

Broken $S_{3L} \times S_{3R}$ flavor symmetry and leptonic CP violation^{*}

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Abstract: In the framework of the canonical seesaw model, we present a simple but viable scenario to explicitly break an $S_{3L} \times S_{3R}$ flavor symmetry in the leptonic sector. It turns out that the leptonic flavor mixing matrix is completely determined by the mass ratios of the charged leptons (i.e., m_e/m_μ and m_μ/m_τ) and those of light neutrinos (i.e., m_1/m_2 and m_2/m_3). The latest global-fit results of the three neutrino mixing angles $\{\theta_{12}, \theta_{13}, \theta_{23}\}$ and two neutrino mass-squared differences $\{\Delta m_{21}^2, \Delta m_{31}^2\}$ at the 3σ level are used to constrain the parameter space of $\{m_1/m_2, m_2/m_3\}$. The predictions for the mass spectrum and flavor mixing are highlighted: (1) the neutrino mass spectrum shows a hierarchical pattern and a normal ordering, e.g., $m_1 \approx 2.2$ meV, $m_2 \approx 8.8$ meV and $m_3 \approx 52.7$ meV; (2) only the first octant of θ_{23} is allowed, namely, $41.8^\circ \lesssim \theta_{23} \lesssim 43.3^\circ$; (3) the Dirac CP -violating phase $\delta \approx -22^\circ$ deviates significantly from the maximal value -90° . All these predictions are ready to be tested in ongoing and forthcoming neutrino oscillation experiments. Moreover, we demonstrate that the cosmological matter-antimatter asymmetry can be explained via resonant leptogenesis, including the individual lepton-flavor effects. In our scenario, leptonic CP violation at low- and high-energy scales is closely connected.

Keywords: canonical seesaw model, $S_{3L} \times S_{3R}$ flavor symmetry, leptonic CP violation, cosmological matter-antimatter asymmetry, resonant leptogenesis

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1 Introduction

Recent neutrino oscillation experiments have firmly established that neutrinos are massive particles and that lepton flavors are significantly mixed [1, 2]. The leptonic flavor mixing is described by a 3×3 unitary matrix V , the

Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix [3, 4], which is conventionally parametrized through three mixing angles $\{\theta_{12}, \theta_{13}, \theta_{23}\}$, one Dirac CP -violating phase δ , and two Majorana CP -violating phases $\{\rho, \sigma\}$. As usual, we take the standard parametrization of the PMNS matrix [5],

$$V = \begin{pmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta} & +c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta} & c_{13}s_{23} \\ +s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta} & c_{13}c_{23} \end{pmatrix} \cdot P_\nu, \quad (1)$$

where $c_{ij} \equiv \cos\theta_{ij}$ and $s_{ij} \equiv \sin\theta_{ij}$ have been defined, and $P_\nu \equiv \text{Diag}\{e^{i\rho}, e^{i\sigma}, 1\}$ is a diagonal matrix of two Majorana CP -violating phases. The latest global analysis of neutrino oscillation data [6] yields the best-fit values of the three mixing angles as $\theta_{12} \approx 33.6^\circ$, $\theta_{13} \approx 8.5^\circ$ and

$\theta_{23} \approx 41.6^\circ$, and those of the two neutrino mass-squared differences as $\Delta m_{21}^2 \equiv m_2^2 - m_1^2 \approx 7.50 \times 10^{-5}$ eV² and $\Delta m_{31}^2 \equiv m_3^2 - m_1^2 \approx 2.52 \times 10^{-3}$ eV². Although the normal ordering of neutrino masses (i.e., $m_1 < m_2 < m_3$) is slightly favored and there is a preliminary hint of a

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nearly-maximal CP -violating phase $\delta \approx 261^\circ$ or equivalently $\delta \approx -99^\circ$, the possibility of an inverted mass ordering (i.e., $m_3 < m_1 < m_2$) and CP conservation in the leptonic sector has not yet been excluded [6]. See also the independent global-fit results from two other groups [7–9]. An unambiguous determination of neutrino mass ordering and a robust discovery of leptonic CP violation in neutrino oscillations are two primary goals of future medium-baseline reactor (e.g., JUNO [10] and RENO-50 [11]) and long-baseline accelerator neutrino experiments (e.g., T2K [12], NO ν A [13] and LBNF-DUNE [14]).

In order to accommodate neutrino masses, we can introduce one right-handed neutrino $N_{\alpha R}$ (for $\alpha=e, \mu, \tau$) for each lepton family and write down the gauge-invariant Lagrangian relevant for lepton masses and flavor mixing as follows:

$$-\mathcal{L}_\ell = \bar{\ell}_L Y_1 H E_R + \bar{\ell}_L Y_\nu \tilde{H} N_R + \frac{1}{2} \bar{N}_R^c M_R N_R + \text{h.c.}, \quad (2)$$

where ℓ_L and $\tilde{H} \equiv i\sigma_2 H^*$ denote respectively the left-handed lepton doublet and the Higgs doublet, and E_R and N_R the right-handed charged-lepton and neutrino singlets. While Y_1 and Y_ν are the 3×3 Yukawa coupling matrices for charged leptons and neutrinos, M_R is a symmetric mass matrix for right-handed Majorana neutrinos. After the Higgs field acquires its vacuum expectation value $\langle H \rangle = v \approx 174$ GeV, the gauge symmetry is spontaneously broken down, and the charged-lepton mass matrix $M_l \equiv Y_1 v$ and Dirac neutrino mass matrix $M_D \equiv Y_\nu v$ can be obtained. Since the Majorana mass term of right-handed neutrinos is not subject to the gauge symmetry breaking, its absolute scale $\mathcal{O}(M_R)$ could be much higher than the electroweak scale $\Lambda_{\text{EW}} \sim 100$ GeV, e.g., $\mathcal{O}(M_R) \sim 10^{14}$ GeV, close to the scale of grand unified theories. At low energies, one can integrate out heavy right-handed neutrinos, and the effective neutrino mass matrix is then given by the seesaw formula $M_\nu \approx M_D M_R^{-1} M_D^T$ [15–19]. In this canonical seesaw model, the smallness of left-handed neutrinos can be attributed to the heaviness of right-handed neutrinos, and the lepton flavor mixing arises from the mismatch between the diagonalizations of M_l and M_ν . However, the seesaw mechanism itself has told us nothing about the flavor structures of the lepton mass matrices M_l , M_D and M_R . Hence the lepton mass spectra and flavor mixing remain to be understood in the canonical seesaw model [20].

One typical approach to constrain the flavor structures is to impose a continuous or discrete flavor symmetry on the generic Lagrangian in the first place, and then a spontaneous or explicit symmetry breaking is introduced to help accommodate realistic fermion mass spectra, flavor mixing angles and CP violation [21, 22]. In Ref. [21], Harari, Haut and Weyers proposed a specific

model for quark mass spectra and flavor mixing based on an $S_{3L} \times S_{3R}$ symmetry, predicting a flavor democracy in the quark sector. Since then, there have been a great number of theoretical works on how to break the flavor democracy or the $S_{3L} \times S_{3R}$ symmetry in order to explain mass spectra and flavor mixing in both quark and lepton sectors [23–62]. In this paper, we put forward a simple but viable scenario for lepton mass spectra and flavor mixing, in which the $S_{3L} \times S_{3R}$ symmetry in the lepton sector is explicitly broken. We assume the following lepton mass matrices:

$$M_l = M_l^0 + \Delta M_l, \quad M_D = M_D^0 + \Delta M_D, \quad M_R = M_R^0 + \Delta M_R, \quad (3)$$

where M_x^0 (for $x=l, D, R$) are determined by the $S_{3L} \times S_{3R}$ flavor symmetry, and the perturbations ΔM_x (for $x=l, D, R$) explicitly break the flavor symmetry and will be specified later. Here the subscripts “L” and “R” of the flavor symmetry $S_{3L} \times S_{3R}$ indicate that the corresponding S_3 symmetry transformation is nontrivially acting only on left-handed or right-handed fermion fields, respectively. As shown in Appendix A, it is straightforward to prove that the lepton mass matrices in the symmetry limit are uniquely given by

$$M_l^0 = \frac{c_l}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \quad M_D^0 = \frac{c_D}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix},$$

$$M_R^0 = \frac{c_R}{3} \left[\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + r_R \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \right], \quad (4)$$

where c_x characterizes the absolute mass scale of M_x^0 (for $x=l, D, R$), and r_R is a dimensionless parameter. The relative size of r_R is able to determine whether the mass spectrum of heavy Majorana neutrinos is hierarchical or degenerate.

The present work differs from previous studies in two aspects. First, we propose a rather simple form of the perturbation matrices ΔM_l , ΔM_D and ΔM_R such that neutrino masses and flavor mixing can be accommodated. Interestingly, the PMNS matrix is fully fixed by the ratios of charged-lepton masses and those of neutrino masses, and a hierarchical pattern of neutrino mass spectrum is favored. Moreover, the prediction for the CP -violating phase is $\delta \approx -22^\circ$, which can be confirmed or disproved in future long-baseline accelerator neutrino oscillation experiments. Second, we calculate the CP asymmetries in the decays of heavy Majorana neutrinos in the early Universe, and find that the observed matter-antimatter asymmetry can be successfully explained via thermal leptogenesis [63]. However, one has to require a mass degeneracy of heavy Majorana neutrinos and implement the mechanism of resonant leptogenesis [64, 65]. A direct connection between the low-energy CP violation

in neutrino oscillations and the high-energy CP asymmetries in heavy Majorana neutrino decays can be established.

The rest of our paper is organized as follows. In Section 2, we specify the perturbation terms ΔM_x (for $x=1, D, R$) and explore their implications for lepton mass spectra, flavor mixing and CP violation. Then, the CP asymmetries from the heavy Majorana neutrino decays are calculated in Section 3. After taking account of resonant enhancement in CP asymmetries and individual lepton-flavor effects, we show that the predicted baryon number asymmetry can be compatible with cosmological observations. We finally summarize our main results in Section 4.

2 Mass spectra, flavor mixing and CP violation

2.1 Analytical results

In the symmetry limit, the lepton mass matrices M_l^0 , M_D^0 and M_R^0 are given in Eq. (4), where the democratic matrix with all matrix elements being one is present. In the Majorana mass matrix of heavy neutrinos, there is an extra term proportional to the identity matrix, which is however not affected by any orthogonal transformation to diagonalize the democratic matrix. As is well known, the democratic matrix can be diagonalized by the following orthogonal matrix

$$V_0 = \frac{1}{\sqrt{6}} \begin{pmatrix} \sqrt{3} & 1 & \sqrt{2} \\ -\sqrt{3} & 1 & \sqrt{2} \\ 0 & -2 & \sqrt{2} \end{pmatrix}. \quad (5)$$

All three matrices M_l^0 , M_D^0 and M_R^0 are to be diagonalized by V_0 , as is the effective neutrino mass matrix $M_\nu^0 = M_D^0 (M_R^0)^{-1} (M_D^0)^T$. Thus, the first two generations of charged leptons and light neutrinos are massless, implying no flavor mixing in the limit of exact $S_{3L} \times S_{3R}$ flavor symmetry.

To account for lepton mass spectra and flavor mixing, we shall introduce the perturbation terms ΔM_l and ΔM_D , which explicitly break the $S_{3L} \times S_{3R}$ flavor symmetry. Note that one can also break the mass degeneracy of heavy neutrinos by assuming a proper ΔM_R , which is not necessary for our discussions on low-energy phenomenology. We find that simple diagonal perturbations will suffice for our purpose:

$$\Delta M_l = \frac{c_l}{3} \begin{pmatrix} +i\delta_l & 0 & 0 \\ 0 & -i\delta_l & 0 \\ 0 & 0 & \varepsilon_l \end{pmatrix}, \quad \Delta M_D = \frac{c_D}{3} \begin{pmatrix} 0 & 0 & 0 \\ 0 & -\delta_D & 0 \\ 0 & 0 & \varepsilon_D \end{pmatrix}, \quad (6)$$

where $0 < \delta_l \ll \varepsilon_l \ll 1$ and $0 < \delta_D \ll \varepsilon_D \ll 1$ are implied. Some discussions on different forms of perturbations can be found in Ref. [58]. Then, we separately diagonalize

the charged-lepton and neutrino mass matrices.

1) **Charged-lepton mass matrix M_l** — It is convenient to work in the hierarchical basis, which is defined by the transformation $V_0^T M_l V_0 = M_l'$. In this basis, one has to further diagonalize the symmetric matrix below [30, 41, 49]

$$M_l' = \frac{c_l}{9} \begin{pmatrix} 0 & i\sqrt{3}\delta_l & i\sqrt{6}\delta_l \\ i\sqrt{3}\delta_l & 2\varepsilon_l & -\sqrt{2}\varepsilon_l \\ i\sqrt{6}\delta_l & -\sqrt{2}\varepsilon_l & 9+\varepsilon_l \end{pmatrix}. \quad (7)$$

The perturbation matrix ΔM_l leads not only to nonzero electron and muon masses, but also to leptonic CP violation at both low- and high-energy scales. Based on the relation of a strong hierarchy $\delta_l \ll \varepsilon_l \ll 1$, we can diagonalize M_l' and obtain the masses of the charged leptons:

$$m_e \approx \frac{\delta_l^2}{6\varepsilon_l} c_l, \quad m_\mu \approx \frac{2\varepsilon_l}{9} c_l, \quad m_\tau \approx c_l. \quad (8)$$

In addition, the unitary matrix V_l , diagonalizing the total charged-lepton mass matrix via $V_l^\dagger M_l V_l^* = \text{Diag}\{m_e, m_\mu, m_\tau\}$, is found to be:

$$V_l \approx V_0 + \frac{i}{\sqrt{6}} \sqrt{\frac{m_e}{m_\mu}} \begin{pmatrix} 1 & +\sqrt{3} & 0 \\ 1 & -\sqrt{3} & 0 \\ -2 & 0 & 0 \end{pmatrix} + \frac{1}{2\sqrt{3}} \frac{m_\mu}{m_\tau} \begin{pmatrix} 0 & \sqrt{2} & -1 \\ 0 & \sqrt{2} & -1 \\ 0 & \sqrt{2} & 2 \end{pmatrix}. \quad (9)$$

A salient feature is that all three model parameters $(c_l, \varepsilon_l, \delta_l)$ in the charged-lepton sector are determined by three charged-lepton masses, as shown in Eq. (8), and the unitary matrix V_l depends only on two mass ratios. By making use of the running charged-lepton masses at the energy scale of $M_Z = 91.2$ GeV [66, 67]: $m_e \approx 0.486570$ MeV, $m_\mu \approx 102.718$ MeV and $m_\tau \approx 1746.17$ MeV, we get $m_e/m_\mu \approx 0.00474$ and $m_\mu/m_\tau \approx 0.0588$. From Eq. (8), one can immediately observe the relations $\varepsilon_l \approx (9/2) \cdot (m_\mu/m_\tau)$ and $\delta_l/\varepsilon_l \approx (2/\sqrt{3}) \cdot (m_e/m_\mu)^{1/2}$, and then arrive at $\varepsilon_l \approx 0.265$ and $\delta_l \approx 0.0211$, which are consistent with the assumption that ε_l and δ_l can be taken as small perturbation parameters.

2) **Effective neutrino mass matrix M_ν** — According to the seesaw formula, the effective neutrino mass matrix is given by

$$M_\nu \approx \frac{c_D^2}{3c_R r_R} \left[r_R \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \frac{1}{3} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 2\delta_D^2 & \delta_D \varepsilon_D \\ 0 & \delta_D \varepsilon_D & 2\varepsilon_D^2 \end{pmatrix} \right], \quad (10)$$

where $0 < r_R \ll \delta_D^2 \ll \varepsilon_D^2 \ll 1$ has been assumed. Diagonalizing the effective neutrino mass matrix via $V_\nu^\dagger M_\nu V_\nu^* = \text{Diag}\{m_1, m_2, m_3\}$ and defining $c_\nu \equiv c_D^2/(3c_R r_R)$, the neutrino masses take on a normal ordering and the three

mass eigenvalues are

$$m_1 \approx r_R c_\nu, \quad m_2 \approx \frac{1}{2} \delta_D^2 c_\nu, \quad m_3 \approx \frac{2}{3} \varepsilon_D^2 c_\nu. \quad (11)$$

The corresponding unitary matrix is

$$V_\nu \approx \begin{pmatrix} 1 & \frac{m_1}{m_2} & \frac{m_1}{m_3} \\ -\frac{m_1}{m_2} & 1 & \frac{1}{\sqrt{3}} \sqrt{\frac{m_2}{m_3}} \\ -\frac{m_1}{m_3} & -\frac{1}{\sqrt{3}} \sqrt{\frac{m_2}{m_3}} & 1 \end{pmatrix}, \quad (12)$$

where only the leading term of each matrix element is kept. Similar to V_1 in the charged-lepton sector, V_ν

is completely fixed by the ratios of the mass eigenvalues. Even with precise values of the three neutrino masses, one cannot calculate all the model parameters $\{c_D, \delta_D, \varepsilon_D\}$ and $\{c_R, r_R\}$ which appear in Eq. (11). The determination of all these parameters calls for more observables related to heavy Majorana neutrinos, such as the baryon number asymmetry, which is discussed in the next section.

Therefore, the PMNS matrix can be calculated via $V = V_1^\dagger V_\nu$, where the unitary matrices V_1 and V_ν can be found in Eq. (9) and Eq. (12), respectively. More explicitly, we have

$$V \approx \frac{1}{\sqrt{6}} \begin{pmatrix} \sqrt{3} & -\sqrt{3} & 0 \\ 1 & 1 & -2 \\ \sqrt{2} & \sqrt{2} & \sqrt{2} \end{pmatrix} - \frac{i}{\sqrt{6}} \sqrt{\frac{m_e}{m_\mu}} \begin{pmatrix} 1 & 1 & -2 \\ \sqrt{3} & -\sqrt{3} & 0 \\ 0 & 0 & 0 \end{pmatrix} + \frac{1}{2\sqrt{3}} \frac{m_\mu}{m_\tau} \begin{pmatrix} 0 & 0 & 0 \\ \sqrt{2} & \sqrt{2} & \sqrt{2} \\ -1 & -1 & 2 \end{pmatrix} \\ + \frac{1}{3\sqrt{2}} \sqrt{\frac{m_2}{m_3}} \begin{pmatrix} 0 & 0 & -\sqrt{3} \\ 0 & 2 & 1 \\ 0 & -\sqrt{2} & \sqrt{2} \end{pmatrix} + \frac{1}{\sqrt{6}} \frac{m_1}{m_2} \begin{pmatrix} \sqrt{3} & \sqrt{3} & 0 \\ -1 & 1 & 0 \\ -\sqrt{2} & \sqrt{2} & 0 \end{pmatrix} + \frac{1}{\sqrt{6}} \frac{m_1}{m_3} \begin{pmatrix} 0 & 0 & \sqrt{3} \\ 2 & 0 & 1 \\ -\sqrt{2} & 0 & \sqrt{2} \end{pmatrix}. \quad (13)$$

The contribution proportional to m_1/m_3 from the neutrino sector has been retained, while that proportional to m_e/m_τ from the charged-lepton sector has been neglected. The main reason is that although neutrino mass spectrum in Eq. (11) is also hierarchical, the mass hierarchy of neutrinos cannot be as strong as that of charged leptons. This observation will become clearer soon when the PMNS matrix in Eq. (13) is confronted with the latest neutrino oscillation data. Some comments on the phenomenological implications for the three flavor mixing angles, CP -violating phases and neutrino masses are in order:

1) Comparing the PMNS matrix V in Eq. (13) and the standard parametrization in Eq. (1), we find:

$$\sin^2 \theta_{13} = |V_{e3}|^2 \approx \frac{1}{6} \frac{m_2}{m_3} - \frac{1}{\sqrt{3}} \frac{m_1}{m_2} \left(\frac{m_2}{m_3} \right)^{3/2} \\ + \frac{1}{2} \left(\frac{m_1}{m_2} \right)^2 \left(\frac{m_2}{m_3} \right)^2 + \frac{2}{3} \frac{m_e}{m_\mu}. \quad (14)$$

The smallest mixing angle θ_{13} is mainly determined by the mass ratio m_2/m_3 , but with sub-leading contributions from both m_e/m_μ and m_1/m_2 . For a rough estimate, we neglect all the terms associated with m_e/m_μ and m_1/m_2 . Given the best-fit value $\theta_{13} \approx 8.5^\circ$ or $\sin^2 \theta_{13} \approx 0.022$ from the latest neutrino oscillation data, we obtain $m_2/m_3 \approx 0.132$. However, as θ_{13} itself is very small, the neglected terms may have a significant impact on the determination of m_2/m_3 . For instance, if $m_1/m_2 \approx 0.25$ is assumed, one can get $m_2/m_3 \approx 0.167$ by numerically solving Eq. (14) with the best-fit value $\sin^2 \theta_{13} \approx 0.022$.

2) Then we proceed to calculate the other two mixing

angles θ_{12} and θ_{23} . Adopting the standard parametrization, we have

$$\sin^2 \theta_{12} = \frac{|V_{e2}|^2}{1 - |V_{e3}|^2} \approx \frac{1}{2} \left(1 - \frac{m_1}{m_2} \right)^2 \left(1 + \frac{1}{6} \frac{m_2}{m_3} \right), \quad (15) \\ \sin^2 \theta_{23} = \frac{|V_{\mu 3}|^2}{1 - |V_{e3}|^2} \approx \frac{2}{3} \left[1 - \frac{1}{2\sqrt{3}} \sqrt{\frac{m_2}{m_3}} - \frac{1}{2} \left(\frac{m_\mu}{m_\tau} + \frac{m_1}{m_3} \right) \right]^2 \\ \times \left(1 + \frac{1}{6} \frac{m_2}{m_3} \right), \quad (16)$$

where only the dominant term of $\sin^2 \theta_{13}$ in Eq. (14) is considered. As indicated by Eq. (15), $\sin^2 \theta_{12}$ is more sensitive to the mass ratio m_1/m_2 than to m_2/m_3 . However, from Eq. (16), the opposite is true for $\sin^2 \theta_{23}$. To estimate the predictions for θ_{12} and θ_{23} , we input $m_1/m_2 = 0.25$ and $m_2/m_3 = 0.167$, and then find $\theta_{12} \approx 32.5^\circ$ or $\sin^2 \theta_{12} \approx 0.289$ from Eq. (15) and $\theta_{23} \approx 43.5^\circ$ or $\sin^2 \theta_{23} \approx 0.474$ from Eq. (16). Both values of θ_{12} and θ_{23} lie in the allowed ranges $31.38^\circ \leq \theta_{12} \leq 35.99^\circ$ and $38.4^\circ \leq \theta_{23} \leq 52.8^\circ$ at the 3σ level.

3) Now let us look at the CP -violating phases from the PMNS matrix. Since the complex term in the PMNS matrix is proportional to $(m_e/m_\mu)^{1/2} \approx 0.069$, as shown in Eq. (13), it is important for V_{e3} but negligible for other matrix elements. Therefore, the Dirac-type CP -violating phase is approximately given as

$$\delta \approx -\arctan \left[2 \sqrt{\frac{m_e}{m_\mu}} \sqrt{\frac{m_3}{m_2}} \left(1 - \sqrt{3} \frac{m_1}{m_2} \sqrt{\frac{m_2}{m_3}} \right)^{-1} \right] \\ \approx -22.3^\circ, \quad (17)$$

where $m_1/m_2 = 0.25$ and $m_2/m_3 = 0.167$ have been used. Recent measurements from T2K and NO ν A

have shown a preliminary hint of $\delta \approx -90^\circ$. The future long-baseline accelerator neutrino oscillation experiments, such as LBNF-DUNE [14] and T2HK [68], and neutrino super-beam experiments, like ESS ν SB [69] and MOMENT [70, 71], are promising to unambiguously discover leptonic CP violation in neutrino oscillations.

Then, considering the freedom of rephasing the charged-lepton fields, we can also extract two Majorana-type CP -violating phases

$$\begin{aligned}\rho &\approx -\arctan \left[\frac{1}{\sqrt{3}} \sqrt{\frac{m_e}{m_\mu}} \left(1 + \frac{m_1}{m_2} \right)^{-1} \right] \approx -1.8^\circ, \\ \sigma &\approx +\arctan \left[\frac{1}{\sqrt{3}} \sqrt{\frac{m_e}{m_\mu}} \left(1 - \frac{m_1}{m_2} \right)^{-1} \right] \approx +3.0^\circ, \quad (18)\end{aligned}$$

where are close to the trivial value π or 0. This is consistent with our previous observation that all the PMNS matrix elements but V_{e3} are real at leading order.

4) Finally, we come to the neutrino mass spectrum. The best-fit value of the neutrino mass-squared difference $\Delta m_{21}^2 \approx 7.50 \times 10^{-5} \text{ eV}^2$, together with $m_1/m_2 \approx 0.25$ and $m_2/m_3 \approx 0.167$, results in a full determination of neutrino masses, i.e.,

$$\begin{aligned}m_1 &= \frac{m_1}{m_2} \sqrt{\frac{\Delta m_{21}^2}{1 - (m_1/m_2)^2}} \approx 2.2 \text{ meV}, \\ m_2 &\approx 8.8 \text{ meV}, \\ m_3 &\approx 52.7 \text{ meV}.\end{aligned} \quad (19)$$

On the other hand, the other neutrino mass-squared difference $\Delta m_{31}^2 \approx 2.77 \times 10^{-3} \text{ eV}^2$ can be computed by using the neutrino masses in Eq. (19), which is slightly larger than the observed value $2.41 \times 10^{-3} \text{ eV}^2 \lesssim \Delta m_{31}^2 \lesssim 2.64 \times 10^{-3} \text{ eV}^2$ in the 3σ range. In order to see if the ansätze of lepton mass matrices in Eqs. (4) and (6) are really consistent with neutrino oscillation data, we shall carry out a complete numerical analysis in the next subsection.

Given the perfectly measured charged-lepton masses, the PMNS matrix is now completely fixed by two neutrino mass ratios m_1/m_2 and m_2/m_3 , which can be determined mainly from the observed values of $\sin^2\theta_{12}$ and $\sin^2\theta_{13}$, respectively. Together with the mass-squared difference Δm_{21}^2 , these two neutrino mass ratios can then be used to predict the mixing angle θ_{23} , the CP -violating phase δ , and three neutrino masses $\{m_1, m_2, m_3\}$, which will be soon tested in future precision data from neutrino oscillation experiments [72]. Moreover, the sum of the three absolute neutrino masses will be probed with unprecedented precision by the observations of cosmic microwave background and large-scale structures [73].

2.2 Numerical analysis

Since a rough estimate of m_1/m_2 and m_2/m_3 leads to predictions for the neutrino mixing angles and mass-squared differences only in marginal agreement with neutrino oscillation data, a complete numerical calculation is necessary to demonstrate the validity of the ansätze of lepton mass matrices in Eqs. (4) and (6). Now we outline the strategy to carry out our numerical calculations.

First, the values of m_1/m_2 and m_2/m_3 are randomly chosen from the range $[0, 1]$, as expected for a normal neutrino mass ordering. Then, we extract the three mixing angles $\{\theta_{12}, \theta_{13}, \theta_{23}\}$ directly from the PMNS matrix elements, which are functions of m_1/m_2 and m_2/m_3 . The ratio of two neutrino mass-squared differences

$$\frac{\Delta m_{21}^2}{\Delta m_{31}^2} = \frac{m_2^2}{m_3^2} \left(1 - \frac{m_1^2}{m_2^2} \right) \left(1 - \frac{m_1^2}{m_2^2} \cdot \frac{m_2^2}{m_3^2} \right)^{-1}, \quad (20)$$

can also be calculated. Both three mixing angles $\{\theta_{12}, \theta_{13}, \theta_{23}\}$ and the ratio $\Delta m_{21}^2/\Delta m_{31}^2$ are then required to lie within their 3σ ranges according to the latest global-fit analysis [6]:

$$\begin{aligned}0.271 &\leq \sin^2\theta_{12} \leq 0.345, \\ 0.01934 &\leq \sin^2\theta_{13} \leq 0.02392, \\ 0.385 &\leq \sin^2\theta_{23} \leq 0.635,\end{aligned} \quad (21)$$

for the mixing angles; and

$$\begin{aligned}7.03 \times 10^{-5} \text{ eV}^2 &\leq \Delta m_{21}^2 \leq 8.09 \times 10^{-5} \text{ eV}^2, \\ 2.407 \times 10^{-3} \text{ eV}^2 &\leq \Delta m_{31}^2 \leq 2.643 \times 10^{-3} \text{ eV}^2,\end{aligned} \quad (22)$$

for the neutrino mass-squared differences [6]. In addition, the lightest neutrino mass m_1 can be found by choosing one value of Δm_{21}^2 in its 3σ range, and then the other mass-squared difference Δm_{31}^2 is computed and confronted with Eq. (22). Finally, the viable values of m_1/m_2 and m_2/m_3 satisfying the above requirements are recorded, and the corresponding predictions for the three mixing angles, the CP -violating phase δ , and the neutrino masses can be obtained.

The numerical results are given in Fig. 1, where we show the allowed regions of the two neutrino mass ratios, the three mixing angles, the Dirac CP -violating phase and the absolute neutrino mass. Some comments on the main features are in order.

1) As indicated in Fig. 1(a), only the ranges $0.19 \lesssim m_1/m_2 \lesssim 0.27$ and $0.165 \lesssim m_2/m_3 \lesssim 0.185$ are allowed by current neutrino oscillation data. Hence, the neutrino mass spectrum is moderately hierarchical, namely, $m_1 : m_2 : m_3 \approx 1 : 5 : 30$. In contrast, the mass hierarchy of charged leptons is extremely strong, i.e., $m_e : m_\mu : m_\tau \approx 1 : 200 : 3400$. With the help of Eq. (11), we obtain $r_R/\delta_D^2 \approx m_1/(2m_2) \approx 0.1$ and $\delta_D^2/\varepsilon_D^2 \approx 4m_2/(3m_3) \approx 0.22$, which are in reasonable agreement with the hierarchical limit $r_R \ll \delta_D^2 \ll \varepsilon_D^2$.

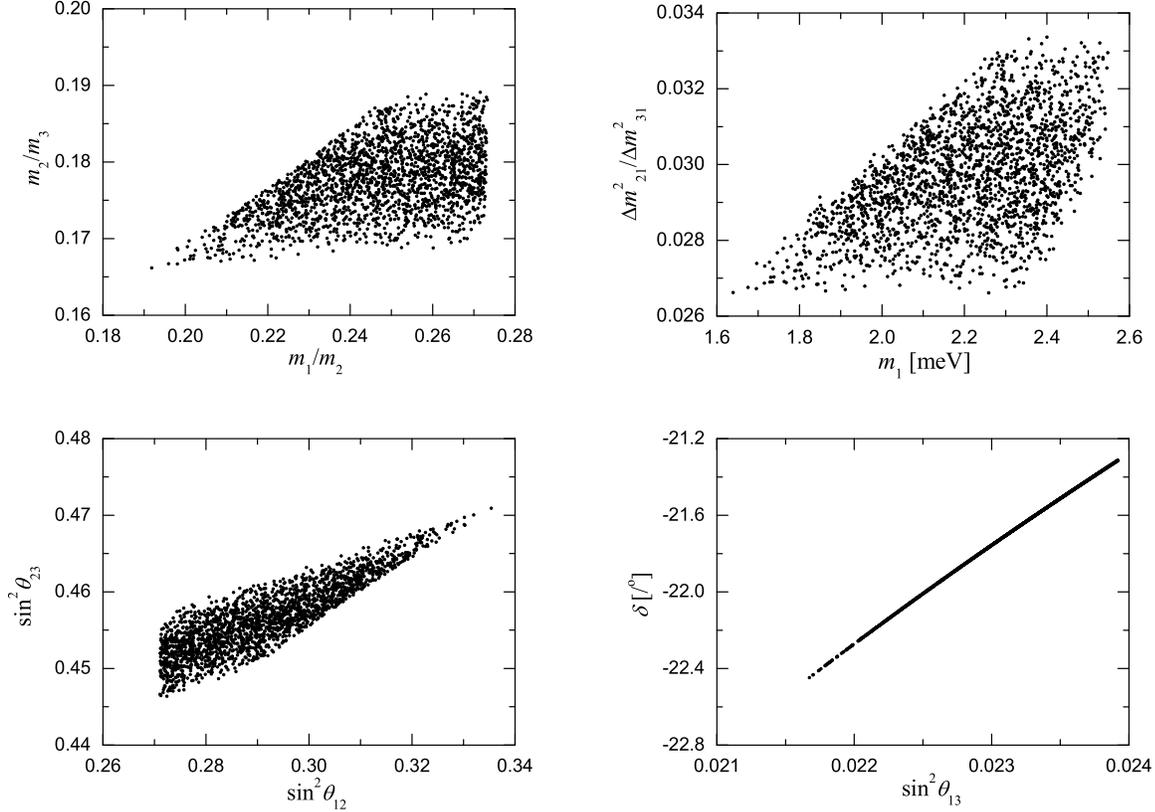


Fig. 1. (a) The allowed regions of neutrino mass ratios ($m_1/m_2, m_2/m_3$), (b) the absolute neutrino mass m_1 versus the ratio of two neutrino mass-squared differences $\Delta m_{21}^2/\Delta m_{31}^2$, and the mixing parameters (c) ($\sin^2 \theta_{12}, \sin^2 \theta_{23}$) and (d) ($\sin^2 \theta_{13}, \delta$), where the 3σ ranges of neutrino mixing parameters in Eqs. (21) and (22) have been used [6].

2) In general, the lightest neutrino mass m_1 is allowed to be zero. However, given the best-fit value of $\Delta m_{21}^2/\Delta m_{31}^2 \approx 0.0298$, we have $m_2/m_3 = (\Delta m_{21}^2/\Delta m_{31}^2)^{1/2} \approx 0.17$ in the limit of $m_1 = 0$, implying $\sin^2 \theta_{13} \approx (1/6) \cdot (m_2/m_3) \approx 0.028$ via Eq. (14). This is obviously in contradiction with the 3σ upper bound on $\sin^2 \theta_{13}$ given in Eq. (21). As a consequence, a nonzero value of m_1 or m_1/m_2 is indispensable to reconcile a larger value of $\Delta m_{21}^2/\Delta m_{31}^2 \approx 0.0298$ and a smaller value of $\sin^2 \theta_{13}$. In Fig. 1(b), the lightest neutrino mass is strictly constrained to $1.6 \text{ meV} \lesssim m_1 \lesssim 2.6 \text{ meV}$, and the other two masses can accordingly be obtained by using the two mass ratios.

3) Fig. 1(c) and (d) give us the allowed regions of $(\sin^2 \theta_{12}, \sin^2 \theta_{23})$ and $(\sin^2 \theta_{13}, \delta)$. The first octant of θ_{23} , i.e., $\theta_{23} < 45^\circ$, is favored, and the maximal mixing angle $\theta_{23} = 45^\circ$ is not reachable. Due to the strong correlation, the allowed regions of three mixing angles are severely constrained: $41.8^\circ \lesssim \theta_{23} \lesssim 43.3^\circ$, $31.4^\circ \lesssim \theta_{12} \lesssim 35.5^\circ$, and $8.45^\circ \lesssim \theta_{13} \lesssim 8.90^\circ$. Compared to the 3σ ranges from current neutrino oscillation data, only a relatively small value of θ_{12} and a relatively large value of θ_{13} can survive. Note that the CP -violating phase $\delta \approx -22^\circ$ does not vary

much, as it is determined by m_2/m_3 and m_1/m_2 , whose values have already been narrowed down to small regions. All these features are ready to be tested in future neutrino oscillation experiments.

Although we have not explored any implications for the effective neutrino mass in tritium beta decays or in neutrinoless double-beta decays, it is straightforward to estimate them by using the PMNS matrix elements in Eq. (13) and the corresponding absolute neutrino masses. A normal mass ordering and the hierarchical mass spectrum imply an effective neutrino mass of a few meV, which seems very difficult to detect in the foreseeable future.

3 Baryon number asymmetry in the universe

In this section, we examine whether the lepton mass matrices given in Eqs. (4) and (6) are also able to explain the baryon number asymmetry in our Universe via thermal leptogenesis [63]. In particular, we focus on the most attractive scenario in which heavy Majorana neutrinos can be thermally produced in the early

Universe, even with a vanishing initial abundance. The central idea of leptogenesis is to first generate lepton number asymmetries from the CP -violating and out-of-equilibrium decays of heavy Majorana neutrinos, which are subsequently converted into baryon number asymmetry via efficient nonperturbative sphaleron processes. See, e.g., Refs. [74–77] for excellent reviews on recent developments in leptogenesis.

3.1 Exactly degenerate masses

To calculate the CP asymmetries from the decays of heavy Majorana neutrinos, we should first diagonalize their mass matrix M_R to derive the mass spectrum. As seen from the previous section, even with the Majorana mass matrix M_R^0 in the limit of exact $S_{3L} \times S_{3R}$ flavor symmetry, one can explain lepton mass spectra, flavor mixing and leptonic CP violation at low energies. Hence, in this subsection, we will not introduce an explicit symmetry-breaking term in the right-handed neutrino sector. In this case, it is evident that $V_0^\dagger M_R V_0^* = \text{Diag}\{M_1, M_2, M_3\}$, where V_0 is the democratic mixing matrix in Eq. (5) and the three mass eigenvalues are given by

$$M_1 = M_2 = c_R r_R / 3, \quad M_3 = c_R (1 + r_R / 3). \quad (23)$$

The mass spectrum is exactly degenerate between M_1 and M_2 , and hierarchical for $r_R \ll 1$, i.e., $M_3 \gg M_2 = M_1$. We shall comment on how to break this mass degeneracy

$$\tilde{M}_D = c_D \begin{pmatrix} -\frac{1}{6}\delta_D & \frac{1}{6\sqrt{3}}\delta_D & \frac{1}{3\sqrt{6}}\delta_D \\ \frac{1}{6\sqrt{3}}\delta_D & \frac{2}{9}\varepsilon_D & -\frac{\sqrt{2}}{9}\varepsilon_D \\ \frac{1}{3\sqrt{6}}\delta_D & -\frac{\sqrt{2}}{9}\varepsilon_D & 1 + \frac{1}{9}\varepsilon_D \end{pmatrix} - i \sqrt{\frac{m_e}{m_\mu}} \begin{pmatrix} \frac{1}{6\sqrt{3}}\delta_D & \frac{2}{9}\varepsilon_D & -\frac{\sqrt{2}}{9}\varepsilon_D \\ -\frac{1}{6}\delta_D & \frac{1}{6\sqrt{3}}\delta_D & \frac{1}{3\sqrt{6}}\delta_D \\ 0 & 0 & 0 \end{pmatrix}, \quad (25)$$

where $m_\mu/m_\tau \ll \delta_D \ll \varepsilon_D \ll 1$ is assumed and only the leading-order terms are retained. As the imaginary parts of the matrix elements in \tilde{M}_D are suppressed by $(m_e/m_\mu)^{1/2} \approx 0.069$, compared to the real parts, small CP asymmetries are generally expected. Taking account of both self-energy and vertex corrections to heavy Majorana neutrino decays, one can get [20, 76]

$$\epsilon_{i\alpha} = \frac{1}{8\pi v^2 \mathcal{H}_{ii}} \sum_{k \neq i} \left\{ \text{Im} \left[(\tilde{M}_D)_{\alpha i}^* (\tilde{M}_D)_{\alpha k} \mathcal{H}_{ik} \right] F(x_{ki}) + \text{Im} \left[(\tilde{M}_D)_{\alpha i}^* (\tilde{M}_D)_{\alpha k} \mathcal{H}_{ik}^* \right] G(x_{ki}) \right\}, \quad (26)$$

with $\mathcal{H} \equiv \tilde{M}_D^\dagger \tilde{M}_D$ and $x_{ki} \equiv M_k^2/M_i^2$. The loop functions are $G(x) \equiv 1/(1-x)$ and

$$F(x) \equiv \sqrt{x} \left[1 + \frac{1}{1-x} + (1+x) \ln \left(\frac{x}{1+x} \right) \right]. \quad (27)$$

in the next subsection.

In thermal leptogenesis, we assume that heavy Majorana neutrinos can be thermally produced at high temperature, and the lepton asymmetries generated from N_3 decays will be completely washed out by the lepton-number-violating processes mediated by two relatively light neutrinos N_1 and N_2 . Therefore, we concentrate only on the CP asymmetries from N_1 and N_2 decays

$$\epsilon_{i\alpha} \equiv \frac{\Gamma(N_i \rightarrow \ell_\alpha H) - \Gamma(N_i \rightarrow \bar{\ell}_\alpha H^\dagger)}{\sum_\alpha [\Gamma(N_i \rightarrow \ell_\alpha H) + \Gamma(N_i \rightarrow \bar{\ell}_\alpha H^\dagger)]}, \quad (24)$$

for $\alpha = e, \mu, \tau$ and $i = 1, 2$, which arise from the interference between the tree and one-loop decay amplitudes. Since the Yukawa interactions of charged leptons are governed by their masses and come into thermal equilibrium at different temperatures, the production and washout of lepton number asymmetries in individual lepton flavors should be taken into account [78]. It has been found that these lepton-flavor effects could significantly modify the final baryon number asymmetry [79, 80]. This is also the reason why we have to compute the CP asymmetries for all distinct lepton flavors in Eq. (24).

In the flavor basis where the charged-lepton mass matrix M_l and the heavy Majorana neutrino mass matrix M_R are diagonal, the Dirac neutrino mass matrix $\tilde{M}_D = V_1^\dagger M_D V_0^*$ can be written as

Some discussion of the CP asymmetries in Eq. (26) is helpful. First, the total CP asymmetries $\epsilon_i = \sum_\alpha \epsilon_{i\alpha} \propto \text{Im}[\mathcal{H}_{ik}^2]$ are vanishing due to the fact that $\mathcal{H} \equiv \tilde{M}_D^\dagger \tilde{M}_D = V_0^\dagger M_D^\dagger M_D V_0$ is a real matrix. Consequently, CP asymmetry exists only for each lepton flavor, and one has to study the generation and evolution of lepton number asymmetry of each lepton flavor. In addition, as shown in Eq. (25), the matrix elements of \tilde{M}_D in the third row are real, so the tau-flavor asymmetries are zero. Second, the exact mass degeneracy requires a more careful treatment of the would-be singularity in $1/(M_2^2 - M_1^2)$ residing in the loop functions. In fact, the contribution to CP asymmetries from the self-energy diagram is vanishing for $M_1 = M_2$. Third, in the limit of $M_3 \gg M_2 = M_1$, the loop function can be simplified as $F(x) \rightarrow -3/(2\sqrt{x})$ and $G(x) \rightarrow -1/x$ for $x \rightarrow +\infty$. With the help of Eqs. (25) and (26), we can obtain

$$\begin{aligned}\epsilon_{1e} &= -\epsilon_{1\mu} \approx -\frac{m_1 M_1}{48\pi v^2} \left[\sqrt{\frac{m_2}{m_1}} (1-2\ln 2) - \frac{3\sqrt{6}}{2} \cdot \sqrt{\frac{M_1}{M_3}} \right] \sqrt{\frac{m_3}{m_1}} \cdot \sqrt{\frac{m_e}{m_\mu}}, \\ \epsilon_{2e} &= -\epsilon_{2\mu} \approx +\frac{m_1 M_1}{48\pi v^2} \left[\sqrt{\frac{m_2}{m_1}} \cdot \sqrt{\frac{m_2}{m_3}} (1-2\ln 2) + \frac{9}{\sqrt{2}} \cdot \sqrt{\frac{M_1}{M_3}} \right] \sqrt{\frac{m_2}{m_1}} \cdot \sqrt{\frac{m_e}{m_\mu}},\end{aligned}\quad (28)$$

and $\epsilon_{1\tau} = \epsilon_{2\tau} = 0$, where the relations in Eq. (11) and (23) have been implemented to express the parameters $\{c_D, \delta_D, \epsilon_D\}$ in terms of mass ratios of light and heavy Majorana neutrinos. Unfortunately, it turns out that the CP asymmetries in Eq. (28) are not sufficient to achieve a successful leptogenesis. The main reasons are summarized below:

1) As we have mentioned before, the total CP asymmetry in N_i decays is vanishing, namely, $\epsilon_i \equiv \epsilon_{ie} + \epsilon_{i\mu} + \epsilon_{i\tau} = 0$ for $i=1,2$. If $M_1 \gtrsim 10^{12}$ GeV, the one-flavor treatment of leptogenesis is valid and only the total CP asymmetry ϵ_i matters. For 10^9 GeV $\lesssim M_1 \lesssim 10^{12}$ GeV, the tau-flavor Yukawa interaction of charged leptons comes into thermal equilibrium, so we have to deal with tau flavor and the other lepton flavors separately. However, the CP asymmetries $\epsilon_i = 0$ in the former case, and $\epsilon_{i\tau} = 0$ and $\epsilon_{ie} + \epsilon_{i\mu} = 0$ in the latter case, indicate that no lepton number asymmetries can be produced.

2) For $M_1 \lesssim 10^9$ GeV, both tau- and muon-flavor Yukawa interactions of charged leptons are in thermal equilibrium, and thus we have to distinguish the production and washout of lepton asymmetries in electron and muon flavors, which will be converted into baryon number asymmetry via sphaleron processes. Since the source of CP violation comes in with a factor of $(m_e/m_\mu)^{1/2}$, both the Dirac CP -violating phase in Eq. (17) and CP asymmetries in Eq. (28) are suppressed. For an order-of-magnitude estimate, we have

$$|\epsilon_{ie}| = |\epsilon_{i\mu}| \approx \frac{m_1 M_1}{48\pi v^2} \sqrt{\frac{m_e}{m_\mu}} \lesssim 3 \times 10^{-11}, \quad (29)$$

where $m_1 \approx 2.2$ meV and $M_1 \lesssim 10^9$ GeV have been used. These CP asymmetries are too small to account for the observed baryon-to-photon ratio $\eta_B \equiv n_B/n_\gamma = (6.09 \pm 0.06) \times 10^{-10}$ at the 95% confidence level [81].

In summary, the flavor structures of lepton mass matrices in Eqs. (4) and (6) cause zero total CP asymmetry, but this is not the case for the CP asymmetries of individual lepton flavors. For the flavor effects to be at work, the lightest heavy neutrino mass is bounded from above, i.e., $M_1 \lesssim 10^9$ GeV, leading to insufficient production of lepton number asymmetries. In the next subsection, we shall go beyond the scenario of the partially degenerate mass spectrum $M_1 = M_2 \ll M_3$, and consider a tiny mass splitting between M_1 and M_2 .

3.2 Nearly degenerate masses

If the $S_{3L} \times S_{3R}$ symmetry is also explicitly broken

down in the right-handed neutrino sector, the mass degeneracy between M_1 and M_2 will be shifted. For illustration, we take the following perturbations for the heavy Majorana neutrino mass matrix:

$$\Delta M_R = \frac{c_R}{3} \begin{pmatrix} -\delta_R & +\delta_R & 0 \\ +\delta_R & -\delta_R & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad (30)$$

where $|\delta_R| \ll 1$ is responsible for the mass splitting between N_1 and N_2 . Even with this nontrivial perturbation matrix, the full mass matrix M_R can be diagonalized in the same way as before, i.e., $V_0^T M_R V_0 = \text{Diag}\{M_1, M_2, M_3\}$, where the mass eigenvalues are given by

$$M_1 = c_R(r_R - 2\delta_R)/3, \quad M_2 = c_R r_R/3, \quad M_3 = c_R(1 + r_R/3). \quad (31)$$

The mass degeneracy parameter, defined by $\Delta \equiv (M_2 - M_1)/M_2 = 2\delta_R/r_R$, can be very small, since it is just the ratio of the symmetry-breaking to symmetry-preserving terms. Another advantage of the perturbations in Eq. (30) can be immediately recognized: the mass spectrum of three light neutrinos and lepton flavor mixing are not affected at all. Hence, all the previous conclusions on low-energy phenomena are still valid.

As pointed out in Refs. [64, 65], the CP asymmetries arising from the self-energy corrections involving N_1 and N_2 will be greatly enhanced when their masses are nearly degenerate. For this reason, we neglect the minor contributions from the vertex correction, and those from the heaviest neutrino N_3 as well, and arrive at [82, 83]

$$\begin{aligned}\epsilon_{i\alpha} &= \frac{1}{8\pi v^2 \mathcal{H}_{ii}} \sum_{k \neq i} \left\{ \text{Im} \left[(\tilde{M}_D)_{\alpha i}^* (\tilde{M}_D)_{\alpha k} \mathcal{H}_{ik} \right] \sqrt{x_{ki}} \right. \\ &\quad \left. + \text{Im} \left[(\tilde{M}_D)_{\alpha i}^* (\tilde{M}_D)_{\alpha k} \mathcal{H}_{ik}^* \right] \right\} G'(x_{ki}),\end{aligned}\quad (32)$$

where $x_{ki} \equiv M_k^2/M_i^2$ has been defined and the regularized loop function reads

$$G'(x_{ki}) = \frac{1 - x_{ki}}{(1 - x_{ki})^2 + r_{ki}^2}. \quad (33)$$

Note that the regulator $r_{ki} \equiv \Gamma_k/M_i$, with $\Gamma_k \equiv \mathcal{H}_{kk} M_k / (8\pi v^2)$ being the total decay width of N_k at the tree level, guarantees the correct behavior of the CP asymmetries in the limit of exact mass degeneracy, i.e., $\epsilon_{i\alpha} \rightarrow 0$ for $M_1 \rightarrow M_2$. With the help of Eq. (25), it is

straightforward to derive

$$\begin{aligned}\epsilon_{1e} &= -\epsilon_{1\mu} = +\frac{1}{3} \cdot \sqrt{\frac{m_2}{m_3}} \cdot \sqrt{\frac{m_e}{m_\mu}} \cdot \frac{\tilde{r}_{21}\Delta}{\Delta^2 + \tilde{r}_{21}^2}, \\ \epsilon_{2e} &= -\epsilon_{2\mu} = +\frac{1}{3} \cdot \sqrt{\frac{m_2}{m_3}} \cdot \sqrt{\frac{m_e}{m_\mu}} \cdot \frac{\tilde{r}_{12}\Delta}{\Delta^2 + \tilde{r}_{12}^2},\end{aligned}\quad (34)$$

where $\tilde{r}_{21} \equiv r_{21}/2 \approx M_1 m_3 / (16\pi v^2)$ and $\tilde{r}_{12} \equiv r_{12}/2 \approx M_1 m_2 / (16\pi v^2)$. Similar to the Dirac CP -violating phase δ , the CP asymmetries are suppressed by the small parameter $(m_e/m_\mu)^{1/2}$. However, they can be resonantly enhanced and the maximum of $\tilde{r}_{21}|\Delta|/(\Delta^2 + \tilde{r}_{21}^2)$ or $\tilde{r}_{12}|\Delta|/(\Delta^2 + \tilde{r}_{12}^2)$ could reach 1/2, when $|\Delta| \approx \tilde{r}_{21}$ or \tilde{r}_{12} is satisfied. Note also that the sign of Δ will be fixed by the observed baryon number asymmetry.

In the strong washout regime, where heavy Majorana neutrinos can be thermally produced, the final baryon number asymmetry is independent of the initial conditions. We assume that the initial abundance of heavy Majorana neutrinos follows the thermal distribution at temperatures much higher than their masses, i.e., $T \gtrsim M_1 \approx M_2$. The lepton number asymmetries generated in heavy neutrino decays will not be completely destroyed by the lepton-number-violating inverse decays and scattering processes. In this case, the final baryon number asymmetry can be approximately computed by [74, 75]

$$\eta_B \approx -0.96 \times 10^{-2} \sum_i \sum_\alpha \epsilon_{i\alpha} \kappa_{i\alpha}, \quad (35)$$

where the efficiency factors $\kappa_{i\alpha}$ measure how efficiently the lepton number asymmetries will be destroyed. To accurately calculate the efficiency factors, one should solve a full set of Boltzmann equations for the evolution of lepton number asymmetries in different flavors. However, we instead follow an approximate and analytical approach by introducing the decay parameter $K_i \equiv \tilde{m}_i/m_*$, where the effective neutrino mass is defined as $\tilde{m}_i \equiv \mathcal{H}_{ii}/M_i$ and the equilibrium neutrino mass is $m_* \approx 1.08 \times 10^{-3}$ eV. If the individual decay parameter $K_{i\alpha} \equiv K_i |(\tilde{M}_D)_{\alpha i}|^2 / \mathcal{H}_{ii}$ happens to be in the range $5 \lesssim K_{i\alpha} \lesssim 100$, $\kappa_{i\alpha} \approx 0.5/K_{i\alpha}^{1.2}$ [84, 85] is a good approximation. In the limit of degenerate masses, i.e., $M_1 \approx M_2$, one has to replace $K_{i\alpha}$ by the sum $K_{1\alpha} + K_{2\alpha}$ in calculating the individual efficiency factor $\kappa_{i\alpha}$. In our case, it is easy to derive $\tilde{m}_1 \approx m_2$ and $\tilde{m}_2 \approx m_3$, and $K_{1e}/K_1 \approx 1/2$, $K_{1\mu}/K_1 \approx 1/6$, and $K_{2e}/K_2 \approx m_2/(6m_3)$, $K_{2\mu}/K_2 \approx 2/3$. For illustration, we take $m_2 = 8.8$ meV and $m_3 = 52.7$ meV as in Eq. (26), and then obtain

$$K_1 \approx \frac{m_2}{m_*} \approx 8.15, \quad K_{1e} \approx 4.07, \quad K_{1\mu} \approx 1.36, \quad (36)$$

and

$$K_2 \approx \frac{m_3}{m_*} \approx 48.8, \quad K_{2e} \approx 1.36, \quad K_{2\mu} \approx 32.5, \quad (37)$$

implying $\kappa_{1e} = \kappa_{2e} \approx 0.5/(K_{1e} + K_{2e})^{1.2} \approx 6.6 \times 10^{-2}$ and $\kappa_{1\mu} = \kappa_{2\mu} \approx 0.5/(K_{1\mu} + K_{2\mu})^{1.2} \approx 7.3 \times 10^{-3}$. Putting it all together, we finally get

$$\begin{aligned}\eta_B &\approx -5.4 \times 10^{-6} \cdot \frac{\tilde{r}_{21}\Delta}{\Delta^2 + \tilde{r}_{21}^2} \left(1 + 0.167 \frac{\Delta^2 + \tilde{r}_{21}^2}{\Delta^2 + \tilde{r}_{12}^2} \right) \\ &= \begin{cases} 6.3 \times 10^{-6} \frac{\tilde{r}_{21}}{|\Delta|}, & |\Delta| \gg \tilde{r}_{21} \\ 3.8 \times 10^{-5} \frac{|\Delta|}{\tilde{r}_{21}}, & |\Delta| \ll \tilde{r}_{21} \end{cases},\end{aligned}\quad (38)$$

where Δ should be negative in order to account for the observed positive η_B , and the parameter

$$\tilde{r}_{21} \approx 3.5 \times 10^{-14} \left(\frac{M_1}{10^3 \text{ GeV}} \right), \quad (39)$$

depends crucially on the heavy Majorana neutrino mass M_1 . For $M_1 = 1$ TeV, the best-fit value $\eta_B = 6.09 \times 10^{-10}$ requires the mass degeneracy parameter to be $\Delta = -3.6 \times 10^{-10}$ or -5.6×10^{-19} . As $\Delta = 2\delta_R/r_R$ arises from the soft breaking of $S_{3L} \times S_{3R}$ flavor symmetry, it is naturally expected to be small. If $M_1 \approx 10^9$ GeV is assumed, we shall obtain $\Delta = -3.6 \times 10^{-4}$ or -5.6×10^{-13} . Therefore, we have demonstrated that the baryon number asymmetry can also be explained in our scenario by implementing the mechanism of resonant leptogenesis, including lepton flavor effects. It is worth stressing that the CP -violating phase $\delta \approx -22^\circ$ leads to a wrong sign of baryon number asymmetry, which however can be corrected by the degeneracy parameter Δ .

4 Summary

In this paper, we have examined a simple but viable scenario to explicitly break the $S_{3L} \times S_{3R}$ flavor symmetry in the canonical seesaw model. In the symmetry limit, both charged-lepton mass matrix M_l and Dirac neutrino mass matrix M_D take on the democratic form, while the heavy Majorana neutrino mass matrix M_R consists of a democratic part and another part proportional to the identity matrix. After introducing the diagonal perturbation matrices ΔM_l and ΔM_D , we have explored their implications for lepton mass spectra, flavor mixing angles and CP -violating phases at low energies, and calculated the baryon number asymmetry in our Universe via the mechanism of thermal leptogenesis.

At the low-energy scale, the effective neutrino mass matrix is given by the famous seesaw formula $M_\nu = M_D M_R^{-1} M_D^T$. The leptonic flavor mixing matrix V , arising from the diagonalizations of M_l and M_ν , is completely determined by the mass ratios of charged leptons, i.e., m_e/m_μ and m_μ/m_τ , and those of neutrinos, i.e., m_1/m_2 and m_2/m_3 . The 3σ ranges of the three neutrino mixing angles and two mass-squared differences from the latest global analysis of neutrino oscillation data have

been used to constrain the parameter space of the neutrino mass ratios. It turns out that our scenario is consistent with current neutrino oscillation data, and the neutrino mass ratios are found to be $0.19 \lesssim m_1/m_2 \lesssim 0.27$ and $0.165 \lesssim m_2/m_3 \lesssim 0.185$. Consequently, a hierarchical pattern of the neutrino mass spectrum (e.g., $m_1 \approx 2.2$ meV, $m_2 \approx 8.8$ meV and $m_3 \approx 52.7$ meV) and a normal mass ordering are favored. The allowed regions of mixing angles are $41.8^\circ \lesssim \theta_{23} \lesssim 43.3^\circ$, $31.4^\circ \lesssim \theta_{12} \lesssim 35.5^\circ$, and $8.45^\circ \lesssim \theta_{13} \lesssim 8.90^\circ$, together with the Dirac CP -violating phase $\delta \approx -22^\circ$, are ready to be tested in future neutrino oscillation experiments.

If the $S_{3L} \times S_{3R}$ symmetry is preserved in the right-handed neutrino sector, the two heavy neutrinos N_1 and N_2 are exactly degenerate in mass. As a consequence of the flavor structures of the lepton mass matrices, the CP asymmetries in the heavy neutrino decays are found to be $\epsilon_{i\tau} = 0$ and $\epsilon_{ie} + \epsilon_{i\mu} = 0$ for $i = 1, 2$. An upper bound $M_1 = M_2 \lesssim 10^9$ GeV should be met for the flavored leptogenesis to work efficiently. However, we find that the CP asymmetries for $M_1 = M_2 \lesssim 10^9$ GeV are insufficient to explain the observed baryon number asymmetry. If the flavor symmetry is explicitly broken

for heavy Majorana neutrinos as well, the CP asymmetries from the mixing between N_1 and N_2 , which are nearly degenerate in mass, will be resonantly enhanced. In this case, even for $M_1 \approx M_2 \approx 1$ TeV, we can successfully explain the observed baryon-to-photon number ratio $\eta_B \approx 6.09 \times 10^{-10}$ by setting a tiny mass degeneracy $\Delta \equiv (M_2 - M_1)/M_2 \approx -3.6 \times 10^{-10}$ or -5.6×10^{-19} . As such a mass splitting comes from the flavor symmetry breaking, it is naturally expected to be small.

Notice that the Dirac CP -violating phase, stemming from the perturbation matrix ΔM_1 , is also responsible for the CP asymmetries in heavy Majorana neutrino decays, which are indispensable for explaining cosmological matter-antimatter asymmetry. Thus, the generation of electron mass, the Dirac CP -violating phase and the baryon number asymmetry are closely connected in this simple scenario. Certainly, further efforts should be devoted to constructing a full model for lepton mass spectra and flavor mixing based on the $S_{3L} \times S_{3R}$ flavor symmetry. The phenomenological studies in this paper are helpful in looking for a viable way of flavor symmetry breaking, and instructive for understanding lepton mass spectra, flavor mixing pattern and CP violation.

Appendix A

In this appendix, we show explicitly how to derive the lepton mass matrices in Eq. (4) in the limit of an exact $S_{3L} \times S_{3R}$ symmetry. First of all, let us summarize the main properties of the symmetry group S_3 of the permutations of three objects. The order of S_3 is equal to $3! = 6$, and the six elements correspond to the following transformations

$$\begin{aligned}
 e: & (x_1, x_2, x_3) \rightarrow (x_1, x_2, x_3), \\
 a_1: & (x_1, x_2, x_3) \rightarrow (x_2, x_1, x_3), \\
 a_2: & (x_1, x_2, x_3) \rightarrow (x_1, x_3, x_2), \\
 a_3: & (x_1, x_2, x_3) \rightarrow (x_3, x_2, x_1), \\
 a_4: & (x_1, x_2, x_3) \rightarrow (x_3, x_1, x_2), \\
 a_5: & (x_1, x_2, x_3) \rightarrow (x_2, x_3, x_1),
 \end{aligned} \tag{A1}$$

which can also be represented by six matrices

$$\begin{aligned}
 P(e): & \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, & P(a_1): & \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, & P(a_2): & \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \\
 P(a_3): & \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}, & P(a_4): & \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, & P(a_5): & \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix},
 \end{aligned} \tag{A2}$$

acting on a reducible triplet $x \equiv (x_1, x_2, x_3)^T$. These six group elements can be categorized into three conjugacy classes

$C_1 = \{e\}$, $C_2 = \{a_1 a_2, a_2 a_1\}$, and $C_3 = \{a_1, a_2, a_2 a_1 a_2\}$, where $a_1 a_2 = a_4$, $a_2 a_1 = a_5$, and $a_1 a_2 a_1 = a_2 a_1 a_2 = a_3$ can be easily identified. The irreducible representations of S_3 contain two singlets $\mathbf{1}$ and $\mathbf{1}'$, and one doublet $\mathbf{2}$. The representations of the singlets $\mathbf{1}$ and $\mathbf{1}'$ are just given by their characters, while those of the doublet $\mathbf{2}$ are found to be

$$\begin{aligned}
 D(e): & \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, & D(a_1): & \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, & D(a_2): & \begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}, \\
 D(a_3): & \begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}, \\
 D(a_4): & \begin{pmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix}, \\
 D(a_5): & \begin{pmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix}.
 \end{aligned} \tag{A3}$$

For a reducible triplet $x = (x_1, x_2, x_3)^T$, it is always possible to find a unitary matrix U which can be used to diagonalize $P(a_1)$ and leads to a block-diagonal form of $P(a_i)$ (for $i = 2, \dots, 5$) in the transformed basis $x' = U^\dagger x$. One can verify

that the new representation matrices $P'(a_i)=U^\dagger P(a_i)U$ turn out to be

$$P'(a_i): \begin{pmatrix} D(a_i) & 0 \\ 0 & 1 \end{pmatrix}, \quad (\text{A4})$$

for $i=0,1,\dots,5$ and $a_0 \equiv e$, and that the unitary matrix U is just the democratic mixing matrix V_0 in Eq. (5). Now that

$$\begin{pmatrix} x'_1 \\ x'_2 \\ x'_3 \end{pmatrix} = V_0^\dagger \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \frac{1}{\sqrt{6}} \begin{pmatrix} \sqrt{3}(x_1-x_2) \\ x_1+x_2-2x_3 \\ \sqrt{2}(x_1+x_2+x_3) \end{pmatrix}, \quad (\text{A5})$$

we can identify the singlet and doublet states

$$S = \frac{1}{\sqrt{3}}(x_1+x_2+x_3), \quad M = \begin{pmatrix} \frac{1}{\sqrt{2}}(x_1-x_2) \\ \frac{1}{\sqrt{6}}(x_1+x_2-2x_3) \end{pmatrix}. \quad (\text{A6})$$

The direct product of two doublets $M_1 = (b_1, c_1)^T$ and $M_2 = (b_2, c_2)^T$ can be decomposed into three irreducible representations

$$\begin{pmatrix} b_1 \\ c_1 \end{pmatrix}_2 \otimes \begin{pmatrix} b_2 \\ c_2 \end{pmatrix}_2 = (b_1 b_2 + c_1 c_2)_{1'} + (b_1 c_2 - b_2 c_1)_{1'} + \begin{pmatrix} b_1 c_2 + b_2 c_1 \\ b_1 b_2 - c_1 c_2 \end{pmatrix}_2. \quad (\text{A7})$$

For more details about the S_3 symmetry group, refer to recent reviews on discrete flavor symmetries and their applications in particle physics [86, 87].

Second, we explain the assignments of fermion fields and write down the Lagrangian for lepton masses under the $S_{3L} \times S_{3R}$ symmetry, following the idea of Ref. [21]. All the left-handed (right-handed) fermion fields are assigned as a reducible three-dimensional representation of S_{3L} (S_{3R}), namely, $(\ell_{eL}, \ell_{\mu L}, \ell_{\tau L})^T$, $(e_R, \mu_R, \tau_R)^T$ and $(N_{eR}, N_{\mu R}, N_{\tau R})^T$, and as a trivial representation of S_{3R} (S_{3L}). The corresponding representations of S_{3L} or S_{3R} group elements are given in Eq. (41). To construct an $S_{3L} \times S_{3R}$ -invariant Lagrangian for lepton masses, we are only allowed to use the left-handed singlet $S_{\ell_L} \equiv (\ell_{eL} + \ell_{\mu L} + \ell_{\tau L})/\sqrt{3}$ and right-handed singlets $S_{E_R} \equiv (e_R + \mu_R + \tau_R)/\sqrt{3}$ for the charged-lepton Yukawa interaction, and S_{ℓ_L} and $S_{N_R} \equiv (N_{eR} + N_{\mu R} + N_{\tau R})/\sqrt{3}$ for the Dirac neutrino Yukawa interaction. However, for the Majorana mass term of right-handed neutrinos, the doublet $M_{N_R} = ((N_{eR} - N_{\mu R})/\sqrt{2}, (N_{eR} + N_{\mu R} - 2N_{\tau R})/\sqrt{6})^T$ is also available. Hence, the invariant Lagrangian with respect to an $S_{3L} \times S_{3R}$ symmetry is given by

$$-\mathcal{L}_\ell = y_l \overline{S_{\ell_L}} H S_{E_R} + y_\nu \overline{S_{\ell_L}} \tilde{H} S_{N_R} + \frac{1}{2} \left[\alpha_R \overline{S_{N_R}^C} S_{N_R} + \beta_R \overline{(M_{N_R}^C)}_1 M_{N_R} \right] + \text{h.c.} \quad (\text{A8})$$

Comparing Eq. (47) and Eq. (2), we can obtain Eq. (4) by identifying $c_1 = y_l v$, $c_D = y_\nu v$, $c_R = \alpha_R - \beta_R$ and $r_R = 3\beta_R/(\alpha_R - \beta_R)$.

References

- 1 T. Kajita, Rev. Mod. Phys., **88**(3): 030501 (2016)
- 2 A. B. McDonald, Rev. Mod. Phys., **88**(3): 030502 (2016)
- 3 B. Pontecorvo, Sov. Phys. JETP, **6**: 429 (1957); Zh. Eksp. Teor. Fiz., **33**: 549 (1957)
- 4 Z. Maki, M. Nakagawa, and S. Sakata, Prog. Theor. Phys., **28**: 870 (1962)
- 5 C. Patrignani et al (Particle Data Group), Chin. Phys. C, **40**(10): 100001 (2016)
- 6 I. Esteban, M. C. Gonzalez-Garcia, M. Maltoni, I. Martinez-Soler, and T. Schwetz, JHEP, **1701**: 087 (2017), arXiv:1611.01514; The latest global-fit results can be found on the webpage of NuFIT 3.0 (2016) at <http://www.nu-fit.org>
- 7 F. Capozzi, E. Lisi, A. Marrone, D. Montanino, and A. Palazzo, Nucl. Phys. B, **908**: 218 (2016), arXiv:1601.07777
- 8 F. Capozzi, E. Di Valentino, E. Lisi, A. Marrone, A. Melchiorri, and A. Palazzo, Phys. Rev. D, **95**(9): 096014 (2017), arXiv:1703.04471
- 9 D. V. Forero, M. Tortola, and J. W. F. Valle, Phys. Rev. D, **90**(9): 093006 (2014), arXiv:1405.7540
- 10 F. An et al (JUNO Collaboration), J. Phys. G **43**(3): 030401 (2016), arXiv:1507.05613
- 11 S. B. Kim, Nuovo Cim. C, **39**(4): 317 (2017)
- 12 K. Abe et al (T2K Collaboration), Nucl. Instrum. Meth. A, **659**: 106 (2011), arXiv:1106.1238
- 13 D. S. Ayres et al (NOvA Collaboration), *NOvA: Proposal to Build a 30 Kiloton Off-Axis Detector to Study $\nu_\mu \rightarrow \nu_e$ Oscillations in the NuMI Beamline*, hep-ex/0503053
- 14 R. Acciarri et al (DUNE Collaboration), *Long-Baseline Neutrino Facility (LBNF) and Deep Underground Neutrino Experiment (DUNE) : Volume 2: The Physics Program for DUNE at LBNF*, arXiv:1512.06148
- 15 P. Minkowski, Phys. Lett. B, **67**: 421 (1977)
- 16 T. Yanagida, In *Proceedings of the Workshop on Unified Theory and the Baryon Number of the Universe*: edited by O. Sawada and A. Sugamoto, (KEK, Tsukuba, 1979), p.95
- 17 M. Gell-Mann, P. Ramond, and R. Slansky, In *Supergravity*: edited by P. van Nieuwenhuizen and D. Z. Freeman, (North-Holland, Amsterdam, 1979), p. 315
- 18 S. L. Glashow, In *Quarks and Leptons*: edited by M. Levy et al, (Plenum, New York, 1980), p. 707
- 19 R. N. Mohapatra, and G. Senjanovic, Phys. Rev. Lett., **44**: 912 (1980)
- 20 Z. Z. Xing and S. Zhou, *Neutrinos in particle physics, astronomy and cosmology*, (Springer-Verlag, Berlin Heidelberg, 2011)
- 21 H. Harari, H. Haut, and J. Weyers, Phys. Lett. B, **78**: 459 (1978)
- 22 C. D. Froggatt and H. B. Nielsen, Nucl. Phys. B, **147**: 277 (1979)
- 23 Y. Koide, Z. Phys. C, **45**: 39 (1989)
- 24 M. Tanimoto, Phys. Rev. D, **41**: 1586 (1990)
- 25 P. Kaus and S. Meshkov, Phys. Rev. D, **42**: 1863 (1990)
- 26 G. C. Branco, J. I. Silva-Marcos, and M. N. Rebelo, Phys. Lett. B, **237**: 446 (1990)
- 27 H. Fritzsch and J. Plankl, Phys. Lett. B, **237**: 451 (1990)
- 28 H. Fritzsch and D. Holtmannspotter, Phys. Lett. B, **338**: 290 (1994), hep-ph/9406241
- 29 G. C. Branco and J. I. Silva-Marcos, Phys. Lett. B, **359**: 166 (1995), hep-ph/9507299
- 30 H. Fritzsch and Z. Z. Xing, Phys. Lett. B, **372**: 265 (1996), hep-ph/9509389

- 31 Z. Z. Xing, *J. Phys. G*, **23**: 1563 (1997), hep-ph/9609204
- 32 M. Fukugita, M. Tanimoto, and T. Yanagida, *Phys. Rev. D*, **57**: 4429 (1998), hep-ph/9709388
- 33 A. Mondragon and E. Rodriguez-Jauregui, *Phys. Rev. D*, **59**: 093009 (1999), hep-ph/9807214
- 34 H. Fritzsch and Z. Z. Xing, *Prog. Part. Nucl. Phys.*, **45**: 1 (2000), hep-ph/9912358
- 35 N. Haba, Y. Matsui, N. Okamura, and T. Suzuki, *Phys. Lett. B*, **489**: 184 (2000), hep-ph/0005064
- 36 G. C. Branco and J. I. Silva-Marcos, *Phys. Lett. B*, **526**: 104 (2002), hep-ph/0106125
- 37 M. Fujii, K. Hamaguchi, and T. Yanagida, *Phys. Rev. D*, **65**: 115012 (2002), hep-ph/0202210
- 38 P. F. Harrison and W. G. Scott, *Phys. Lett. B*, **557**: 76 (2003), hep-ph/0302025
- 39 J. Kubo, A. Mondragon, M. Mondragon, and E. Rodriguez-Jauregui, *Prog. Theor. Phys.*, **109**: 795 (2003); *Prog. Theor. Phys.*, **114**: 287 (2005), hep-ph/0302196
- 40 S. L. Chen, M. Frigerio, and E. Ma, *Phys. Rev. D*, **70**: 073008 (2004); *Phys. Rev. D* **70**: 079905 (2004), hep-ph/0404084
- 41 H. Fritzsch and Z. Z. Xing, *Phys. Lett. B*, **598**: 237 (2004), hep-ph/0406206
- 42 W. Rodejohann and Z. Z. Xing, *Phys. Lett. B*, **601**: 176 (2004), hep-ph/0408195
- 43 T. Araki, J. Kubo, and E. A. Paschos, *Eur. Phys. J. C*, **45**: 465 (2006), hep-ph/0502164
- 44 T. Teshima, *Phys. Rev. D*, **73**: 045019 (2006), hep-ph/0509094
- 45 T. Kimura, *Prog. Theor. Phys.*, **114**: 329 (2005)
- 46 R. N. Mohapatra, S. Nasri, and H. B. Yu, *Phys. Lett. B*, **639**: 318 (2006), hep-ph/0605020
- 47 Y. Koide, *Eur. Phys. J. C* **50**: 809 (2007), hep-ph/0612058
- 48 A. Mondragon, M. Mondragon, and E. Peinado, *Phys. Rev. D*, **76**: 076003 (2007), arXiv:0706.0354
- 49 Z. Z. Xing, D. Yang, and S. Zhou, *Phys. Lett. B*, **690**: 304 (2010), arXiv:1004.4234
- 50 T. Teshima and Y. Okumura, *Phys. Rev. D*, **84**: 016003 (2011), arXiv:1103.6127
- 51 S. Zhou, *Phys. Lett. B*, **704**: 291 (2011), arXiv:1106.4808
- 52 P. V. Dong, H. N. Long, C. H. Nam, and V. V. Vien, *Phys. Rev. D*, **85**: 053001 (2012), arXiv:1111.6360
- 53 S. Dev, R. R. Gautam, and L. Singh, *Phys. Lett. B*, **708**: 284 (2012), arXiv:1201.3755
- 54 F. Gonzalez Canales, A. Mondragon, and M. Mondragon, *Fortsch. Phys.*, **61**: 546 (2013), arXiv:1205.4755
- 55 R. Jora, J. Schechter, and M. N. Shahid, *Int. J. Mod. Phys. A*, **28**: 1350028 (2013), arXiv:1210.6755
- 56 F. González Canales, A. Mondragón, M. Mondragón, U. J. Salda na Salazar, and L. Velasco-Sevilla, *Phys. Rev. D*, **88**: 096004 (2013), arXiv:1304.6644
- 57 C. Arbeláez, A. E. Cárcamo Hernández, S. Kovalenko, and I. Schmidt, arXiv:1602.03607
- 58 M. J. S. Yang, *Phys. Lett. B*, **760**: 747 (2016), arXiv:1604.07896
- 59 J. Xu, X. Yang, and D. Yang, *Mod. Phys. Lett. A*, **31**(24): 1650131 (2016)
- 60 S. Pramanick and A. Raychaudhuri, *Phys. Rev. D*, **94**(11): 115028 (2016), arXiv:1609.06103
- 61 E. Barradas-Guevara, O. Félix-Beltrán, F. Gonzalez-Canales, and M. Zeleny-Mora, arXiv:1704.03474
- 62 H. Fritzsch, Z. Z. Xing, and D. Zhang, arXiv:1705.01391
- 63 M. Fukugita and T. Yanagida, *Phys. Lett. B*, **174**: 45 (1986)
- 64 A. Pilaftsis, *Phys. Rev. D*, **56**: 5431 (1997), hep-ph/9707235
- 65 A. Pilaftsis and T. E. J. Underwood, *Nucl. Phys. B*, **692**: 303 (2004), hep-ph/0309342
- 66 Z. Z. Xing, H. Zhang and S. Zhou, *Phys. Rev. D*, **77**: 113016 (2008), arXiv:0712.1419
- 67 Z. Z. Xing, H. Zhang, and S. Zhou, *Phys. Rev. D*, **86**: 013013 (2012), arXiv:1112.3112
- 68 K. Abe et al (Hyper-Kamiokande Working Group), arXiv:1412.4673
- 69 E. Wildner et al, *Adv. High Energy Phys.*, **2016**: 8640493 (2016), arXiv:1510.00493
- 70 J. Cao et al, *Phys. Rev. ST Accel. Beams*, **17**: 090101 (2014), arXiv:1401.8125
- 71 M. Blennow, P. Coloma, and E. Fernández-Martínez, *JHEP*, **1603**: 197 (2016), arXiv:1511.02859
- 72 Y. Wang and Z. Z. Xing, *Adv. Ser. Direct. High Energy Phys.*, **26**: 371 (2016), arXiv:1504.06155
- 73 K. N. Abazajian et al (CMB-S4 Collaboration), *CMB-S4 Science Book*, First Edition, arXiv:1610.02743
- 74 W. Buchmuller, P. Di Bari, and M. Plumacher, *Annals Phys.*, **315**: 305 (2005), hep-ph/0401240
- 75 W. Buchmuller, R. D. Peccei, and T. Yanagida, *Ann. Rev. Nucl. Part. Sci.*, **55**: 311 (2005), hep-ph/0502169
- 76 S. Davidson, E. Nardi, and Y. Nir, *Phys. Rept.*, **466**: 105 (2008), arXiv:0802.2962
- 77 T. Hambye, *New J. Phys.*, **14**: 125014 (2012), arXiv:1212.2888
- 78 R. Barbieri, P. Creminelli, A. Strumia, and N. Tetradis, *Nucl. Phys. B*, **575**: 61 (2000), hep-ph/9911315
- 79 A. Abada, S. Davidson, F. X. Josse-Michaux, M. Losada, and A. Riotto, *JCAP*, **0604**: 004 (2006), hep-ph/0601083
- 80 E. Nardi, Y. Nir, E. Roulet, and J. Racker, *JHEP*, **0601**: 164 (2006), hep-ph/0601084
- 81 P. A. R. Ade et al (Planck Collaboration), *Astron. Astrophys.*, **594**: A13 (2016), arXiv:1502.01589
- 82 Z. Z. Xing and S. Zhou, *Phys. Lett. B*, **653**: 278 (2007), hep-ph/0607302
- 83 J. Zhang and S. Zhou, *JHEP*, **1509**: 065 (2015), arXiv:1505.04858
- 84 S. Blanchet and P. Di Bari, *JCAP*, **0606**: 023 (2006), hep-ph/0603107
- 85 S. Blanchet and P. Di Bari, *JCAP*, **0703**: 018 (2007), hep-ph/0607330
- 86 H. Ishimori, T. Kobayashi, H. Ohki, Y. Shimizu, H. Okada, and M. Tanimoto, *Prog. Theor. Phys. Suppl.*, **183**: 1 (2010), arXiv:1003.3552
- 87 W. Grimus and P. O. Ludl, *J. Phys. A*, **45**: 233001 (2012), arXiv:1110.6376