On the decays of $d^*(2380)$ in a constituent chiral quark model *

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Abstract: After summarizing the experimental results and present status of the $d^*(2380)$ observed at WASA@COSY, two “extreme” models for explaining its structure, a compact hexaquark dominated model and a loose $\Delta\Delta'-\mathcal{D}_{12}\pi$ model, are briefly discussed, especially the former. By comparing their results with the corresponding data, the differences of the two models are addressed. As a remedy for the latter model, a mixing model and its result are also quoted for a comparison. It is shown that the compact hexaquark dominated structure might be more promising. However, the mixing model is also a possible structure, and more accurate $\Gamma_{d^*\to NN\pi}$ data are needed for confirmation.

Keywords: $d^*(2380)$, chiral quark model, strong pionic decays

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1 Introduction

In the last ten years, exotic hadronic states have attracted much attention among physicists. Many exotic new resonances, the so-called XYZ resonances, like $X(3872)$, $Z_c(3900)$, $Y(4260)$, $Z_b(10610)$, $P_c(4380)$, etc., have been observed. Most of their masses are very close to the relevant thresholds of two mesons (or one baryon and one meson), and their widths are very narrow compared to the ordinary mesons (or baryons), so usually one can neither regard them as conventional $q\bar{q}$ (or qqq) systems, nor pin down their exact structures from the interpretations of the hadronic molecular states, tetraquark (or pentaquark) states, or the cusp and triangle-singularity effects [1–5].

Besides those exotic states in the meson-like and baryon-like sectors, the dibaryon systems, like the H particles, d^*, and d' have been known for many years as interesting multi-quark (six-quark) systems with a baryon number of 2 (see for example Ref. [6] for a review). In 2009, the CELSIUS/WASA Collaboration [7] reported that the data on the ABC effect in the $p+n\to d\pi^0\pi^0$ reaction cannot be simply explained by either the intermediate Roper excitation contribution or by the t-channel $\Delta\Delta$ process. An $s-$channel resonance, lately called d^*, has to be imposed. In order to confirm the existence of such a resonance, the WASA@COSY Collaboration [8, 9] checked all the possible double pion fusion reaction channels, including 3-body and 4-body $\pi\pi$ channels, like $d^*\to d\pi^0\pi^0$, $d\pi^+\pi^-$, $pn\pi^+\pi^-$, $nn\pi^+\pi^0$, $pp\pi^0\pi^-$, etc. [9, 10], and found the trail of the resonance, with a mass of $m=2370$ MeV, width of 70 MeV, and quantum numbers of $I(J^P)=0(3^+)$, in all the channels. They further measured the np analyzing power A_y, incorporated the data into the SAID analysis, and produced a pole of $(2380\pm10)-i(40\pm5)$ MeV in the ${}^3D_3-{}^3G_3$ wave [11]. Then, the report of the discovery of $d^*(2380)$, whose quantum numbers, mass, and width are $I(J^P)=0(3^+)$, $M\approx2370$ MeV, and $\Gamma\approx70$ MeV (see also their recent paper [10], the averaged mass and width are $M\approx2375$ MeV and $\Gamma\approx75$ MeV, respectively), was released [12]. The most important characteristics of this state are that its mass is about 80 MeV below the $\Delta\Delta$ threshold and about 70 MeV above the $\Delta\pi N$ threshold, so that the threshold (or cusp) effect may not be as significant as for the XYZ particles [1–5]. Moreover, its width is only about 70 MeV, which is much smaller than the width of 2 Δs or even a single Δ, and is completely different from a conventional hadron. Therefore, Ref. [13] used Harvey's relation, which describes the relation between the physi-

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cal states and the symmetry states on the basis of group theory, to argue that this state might have a large hidden color component. This was, in fact, proposed and calculated by us in 1999 [14]. This implies that this state might have a compact structure. However, others believe that this state might be a $\Delta N\pi$ three body molecular-like state [15, 16]. It means that the observed state might have a relatively large size. These two assumed structures are quite different. Discriminating the structure of $d^*(2380)$ has become one of the important problems in the study of $d^*(2380)$. From the latter structure, if the size of the system is relatively large, the structure of a pion with a weakly bound D_{12} would not be compact. Consequently, the pion might be relatively easy to release, and the single pion decay cross section might have a sizeable value. Therefore, the single pion decay process $d^*(2380) \rightarrow NN\pi$ might be a criterion for the different structures of d^*. After carefully analyzing the data for such a decay channel, the WASA@COSY Collaboration reported an upper limit of 9% for the branching ratio of $d^*(2380) \rightarrow NN\pi$ [17]. Because the present data only give an upper limit, more accurate data would be appreciated. Of course, finding $d^*(2380)$ in other types of physical processes is also necessary and would be a cross check for the existence of this state.

This $d^*(2380)$ state was studied for several decades even before WASA's observation [14, 18–27] (refer also to the review in Ref. [6]). After the experimental observation of $d^*(2380)$, there have been many calculations with various approaches, for instance, the chiral $SU(2)$ quark model [27, 28], chiral $SU(3)$ quark model [29, 30], three body hadronic molecular-like model [15, 16, 31–35], QCD sum rules [36], etc. [37, 38]. Of these, only two structural models can simultaneously provide a mass and a width of $d^*(2380)$ which are consistent with the observed data. However, the structures in these two models are quite different. One is based on the quark-gluon degrees of freedom and the other on the framework of the hadronic degrees of freedom. The structure of d^* in these two approaches corresponds to a compact structure and a hadronic molecular-like structure, respectively. A typical calculation for the former is based on the chiral $SU(3)$ constituent quark model with a coupled-channel trial function of $\Delta\Delta{-}CC$ in the framework of the Resonating Group Method (RGM), where C denotes a color octet cluster [29, 30]. A typical study for the latter approach is done by performing a three-body Faddeev equation for a $\Delta N\pi$ system [15, 16]. In this work, we will introduce how these two calculations are carried out, what results they can give, and whether the obtained results can explain the data. In addition, it should be noted that a compromised mixing model, where a core with a large fraction of a compact structure is surrounded by an extended structure of $D_{12}\pi$, has already been proposed recently [39].

The paper is organized as follows. In Section 2, the calculation for a compact structure of $d^*(2380)$ by employing our chiral $SU(3)$ constituent quark model is briefly discussed. The study for a pure $\Delta N\pi$ structure of $d^*(2380)$ and the mixing model are introduced in Section 3. A comparisons of the results in both structural models with the data and a brief conclusion are given in Section 4.

2 A compact hexaquark dominated structure model

In the quark-gluon degrees of freedom, the fundamental interaction is the interaction between quarks. To study the inner structure of $d^*(2380)$ at this level, we employ the chiral $SU(3)$ constituent quark model. In this model, the quark-quark interaction includes three parts: the one-gluon-exchange interaction describing the short-range perturbative effect of QCD, the chiral field interaction representing the medium- and long-range nonperturbative QCD effect, and a confining potential governing the long-range nonperturbative effect of QCD [40]. In the constituent quark model, because the quarks get their constituent masses through spontaneous symmetry breaking of the vacuum, Goldstone bosons appear. The Goldstone bosons obtain their physical masses through the apparent chiral symmetry breaking due to the nonzero masses of the current quarks. The interactions caused by the chiral fields, or Goldstone bosons, must therefore be considered in the strong interaction between the constituent quarks. Even more, in the extended chiral $SU(3)$ constituent quark model, the potentials caused by the vector meson exchange are also taken into account to better describe the short-range nonperturbative QCD effect [40]. In this approach, the model parameters are determined by the mass difference between N and Δ, the stability condition of nucleon, the measured coupling constant $g_{NN\pi}$, the coupling constants $G_{NN\rho}$ and $f_{NN\rho}$ from well-established strong interaction models, etc. [40]. The properties of ground state baryons, the properties of two-baryon systems (like the root-mean-square (RMS) radius, binding energy, and $S-$ and $D-$wave admixture in the wave function of deuteron, and even the mass of $2225\sim2234$ MeV for the H particle [41]), the N-N phase shifts, and the hyperon-nucleon interactions, etc, can therefore be well described, and all the model parameters are fixed [40, 41]. In particular, in the latter approach, the effect of the vector meson exchange almost substitutes for the effect of the one gluon exchange, and there is no double counting problem between the one-gluon-exchange and meson-exchange potentials. The success of the model implies that it has a very good chance of making meaningful predictions for other systems, like $d^*(2380)$.

Since d^* was observed in the n+p reaction, the baryon number of the system is B=2, i.e. this is a six-quark system. Then, the six-quark system can be solved in the two cluster approximation in the framework of the Resonanting Group Method (RGM). In this calculation, we assume that $d^*(2380)$ has two components. One is the $\Delta\Delta$ component, due to its quantum numbers $I(J^P)=0(3^+)$, and the other is a "hidden-color" component of "CC". Therefore, the trial wave function of the system can be written as

$$\Psi_{6q} = \mathcal{A}\Big[\phi_\Delta(\vec{\xi}_1,\vec{\xi}_2)\phi_\Delta(\vec{\xi}_4,\vec{\xi}_5)\eta_{\Delta\Delta}(\vec{r}) +\phi_C(\vec{\xi}_1,\vec{\xi}_2)\phi_C(\vec{\xi}_4,\vec{\xi}_5)\eta_{CC}(\vec{r})\Big]_{S=3,I=0,C=(00)}, \quad (1)$$

where $\mathcal{A}=1-9P_{36}$ is the anti-symmetrizer in the orbital (o), spin (s), isospin (f), and color (c) spaces due to the Pauli exclusion principle, $\phi_{\Delta,C}$ are the internal wave functions of the 3-quark clusters, with $\xi_i(i=1,2\ (4,5))$ being the internal Jacobi coordinates, and $\eta_{\Delta\Delta,CC}$ stand for the relative wave functions between the two clusters. Based on the fixed model parameters, which can be found in Refs. [29, 30], the bound state problem of the six-quark system with $I(J^P)=0(3^+)$ can be solved and the relative wave functions can be obtained.

Due to the non-orthogonality of the basis wave function, the two components in Eq. (1) are not orthogonal to each other. By making a projection and integrating out the internal Jacobi coordinates of $\xi_{1,2,3,4}$

$$\begin{aligned}\chi_{\Delta\Delta}(\vec{r}) &= <\phi_\Delta(\vec{\xi}_1,\vec{\xi}_2)\phi_\Delta(\vec{\xi}_4,\vec{\xi}_5)|\Psi_{6q}> \\ \chi_{CC}(\vec{r}) &= <\phi_C(\vec{\xi}_1,\vec{\xi}_2)\phi_C(\vec{\xi}_4,\vec{\xi}_5)|\Psi_{6q}>,\end{aligned} \quad (2)$$

the two channel wave functions in the hadronic level are expressed in the following equation as

$$\begin{aligned}\Psi_{d^*} &= \Psi_{d^*;\Delta\Delta}+\Psi_{d^*;CC} \\ &= |\Delta\Delta>\chi_{\Delta\Delta}+|CC>\chi_{CC},\end{aligned} \quad (3)$$

where the two channel wave functions, $\Psi_{d^*;\Delta\Delta}$ and $\Psi_{d^*;CC}$, are orthogonal to each other and include all the effects of anti-symmetrization.

The obtained mass of $d^*(2380)$ and the fractions of the S- and D-waves in the d^* wave function, as well as the deuteron properties, are listed in Table 1.

Table 1. The mass (the binding energy (BE) with respect to the threshold of the $\Delta\Delta$) of $d^*(2380)$ and the fractions of the S- and D-waves in the $d^*(2380)$ wave function, as well as the deuteron (d) properties, in the chiral $SU(3)$ constituent quark model and its extended version with f/g=0.

model	systems	m_{d^*} (BE)/MeV	S-wave		D-wave	
$SU(3)$ quark model	d	1876 (2.09)	93.68%		6.32%	
	$d^*(2380)$ in coupled-channel of $\Delta\Delta$+CC	2417 (47.27)	$\Delta\Delta$ 33.11%	CC 66.25%	$\Delta\Delta$ 0.62%	CC 0.02%
	$d^*(2380)$ in single channel of $\Delta\Delta$	2435 (28.96)	97.18%		2.82%	
extended $SU(3)$ quark model	d	1876 (2.24)	94.66%		5.34%	
	$d^*(2380)$ in coupled-channel of $\Delta\Delta$+CC	2380 (83.66)	$\Delta\Delta$ 31.22%	CC 68.33%	$\Delta\Delta$ 0.45%	CC 0.00%
	$d^*(2380)$ in single channel of $\Delta\Delta$	2402 (62.28)	98.01%		1.99%	

The extracted channel wave functions for the $\Delta\Delta$ and CC channels of the d^* state in the coupled channel case and the wave function in the single $\Delta\Delta$ case in momentum space are plotted in Fig. 1. It is shown from Table 1 that the obtained mass of $d^*(2380)$ in the coupled channel case is in a region of about 2.38∼2.42 GeV, which is consistent with the observed data, and the inclusion of the CC channel will suppress the mass of d^* by about 20 MeV. It is also found that the $S-$wave is dominant in both the single and coupled channel cases. An important characteristic of the wave function is that the fraction of the CC component in the coupled channel case is about 2 times larger than that of the $\Delta\Delta$ component.

Moreover, from the wave functions in Fig. 1, the root-mean-square radii (RMS) of the $d^*(2380)$ system for both the single channel structure and the coupled $\Delta\Delta-CC$ structure are 1.1 fm and 0.72 fm, respectively. The $d^*(2380)$ wave function in the single-channel case is broader than the one in the coupled-channel case. Especially, the wave function of the CC component is very compact. It is likely a single Gaussian-type wave function with the size parameter b_{CC}=0.45 fm.

Now, the decay width of the d^* state can be calculated by using the obtained channel wave function, which is a function of the relative momentum between the two Δs. In the leading order approximation, it is found that in the coupled channel case, only the $\Delta\Delta$ component contributes, not the CC component. Since the fractions

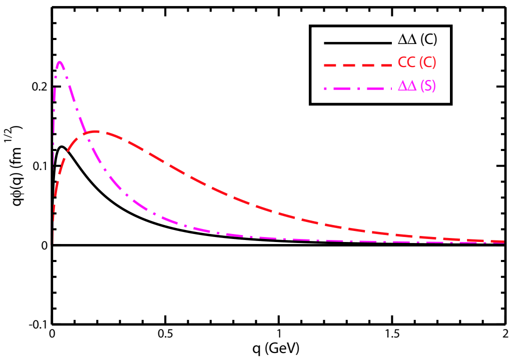

Fig. 1. (color online) Wave functions of $d^*(2380)$ and its components in momentum space and in the framework of the extended chiral $SU(3)$ constituent quark model. The pink dashed-dotted curve denotes the relative wave function between Δ and Δ in the single (denoted by "(S)") $\Delta\Delta$ channel only and the black solid and red dashed curves represent the wave functions of the $\Delta\Delta$ and CC components in the coupled $\Delta\Delta$+CC channel (denoted by "(C)"), respectively [29, 30].

of the $\Delta\Delta$ and CC components are about 31.22% and 68.33%, respectively, a smaller decay width could be expected. In time-order perturbation theory, the diagrammatic sketches of Feynman diagrams for the 3-body double-pion decay of $d^*(2380)\rightarrow\pi\pi d$ are drawn in Fig. 2.

The decay width of this process in the non-relativistic approximation, for example for $d\pi^0\pi^0$, can be formally written as

$$\Gamma_{d^*\rightarrow d\pi^0\pi^0} = \frac{1}{2!}\int \mathrm{d}^3k_1\mathrm{d}^3k_2\mathrm{d}^3p_d\delta^3(\vec{k_1}+\vec{k_2}+\vec{p}_d)(2\pi) \times\delta\left(\omega_{k_1}+\omega_{k_2}+\omega_{p_d}-M_{d^*}\right)\left|\overline{\mathcal{M}_{ij}^{d\pi^0\pi^0}}\right|^2, \quad (4)$$

where $k_{1,2}$ and p_d are the three-momenta of the two outgoing pions and the deuteron, respectively, $\omega_{k_{1,2}}$ and ω_{p_d} represent the energies of the two pions and deuteron, respectively, and $\left|\overline{\mathcal{M}_{if}^{d\pi^0\pi^0}}\right|^2$ stands for the squared transition matrix element with the sum over the final states and average over the initial states. According to the Feynman rule, it is easy to work out the transition matrix element as follows:

$$\begin{aligned}\mathcal{M}_{if}^{d\pi^0\pi^0} =& \frac{1}{\sqrt{3}}\sum F_1F_2k_{1,\mu}k_{2,\nu}I_S^0I_I^0C_{1\nu,1\mu}^{jm_j}C_{3m_{d^*},jm_j}^{1m_d}\\ &\times\int\mathrm{d}^3q\,\Big[\frac{\chi_d^*(\vec{q}-\frac{1}{2}\vec{k}_{12})}{E_\Delta(q)-E_N(q-k_1)-\omega_1}\\ &+\frac{\chi_d^*(\vec{q}+\frac{1}{2}\vec{k}_{12})}{E_\Delta(q)-E_N(q-k_2)-\omega_2}\\ &+\frac{\chi_d^*(\vec{q}+\frac{1}{2}\vec{k}_{12})}{E_\Delta(-q)-E_N(-q-k_1)-\omega_1}\\ &+\frac{\chi_d^*(\vec{q}-\frac{1}{2}\vec{k}_{12})}{E_\Delta(-q)-E_N(-q-k_2)-\omega_2}\Big]\chi_{d^*}(\vec{q}), \quad (5)\end{aligned}$$

where i and f stand for the initial d^* state with quantum numbers $((Sm_S)=(3m_{d^*}))$ and the final deuteron state with $((Sm_S)=(1m_d))$, respectively, and $I_{S(I)}^0$ is the spin (isospin) factor [42], $F_{1,2}=F(k_{1,2}^2)=\frac{4G}{(2\pi)^{3/2}\sqrt{\omega_{1,2}}}$, $\vec{k}_{12}=\vec{k}_1-\vec{k}_2$, $\omega_{1,2}=\sqrt{m_\pi^2+\vec{k}_{1,2}^2}$. $\chi_d(\vec{q})$ and $\chi_{d^*}(\vec{q})$ are, respectively, the relative wave functions of the final deuteron (between the two nucleons) and the initial d^* (between the two Δs) where $\vec{q}=\frac{1}{2}(\vec{p}_1+\vec{p}_2+\vec{p}_3-\vec{p}_4-\vec{p}_5-\vec{p}_6)$ with $\vec{p}_i$ being the momentum of the i-th quark. By using the coupling constant of Δ-π-N obtained through the $\Gamma_{\Delta\rightarrow N\pi}$ data fitting, the partial width for the $d^*\rightarrow d\pi\pi$ decay can be reached [42]. The obtained partial widths

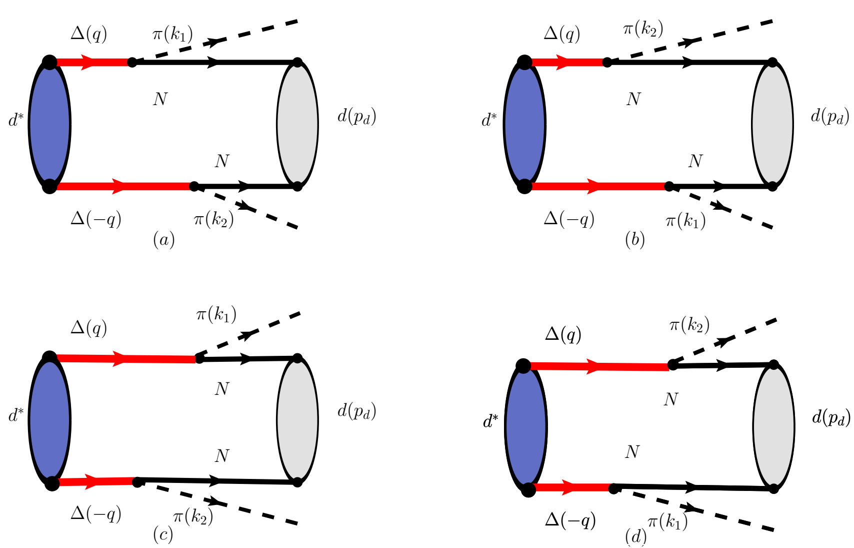

Fig. 2. (color online) Illustration of $d^*(2380)\rightarrow d\pi\pi$ decay.

in the $d^* \to d\pi^0\pi^0$ and $d^* \to d\pi^+\pi^-$ processes in the coupled channel case are 9.2 MeV and 16.8 MeV, respectively, which are shown in Table 2. The ratio of the partial width for the charged pion process to that for the neutral pion processes is about 1.83. These partial widths agree with the observed data of 10.2 MeV and 16.7 MeV quite well.

The 4-body double pion decay of $d^*(2380) \to NN\pi\pi$ can also be calculated in a similar way. In time-order perturbation theory, the diagrammatic sketches of Feynman diagrams for the $d^* \to NN\pi\pi$ decays are plotted in Fig. 3.

Table 2. Theoretical calculations (in units of MeV) for the double-pion decays of $d^*(2380)$ in the coupled-channel csae.

mode	our theor./MeV	expt./MeV
$d^* \to d\pi^+\pi^-$	16.8	16.7
$d^* \to d\pi^0\pi^0$	9.2	10.2
$d^* \to pn\pi^+\pi^-$	20.6	21.8
$d^* \to pn\pi^0\pi^0$	9.6	8.7
$d^* \to pp\pi^0\pi^-$	3.5	4.4
$d^* \to nn\pi^0\pi^+$	3.5	4.4
$d^* \to pn$	8.7	8.7
total	71.9	74.9

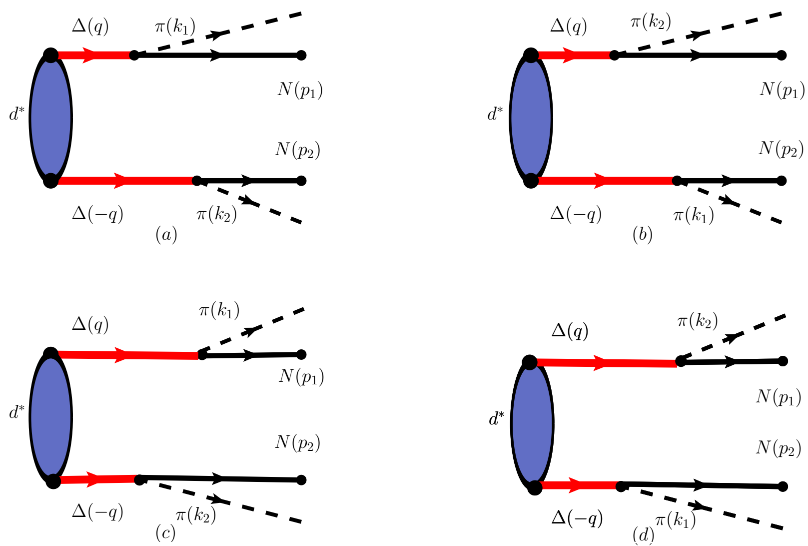

Fig. 3. (color online) Illustration of $d^*(2380) \to pn\pi\pi$ decay.

The decay widths of those processes in the non-relativistic approximation, for example for the $pn\pi^0\pi^0$ channel, can be written as [43]

$$\begin{aligned}\Gamma_{d^* \to pn\pi^0\pi^0} = & \frac{1}{2!2!}\int \mathrm{d}^3k_1\mathrm{d}^3k_2\mathrm{d}^3p_1\mathrm{d}^3p_2\delta^3(\vec{k_1}+\vec{k_2}+\vec{p}_1+\vec{p}_2) \\ & \times(2\pi)\delta\left(\omega_{k_1}+\omega_{k_2}+\omega_{p_1}+\omega_{p_2}-M_{d^*}\right) \\ & \times\left|\overline{\mathcal{M}_{ij}^{pn\pi^0\pi^0}}\right|^2, \end{aligned} \quad (6)$$

where $p_{1,2}$ and $\omega_{p_{1,2}}$ are the three-momenta and energy of the two outgoing nucleons, respectively, and $\left|\overline{\mathcal{M}_{ij}^{pn\pi^0\pi^0}}\right|^2$ is the squared transition matrix element with the sum over the final states and average over the initial states. The final state interaction (FSI) between the two outgoing nucleons must be taken into account for the four-body decay processes. The FSI can be formally written as

$$\mathcal{M}_{ij}^{pn\pi^0\pi^0} = \mathcal{M}_{if}^{pn\pi^0\pi^0(bare)} \times \mathcal{I}, \quad (7)$$

where $\mathcal{M}_{ij}^{pn\pi^0\pi^0(bare)}$ stands for the transition matrix element without FSI, and $\mathcal{I}$ denotes the enhancement factor caused by FSI. In the low energy region, the S-wave approximation can be taken, then $\mathcal{I}$ can be expressed by the Jost function. Detailed discussions for FSI between the proton and neutron can be found in Refs. [44–48]. The transition matrix element $\mathcal{M}_{if}^{pn\pi^0\pi^0(bare)}$ can also be calculated by using the relevant Feynman rule in non-relativistic time-order perturbation theory. For example, the explicit expression for Fig. 3(a) can be written as

$$\begin{aligned} & \mathcal{M}^a(k_1,k_2;p_1) \\ = & \int \mathrm{d}^3p_2\mathrm{d}^3q\left[\mathcal{H}\mathcal{S}_f\mathcal{H}\right]\Psi_{d^*}(q)\delta^3(\vec{p_1}+\vec{k}_1-\vec{q})\delta(\vec{p}_2+\vec{k}_2+\vec{q}) \\ = & \int \mathrm{d}^3p_2\delta^3(\vec{p}_1+\vec{p}_2+\vec{k}_1+\vec{k}_2)\left[\mathcal{H}\mathcal{S}_f\mathcal{H}\right]\psi_{d^*}(-\vec{p}_2-\vec{k}_2), \end{aligned} \quad (8)$$

where $\mathcal{S}_f$ is the propagator of the intermediate state, and $\mathcal{H}$ is the effective Hamiltonian for the pseudo-scalar interaction among quark, pion, and quark in the non-relativistic approximation. Ψ_{d^*} represents the d^* wave function in momentum space which can be obtained by

Fourier transforming the d^* wave functions in coordinate space [43]. The resultant partial widths for the decay processes $d^*\to pn\pi^0\pi^0$, $d^*\to pn\pi^+\pi^-$, $d^*\to pp\pi^0\pi^-$, and $d^*\to nn\pi^0\pi^+$ are 9.6 MeV, 20.6 MeV, 3.5 MeV, and 3.5 MeV, respectively, which are also tabulated in Tab. 2. Clearly, these values are roughly consistent with the observed values of 8.7 MeV, 21.8 MeV, 4.4 MeV, and 4.4 MeV, respectively. The ratio of the partial width for the $pn\pi^+\pi^-$ (charged pion) process to that for the $pn\pi^0\pi^0$ (neutral pion) processes is about 2.15.

By estimating the partial width for $d^*\to pn$ in terms of branching ratio data, we finally end up with the total width of the observed $d^*(2380)$ state. In our coupled $\Delta\Delta$+CC channel case, the total width of d^* is 71.9 MeV (see Table 2). Comparing this value with the observed value of 74.9 MeV, the total width from the current structural model, where a hexaquark state dominates, can explain the data very well.

As mentioned in the first section, because the single pion decay process $d^*(2380)\to NN\pi$ becomes one of the structure discriminators, it is necessary to study this decay process in the same framework. The diagrammatic Feynman diagrams in time-order perturbation theory for the $d^*\to NN\pi$ decays are sketched in Fig. 4.

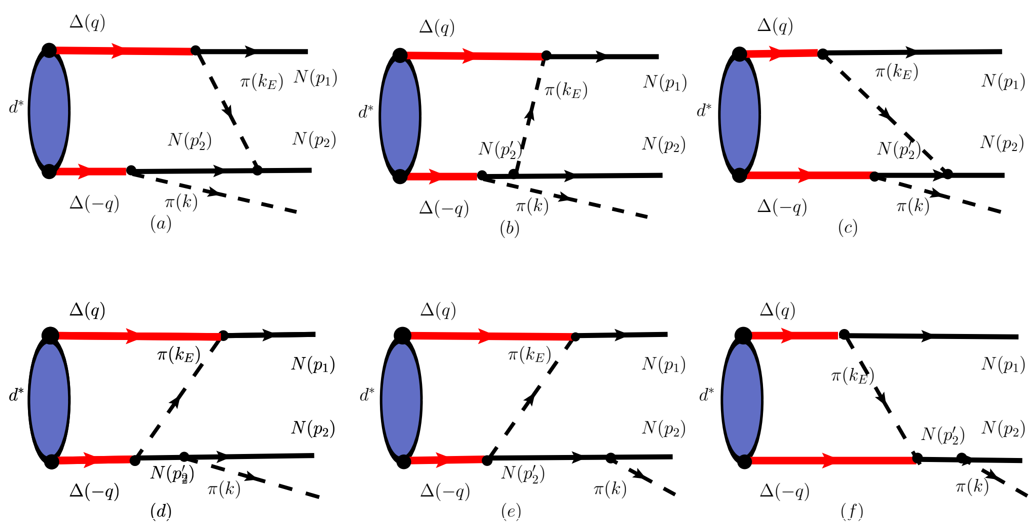

Fig. 4. (color online) Illustration of $d^*(2380)\to pn\pi$ decay, where the outgoing pion is emitted from the second Δ resonance.

In the leading order approximation, in our double-pion decay calculation mentioned above, two outgoing pions are emitted directly from two Δ clusters in the $\Delta\Delta$ component of $d^*(2380)$. However, in the single-pion decay calculation, one pion is emitted from one of the Δ clusters in the $\Delta\Delta$ component of d^*, and another is emitted from another Δ cluster, and is then absorbed by the system. Of course, apart from the diagrams in Fig. 4, there are also some Feynman diagrams where the outgoing pion is emitted from the second Δ cluster, which should be considered in the calculation as well.

The partial decay width of such a process is usually written as [49]

$$\Gamma_{d^*\to NN\pi}=\frac{1}{2!}\int \mathrm{d}^3p_1\mathrm{d}^3p_2(2\pi)\delta(\Delta E)\left|\overline{\mathcal{M}(\vec{p}_1,\vec{p}_2)}\right|^2, \quad (9)$$

where $\left|\overline{\mathcal{M}(\vec{p}_1,\vec{p}_2)}\right|^2$ stands for the squared transition matrix element with a sum over the polarizations of the final three body states and an average over the initial state d^*, and $\delta(\Delta E)$ denotes the energy conservation with $\Delta E = M_{d^*}-\omega_\pi(k)-E_N(p_1)-E_N(-p_1-k)$, where $\omega_\pi(k)$ and E_N represent the energies of the pion and nucleon, respectively. In terms of the Feynman rule, the involved matrix element can be obtained. For instance, the matrix element for Fig. 4(a) reads

$$\begin{aligned}\mathcal{M}^{(a)}_{d^*\to NN\pi} = &\int \mathrm{d}^3q\frac{\Psi_{d^*}(q)}{2\omega_{k_E}\sqrt{2\omega_k}(2\pi)^6}\delta^3\left(p_{N'_2}+p_{N'_1}+k-p_{\Delta_1}-p_{\Delta_2}\right)\\ &\times\tilde{\mathcal{M}}_{\pi(k_E)N(p'_2)\to N(p_2)}\mathcal{D}_{af}\tilde{\mathcal{M}}_{\Delta_1\to\pi(k_E)N(p_1)}\mathcal{D}_{ai}\tilde{\mathcal{M}}_{\Delta_2\to\pi(k)N(p'_2)},\end{aligned} \quad (10)$$

where Ψ_{d^*} represents the d^* wave function in momentum space, $\tilde{\mathcal{M}}_{\pi(k_E)N(p_2')\to N(p_2)}$, $\tilde{\mathcal{M}}_{\Delta_1\to\pi(k_E)N(p_1)}$, $\tilde{\mathcal{M}}_{\Delta_2\to\pi(k)N(p_2')}$ denote the transitions of $\pi(k_E)N(p_2')\to N(p_2)$, $\Delta_1\to\pi(k_E)N(p_1)$, and $\Delta_2\to\pi(k)N(p_2')$, respectively, and $\mathcal{D}_{ai(af)}$ is a non-relativistic energy propagator with the form

$$\begin{aligned}\mathcal{D}_{af} &= \frac{1}{M_{d^*}-\omega(\vec{k})-\omega(\vec{k}_E)-E_N(\vec{p}_1)-E_N(\vec{p}_2{}')}\\ \mathcal{D}_{ai} &= \frac{1}{M_{d^*}-\omega(\vec{k})-E_{\Delta_1}(\vec{q})-E_N(\vec{p}_2{}')}.\end{aligned} \quad (11)$$

As for the effect of FSI between the two outgoing nucleons, it should be noted that for such a system, due to the conservation of P and J, either the orbital angular momentum between the outgoing pion and nucleon is at least equal to 3, or the orbital angular momentum between the two outgoing nucleons is at least equal to 2. Then, in both cases, the effect of FSI will be suppressed by the higher partial wave. Therefore, in this calculation, we assume the enhancement factor from FSI is close to 1. By considering other Feynman diagrams where the outgoing pion is emitted from the second Δ cluster, the decay width of the $d^*\to NN\pi$ is obtained and shown in Table 3. From this table, the branching ratio of about 1% for this channel is quite small.

Table 3. The calculated single pion decay (in units of MeV) of $d^*(2380)$ in the extended chiral $SU(3)$ constituent quark model.

case	width/MeV
coupled-channel ($\Delta\Delta+CC$)	0.670
single-channel ($\Delta\Delta$)	2.276

3 Brief introduction of a $\Delta N\pi$ molecular-like structure model

For comparison, we briefly introduce another structural model based on baryon-baryon and baryon-pion interactions in the hadronic degrees of freedom [15, 16]. We first introduce a "pure" $\Delta N\pi$ molecular-like structure, where N stands for a nucleon. The general idea of that structure is that by solving a $\Delta' N\pi$ three-body Faddeev equation, where Δ' denotes a stable $\Delta(1232)$ and the $N\Delta'$ interaction is dominated by the $\mathcal{D}_{12}$ dibaryon, the $\Delta\Delta'$ structure could couple to a $\mathcal{D}_{12}\pi$ structure with the assistance of a pion.

In Refs. [15, 16], one needs the interactions between N and Δ', between N and π, and between Δ' and π. In order to get the N-Δ' interaction, they again employed the Faddeev equation to solve the $NN\pi$ three-body problem. In the calculation, the $N\pi$ interaction, which is dominated by the P_{33} channel, is taken to be a rank-one separable potential, and the NN interaction is described by a rank-two separable potential. The numerical result showed a $\mathcal{D}_{12}$ resonance with a S-matrix pole of 2147–i60 MeV in the N–Δ' channel, and consequently a separable potential for the $N-\Delta'$ interaction was obtained. As for the $\Delta'-\pi$ interaction, it was neglected due to the lack of a known Δ resonance to dominate it. Solving the $\Delta\Delta'$–$\mathcal{D}_{12}\pi$ channel $\Delta N\pi$ three-body Faddeev equation, a S-matrix pole of $(2363\pm20)-i(33\pm8)$ MeV for $\mathcal{D}_{03}$ or d^* was obtained [15]. These values are also consistent with the reported data.

However, the obtained partial width for the $d^*\to NN\pi$ decay in a model-dependent way showed a value of about 11.5 MeV (or branching ratio of 15.4%). This value is much larger than the recently reported upper limit of 9% [17]. Then, for this molecular-like model, a complementary mixing model, where a compact core is surrounded by an extended $\mathcal{D}_{12}\pi$ structure, was proposed [39]. In that paper, the decay width of the $d^*\to NN\pi\pi$ was written as

$$\Gamma_{NN\pi\pi}=\alpha\Gamma_<+(1-\alpha)\Gamma_> \quad (12)$$

with $\Gamma_<=44$ MeV and $\Gamma_>=100$ MeV being the widths for a compact structure and a $\mathcal{D}_{12}\pi$ structure, respectively, and $\alpha=5/7$, so a value of $\Gamma_{d^*\to NN\pi}=6.2$ MeV can be reached.

4 Comparison with data and concluding remarks

We compare the data with the model results in the following three aspects.

(1) **Mass:** From Table 1, we find that in the cases with and without the CC component, our obtained masses for the six-quark state with $I(J^P)=0(3^+)$ are $2402\sim2435$ MeV and $2380\sim2417$ MeV, respectively. This means that no matter whether a CC component is included, all the obtained masses of the state are generally consistent with the observed value of 2380 MeV. Inclusion of the CC channel will lower the mass by about 20 MeV. Especially, the agreement between the mass obtained in the coupled $\Delta\Delta+CC$ channel calculation in the extended $SU(3)$ constituent quark model and the observed value, and the large CC component of about 68~66%, implies that the observed state prefers a compact hexaquark dominated structure.

On the other hand, the mass from the $\Delta N\pi$-$\mathcal{D}_{12}\pi$ molecular-like model is about 2363±20 MeV. This value also explains the observed value. So as far as the mass concerned, both the compact hexaquark dominated structure and the $\Delta N\pi$-$\mathcal{D}_{12}\pi$ molecular-like structure can be the candidates for the possible structure.

(2) **Total width and partial widths for the double pion decays:** By looking at Table 2, the total width of $d^*(2380)$ with a compact structure is about

71.9 MeV, which agrees with the averaged observed value of 75 MeV. Superficially, this is because all the partial widths for double-pion decays, which are dynamically calculated directly from the Feynman diagrams, are consistent with the observed values, except that the partial width for $d^* \to pn$ is extracted by using the branching ratio data. By its nature, this should be a consequence of its compact hexaquark dominated structure.

On the other hand, the total width with a "pure" $\Delta N\pi$-$\mathcal{D}_{12}\pi$ molecular-like structure is about 65±17 MeV, which also agrees with the data. The partial widths for various decay channels, which are extracted by comparing with the relevant data and using the isospin breaking factor of 1.83 from our calculation [42], can also fit to the data. Thus, from the viewpoint of the decay width, the data allow the existences of both structures.

(3) **Partial width for the single pion decays:** The results in Table 3 were also dynamically calculated from various Feynman diagrams, although some approximations were made in the calculation. The obtained partial widths of the decay process $d^* \to NN\pi$ with and without the CC component are about 0.67 MeV and 2.28 MeV, respectively. The corresponding branching ratios are about 0.9% and 3.2%, which do not contradict the experimental upper limit of 9% reported recently [17].

On other hand, if a "pure" $\Delta N\pi$-$\mathcal{D}_{12}\pi$ molecular-like structure is considered, the partial width is about 15.4 MeV, which is much larger than the observed upper limit of about 9%. Clearly, at least this "pure" molecular-like structure is not the exact structure for the observed $d^*(2380)$. To overcome this defect, a compensatory mixing model is proposed [39]. Apparently, the idea of the two "extreme" structures mixing is meaningful. By taking $\Gamma_< = 44$ MeV and $\Gamma_> = 100$ MeV, estimating $\Gamma_{NN\pi\pi}=60$ MeV, and solving Eq. 12, a value of $\alpha = 5/7$ and a partial width of 6.2 MeV (corresponding branching ratio is 8.3%) for the $d^* \to NN\pi$ decay are obtained. By this phenomenological mixing treatment, this new branching ratio is below the upper limit reported by the experimental measurement. So the mixing structure can also be a more reasonable structural candidate for $d^*(2380)$. However, the physical picture of the mixing model looks like a compact structure in the center as a core and a much larger sized molecular-like structure surrounding the compact structure. Since the mixing parameter α has a large value of 5/7, it implies that $d^*(2380)$ is a compact structure dominated state. Moreover, the value of α is sensitive to the input value of $\Gamma_<$. Consequently, the partial width for $d^* \to NN\pi$ will also change if $\Gamma_<$ varies. The final α value is still waiting for the accurate measurement of the $NN\pi$ channel.

In summary, we believe that the compact hexaquark dominated structure is a good candidate for the structure of $d^*(2380)$, because not only the mass but also the total width and all the partial widths for various double-pion decay channels are consistent with the observed data. Especially, the partial width for the decisive decay channel $d^* \to NN\pi$ does not contradict the observed upper limit. The constituent quark model used is predictive, and no additional free parameters are employed. The important thing here is that our proposed structure is more a dominant structure than a complete structure in $d^*(2380)$. Other minor structures with the same quantum numbers can, of course, be mixed in. However, this would be another complicated calculation. The mixing model, where a compact system of size 0.5∼1 fm ($\Delta\Delta$) mixes with a loose system of size typically 1.5 ∼ 2 fm ($\mathcal{D}_{12}\pi$), can also be a possible structure. To identify this structure, a more accurate $\Gamma_{d^*\to NN\pi}$ measurement is needed to fix the phenomenological parameter α. Nevertheless, the existence of such a $d^*(2380)$ state should be further checked in other types of experiments, for instance, the $\gamma+d$ process, the Υ decay process, charge distribution function measurement, etc. More accurate measurements of the partial decay widths of various decay channels, especially the single pion decay channel, should be carried out in future. Although our proposed compact hexaquark dominated structure is promising, up to now, the structure of $d^*(2380)$ is still an open question. Any experimental effort and theoretical calculations by any models are welcome.

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