Analysis of the energy spectra of ground states of deformed nuclei in the rare-earth region*

A. A. Okhunov[1,3;1)] G. I. Turaeva[1] H. A. Kassim[2] M. U. Khandaker[2] Noora B. Rosli[2]

[1] Department of Science in Engineering, International Islamic University Malaya, Kuala Lumpur 50728, Malaysia
[2] Department of Physics, University of Malaya, Kuala Lumpur 50603, Malaysia
[3] Institute of Nuclear Physics, Academy of Science Republic of Uzbekistan, Tashkent 100214, Uzbekistan

Abstract: The $_{62}$Sm, $_{64}$Gd, $_{66}$Dy, $_{70}$Yb, $_{72}$Hf and $_{74}$W nuclei are classified as deformed nuclei. Low-lying bands are one of the most fundamental excitation modes in the energy spectra of deformed nuclei. In this paper a theoretical analysis of the experimental data within the phenomenological model is presented. The energy spectra of ground states are calculated. It is found that the low-lying spectra of ground band states are in good agreement with the experimental data.

Key words: ground state, energy spectra, rare-earth, deformed nuclei

PACS: 21.10.-k, 21.60.-n **DOI:** 10.1088/1674-1137/39/4/044101

1 Introduction

Though the structure of rotational deformed nuclei and the nature of low excited levels have been studied for 50 years, this problem is still relevant today.

Nowadays, there are accumulations of rich experimental data on the excited and low excited states of deformed nuclei, which are needed to develop their theoretical research.

The phenomenological nuclear adiabatic model described by Bohr and Mottelson [1] has played a big role in understanding the properties of deformed nuclei. According to this model, low excitation states of even-even deformed nuclei are connected with a rotation of the axial-symmetric nucleus as a whole. Such a simple phenomenological explanation allows a description of a large number of experimental data on deformed nuclei and predicts many new properties of these nuclei.

In the present paper the deviations from the adiabatic theory which appear in energies of ground band states is analyzed within the phenomenological model [2, 3], which take into account the Coriolis mixture of low-lying state bands. The objects of calculation are $^{152-156}$Sm, $^{156-166}$Gd, $^{156-166}$Dy, $^{166-176}$Yb, $^{170-180}$Hf and $^{174-184}$W, isotopes. These nuclei have been quite well studied experimentally, such as in nuclear reactions and Coulomb excitation [4–6]. Also in Ref. [7] theoretically investigated the properties of ground-state in the rare-earth even-even nuclei, applying the deformed relativistic mean-field model with the FSUGold.

The experiments obtain the systematical measurement of the properties of low-lying states.

2 Energy spectra of gr- state bands

The energy of the rotational core $E_{\rm rot}(I)$ is in agreement with the energy of the ground state of rotational bands of even-even deformed nuclei in the lower value of spin I.

The effective angular frequency of the rotating nucleus is defined as follows:

$$\omega_{\rm eff}(I)=\frac{E^{\rm exp.}(I+1)-E^{\rm exp.}(I-1)}{2}, \qquad (1)$$

and hence the effective moment of inertia for states $\Im_{\rm eff}(I)$ in terms of the angular frequency of rotation $\omega_{\rm rot}(I)$ is

$$\Im_{\rm eff}(I)=\frac{d(I(I+1))^{1/2}}{\omega_{\rm eff}(I)}. \qquad (2)$$

From Eq. (2), we can calculate the effective moment of inertia $\Im_{\rm eff}(I)$ and the effective angular frequency of rotation of the nuclei $\omega_{\rm eff}(I)$ is obtained by Eq. (1), with the energy $E^{\rm exp.}(I)$ from experiment [8]. At a low angular frequency of rotation, i.e. in low spin $I\leqslant 8\ \hbar$ the dependency is linear. We parameterize this dependency as follows:

$$\Im_{\rm eff}=\Im_0+\Im_1\omega^2_{\rm eff}(I). \qquad (3)$$

Received 26 May 2014

* Supported by IIUM University Research Grant Type B (EDW B13-034-0919) and MOHE Fundamental Research Grant Scheme (FRGS13-074-0315)

1) E-mail: aokhunov@yahoo.com

Equation (3) defines the parameters of inertia $\Im_0$ and $\Im_1$, for the effective moment of inertia $\Im_{\text{eff}}(I)$ when $I\leqslant 8\hbar$. The numerical values for the parameters $\Im_0$ and $\Im_1$ are determined using the least squares method in Eq. (3).

The energy of the rotational ground band state is calculated using the parameters $\Im_0$ and $\Im_1$ by Harris parametrization for the energy and angular momentum [9].

$$E_{\text{rot}}(I) = \frac{1}{2}\Im_0\omega_{\text{rot}}^2(I)+\frac{3}{4}\Im_1\omega_{\text{rot}}^4(I). \quad (4)$$

$$\sqrt{I(I+1)} = \Im_0\omega_{\text{rot}}(I)+\Im_1\omega_{\text{rot}}^3(I). \quad (5)$$

Where $\omega_{\text{rot}}(I)$ is the rotational angular frequency of nuclei, which is defined by solving cubic Eq. (5) [3].

3 Results and discussion

To analyze the properties of states of the ground band in deformed nuclei, the phenomenological model of [2] has been utilized.

The numerical values for the parameters $\Im_0$ and $\Im_1$ are determined using the least squares method in Eq. (3). These results are shown in Table 1, for the isotopes ^{152,154}Sm, $^{158-162}$Dy and $^{172-176}$Yb respectively ($\Im_0$ in MeV^{-1} and in MeV^{-3}).

The calculated values of the angular frequency and energy spectra of Sm, Dy and Yb isotopes are illustrated in Table 2 respectively. All of the predicted data seem to agree with the experimental data.

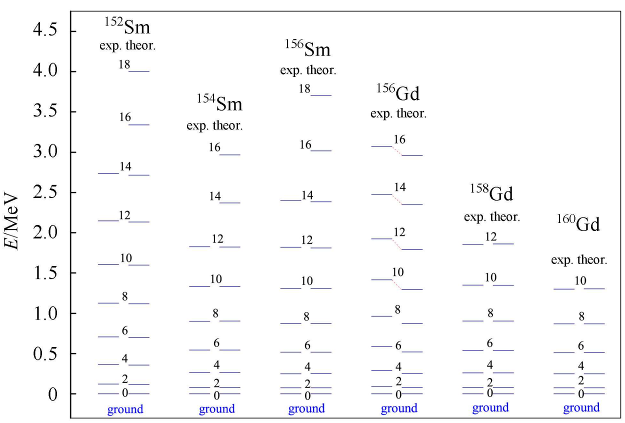

Fig. 1. (color online) Comparison of the experimental and theoretical spectra energy of ground bands for isotopes Sm and Gd, correspondingly.

The comparison between the calculated values of energy of the ground band state with experimental data [8] is given for the isotopes $_{62}$Sm, $_{64}$Gd, $_{66}$Dy, $_{70}$Yb, $_{72}$Hf and $_{74}$W in Figs. 1–5, respectively. From the figures, we see that the energy difference $\Delta E(I)=E^{\text{theor}}(I)-E^{\text{exp.}}(I)$, increases with the increase in the angular momentum I. This is due to the occurrence of the non-adiabaticity of the energy rotational bands in large spin.

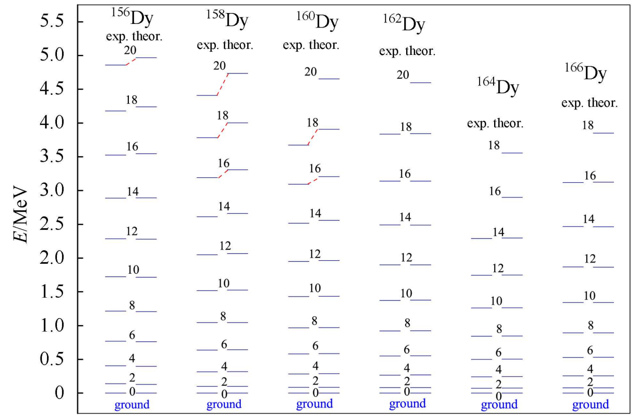

Fig. 2. (color online) Comparison of the experimental and theoretical spectra energy of ground bands for isotopes Dy.

Table 1. The values of parameters $\Im_0$, $\Im_1$, E_{2+} (MeV) and intrinsic quadrupole moment Q_0 for isotopes Sm, Gd, Dy, Yb, Hf and W.

A	$\Im_0$	$\Im_1$	$E_{2+}^{\text{exp.}}$	E_{2+}^{theory}	Q_0 [8]
^{152}Sm	24.74	256.57	0.1218	0.1234	5.83 (057)
^{154}Sm	36.07	178.88	0.0724	0.0841	6.53 (15)
^{156}Sm	39.22	98.36	0.0760	0.0778	7.85 (79)
^{156}Gd	38.74	95.25	0.0890	0.0769	6.76 (34)
^{158}Gd	37.52	107.00	0.0795	0.0795	7.03 (4)
^{160}Gd	39.72	83.49	0.0755	0.0752	7.16 (2)
^{156}Dy	21.93	238.14	0.1370	0.1290	6.12 (6)
^{158}Dy	29.69	174.26	0.0989	0.0990	6.85 (8)
^{160}Dy	33.96	131.07	0.0898	0.0870	6.91 (20)
^{162}Dy	36.61	105.77	0.0806	0.3350	7.13
^{164}Dy	40.25	121.09	0.0734	0.0740	7.49
^{166}Dy	38.68	73.82	0.0766	0.0770	–
^{166}Yb	28.75	132.20	0.1024	0.1032	7.26 (14)
^{168}Yb	33.34	148.77	0.0877	0.0890	7.62 (14)
^{170}Yb	35.06	87.82	0.0843	0.0850	7.80 (30)
^{172}Yb	37.60	81.31	0.0788	0.0790	7.91 (18)
^{174}Yb	38.71	76.65	0.0765	0.0770	7.82 (24)
^{176}Yb	36.00	63.96	0.0821	0.0830	7.59 (3)
^{170}Hf	27.93	248.95	0.1008	0.1007	7.14 (30)
^{172}Hf	30.02	172.45	0.0953	0.0953	6.87 (18)
^{174}Hf	31.09	134.03	0.0900	0.0901	7.29 (24)
^{176}Hf	33.70	92.27	0.0884	0.0877	7.23 (8)
^{178}Hf	31.95	71.88	0.0932	0.0928	6.98 (4)
^{180}Hf	32.06	36.53	0.0933	0.0931	6.93 (3)
^{174}W	25.06	212.67	0.1130	0.1136	–
^{176}W	25.91	177.61	0.1091	0.1097	–
^{178}W	26.87	143.47	0.1061	0.1069	–
^{180}W	27.51	127.14	0.1036	0.1030	6.24 (11)
^{182}W	28.31	110.50	0.1001	0.1045	6.57 (8)
^{184}W	29.22	98.36	0.1112	0.1015	6.27 (8)

Table 2. The value of angular frequency and energy spectra for isotopes Sm, Dy and Yb (E in MeV).

I	^{152}Sm			^{154}Sm			^{156}Sm		
	$\omega_{\rm rot}^{\rm theor}(I)$	$E_{\rm rot}^{\rm exp.}(I)$	$E_{\rm rot}^{\rm theor}(I)$	$\omega_{\rm rot}^{\rm theor}(I)$	$E_{\rm rot}^{\rm exp.}(I)$	$E_{\rm rot}^{\rm theor}(I)$	$\omega_{\rm rot}^{\rm theor}(I)$	$E_{\rm rot}^{\rm exp.}(I)$	$E_{\rm rot}^{\rm theor}(I)$
2^+	0.091	0.122	0.116	0.066	0.082	0.082	0.062	0.076	0.076
4^+	0.147	0.366	0.360	0.116	0.267	0.268	0.111	0.250	0.251
6^+	0.190	0.707	0.701	0.160	0.544	0.546	0.156	0.517	0.519
8^+	0.225	1.126	1.119	0.197	0.903	0.904	0.197	0.872	0.874
10^+	0.254	1.609	1.599	0.230	1.333	1.333	0.235	1.307	1.307
12^+	0.280	2.149	2.134	0.260	1.826	1.824	0.269	1.819	1.812
I	^{156}Dy			^{158}Dy			^{160}Dy		
2^+	0.101	0.138	0.129	0.080	0.099	0.100	0.071	0.087	0.087
4^+	0.160	0.404	0.396	0.136	0.317	0.319	0.124	0.284	0.286
6^+	0.204	0.770	0.763	0.183	0.638	0.640	0.171	0.581	0.584
8^+	0.239	1.216	1.207	0.222	1.044	1.046	0.213	0.967	0.970
10^+	0.268	1.725	1.716	0.255	1.520	1.525	0.249	1.429	1.433
12^+	0.294	2.286	2.280	0.285	2.049	2.067	0.282	1.951	1.965
I	^{162}Dy			^{164}Dy			^{166}Dy		
2^+	0.066	0.081	0.081	0.060	0.073	0.074	0.063	0.077	0.077
4^+	0.117	0.266	0.268	0.107	0.242	0.244	0.113	0.254	0.255
6^+	0.164	0.549	0.551	0.151	0.501	0.504	0.160	0.527	0.530
8^+	0.206	0.921	0.924	0.190	0.844	0.846	0.203	0.892	0.894
10^+	0.244	1.375	1.376	0.226	1.261	1.264	0.244	1.341	1.342
12^+	0.279	1.901	1.900	0.258	1.745	1.749	0.281	1.868	1.867
I	^{166}Yb			^{168}Yb			^{170}Yb		
2^+	0.083	0.102	0.103	0.072	0.088	0.089	0.069	0.084	0.085
4^+	0.142	0.331	0.332	0.125	0.287	0.289	0.123	0.277	0.280
6^+	0.193	0.668	0.670	0.172	0.585	0.589	0.172	0.574	0.577
8^+	0.235	1.098	1.100	0.212	0.970	0.975	0.217	0.964	0.967
10^+	0.272	1.606	1.610	0.247	1.426	1.435	0.257	1.438	1.442
12^+	0.305	2.176	2.188	0.278	1.936	1.962	0.293	1.984	1.993
I	^{172}Yb			^{174}Yb			^{176}Yb		
2^+	0.065	0.079	0.079	0.063	0.076	0.077	0.067	0.082	0.083
4^+	0.116	0.260	0.262	0.113	0.253	0.255	0.121	0.272	0.274
6^+	0.163	0.540	0.543	0.159	0.526	0.529	0.171	0.565	0.568
8^+	0.207	0.912	0.914	0.203	0.890	0.892	0.217	0.955	0.958
10^+	0.247	1.370	1.368	0.243	1.336	1.339	0.260	1.431	1.437
12^+	0.283	1.907	1.899	0.279	1.861	1.862	0.299	1.985	1.998

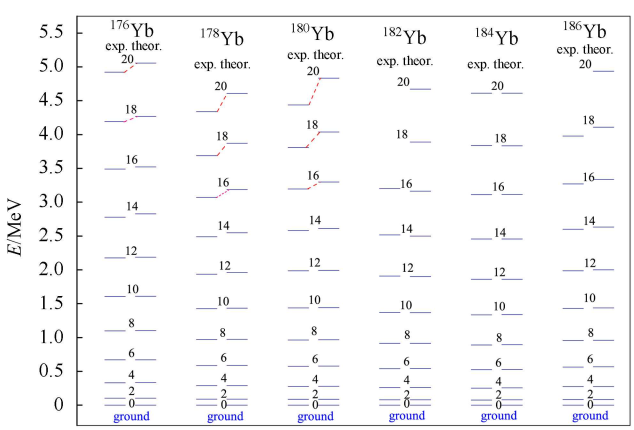

Fig. 3. (color online) Comparison of the experimental and theoretical spectra energy of ground bands for isotopes Yb.

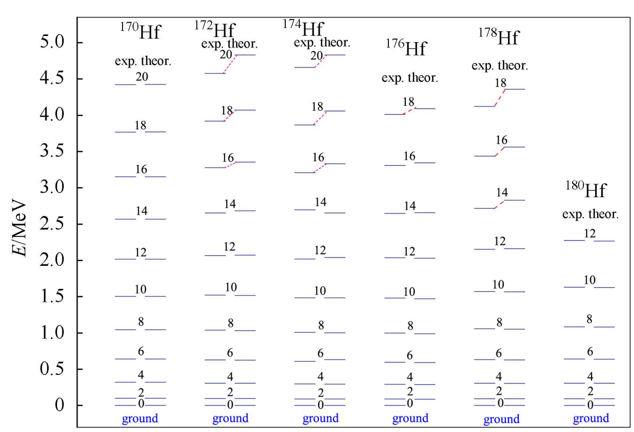

Fig. 4. (color online) Comparison of the experimental and theoretical spectra energy of ground bands for isotopes Hf.

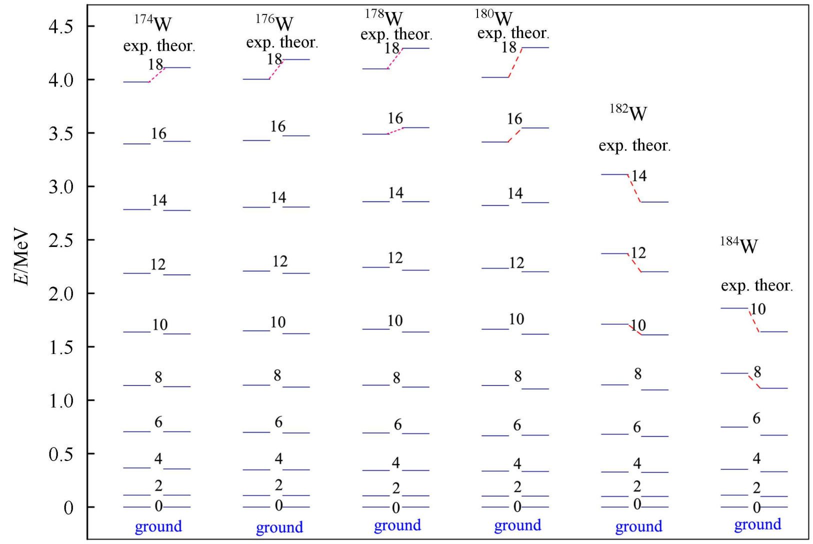

Fig. 5. (color online) Comparison of the experimental and theoretical spectra energy of ground bands for isotopes W.

4 Conclusions and further studies

This work is based on the phenomenological model [2, 3], which clearly describes a large number of experimental data by the deviation properties of the positive parity in even-even deformed nuclei from the role of adiabatic theory. Spectral energy of ground states for the isotopes $^{152-156}$Sm, $^{156-166}$Gd, $^{156-166}$Dy, $^{166-176}$Yb, $^{172-182}$Hf and $^{172-176}$W were calculated which show the violation in the $E(I)\sim I(I+1)$ law. This is explained by the fact that the nuclear core is under rotation with the large mixture frequency of ground-state bands with other rotational bands that have vibrational characters. The calculation takes into account the Coriolis mixing of positive parity states which has good agreement with experimental data.

The author A. A. Okhunov is grateful to Prof. Ph.N. Usmanov for useful discussion and exchange of ideas.

References

1 Bohr A, Mottelson B R. Nuclear Structure. Benjamin, New York, 1997. 1.2
2 Usmanov Ph N, Mikhailov I N. Phys. Part. Nucl. Lett., 1997, **28**: 348
3 Usmanov Ph N, Okhunov A A, Salikhbaev U S, Vdovin A I. Phys. Part. Nucl. Lett., 2010, **7**(3): 185
4 Reich C W. Nucl. Data Sheets, 2003, **99**: 753
5 Helmer R G. Nucl. Data Sheets, 1996, **77**: 481
6 Reich C W. Nucl. Data Sheets, 2005, **105**: 555
7 SHENG Z et al. Nucl. Phys. A, 2010, **832**: 49
8 Begzhanov R B, Belinkiy V M, Zalyubovskiy I I, Kuznichenko A B. Handbook on Nuclear Physics: Tashkent, FAN, 1989, 1.2
9 Harris S M. Phys. Rev. B, 1965, **138**: 509