the SM result, by at least one order of magnitude for some points in the parameter space with

$y_{\star} = 1.5$, which leaves this decay free for searches for new physics in future experiments.

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