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Study of an unfolding algorithm for D-T neutron energy spectra measurement using a recoil proton method *

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Abstract: A proton recoil method for measuring D-T neutron energy spectra using polyethylene film and a Si(Au) surface barrier detector is presented. An iteration algorithm for unfolding the recoil proton energy spectrum to the neutron energy spectrum is investigated. The response matrices R of the polyethylene film at angles of 0° and 45° were obtained by simulating the recoil proton energy spectra from mono-energetic neutrons using the MCNPX code. With an assumed D-T neutron spectrum, the recoil proton spectra from the polyethylene film at angles of 0° and 45° were also simulated using the MCNPX code. Based on the response matrices R and the simulated recoil proton spectra at 0° and 45°, the respective unfolded neutron spectra were obtained using the iteration algorithm, and compared with the assumed neutron spectrum. The results show that the iteration algorithm method can be applied to unfold the recoil proton energy spectrum to the neutron energy spectrum for D-T neutron energy spectra measurement using the recoil proton method.

Key words: D-T neutron source, recoil proton energy spectrum, D-T neutron energy spectrum, iteration algorithm method

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1 Introduction

In 1988, a 3.3×10^{12} n/s neutron generator (ZF-300) based on T(d, n)^{4}He (D-T) reactions with a rotating target was built and developed at Lanzhou University [1]. It has been applied in the research fields of nuclear data measurements, radiation hardening and radioactive breeding [2]. In applications of the D-T neutron generator, the neutron energy spectrum is one of the most important parameters. In previous studies, a mathematical method was developed to calculate the energy spectrum from D-T reactions in a thick tritium-titanium target for incident deuteron energy lower than 1.0 MeV [3]. In addition, Monte-Carlo simulation research and experimental measurements using a nuclear emulsion detector have been carried out for the neutron energy spectrum of the ZF-300 D-T neutron generator [4]. However, because the energy resolution of nuclear emulsion detectors is poor, the agreement between the simulation results and the experimental data is not very good.

In recent years, scintillation detectors have been used to measure D-T fast neutron energy spectra [5–7]. However, scintillation detectors need complex electronic systems to distinguish the neutron signals and γ-ray signals [8]. In order to avoid the use of complex electronic systems, we put forward a proton recoil method for measuring the D-T neutron energy spectrum using polyethylene film and a Si(Au) surface barrier detector. An iteration algorithm for unfolding the recoil proton spectrum to obtain the neutron energy spectrum will be investigated in this paper.

2 Principles

The basic principle for measuring the D-T fast neutron spectrum using the recoil proton method is shown in Fig. 1. Fast neutrons from the D-T reaction in the θ direction are collimated to bombard a polyethylene (PE) film. A thick depletion layer Si(Au) surface barrier detector is installed in the α direction to measure the recoil

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proton energy spectrum. The measured recoil proton energy spectrum is unfolded to obtain the fast neutron energy spectrum.

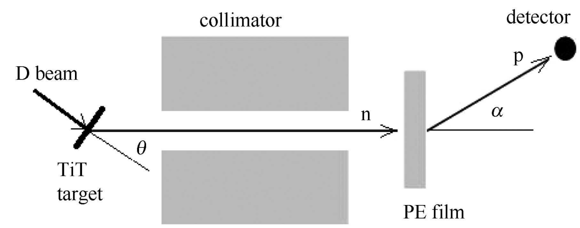

Fig. 1. Principle of D-T neutron energy spectrum measurement using a recoil proton method.

3 Unfolding algorithm

An iteration algorithm based on the SAND-II method [9, 10] is used to unfold the recoil proton energy spectrum to the fast neutron energy spectrum. Let the matrix N represent the recoil proton energy spectrum data achieved from the experiment,

$$N=\begin{bmatrix} N_1 \\ N_2 \\ \vdots \\ N_n \end{bmatrix}, \tag{1}$$

where n is the energy bin number of the recoil proton spectrum data and N_n is the proton count in the n-th bin, so N should be the linear superposition of all recoil proton energy spectra produced by mono-energetic neutrons in the incident neutron beam bombarding the PE film. It is assumed that the number of mono-energetic neutron sources with different neutron energies is m. The recoil proton spectrum data produced by every mono-energetic neutron on PE film will form a column matrix. All of the recoil proton spectrum data will then form a matrix R with n lines and m columns. The matrix R, which is called the response matrix or response function of the polyethylene film, can be expressed by the following equation,

$$R=\begin{bmatrix} R_{11} & R_{12} & \cdots & R_{1m} \\ R_{21} & R_{22} & \cdots & R_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ R_{n1} & R_{n2} & \cdots & R_{nm} \end{bmatrix}, \tag{2}$$

where n is the energy bin number of the recoil proton spectrum data and m is the number of mono-energetic neutron sources. According to the principle of linear superposition, the relationship between the matrix N and the matrix R can be written as the following equation,

$$\begin{bmatrix} N_1 \\ N_2 \\ \vdots \\ N_n \end{bmatrix}=\begin{bmatrix} R_{11} & R_{12} & \cdots & R_{1m} \\ R_{21} & R_{22} & \cdots & R_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ R_{n1} & R_{n2} & \cdots & R_{nm} \end{bmatrix}\cdot\begin{bmatrix} \phi_1 \\ \phi_2 \\ \vdots \\ \phi_m \end{bmatrix}, \tag{3}$$

where ϕ_1, $\phi_2\cdots\phi_m$ are the coefficients of linear superposition. They represent the relative contribution to the recoil proton spectra (N) of different energy neutrons in the incident neutron beam, and can be regarded as the neutron energy spectrum data. However, it is difficult to solve Eq. (3) accurately because R $(n\times m)$ is an ill-conditioned matrix.

An iteration algorithm will be used to obtain the neutron energy spectrum based on the recoil proton spectrum. Eq. (3) can be derived into the following form

$$N_i=\sum_{j=1}^{m}(R_{ij}\cdot\phi_j)=\sum_{j=1}^{m}\{R_{ij}\cdot\exp(\ln\phi_j)\}\ i=1,2,\cdots,n, \tag{4}$$

where R_{ij} is the i, j-th element of the polyethylene film response matrix R and ϕ_j is the neutron count in the j-th energy bin. Under an assumed initial neutron energy spectrum $\phi_j^{(1)}$, $\ln N_i$ in Eq. (4) can be expanded into the following Taylor series, truncated by ignoring higher order terms after the second term:

$$\ln N_i=\ln N_i^{(1)}+\sum_{j=1}^{m}w_{ij}^{(1)}(\ln\phi_j-\ln\phi_j^{(1)})+\cdots\ i=1,2,\cdots,n, \tag{5}$$

where

$$N_i^{(1)} = \sum_{j=1}^{m}\{R_{ij}\cdot\exp(\ln\phi_j^{(1)})\}, \tag{6}$$

$$w_{ij}^{(1)} = \frac{R_{ij}\cdot\exp(\ln\phi_j^{(1)})}{N_i^{(1)}}. \tag{7}$$

In the iteration algorithm, χ^2 is defined to restrain the iteration results, and is written as

$$\chi^2=\sum_{i=1}^{n}\left\{(\ln N_{0i}-\ln N_i)\cdot\frac{1}{\rho_i^2}\right\}, \tag{8}$$

where N_{0i} is the measured proton spectrum data in the i-th energy bin and ρ_{0i} is the relative standard deviation of N_{0i}, $\rho_{0i}=\dfrac{1}{\sqrt{N_{0i}}}$. To find the minimum value of χ^2 in Eq. (4), let

$$\frac{\partial\chi^2}{\partial\ln N_i}=0, \tag{9}$$

Then, an iterative equation for computing the neutron energy spectrum can be derived as in [11]:

$$\ln\phi_j^{(k+1)}-\ln\phi_j^{(k)} = \frac{\sum_{i=1}^{n}\left(\ln N_{0i}-\ln\sum_{j=1}^{m}[R_{ij}\cdot\exp(\ln\phi_j^{(k)})]\right)}{\sum_{i=1}^{n}\frac{w_{ij}^{(k)}}{\rho_{0i}^2}}\cdot\frac{w_{ij}^{(k)}}{\rho_{0i}^2}, \quad (10)$$

where, $\phi_j^{(k)}$ and $\phi_j^{(k+1)}$ are the neutron spectra corresponding to the k-th and (k+1)-th iteration steps, respectively. After selecting an initial input neutron spectrum $\phi_j^{(1)}$, the iterative step will be repeated until the iterative spectrum data in every energy interval satisfy the following criterion:

$$\left|\phi_j^{(k+1)}-\phi_j^{(k)}\right|\leqslant\delta, \quad (11)$$

where, δ is set to a small value to control the precision of the iterative result. The final iterated value will be regarded as the solution of the neutron energy spectrum.

4 Simulation and test

In this section, a simulation method and the simulated recoil proton spectrum data are used to verify the feasibility of the above iterative method. Firstly, the response matrix R of the mono-energetic neutrons at the polyethylene film should be confirmed according to the above iteration algorithm.

4.1 Response matrix *R*

In fact, the experimental verification of the matrix data is difficult because of the lack of mono-energetic neutron sources. The elastic scattering cross section for the n-p reaction is well known, however. In this investigation, the response matrix data will be determined by Monte-Carlo simulation using the MCNPX code [12].

To simulate the response matrix R, a MCNPX model is established. Let a mono-energetic neutron beam with a diameter of 2 μm be incident on the polyethylene film. The thickness of the polyethylene film is selected as 20 mg·cm^{-2}. Based on the energy region of D-T neutron energy spectra, the variation range of energy of the mono-energetic neutron beam is set from 1 MeV to 16 MeV with intervals of 0.1 MeV. The F4 card in the MCNPX code is laid out in the α degree direction to record the recoil proton energy spectra. The energy range of the recoil proton energy spectra is set from 1 MeV to 17 MeV with intervals of 0.1 MeV. Result data for the recoil proton energy spectra are shown in Fig. 2(a) and in Fig. 2(b), corresponding to the directions α=0° and α=45°, respectively. The response matrices of PE film with thickness of 20 mg·cm^{-2} corresponding to the α=0° and α=45° directions can be made up with the respective simulated proton spectra results. The response matrix corresponding to an arbitrary angle can also be structured with the same simulation method.

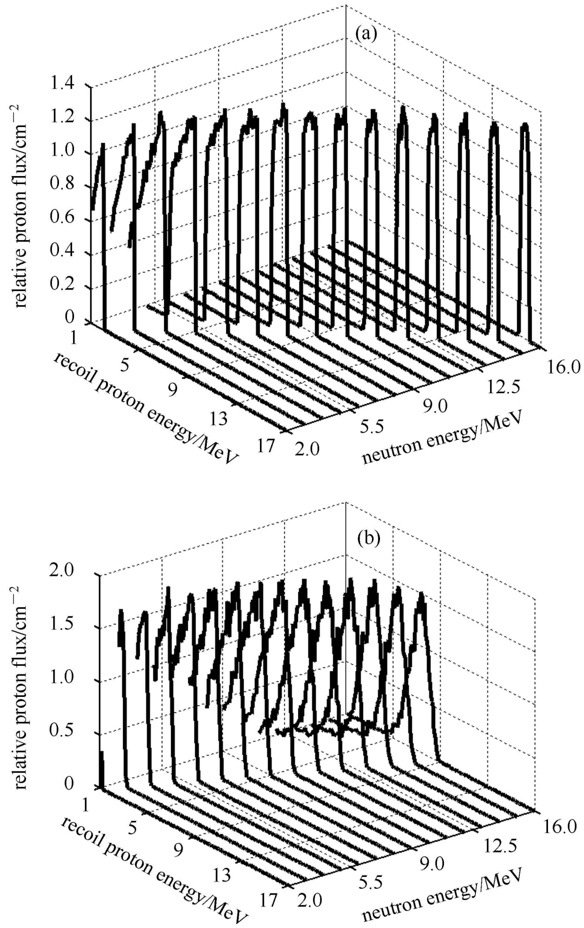

Fig. 2. Response data of the recoil proton energy spectra for (a) α=0°, (b) α=45°.

4.2 Method test and result

In order to verify the feasibility of the above iterative method, a MCNPX simulation model is built. As shown in Fig. 1, a D-T neutron beam with a diameter of 2 μm is assumed to be incident on PE film with a thickness of 20 mg·cm^{-2}. Fig. 3 shows the energy spectrum of the D-T neutron beam, based on the previous calculation result [3]. The F4 card in the MCNPX code is laid out in the α=0° and α=45° directions to record the energy spectra of the recoil protons. The energy interval of the recoil proton energy spectra is also set to 0.1 MeV. The simulation results are shown in Fig. 3.

Based on the iterative algorithm shown in Eq. (10), a computer program was developed using the C^{++} code. The response matrix (R) data in Fig. 2 and the recoil proton energy spectrum data in Fig. 3 at 0° and 45° are input into the program to obtain the neutron energy spectra by the iterative algorithm. The δ value is set as 10^{-5}. The unfolded neutron energy spectra using the recoil proton energy spectrum data corresponding to the detector at α=0° (after 12 iteration steps) and α=45° (after 150 iteration steps) are shown in Fig. 4 and Fig. 5 respectively. The assumed incident neutron spectrum is also shown in Fig. 4 and Fig. 5 for comparison.

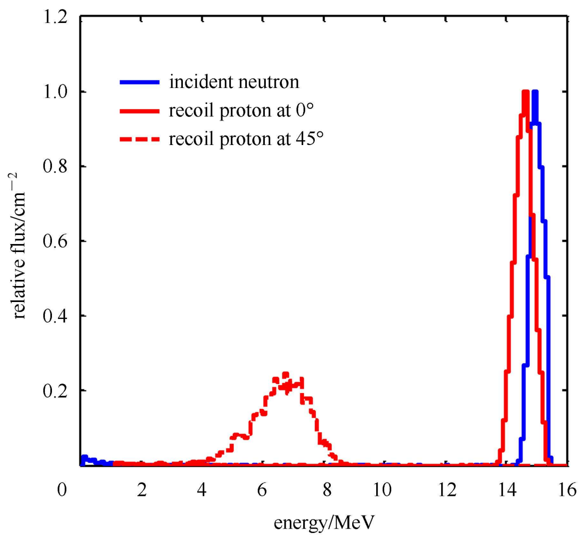

Fig. 3. (color online) The incident neutron energy spectrum (blue line) and the recoil proton energy spectrum (red line) at α=0° and α=45°.

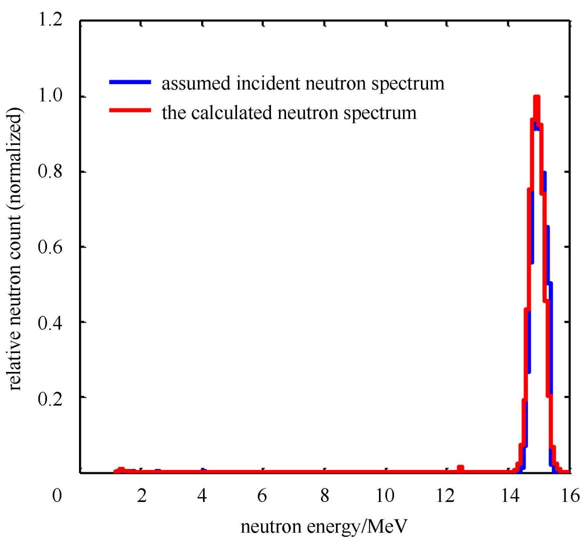

Fig. 4. (color online) Comparison between the unfolded neutron spectrum corresponding to the detector at α=0° and the assumed incident neutron spectrum.

Figure 4 shows that the unfolded neutron spectrum is in perfect agreement with the assumed incident neutron spectrum when the recoil proton energy spectrum corresponding to the detector at α=0° is employed. The full width at half maximum (FWHM) of the incident neutron spectrum and the unfolded neutron spectrum are respectively 0.542 MeV and 0.536 MeV. From Fig. 5, the FWHM of the unfolded neutron spectrum for using the recoil proton energy spectrum corresponding to the detector at α=45° is 0.542 MeV, which agrees with the FWHM of the incident neutron spectrum. However, the width of the unfolded neutron spectrum in Fig. 5 is significantly greater than the incident spectrum at the tail region. One reason may be that the response matrix (R) and recoil proton spectrum themselves are broadening, another reason may be that the iteration algorithm still needs to be optimized. In a word, the calculated neutron spectrum result can be accepted for a neutron spectrum experiment from the D-T reaction.

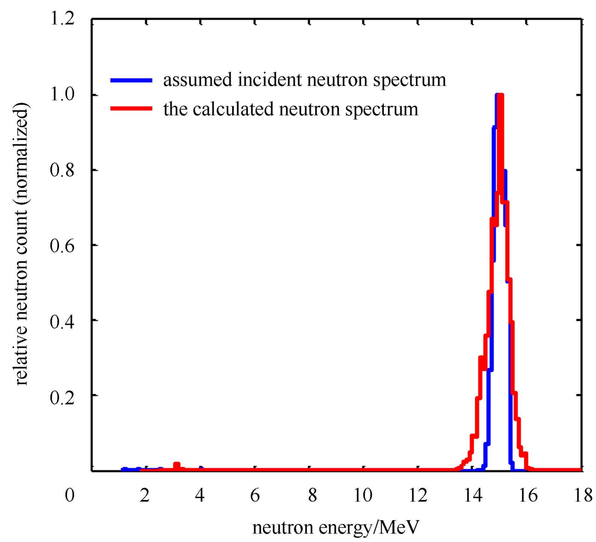

Fig. 5. (color online) Comparison between the unfolded neutron spectrum corresponding to the detector at α=45° and the assumed incident neutron spectrum.

5 Conclusions

A proton recoil method for measuring D-T neutron energy spectra using polyethylene film and a Si(Au) surface barrier detector has been proposed. The unfolding study results show that the developed iteration algorithm can be applied to obtain better neutron energy spectra. The unfolding neutron energy spectrum for the detector located at α=0° is obviously better than the unfolding neutron energy spectrum for the detector located at α=45°. However, when the detector is located at α=0°, the collimated fast neutron beam will be directly incident on the Si detector and lead to damage of the detector. Additionally, protons and alpha particles from (n, p) and (n, α) reactions produced by fast neutrons in the Si detector will cause interference for the recoil proton energy spectrum measurement. Based on the above reasons, the

Si detector should be placed in the shadow zone of the collimator to achieve good shielding for the Si detector (see Fig. 1). It may be a good choice to locate the Si detector at α=45°.

The main purpose of this research is to confirm the practicability of the unfolding algorithm, and the energy resolution of the Si detector is not considered when it is employed to measure the recoil proton energy spectrum. The energy resolution of a Si detector with 2000 μm depletion layer is lower than 1% for ^{241}Am 5.48 MeV alpha particles. A better energy resolution should be obtained for protons in the energy region of 4–16 MeV.

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