Chinese Physics C Vol. 39, No. 7 (2015) 077001

A multi-length bunch design for electron storage rings with odd buckets*

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Abstract: A scheme with two superconducting RF cavities is designed to upgrade electron storage rings with odd buckets to multi-length bunches. In this paper, the Hefei Light Source Ⅱ (HLS Ⅱ) is given as an example for odd buckets. As it is designed for 45 buckets, which is a multiple of 3, simultaneous generation of three different lengths of bunches is proposed with the presently applied user optics. The final result, without low-α optics, is to fill HLS Ⅱ with long bunches of length 50 ps, medium bunches of 23 ps and short bunches of 6 ps. Every third bucket can be filled with short bunches, of which the current limit is up to 6.6 mA, more than 60 times the limit for low-α mode. Moreover, particle tracking simulations to examine the beam dynamics, performed by ELEGANT, and calculations of the beam instabilities are presented in this paper.

Key words: storage ring, bunch length, odd buckets, superconducting cavities, HLS

PACS: 29.20.db, 29.27.Bd **DOI:** 10.1088/1674-1137/39/7/077001

1 Introduction

An increasing interest in short X-ray pulses requires short electron bunches in storage rings. Over the years, the traditional way to get short bunches is to decrease the momentum compaction factor by low-α optics [1]. However, the average current per bunch will decrease to the order of μA magnitude with low-α, limited by beam instability and microbunching due to coherent synchrotron radiation and other collective effects [2].

Berlin Electron Storage Ring Society for Synchrotron Radiation (BESSY) II presented an idea to produce long and short bunches alternately in their storage ring: with two superconducting cavities (3rd harmonic and 3.5th harmonic of the fundamental RF cavity), the voltage gradient produced by two superconducting RF cavities (sc-cavities) could add up at even points for bunch focusing to get short bunches, and cancel each other at odd points to get long bunches [3]. The new method greatly improves the capacity for storing current in the ring. Nonetheless, BESSY II is filled with 400 buckets, which is a multiple of 2, leading to the result of choosing the second sc-cavity as a 1/2 times higher harmonic. What if the ring has an odd number of buckets, such as a multiple of 3, 5 or 7? In this paper, we find the problem can be solved by choosing the frequency of the second sc-cavity (sub-harmonic cavity) in a different way. For a ring whose minimum common factor for the number of buckets is k ($k>1$), the frequency of the sub-harmonic cavity can be chosen to be $1/k$ times higher than the harmonic one. Several typical situations are listed in Table 1, in which N_i(i=1, 2, 3, 4) is chosen according to the voltage and frequency of the original cavity in the different rings.

Table 1. Harmonic number with different number of buckets.

common factor	harmonic	sub-harmonic
2	N_1	N_1+1/2
3	N_2	N_2+1/3
5	N_3	N_3+1/5
7	N_4	N_4+1/7

In addition, because of the high original cavity voltage, a multi-cell structure is required for BESSY Ⅱ and other rings to achieve the 20 MV voltage of new sc-cavities. However, a multi-cell structure may lead to some high order mode (HOM) problems. So, if we can

Received 11 September 2014, Revised 17 February 2015
∗ Supported by National Natural Science Foundation of China (11327902, 11175180, 11175182) and U.S. DOE (DE-AC02-76SF00515)
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find a ring with low voltage, the HOM problems may be much simpler.

In this paper, we present a new scheme for HLS Ⅱ which is a good example of how to solve the odd buckets problem and simplify the HOM problem at the same time. A ring with two 2 MV, 1.3 GHz sc-cavities is designed for HLS Ⅱ, and a 1/3 times higher harmonic sc-cavity is expected to be applied. With careful phase tuning, medium, long and short bunches can be generated simultaneously. Moreover, because of the low voltage and frequency of the fundamental cavity, it is easy to increase the voltage gradient in the ring by a factor of 100 with a single-cell or two-cell structure.

2 HLS Ⅱ storage ring lattice

The Hefei Light Source Ⅱ [4] is an 800 Mev electron storage ring, whose main parameters are given in Table 2. The lattice structure of HLS Ⅱ was chosen to be the double bend achromat (DBA) lattice with 4 periods and eight straight sections, which include four 4.0 m long straight sections and four 2.3 m short chromatic straight sections. In each half cell, there are four quadrupoles and four combined function sextupoles. The magnet layout of the ring is shown in Fig. 1 and Fig. 2.

Table 2. General parameters of HLS Ⅱ.

nominal energy/MeV	800
circumference/m	66.13
number of cells	4
number of dipoles	8
bending radius/m	2.1645
RF frequency/GHz	0.204
energy spread	0.00047
emittance/(nm·rad)	36.38
beam current/mA	>300
momentum compaction	0.0205
damping time (τ_x, τ_y, τ_s)/ms	(20.00, 21.08, 10.84)
tunes (ν_x, ν_y)	(4.44, 2.80)
natural chromaticity (ξ_x, ξ_y)	(−9.89, −4.66)
energy loss per turn/keV	16.74

Table 3. The main lattice parameters (half cell).

start point	midpoint of DL
DL/2	2.003175 m
S1	0.00 m^{-3}
Q1	3.8807 m^{-2}
S2	0.00 m^{-3}
Q2	−3.2031 m^{-2}
DBQ1	0.30 m
B	1.7 m / 1.2336 T
DBQ2	0.70 m
Q3	3.7871 m^{-2}
S3	49.36 m^{-3}
Q4	−3.3874 m^{-2}
S4	−79.07 m^{-3}
DM/2	1.163175 m
symmetric point	midpoint of DL

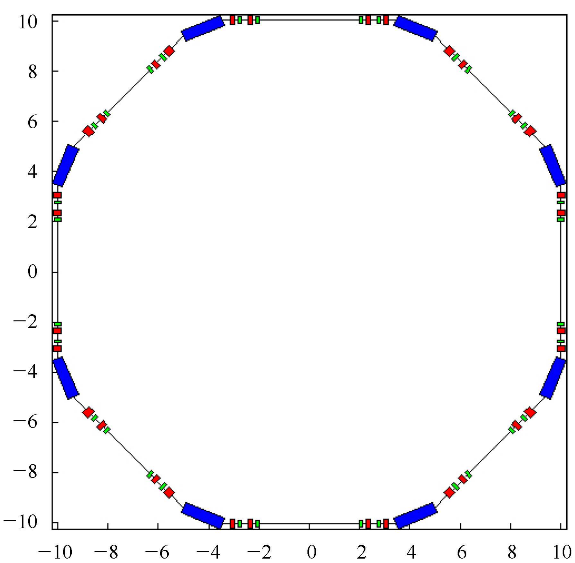

Fig. 1. (color online) Magnet layout of HLS Ⅱ. The HLS Ⅱ ring is a typical octagonal structure.

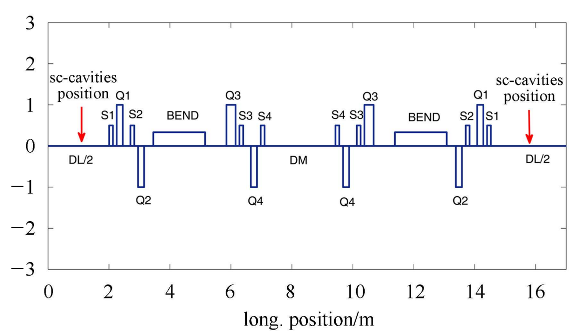

Fig. 2. (color online) The lattice structure per cell in HLS Ⅱ.

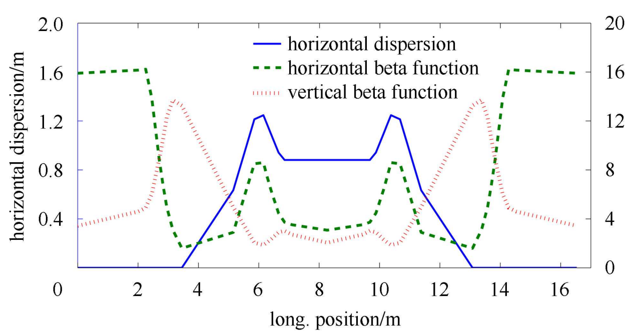

Fig. 3. (color online) β function and dispersion function of HLS Ⅱ. The new superconducting cavities are required to be located at the zero dispersion position.

There are two operating modes in HLS Ⅱ. Mode A is an achromatic mode whose dispersion in the long straight sections is zero, and Mode B is a distributed dispersion mode whose emittance is smaller than Mode A. In this paper, we choose the achromatic mode (Mode A). The main parameters of HLS Ⅱ are summarized in Table 3, and Fig. 3 shows the β and dispersion functions per cell.

3 Medium-long-short bunches scheme

In an electron storage ring, the equilibrium bunch length is [5]

$$\sigma_{\rm s}=\frac{c\delta_\varepsilon}{2\pi}\sqrt{\frac{4\pi^2\alpha E_0}{cef_{\rm rev}V'}}, \tag{1}$$

where $f_{\rm rev}$ is the revolution frequency, α is the momentum compaction factor, V' is the voltage gradient and δ_ε is the equilibrium energy spread. From this relation, the bunch length can be shortened by decreasing the momentum compaction factor or increasing the voltage gradient for $\sigma_{\rm s}\propto\sqrt{\alpha/V'}$. In this section, we just focus on increasing the voltage gradient.

For the original cavity , the rf-frequency f_0 is 0.204 GHz, the voltage V_0 is 0.25 MV and the phase $\psi_{\rm s0}$ is 3.0747 rad. It is used to replenish the energy loss by synchrotron radiation, and the frequency also leads to the fill pattern of 45 buckets. Because of the high gradient for bunch focusing, two superconducting cavities are required here. The first is a sixth harmonic cavity f_1 = 1.2240 GHz ($f_1=6f_0$), and considering the 45 buckets, the second cavity is chosen as $\left(f_2=\left(6+\frac{1}{3}\right)f_0\right)$, a 1/3 times higher harmonic in rf-frequency. The two sc-cavities can be used for three kinds of bunches.

In addition to the rf-frequency, finite voltage and phase for the sc-cavities must be chosen for particle acceleration and stable buckets. The sum voltage gradient in the longitudinal direction is

$$V'_{\rm sum}=\frac{2\pi}{c}\cdot[fv]_{\rm focusing}, \tag{2}$$

$[fv]_{\rm focusing}$ is defined as an rf-focusing parameter which directly dominates the bunch length:

$$\begin{aligned}[fv]_{\rm focusing} = &\left[f_0V_0\cos\left(\frac{2\pi f_0}{c}s+\psi_{\rm s0}\right)\right.\\ &+f_1V_1\cos\left(\frac{2\pi f_1}{c}s+\psi_{\rm s1}\right)\\ &\left.+f_2V_2\cos\left(\frac{2\pi f_2}{c}s+\psi_{\rm s2}\right)\right],\end{aligned} \tag{3}$$

where $\psi_{\rm s0}$, $\psi_{\rm s1}$, $\psi_{\rm s2}$ are the phases of the original, harmonic and sub-harmonic cavity respectively. For a proper cancelation to get long bunches, the relation between frequency and voltage amplitudes must follow $f_1V_1 = f_2V_2$. Although higher voltage could shorten bunches more, taking the HOM problem and cavity design into account, voltages are chosen as V_1=1.9 MV and V_2=1.8 MV.

To get $\psi_{\rm s1}$ and $\psi_{\rm s2}$, the 45 buckets are divided into three groups: Bucket$_{(3m)}$, Bucket$_{(3m+1)}$, and Bucket$_{(3m+2)}$, (m=0, 1, 2, 3,$\cdots$,14). Two conditions are made here to determine the phase for the cavities. First, only the original cavity is used for energy recovery, because of the expensive cooling system for superconducting cavities. Second, the voltage gradients produced by the two cavities are expected to cancel each other at the position of Bucket$_{(3m+1)}$ to get long bunches. These two conditions can be summarized as follows

$$\begin{aligned}&\text{condition1 } V_1\sin(\psi_{\rm s1})+V_2\sin(\psi_{\rm s2})=0,\\ &\text{condition2 } \psi_{\rm s1}+\psi_{\rm s2}=\tfrac{1}{3}\pi+k\pi,\end{aligned} \tag{4}$$

where k is an integer.

By solving these conditions, the phases are chosen as $\psi_{\rm s1}=2.1412$ rad and $\psi_{\rm s2}=-1.0940$ rad. The main parameters of the three cavities are given in Table 4. The two sc-cavity positions, shown in Fig. 2, require zero dispersion to avoid bunch lengthening by coupling effects.

After phase tuning, the voltage gradient produced by the two sc-cavities cancel each other at Bucket$_{(3m+1)}$ to get long bunches, add up at Bucket$_{(3m+2)}$ to get short ones, and Bucket$_{(3m)}$ can be filled with medium bunches.

Table 4. Main parameters of the three cavities.

cavity	harmonic number	frequency f/GHz	voltage V/MV	phase $\psi_{\rm s}$
original	1	0.204	0.25	176.17°
first	6	1.2240	1.9	122.68°
second	$6\frac{1}{3}$	1.2920	1.8	−62.68°

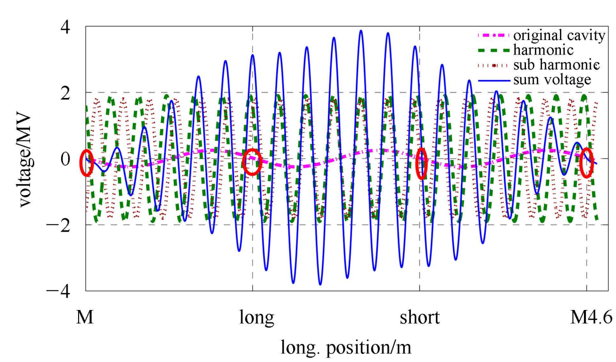

Fig. 4. (color online) Voltage of the three cavities. The sum voltage in MV as a function of the longitudinal distance in m is shown as the solid line. The voltages of the original, harmonic and sub-harmonic cavities are shown as the broken lines.

Figure 4 shows the sum voltage of the three cavities as a function of the longitudinal position. As $\sigma_s \propto \sqrt{\alpha/V'}$, the sum voltage gradient as a function of longitudinal position is shown in Fig. 5. From Fig. 5, it is clear that the voltage gradient is increased a great deal by the two sc-cavities, with the three buckets in one period. The first bucket is filled with a medium bunch, the second is long, and the third is short, with the following buckets in the same pattern, which are indicated by the circles in Fig. 4 and Fig. 5.

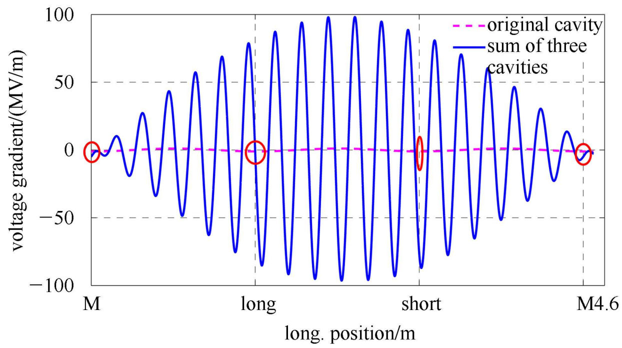

Fig. 5. (color online) Sum voltage gradient of the three cavities as a function of longitudinal distance, compared to the voltage gradient of the original cavity.

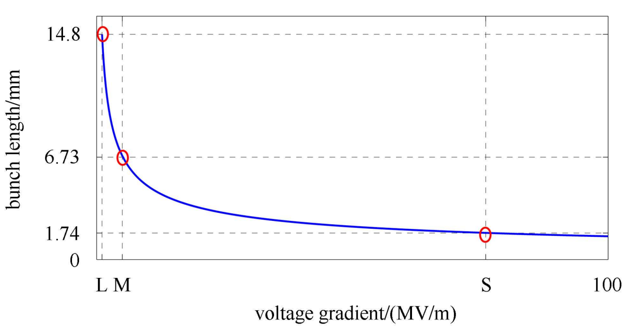

Fig. 6. (color online) The bunch length in mm as a function of voltage gradient in MV/m. Three different bunches are indicated in the circles marked L, M, S for long, medium and short.

Actually, by applying sc-cavities in HLS Ⅱ, the total number of buckets is multiplied by 6 (to 270 buckets). However, we are only interested in bunches at the position of the original buckets. Long bunches are placed at $3m+1$ multiples of 1.4696 m, short bunches are at $3m+2$ multiples of 1.4696 m, and bunches at $3m$ multiples of 1.4696 m are about 2.2 times shorter than the long bunch length. Without low-α mode, we can get long bunches at 14.8 mm (50 ps), short bunches at 1.74 mm (6 ps) and medium bunches at 6.73 mm (23 ps). The relationship between bunch length and voltage gradient are shown in Fig. 6.

4 Dynamic aperture

Four different groups of sextupoles per cell shown in Fig. 1 are used to correct chromaticity and harmonics in HLS Ⅱ. In Mode A, the operation mode considered in this paper, only the sextupoles in the short sections are useful for chromaticity corrections because sextupoles must be placed at locations where the dispersion function does not vanish, $\eta_x \neq 0$.

Simulation of the dynamic aperture by tracking 1000 turns was performed by ElegantRingAnalysis [6]. Fig. 7 shows the dynamic aperture (DA) and frequency map for different energy spreads at $\delta=0$, $\pm2\%$, which proves that the DA in the horizontal plane can reach about 40 mm. The aperture is large enough to ensure no particle loss.

5 Particle tracking results

Particle tracking was performed by ELEGANT [7] to simulate the longitudinal bunch length in the new system with two sc-cavities added. The equilibrium length of long bunches (top left), medium bunches (top right) and short bunches (bottom) are shown in Fig. 8.

Table 5. Comparison of fitted and theoretical equilibrium length.

	theoretical	fitted length	error
long	14.8 mm	14.7 mm	0.7%
medium	6.73 mm	6.59 mm	2.1%
short	1.74 mm	1.69 mm	2.8%

A long term particle tracking simulation of 1000 particles with three damping times (about 150000 turns) at the same initial length of 20 mm was also performed with ELEGANT, to simulate the process of shortening bunches. Fig. 9, Fig. 10 and Fig. 11 indicate the length variation of the three kinds of bunches, where the particles start at a long, medium and short bucket location.

The data of the three damping times can be fitted to get the equilibrium length, and the comparison to theoretical values is given in Table 5.

6 Current limit

The increase in the bunch length and energy spread is obvious when the beam current exceeds the threshold current. To avoid bunch lengthening, calculation of the current threshold is important.

For an electron beam of energy E_0 and relative energy spread σ_δ, the beam is described by the longitudinal distribution function ϕ, which is the sum of the equilibrium distribution function ϕ_0 and a perturbation $\phi_1=\widehat{\phi}_1\exp[-\mathrm{i}\omega s/c+\mathrm{i}kz]$, where k is the wave number of the perturbation. From the one-dimensional Vlasov

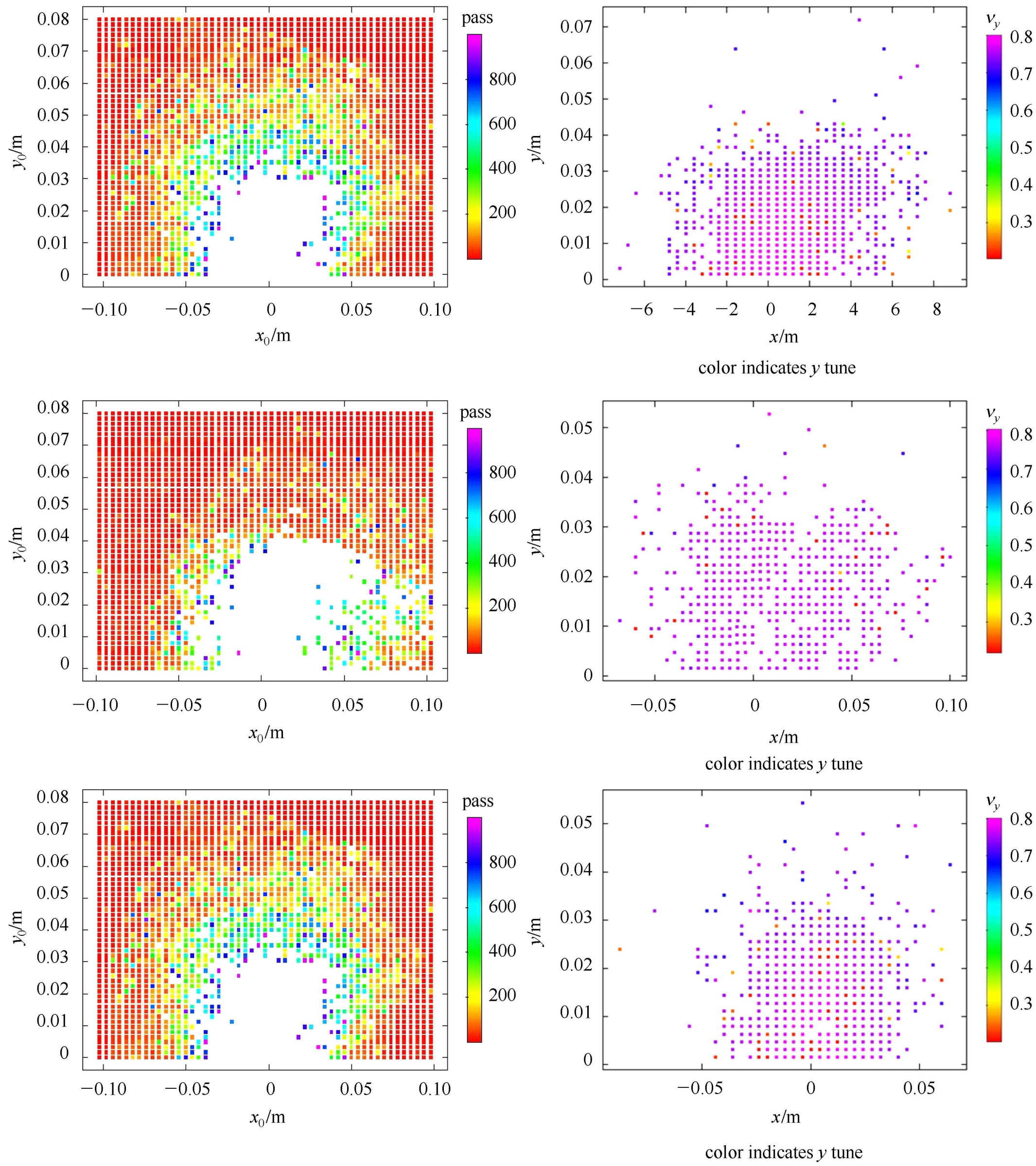

Fig. 7. (color online) Dynamic aperture and frequency map analysis for δ=0, ±2%.

equation for the distribution function [8], one can derive

$$1=\frac{\mathrm{i}r_0cZ(k)}{\gamma}\int\frac{\mathrm{d}\delta(\mathrm{d}\phi_0/\mathrm{d}\delta)}{\omega+ck\alpha\delta},\tag{5}$$

where $Z(k)$ is the impedance, δ is the relative energy offset of a particle and a Gaussian distribution can be used for ϕ_0.

Consider the Coherent Synchrotron Radiation (CSR) wakefield generated by an electron moving in a circular orbit of radius ρ in the middle of two parallel metal plates separated by a distance of $2h$. The shielding parameter is given by [9]

$$\Pi=\sigma_{\mathrm{s}}\rho^{1/2}/h^{3/2},\tag{6}$$

where σ_{s} is the bunch length. The longitudinal distribution varies for different bunch lengths. In HLS Ⅱ, the radius ρ is 2.1645 m and the gap $2h$ is 4 cm.

For a long bunch, with shielding parameter $\Pi>3$, the CSR impedance with shielding was calculated by Warnock [10], and analysis of Eq. (6) was carried out for various values of scaled current [11]

$$S=2\sqrt{2\pi}Ih/\alpha\gamma{\sigma_\delta}^2I_{\mathrm{A}}\sigma_{\mathrm{s}},\tag{7}$$

where σ_δ is the rms relative energy spread and I_{A}=17045 A is Alfven current. The beam is unstable when $S>6/\pi$.

In HLS Ⅱ, shielding parameter $\Pi>3$ corresponds to a bunch length $\sigma_{\mathrm{s}}>5.8$ mm, and both medium and long bunches are in that range. For bunches whose length is larger than 5.8 mm in HLS Ⅱ, the current threshold is

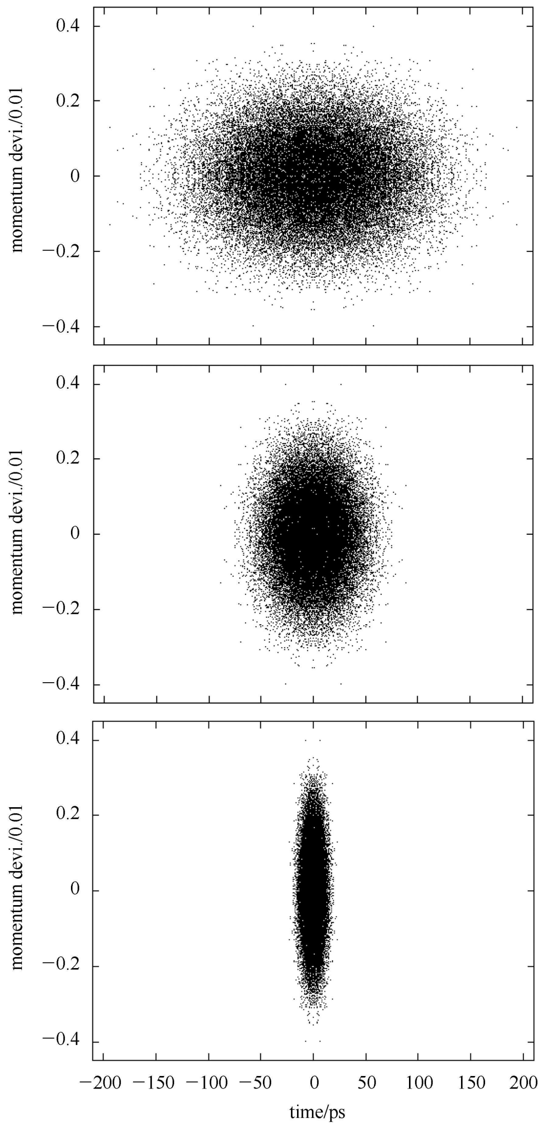

Fig. 8. Tracking results of the equilibrium length. The vertical axis indicates the momentum deviation of the bunch (central momentum of the bunch is about 1566), the horizontal axis indicates the longitudinal bunch length in picoseconds.

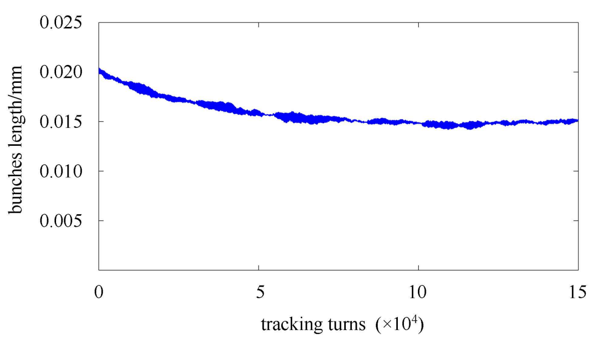

Fig. 9. (color online) Results of tracking for three damping times for long bunch length variation. The final tracking length is about 15.1 mm.

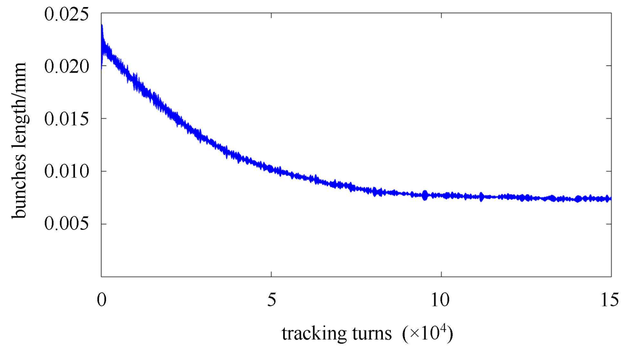

Fig. 10. (color online) Results of tracking for three damping times for medium bunch length variation. The final tracking length is about 7.4 mm.

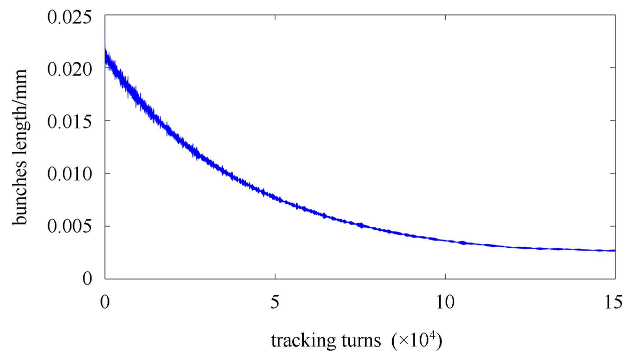

Fig. 11. (color online) Results of tracking for three damping times for short bunch length variation. The final tracking length is about 2.6 mm.

$$I_{\rm th}=I_{\alpha\sigma}\alpha\sigma_{\rm s}, \tag{8}$$

where

$$I_{\alpha\sigma}=\frac{3\sqrt{2}\gamma\sigma_\delta^2 I_{\rm A}}{2\pi^{3/2}h}, \tag{9}$$

$I_{\rm th}$ is the threshold current, and the beam becomes unstable when the current is above $I_{\rm th}$. The long bunch current limit is easy to predict, as the threshold is proportional to the momentum compaction factor and bunch length. The physical meaning of $I_{\alpha\sigma}$ can be thought of as the current per momentum compaction factor per bunch length, and is about 112 mA/mm in HLS Ⅱ. As a result, the threshold currents of the long and medium bunches are about 34.0 mA and 15.5 mA respectively, which are both large values.

When a bunch is short, with a length that is smaller than 5.8 mm in HLS Ⅱ, the longitudinal wakefield requires special care in the ring because the equilibrium becomes a Haissinski distribution [12]. The bunch beam theory should be applied, and the current threshold should be amended as

$$I_{\rm th}=\frac{8\pi^2(0.5+0.12\varPi)\sigma_{\rm s}^{7/3}[fv]_{\rm focusing}f_{\rm rev}}{c^2 Z_0\rho^{1/3}}, \tag{10}$$

where $Z_0=120\ \pi\Omega$, which is the impedance of free space. BESSY Ⅱ also provides an empirical equation for bursting threshold calculation [13], which agrees well with the Eq. (10). So the current threshold of short bunches is about 6.6 mA.

Consider the microwave instability from the longitudinal impedance, which leads to internal bunch oscillations resulting in bunch lengthening and an increase in energy spread. The threshold current per bunch for this instability to appear is given by

$$I_{||\mathrm{th}}=\frac{\sqrt{2\pi}\alpha(E/e)\sigma_{\mathrm{s}}}{\rho(Z_{||}/n)_{\mathrm{eff}}}\sigma_{\delta}^{2}. \tag{11}$$

The $(Z_{||}/n)_{\mathrm{eff}}$ in HLS Ⅱ is designed to be around 1 Ω. Comparing Eq. (8) and Eq. (10) to Eq. (11), we can find that the CSR instability is the dominant instability in the system.

In comparison with low α, using superconducting RF cavities (SRF) to get short bunches can improve the current stored in the ring. If low-α optics are used to shorten the bunch at the same length to 1.74 mm, the threshold is just about 0.1 mA. To make this clearer, we assume the threshold I_0 is 10 mA at bunch length $\sigma_{z0}=6$ mm. Then the current limit difference between the two methods below 6 mm is shown in Fig. 12.

Combined with low-α, bunches can be made even shorter. For the new superconducting system above, the momentum compaction factor is about α=0.0205, and if α is adjusted to 0.000205, all bunches are expected to be shorter by a factor of 10. In this way, the short bunches can reach about 0.174 mm (about 0.6 ps), and the total current about 2 mA. The current is small, but it is still much better than the current with low-α mode. However, for users, a suitable value of α could be chosen for good current and short bunch length at the same time. Table 6 shows typical examples of expected bunch length and total current limit.

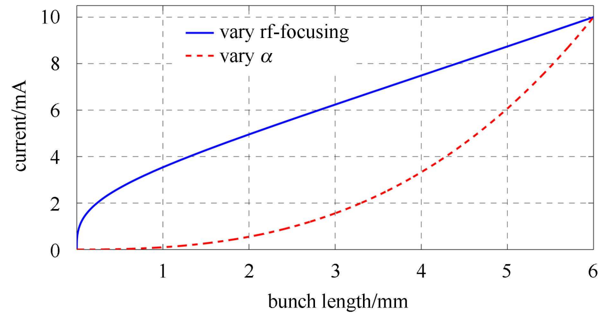

Fig. 12. (color online) Current threshold with increased rf-focusing and decreased α.

Table 6. Typical values of short bunch length and total current limit.

α	σ_{s}/mm	total current/mA
0.0205	1.74	> 300
0.0068	1.00	166
0.0024	0.6	41
0.0017	0.4	15
0.0002	0.174	2

7 Conclusion

A scheme is presented to operate the HLS Ⅱ ring with simultaneous medium, long and short bunches. The current limit of short bunches can be increased more than 60 times compared to the low-α mode. The frequencies of the two superconducting cavities are both near 1.3 GHz, which is easy to achieve by tuning, as 1.3 GHz is a common frequency for sc-cavities [14]. The voltages are about 2 MV, which can easily be realized by a single-cell or two-cell structure, so the HOM problems will be simplified.

We thank A. Chao and J. Wu for useful discussions and encouragements.

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