Meson polarized distribution function and mass dependence of the nucleon parton densities

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Abstract: The polarized distribution functions of mesons, including pion, kaon and eta, using the proton structure function, are calculated. We are looking for a relationship between the polarized distribution of mesons and the polarized structure of nucleons. We show that the meson polarized parton distributions leads to zero total spin for the concerned mesons, considering the orbital angular momentum of quarks and gluons inside the meson. Two separate Monte Carlo algorithms are applied to compute the polarized parton distributions of the kaon. Via the mass dependence of quark distributions, the distribution function of the eta meson is obtained. A new method by which the polarized sea quark distributions of protons are evolved separately – which cannot be performed easily using the standard solution of DGLAP equations – is introduced. The mass dependence of these distributions is obtained, using the renormalization group equation which makes their evolutions more precise. Comparison between the evolved distributions and the available experimental data validates the suggested solutions for separated evolutions.

Key words: Valon model, polarized chiral quark model, constituent quark, renormalization group equation, evolution operator

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1 Introduction

The effective Lagrangian has been written by Manohar [1] to justify the chiral symmetry of quarks. This model can be extended to polarized parton distributions and many studies have used it to find the distributions of quarks inside nucleons [2]. There are also many studies which calculate the polarized parton distribution function (PPDF) of mesons, based on lattice QCD computations [3] or other approaches [4, 5]. We know that due to the orbital angular momentum of quarks and gluons inside hadrons, the + and − helicity distributions do not compensate each other exactly [6]. If we extend our theoretical framework to the case where the meson mass corrections and higher twist effects are considered, then it is possible to consider the longitudinal polarization for pion parton distribution. It turns out that the transverse polarization, which is denoted by g_2 [7], is connected with the longitudinal polarization structure g_1 via the Wandzura-Wilczek relation [8]. In a similar fashion, we can also consider some extra effects due to meson-mass correction which will lead us to additional longitudinal polarization for the partons of pseudo-scalar mesons. The structure of meson-mass corrections in inclusive processes is in general more complicated than that of target-mass corrections in deep inelastic scattering, which can be re-summed using the Nachtmann variable. The twist approximation which is used for the amplitude distribution of pions can also be employed in deep inelastic scattering processes [9]. In addition to this theoretical justification for assigning longitudinal polarization to the partons of pseudo-scalar mesons, we can also consider the diffraction effect, using the factorization theorem for the hard exclusive electro-production of mesons in QCD. The full theorem applies to all kinds of meson and not just to vector mesons. The parton densities used include not only the ordinary parton densities, but also the helicity densities.

In this article we try to calculate the PPDF of mesons using the (definite) PPDF of nucleons. This work contains two separate parts:

1) Computing the polarization densities and orbital angular momentum of quarks and gluon inside the meson.

2) Evolving the sea quark distributions of nucleons (in which their symmetry is broken) separately, using the renormalization group equation for the running mass of quarks.

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In Part 1, we calculate the bare quark distributions using the proton polarized structure function $g_1^{\mathrm{p}}(x)$, using data from Ref. [10]. Then we compute the ratio of the polarized valence data of kaons to that of pions, $\delta q^{\mathrm{K}}_{\mathrm{val}}/\delta q^{\pi}_{\mathrm{val}}$, using the data for their unpolarized ratio, $q^{\mathrm{K}}_{\mathrm{val}}/q^{\pi}_{\mathrm{val}}$ [11], based on two separate Monte Carlo algorithms. We also calculate the polarized valence ratio of eta mesons to pions, $\delta q^{\eta}_{\mathrm{val}}/\delta q^{\pi}_{\mathrm{val}}$, using the mass dependence of the valence quark distribution inside the meson. Substituting these ratios into the chiral quark model (χQM) equations and fitting with experimental data (or any reasonable phenomenological model), the polarized distribution functions in pion, kaon and eta mesons at low energy scales will be obtained. Following that, the evolution of the parton distribution functions (PDFs), employing the DGLAP equations, can be done straightforwardly [12–15]. Using these evolved PDFs, we can extract the values of the orbital angular momentum of quarks and gluons inside mesons [6].

In Part 2, the PPDFs of the proton, using the distributions extracted for mesons, are calculated. The valence PPDF of the proton can be evolved easily using the non-singlet moment δM_{NS}. Since the DGLAP equations can thoroughly evolve only the sea quark distribution, however, the evolution of the separated sea quarks is more complicated. There are reasonable methods to separate the evolution of sea quarks [16] but in this work we use the running mass and renormalization equation to make the sea quark distribution functions depend on the quark masses. Thereby, the eigenvalues of the evolution operator become non-degenerate. The sea quark distributions at low scale Q_0^2, arising from χQM, are unsymmetrized. The different eigenvalues of the evolution operator, which are obtained as a result of the new method introduced in this paper, makes the evolution of sea quark densities more distinctive than the result obtained in Ref. [17]. Two boundary conditions at low- and high-energy scales are applied to the equations to test the sea quark spectrum. Finally, a comparison to experimental data is carried out for sea and valence distributions [18–20].

This paper is organised as follows. In Section 2 we review the basic concepts of χQM in the unpolarized case. The extension of this model to the polarized case is done in Section 3. In Section 4 we deal with a method to extract the polarized bare quark distributions inside the proton. In Section 4.1, two Monte Carlo algorithms are introduced which give us the polarized valence distributions of the kaon and in Section 4.2 we calculate the distribution function of the eta meson, using the fact that the masses of the quarks gives different distributions for the various quark flavours. The parton orbital angular momentum inside the meson and the spin of the meson is discussed in Section 5. In Section 6 we use the renormalization group equation for the running mass of quarks to get the separated evolution operators for nucleon sea quark densities. We give our conclusions in Section 7.

2 Unpolarized chiral quark model

Our calculations are based on the constituent quark Fock state using the chiral quark model, χQM [1]. According to this model, spontaneous chiral symmetry breaking creates Goldstone (GS) bosons which couple to the constituent quarks. The low-energy dynamics ($\mu \leqslant 4\pi f_\pi \sim 1$ GeV where $f_\pi \approx 93$ MeV is the pion decay constant) is governed by the GS bosons, in particular the pion, which is the approximate zero mode of the QCD vacuum [1, 2, 21]. The diagrams responsible for this process are as shown in Fig. 1.

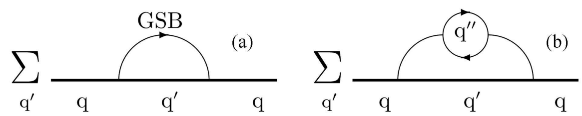

Fig. 1. At low energy, the bare quarks are dressed as indicated by diagram (a). At higher energies, probing reveals the structure of GS bosons (pion, kaon and eta) as is shown in diagram (b). This figure has been adapted from Ref. [17].

The interaction Lagrangian of the effective chiral quark theory in the leading order of an expansion in Π/f is given by [1]:

$$\mathcal{L}=-\frac{g_{\mathrm{A}}}{f}\bar{\psi}(\partial_\mu \Pi)\gamma^\mu\gamma_5\psi, \tag{1}$$

while the GS boson matrix field is written as:

$$\Pi=\frac{1}{\sqrt{2}}\begin{pmatrix} \frac{\pi^0}{\sqrt{2}}+\frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}}+\frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta}{\sqrt{6}} \end{pmatrix}. \tag{2}$$

Using the notation of Ref. [2], we can write the constituent U and D quark Fock-state as:

$$|q\rangle=\sqrt{Z}\,|q_0\rangle+\sum_M \alpha_M\,|qM\rangle, \tag{3}$$

where Z is the renormalization constant for a "bare", $|q_0\rangle$, constituent quark and we have absorbed all coefficients in α_M. The $|qM\rangle$ indicates the quark states dressed by GS bosons. Eq. (3) will yield the one-point Fock state contribution and ignores higher order approximations (see Fig. 1).

In the unpolarized case, the splitting function which gives the probability to convert a parent constituent quark q into a constituent quark q′ carrying the light-cone momentum fraction $(1-x_{\mathrm{M}})$, and a spectator GS boson (pion, koan, eta) carrying the momentum fraction

$x_{\rm M}$, is given by [2]:

$$f_{\rm q/q'M} = \left(\frac{g_{\rm q/q'M}}{4\pi}\right)^2 \frac{1}{x_{\rm M}(1-x_{\rm M})^2} \times \int_0^\infty {\rm d}k_\perp^2 \, |G_{\rm q/q'M}|^2 \frac{[(1-x_{\rm M})m_{\rm q}-m_{\rm q'}]^2+k_\perp^2}{(m_{\rm q}^2-M_{\rm q'M}^2)^2}, \quad (4)$$

where

$$g_{\rm q/q'M}=\frac{g_{\rm A}}{f}\bar{m}, \qquad \bar{m}=\frac{m_{\rm q}+m_{\rm q'}}{2}, \quad (5)$$

and the vertex function can be written as:

$$G_{\rm q/q'M}=\exp\left(\frac{m_{\rm q}^2-M_{\rm q'M}^2}{2\Lambda^2}\right). \quad (6)$$

In Eq. (5), f is the pseudo-scalar decay constant and is taken to be equal to the pion decay constant, so $f\approx 93$ MeV. The quark axial-vector coupling is represented by $g_{\rm A}$ and it can be taken to be 1, as suggested in Ref. [22], or 0.75, as suggested in Ref. [1]. In our calculations below we choose the former value. The cut-off parameter is Λ and is usually determined phenomenologically [1, 2, 21, 23]. We use its previously determined value, Λ=1.4 GeV [2].

In Eq. (6), $G_{\rm q/q'M}$ is the vertex function and accounts for the extended structure of the GS bosons and the constituent quark. $M_{\rm q'M}^2$ is the invariant mass squared of the "meson + constituent quark" system [2, 21]:

$$M_{\rm q'M}^2=\frac{M_{\rm M}^2+k_\perp^2}{x_{\rm M}}+\frac{m_{\rm q'}^2+k_\perp^2}{1-x_{\rm M}}. \quad (7)$$

In Eq. (7), $m_{\rm q'}$ denotes the mass of constituent quark q′. In our calculations we use $m_{\rm u}=m_{\rm d}=360$ MeV and $m_{\rm s}$=570 MeV as typical values guided by the NJL model calculations [24].

3 Chiral quark model in the polarized case

Although the parton picture only applies to high energy processes, it is possible to obtain the parton densities at low energy scales using the effective Lagrangian. In the following, we need to employ the chiral quark model in which the bare quarks are surrounded by meson clouds. The result of this approach is that we can access the constituent U and D quarks, which lead us to achieve the parton densities of the nucleon at low energy scales. To calculate the nucleon PPDFs in the chiral quark model, the polarized splitting function is needed [1, 2, 21, 23]:

$$\delta f_{\rm q/q'M} = \left(\frac{g_{\rm q/q'M}}{4\pi}\right)^2 \frac{1}{x_{\rm M}(1-x_{\rm M})^2} \times \int_0^\infty {\rm d}k_\perp^2 \, |G_{\rm q/q'M}|^2 \frac{[(1-x_{\rm M})m_{\rm q}-m_{\rm q'}]^2-k_\perp^2}{(m_{\rm q}^2-M_{\rm q'M}^2)^2}, \quad (8)$$

which is analogous to Eq. (4) with the exception of the minus sign before $k_\perp^2$. The expression $f_{\rm q/q'M} = f_{\rm q/q'M}\uparrow + f_{\rm q/q'M}\downarrow$ is the sum of probabilities to find +1/2 and −1/2 helicities for quarks; while $\delta f_{\rm q/q'M} = f_{\rm q/q'M}\uparrow - f_{\rm q/q'M}\downarrow$ is the difference of probabilities to find +1/2 minus −1/2 helicity for quarks; the quarks being emitted from a parent quark with a specific helicity. For comparison, $\delta f_{\rm q/q'\pi}(x_\pi)$, $\delta f_{\rm q/q'K}(x_{\rm K})$ and $\delta f_{\rm q/q'\eta}(x_\eta)$ are plotted in Fig. 2. The polarized quark densities inside the proton can then be obtained using the following relations [2]:

$$\begin{aligned}\delta u(x) =\ & Z\delta u_0(x)+\delta f_{\rm d/u\pi}\otimes\delta d_0+\delta u_{\rm val}^{\pi}\otimes\delta f_{\rm u/d\pi}\otimes\delta u_0 \\ & +\frac{1}{2}\delta f_{\rm u/u\pi}\otimes\delta u_0+\frac{1}{4}\delta u_{\rm val}^{\pi}\otimes\delta f_\pi\otimes(\delta u_0+\delta d_0) \\ & +\delta u_{\rm val}^{\rm K}\otimes\delta f_{\rm K}\otimes\delta u_0+\frac{1}{6}\delta f_\eta\otimes\delta u_0 \\ & +\frac{1}{36}\delta u_{\rm val}^{\eta}\otimes\delta f_\eta\otimes(\delta u_0+\delta d_0), \\ \delta d(x) =\ & Z\delta d_0(x)+\delta f_\pi\otimes\delta u_0+\delta d_{\rm val}^{\pi}\otimes\delta f_\pi\otimes\delta d_0+\frac{1}{2}\delta f_\pi\otimes\delta d_0 \\ & +\frac{1}{4}\delta d_{\rm val}^{\pi}\otimes\delta f_\pi\otimes(\delta u_0+\delta d_0)+\delta d_{\rm val}^{\rm K}\otimes\delta f_{\rm K}\otimes\delta d_0 \\ & +\frac{1}{6}\delta f_\eta\otimes\delta d_0+\frac{1}{36}\delta d_{\rm val}^{\eta}\otimes\delta f_\eta\otimes(\delta u_0+\delta d_0), \end{aligned} \quad (9)$$

where $\delta f_{\rm d/u\pi} = \delta f_{\rm u/d\pi} = \cdots = \delta f_\pi$ (and so on) and are defined by Eq. (8) as polarized splitting functions. In Eq. (9), δu_0 and δd_0 denote the bare quark distributions inside the proton, and $\delta u_{\rm val}^{\pi}$, $\delta d_{\rm val}^{\pi}$ and so on are the polarized quark distributions of mesons, relating to the cloud which surrounds the bare quarks. The '⊗' symbol corresponds to the convolution integral, which is defined as:

$$\begin{aligned} p\otimes q &= \int_x^1 \frac{{\rm d}y}{y} p(y) q\left(\frac{x}{y}\right), \\ p\otimes q\otimes r &= \int_x^1 \frac{{\rm d}y}{y}\int_y^1 \frac{{\rm d}y'}{y'} p(y') q\left(\frac{y}{y'}\right) r\left(\frac{x}{y}\right). \end{aligned} \quad (10)$$

Note that the Mellin transform of these equations causes all ⊗ products to convert to ordinary products. Z is the renormalization constant and in the polarized case it

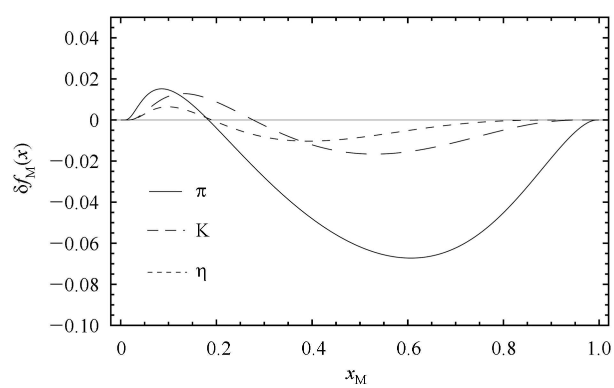

Fig. 2. The polarized splitting functions $\delta f_{\rm q/q'M}$ for pion, kaon and eta.

should be defined by (see Eq. (8) of Ref. [2]):

$$Z=H-\frac{3}{2}\Delta f_{\pi}-\Delta f_{\mathrm{K}}-\frac{1}{6}\Delta f_{\eta}, \tag{11}$$

where Δ is defined as the first Mellin moment of the splitting functions $\delta f_{\mathrm{q/q'M}}$. In the unpolarized formulation, H can be determined using the momentum and number sum rules and equals 1. In the polarized case, it can be determined using the Jaffe-Ellis sum rule [25]:

$$\int_0^1 x g_1^{\mathrm{p}}(x)\mathrm{d}x=0.185\pm0.010, \tag{12}$$

where the polarized proton structure function, $g_1^{\mathrm{p}}(x)$, in the leading order (LO) approximation is given by:

$$g_1^{\mathrm{p}}(x)=\frac{1}{2}\left(\frac{4}{9}\delta u_{\mathrm{val}}(x)+\frac{1}{9}\delta d_{\mathrm{val}}(x)\right). \tag{13}$$

We can substitute $\delta u_{\mathrm{val}}(x)$ and $\delta d_{\mathrm{val}}(x)$ with the values given by Eqs. (11, 15) from Ref. [17], which involve H, into Eq. (13) and the result into Eq. (12) to find the H value. We get the numerical value $H=0.909$. Substituting this value into Eq. (11) yields $Z=0.987$.

In further steps of our calculations, we need to use the sea quark distributions in the chiral quark model, which are given by [2]:

$$\begin{aligned}
\delta\bar{u}(x) &= \delta u_{\mathrm{val}}^{\pi}\otimes\delta f_{\pi}\otimes\delta d_0+\frac{1}{4}\delta u_{\mathrm{val}}^{\pi}\otimes\delta f_{\pi}\otimes(\delta u_0+\delta d_0)\\
&\quad+\frac{1}{36}\delta u_{\mathrm{val}}^{\eta}\otimes\delta f_{\eta}\otimes(\delta u_0+\delta d_0),\\
\delta\bar{d}(x) &= \delta d_{\mathrm{val}}^{\pi}\otimes\delta f_{\pi}\otimes\delta u_0+\frac{1}{4}\delta d_{\mathrm{val}}^{\pi}\otimes\delta f_{\pi}\otimes(\delta u_0+\delta d_0)\\
&\quad+\frac{1}{36}\delta d_{\mathrm{val}}^{\eta}\otimes\delta f_{\eta}\otimes(\delta u_0+\delta d_0),\\
\delta s(x) &= \delta f_{\mathrm{K}}\otimes(\delta u_0+\delta d_0)+\frac{4}{9}\delta s_{\mathrm{val}}^{\eta}\otimes\delta f_{\eta}\otimes(\delta u_0+\delta d_0)\\
\delta\bar{s}(x) &= \delta s_{\mathrm{val}}^{\mathrm{K}}\otimes\delta f_{\mathrm{K}}\otimes(\delta u_0+\delta d_0)\\
&\quad+\frac{4}{9}\delta s_{\mathrm{val}}^{\eta}\otimes\delta f_{\eta}\otimes(\delta u_0+\delta d_0).
\end{aligned} \tag{14}$$

4 Polarized distribution of the bare quarks

In order to use Eqs. (9) and (14) in practice, we need to determine the polarized distributions of bare quarks inside the proton which, in these equations, are denoted by $\delta u_0(x)$ and $\delta d_0(x)$. To extract these distributions, we do the following.

We fit two simple functions with $\delta q_{\mathrm{val}}^{\mathrm{p}}$ data from Refs. [18–20] and find their ratio, $\delta u_{\mathrm{val}}/\delta d_{\mathrm{val}}$, and also suppose that this ratio is unchanged when $Q^2\rightarrow Q_0^2$. This assumption is valid because when we move from Q_0^2 to Q^2, all valence quarks share an equal proportion of their momentum with gluons and sea quarks. Then we can rewrite Eq. (13) as:

$$g_1^{\mathrm{p}}(x)=\frac{1}{2}\left(\frac{4}{9}\frac{\delta u_{\mathrm{val}}}{\delta d_{\mathrm{val}}}\delta d_{\mathrm{val}}+\frac{1}{9}\delta d_{\mathrm{val}}\right). \tag{15}$$

The only unknown function is then $\delta d_{\mathrm{val}}(x)$, which can be determined by fitting the right hand side of Eq. (15) with the available experimental data for $g_1^{\mathrm{p}}(x)$ at high energy scales [10, 26]. We evolve it down to Q_0^2 to find $\delta d_0(x)$. We are also able to find $\delta u_0(x)$ from the known ratio $\delta u_{\mathrm{val}}/\delta d_{\mathrm{val}}$. The final results are given by:

$$\begin{aligned}
x\delta u_0(x)&=2.313x^{1.100}(1-x)^{1.908},\\
x\delta d_0(x)&=-0.852x^{0.964}(1-x)^{2.485}.
\end{aligned} \tag{16}$$

The only functions that remain unknown in the rest of our calculations are the PPDFs of the mesons, denoted by $\delta q_{\mathrm{val}}^{\mathrm{M}}$ in Eqs. (9) and (14). We use the following strategy to find the polarized valence densities in all mesons. We first consider the following functions for the polarized valence distribution in mesons:

$$\delta q_{\mathrm{val}}^{\mathrm{M}}=ax^{\mathrm{b}}(1-x)^{\mathrm{c}}P_{\mathrm{M}}(x), \tag{17}$$

where the superscript M denotes meson and for the pion we have $P_{\pi}(x)=1$. To calculate the other functions $P_{\mathrm{K}}(x)$ and $P_{\eta}(x)$ for the kaon and eta, we need to resort to a method which will be explained in the following subsections.

4.1 Monte Carlo simulation – meson polarized quark densities

Meson polarized distributions for kaon and eta will be specified if we determine $P_{\mathrm{K}}(x)$ and $P_{\eta}(x)$ in Eq. (17). The quantity $P_{\mathrm{K}}(x)$ can be determined using Monte Carlo (MC) simulation and $P_{\eta}(x)$ can be obtained by expanding the meson PDFs as a function of quark mass.

In the MC simulation which we introduce, $P_{\mathrm{K}}(x)=\delta q_{\mathrm{val}}^{\mathrm{K}}/\delta q_{\mathrm{val}}^{\pi}$ should be found. This ratio is related to the unpolarized $q_{\mathrm{val}}^{\mathrm{K}}/q_{\mathrm{val}}^{\pi}$ data [11]. Two distinct MC algorithms are employed. In one of them we use the unpolarized values and their errors directly and in the other algorithm, these values are used as controlling conditional parameters to generate random numbers to calculate and estimate the polarized values. The first algorithm can be expressed as follows. There are experimental data for the ratio [11]:

$$\frac{q_{\mathrm{val}}^{\mathrm{K}}}{q_{\mathrm{val}}^{\pi}}=\frac{q_{\mathrm{val}}^{\mathrm{K}}\uparrow+q_{\mathrm{val}}^{\mathrm{K}}\downarrow}{q_{\mathrm{val}}^{\pi}\uparrow+q_{\mathrm{val}}^{\pi}\downarrow}=r\pm\delta r. \tag{18}$$

Experimental data for the unpolarized pion valence distributions are also available [27]:

$$q_{\mathrm{val}}^{\pi}\uparrow+q_{\mathrm{val}}^{\pi}\downarrow=d\pm\delta d. \tag{19}$$

We can therefore find that $q_{\mathrm{val}}^{\mathrm{K}}\uparrow+q_{\mathrm{val}}^{\mathrm{K}}\downarrow=r.d\pm\delta(r.d)$ where $\delta(r.d)=d.\delta r+r.\delta d$ (we ignore the product term $\delta r.\delta d$). On the other hand, we know that the difference of two numbers between 0 and 1 should lie between -1 and their sum, hence:

$$-1\leqslant\delta q^{\pi}_{\rm val}\leqslant n q^{\pi}_{\rm val}, \tag{20}$$

where we have inserted n in Eq. (20) for some other possible theoretical and/or experimental considerations. Nevertheless we take $n=1$ in our calculations. From Eq. (20) we have:

$$-1\leqslant q^{\pi}_{\rm val}\uparrow-q^{\pi}_{\rm val}\downarrow\leqslant n(q^{\pi}_{\rm val}\uparrow+q^{\pi}_{\rm val}\downarrow). \tag{21}$$

From Eq. (19) and the upper limit of its right hand side we get:

$$-1\leqslant d\pm(\delta d)-2q^{\pi}_{\rm val}\downarrow\leqslant n(d+\delta d),$$

$$\Rightarrow\ \frac{1+d}{2}\geqslant q^{\pi}_{\rm val}\downarrow\mp\left(\frac{\delta d}{2}\right)\geqslant-\frac{n\delta d+(n-1)d}{2}. \tag{22}$$

We can generate random numbers between these two limits and find $q^{\pi}_{\rm val}\downarrow$ while the uncertainty δd is known. The same method can be used to find $q^{\rm K}_{\rm val}\downarrow$ and hence we can calculate $\delta q^{\rm K}_{\rm val}/\delta q^{\pi}_{\rm val}=(r.d-2q^{\rm K}_{\rm val}\downarrow)/(d-2q^{\pi}_{\rm val}\downarrow)$ where we use Eqs. (18) and (19). As a result $P_{\rm K}(x)$ is calculated. One can determine the maximum value of n using the theoretical and experimental values of $\delta q^{\rm M}_{\rm val}/q^{\rm M}_{\rm val}$. This could be a question for further research activity.

The second method is based on direct generation of random values between 0 and 1 including all $q^{\rm M}_{\rm val}\uparrow$ and $q^{\rm M}_{\rm val}\downarrow$ for both pion and kaon, and eliminating the results for $q^{\pi}_{\rm val}$ which lie outside the interval $d\pm\delta d$ (Eq. (19)) and those for $q^{\rm K}_{\rm val}$ which lie outside the interval $r.d\pm\delta(r.d)$. The difference between the outputs of these two algorithms, plus the uncertainty that we mentioned before, can be used to calculate the error in the calculations.

Although the results of MC algorithms always depend on the running duration of the program (because of their probabilistic structure), we increase the number of random values in order that the results vary less than 10 percent in two consecutive runs of the program. The unpolarized data and polarized values produced by MC simulation for the ratio concerned, together with the functions which have been fitted to them, are depicted in Fig. 3; consequently we get:

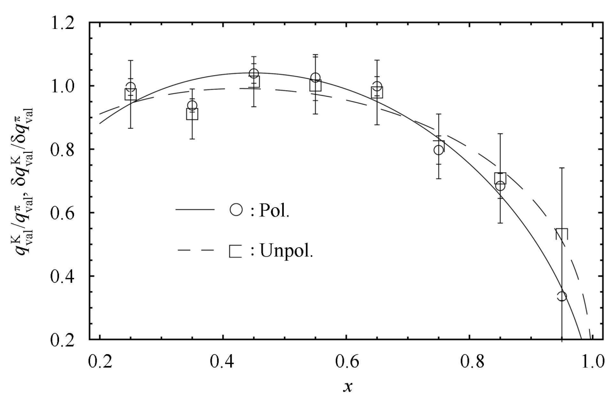

Fig. 3. The unpolarized data [11] and the polarized Monte Carlo results for the ratio $\delta q^{\rm K}_{\rm val}/\delta q^{\pi}_{\rm val}$.

$$P_{\rm K}(x)=2.170x^{0.478}\ (1-x)^{0.591}. \tag{23}$$

By obtaining $P_{\rm K}(x)$, we are able to determine $P_{\eta}(x)$ and finally to extract the valence quark distribution of the eta meson, which we explain in the following subsection.

4.2 Valence quark distribution for eta meson

To find $P_{\eta}(x)$ we need to have the quark distribution functions depend on their masses. If we consider the mass of the quarks as a factor that causes their densities to be different, we can obtain:

$$\begin{aligned}\delta q^{\pi}_{\rm val}\ &=\ f(m,\cdots)|_{m=m_{\rm l}}\times f(m,\cdots)|_{m=m_{\rm l}},\\ \delta q^{\rm K}_{\rm val}\ &=\ f(m,\cdots)|_{m=m_{\rm l}}\times f(m,\cdots)|_{m=m_{\rm s}},\\ \delta q^{\eta}_{\rm val}\ &=\ f(m,\cdots)|_{m=m_{\rm s}}\times f(m,\cdots)|_{m=m_{\rm s}},\end{aligned} \tag{24}$$

where f could be any function of quark mass, m, and all other QCD parameters (x, Q_0, $\Lambda_{\rm QCD}$, $\cdots$). The concealed logic in Eq. (25) is that the pion contains two light quarks, the kaon contains one light and one strange quark, and so forth. The product of two f functions in Eq. (25) is justifiable by the probabilistic nature of the distributions. The mass of light quarks is denoted by $m_{\rm l}=m_{\rm u}\approx m_{\rm d}$ and $m_{\rm s}$ is the mass of the strange quark. The expansion of Eq. (24) yields:

$$\begin{aligned}\delta q^{\pi}_{\rm val}\ &=\ \left(f|_{m=0}+m_{\rm l}\frac{\partial f}{\partial m}|_{m=0}+\mathcal{O}(m^2)\right)\\ &\quad\times\left(f|_{m=0}+m_{\rm l}\frac{\partial f}{\partial m}|_{m=0}+\mathcal{O}(m^2)\right),\\ &=\ f_0^2+2m_{\rm l}f_0f_0'+\mathcal{O}(m^2),\end{aligned} \tag{25}$$

where $f_0=f|_{m=0}$ and $f_0'=(\partial f/\partial m)|_{m=0}$. Doing the same calculations for the kaon and eta meson we find:

$$\begin{aligned}\delta q^{\rm K}_{\rm val}\ &=\ (f_0+m_{\rm s}f_0'+\mathcal{O}(m^2))(f_0+m_{\rm l}f_0'+\mathcal{O}(m^2))\\ &=\ f_0^2+(m_{\rm s}+m_{\rm l})f_0f_0'+\mathcal{O}(m^2),\end{aligned} \tag{26}$$

$$\delta q^{\eta}_{\rm val}=f_0^2+2m_{\rm s}f_0f_0'+\mathcal{O}(m^2). \tag{27}$$

Defining $D=f_0'/f_0$ we have:

$$\frac{\delta q^{\rm K}_{\rm val}}{\delta q^{\pi}_{\rm val}}\ =\ P_{\rm K}(x)=\frac{f_0^2+(m_{\rm s}+m_{\rm l})f_0f_0'}{f_0^2+2m_{\rm l}f_0f_0'}=\frac{1+(m_{\rm s}+m_{\rm l})D}{1+2m_{\rm l}D}, \tag{28}$$

$$\frac{\delta q^{\eta}_{\rm val}}{\delta q^{\pi}_{\rm val}}\ =\ P_{\eta}(x)=\frac{f_0^2+2m_{\rm s}f_0f_0'}{f_0^2+2m_{\rm l}f_0f_0'}=\frac{1+2m_{\rm s}D}{1+2m_{\rm l}D}. \tag{29}$$

Consequently, by writing D in terms of $P_{\rm K}$ from Eq. (28) and substituting it into Eq. (29) we find:

$$P_{\eta}(x)=2P_{\rm K}(x)-1. \tag{30}$$

This equation allows us to determine the valence quark distributions of the eta meson, principally. Now we should determine the unknown parameters (a, b and c) in Eq. (17).

Substituting Eqs. (30) and (23) into Eq. (17) and the obtained result into Eqs. (9, 14) and then fitting the right hand side of this equation with the experimental data for proton parton distributions (in the polarized case, their large errors have made them unusable for fitting processes) or the results of phenomenological collaborations, we can find parameters a, b and c from Eq. (17). We choose the average of GRSV [16, 28, 29] and AAC [30] for fitting. Due to the slightly imprecise results for $Q^2<4$ GeV^2, we excluded the BB model from our calculations . The results are:

$$\begin{aligned} a &= 1.100\pm0.235,\\ b &= 0.686\pm0.118,\\ c &= 1.073\pm0.274. \end{aligned} \tag{31}$$

Taking these parameters into account and using the results of Section 3.1 of Ref. [23], evolution of the PPDFs based on the constituent valon model (for mesons) is straightforward [23, 31]. The results for the polarized valence distribution functions at $Q^2=3$ GeV^2 are depicted in Fig. 4 and compared with unpolarized distribution. Other meson densities, extracted from the valon model, are listed in the Appendix A.

As an adjunct to this study and to complete the discussion, let us review how to get the numerical values which are listed in the appendix. In analogue to Eq. (33) from Ref. [23], but for the polarized case, we can write:

$$\delta M_{\rm val}(n,Q^2)=\delta V(n)\times\delta M_{\rm NS}(n,Q^2), \tag{32}$$

where $\delta V(n)$ is the moment of the polarized valon distributions. Note that in the unpolarized case we had two valons with corresponding distributions which were generally different. In Appendix A of Ref. [23], we have shown that their difference can be obtained using the number and momentum sum rules. For the pion – which consists of two light valence quarks – the calculations of Ref. [23] showed us that we can take their valon distributions to be equal to each other. But in the polarized case, the lack of sufficient theoretical sum rules force us to suppose that all valence distributions inside each meson are equal. Hence, instead of $V_1(n)$ and $V_2(n)$ from Eq. (33) of Ref. [23] we have only $\delta V(n)$, which is assumed to have the following form:

$$\delta V(n)=\frac{B(p+n,q+1)}{B(p+1,q+1)}, \tag{33}$$

where B is the Euler beta function and p and q are two (valon) free parameters. There are also two (QCD) free parameters, i.e. Q_0 and $\Lambda_{\rm QCD}$, which exist in the definition of $\delta M_{\rm NS}(n,Q^2)$. These four parameters can be determined by fitting the right hand side of Eq. (32) to Eq. (17), using Eqs. (23), (30) and (32) for mesons. Having these four parameters, we can then calculate the distributions of $\delta M_{\Sigma}(n,Q^2)$ (the moment of singlet sector of the distributions) and $\delta M_{\rm g}(n,Q^2)$ (the moment of the gluon distribution):

$$\delta M_{\Sigma}(n,Q^2) = 2\delta V(n)\times\delta M_{\rm s}(n,Q^2), \tag{34}$$

$$\delta M_{\rm g}(n,Q^2) = 2\delta V(n)\times\delta M_{\rm qg}(n,Q^2). \tag{35}$$

To find the required distribution functions ($\delta\Sigma(x,Q^2)$ and $\delta g(x,Q^2)$), we can use the inverse Mellin transform (Eq. (34) from Ref. [23]):

$$xq(x,Q^2)=\frac{1}{2\pi{\rm i}}\int_{{\rm c}-{\rm i}\infty}^{{\rm c}+{\rm i}\infty}\frac{{\rm d}x}{x^{n-1}}M(n,Q^2). \tag{36}$$

A numerical method to cope with the integral in Eq.(36) has been introduced in Ref. [31]. In all of the numerical methods, we simply fit the moment of a definite function (which contains free parameters) to the experimental data or the results of a known function – for example, Eqs. (32), (34) or (35) – to find the free parameters. In our fitting procedure, we assume the unknown function to be $ax^b(1-x)^c$ with free parameters a, b and c. Because Eqs. (32), (34) and (35) depend on Q^2, the free parameters (a, b and c) take different values at each energy scale. We compute them for a wide range of energies ($Q^2=0.7$ to 100 GeV^2) and categorize them in the appendix for any possible practical usage.

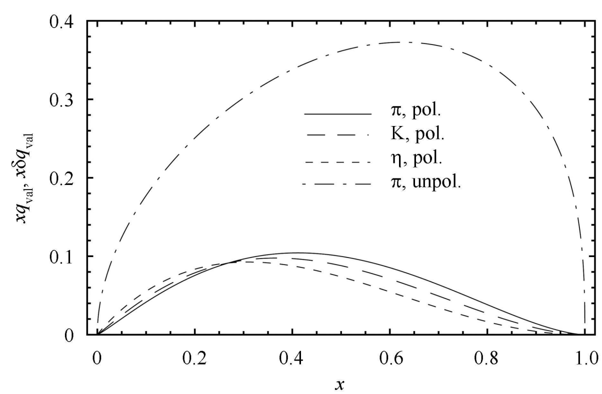

Fig. 4. The polarized valence densities in mesons. A corresponding unpolarized PDF [32, 33] is also included for comparison.

5 Parton orbital angular momentum

Since we now have the polarized parton densities for mesons, we can investigate the first Mellin moment of the singlet, non-singlet and gluon sectors of the meson and finally determine its spin. To avoid spin crises, we need to calculate the gluon and quark orbital momenta. Their analytical calculations are fully discussed in Ref. [6]. The leading-log evolution of the quark and gluon orbital angular momenta are:

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix} L_{\mathrm{q}} \\ L_{\mathrm{g}} \end{pmatrix} = \frac{\alpha_{\mathrm{s}}(t)}{2\pi}\begin{pmatrix} -\frac{4}{3}C_{\mathrm{F}} & \frac{n_{\mathrm{f}}}{3} \\ \frac{4}{3}C_{\mathrm{F}} & -\frac{n_{\mathrm{f}}}{3} \end{pmatrix}\begin{pmatrix} L_{\mathrm{q}} \\ L_{\mathrm{g}} \end{pmatrix} + \frac{\alpha_{\mathrm{s}}(t)}{2\pi}\begin{pmatrix} -\frac{2}{3}C_{\mathrm{F}} & \frac{n_{\mathrm{f}}}{3} \\ -\frac{5}{6}C_{\mathrm{F}} & -\frac{11}{2} \end{pmatrix}\begin{pmatrix} \Delta\Sigma \\ \Delta g \end{pmatrix}, \quad (37)$$

where L_{q} and L_{g} are the orbital angular momentum of quarks and gluons respectively; $t=\ln(Q^2/\Lambda^2_{\mathrm{QCD}})$; $C_{\mathrm{F}}=4/3$ and $\Delta\Sigma$ and Δg are the first Mellin moment of the distributions $\delta\Sigma$ and δg, i.e. :

$$\begin{pmatrix} \Delta\Sigma \\ \Delta g \end{pmatrix} = \int_0^1 \begin{pmatrix} \delta\Sigma(x) \\ \delta g(x) \end{pmatrix} \mathrm{d}x \ . \quad (38)$$

As Q^2 increases, $\Delta\Sigma$ decreases very slightly. It is therefore considered constant in Ref. [6] and also in our calculations. The dependence of Δg on Q^2 can be obtained via:

$$\Delta g(t) = -\frac{4}{\beta_0}\Delta\Sigma + \frac{t}{t_0}\left(\Delta g_0 + \frac{4}{\beta_0}\Delta\Sigma\right), \qquad \Delta\Sigma = \text{const}, \quad (39)$$

where $t_0=t(Q_0^2)$ and the first universal coefficient of the QCD β-function is $\beta_0=11-2n_{\mathrm{f}}/3$. If we solve Eq. (37) for a meson and use the following boundary condition:

$$0=\frac{1}{2}\Delta\Sigma+\Delta g(0)+L(0), \quad (40)$$

we will get [17]:

$$L_{\mathrm{q}}(t) = -\frac{1}{2}\Delta\Sigma+(t/t_0)^{-2(16+3n_{\mathrm{f}})/(9\beta_0)}\left(L_{\mathrm{q}}(0)+\frac{1}{2}\Delta\Sigma\right),$$
$$L_g(t) = -\Delta g(t)+(t/t_0)^{-2(16+3n_{\mathrm{f}})/(9\beta_0)}\left(L_g(0)+\Delta g(0)\right). \quad (41)$$

By summing up the two equations in (41), the total orbital angular momentum is obtained:

$$L(t)=L_{\mathrm{q}}(t)+L_{\mathrm{g}}(t). \quad (42)$$

The initial value of parton angular momentum for a meson, i.e., $L(0)=L_{\mathrm{q}}(0)+L_g(0)$, can be obtained using the first Mellin moment of the total quark and gluon helicity distributions and their orbital angular momentum respectively, i.e. Eq. (40). Knowing $L(0)$ and using Eqs. (41) and (42) we can calculate $L(t)$ at all energy ranges. The results for mesons are shown in Fig. 5. They are in good agreement with those in Ref. [17], in which another aspect of χQM was used. This agreement confirms the validity of our recent calculations. The total spin of hadrons at all energies can be obtained from:

$$S(t)=\frac{1}{2}\Delta\Sigma+\Delta g(t)+L_{\mathrm{qg}}(t)\ , \quad (43)$$

which easily leads to a value of zero, using Eqs. (39) and (41), as expected.

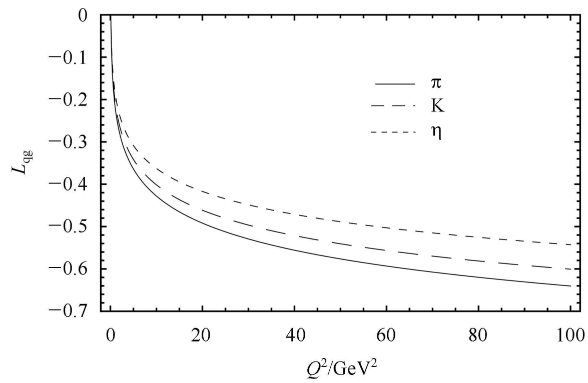

Fig. 5. The orbital angular momentum $L(t)$ for mesons with respect to Q^2.

6 Mass dependence of the proton quark distribution – evolution of sea quark densities

Accessing the bare quark distributions inside the proton, using Eq. (16) and the valence densities of mesons with Eq. (17), we can obtain the polarized quark distribution inside the proton using Eqs. (9) and (14). To evolve the valance density to high energies, we use the evolution of non-singlet moments, based on the DGLAP equations. The required relation to evolve the non-singlet moments is as follows:

$$\delta M_{\mathrm{val}}(n,Q^2)=\delta M_{\mathrm{val}}(n,Q_0^2)\times\delta M_{\mathrm{NS}}(n,Q^2). \quad (44)$$

The term $\delta M_{\mathrm{NS}}(n,Q^2)$ is available from QCD calculations and $\delta M_{\mathrm{val}}(n,Q_0^2)$ is the Mellin moment of the valence distribution, which has been previously obtained based on the χQM at low Q^2 (see Eq.(9) and Eq.(14)). The evolved valence densities inside the proton are indicated in Fig. 6 and compared with available experimental data.

The evolution of sea quark densities inside the proton is not as simple as that of valence quarks. In this case we need the singlet moment which does not relate individually to the moment of sea quarks but relates to the summation of all quark moments ($\delta\Sigma$). Although there are some methods that solve this difficulty [16], we are looking for a different method by applying the mass of

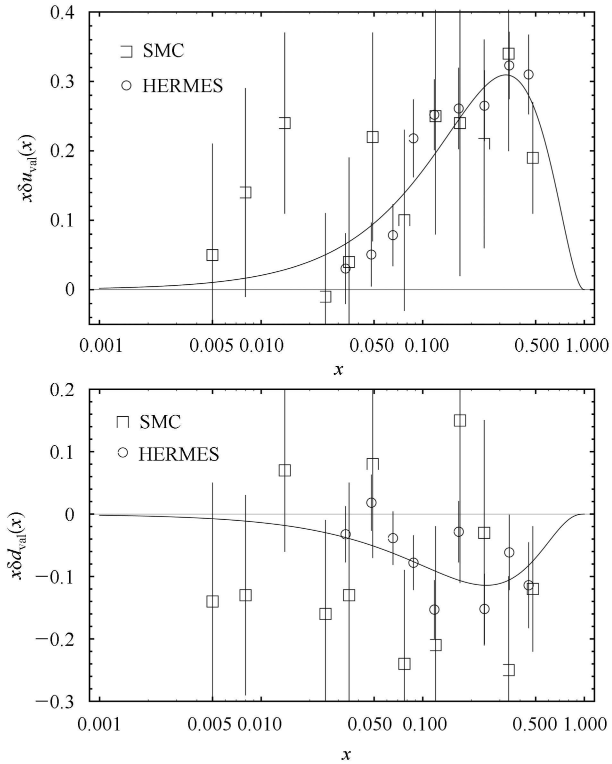

Fig. 6. The polarized valence densities inside a proton at $Q^2=3$ GeV^2. Note that HERMES data [18, 19] are at $Q^2=2.5$ GeV^2 and the SMC data [20] are at $Q^2=3$ GeV^2.

the quarks in the calculations. We first assume that all four light sea quarks are eigenstates of the evolution operator:

$$|\delta q_{\rm sea};Q^2\rangle = E|\delta q_{\rm sea};Q_0^2\rangle$$
$$\Rightarrow |\delta q_{\rm sea};Q^2\rangle = E_{\rm q}(x,Q^2)|\delta q_{\rm sea};Q_0^2\rangle, \tag{45}$$

in which E is the evolution operator and $E_{\rm q}$ is its eigenvalue. Two distinct cases can occur:

1) Degenerate state

In this case we have:

$$E_{\bar{\rm u}}=E_{\bar{\rm d}}=E_{\rm s}=E_{\bar{\rm s}}=E(x,Q^2), \tag{46}$$

and subsequently:

$$\delta q_{\rm sea}(x,Q^2)=E(x,Q^2)\times\delta q_{\rm sea}(x,Q_0^2). \tag{47}$$

By summing both sides of Eq. (47) for the four light quarks and then factorizing $E(x,Q^2)$, we will finally reach the following relation:

$$E(x,Q^2) = \frac{[\delta\bar{u}+\delta\bar{d}+\delta s+\delta\bar{s}](x,Q^2)}{[\delta\bar{u}+\delta\bar{d}+\delta s+\delta\bar{s}](x,Q_0^2)}$$
$$= \frac{[\delta\Sigma-\delta u_{\rm val}-\delta d_{\rm val}](x,Q^2)}{[\delta\bar{u}+\delta\bar{d}+\delta s+\delta\bar{s}](x,Q_0^2)}. \tag{48}$$

In the second fraction of Eq. (48), the evolved valence densities can be obtained from Eq. (44) and the evolved distributions for $\delta\Sigma$ can be obtained, using the notation of Ref. [12], as:

$$\delta M_\Sigma(n,Q^2) = [\delta M u_{\rm val}(n,Q_0^2)+\delta M d_{\rm val}(n,Q_0^2)] \times\delta M_{\rm s}(n,Q^2). \tag{49}$$

The denominator in Eq. (48) can be obtained from χQM (see Eq. (14)). Having the functional form of $E(x,Q^2)$, the evolution of individual sea quark densities will be possible. This is also the method which has been used in Ref. [17] based on another aspect of χQM.

2) Non-degenerate state

In this case, we cannot factorize $E_{\rm q}$ in Eq. (47). Assuming that the eigenvalues in this equation depend on Q^2 through the running mass of the quarks, we can write:

$$[\delta\Sigma-\delta u_{\rm val}-\delta d_{\rm val}](x,Q^2)$$
$$= E_{\rm u}(x,m_{\rm u}(Q^2))\times\delta\bar{u}(x,Q_0^2)$$
$$+E_{\rm d}(x,m_{\rm d}(Q^2))\times\delta\bar{d}(x,Q_0^2)$$
$$+E_{\rm s}(x,m_{\rm s}(Q^2))\times[\delta s(x,Q_0^2)+\delta\bar{s}(x,Q_0^2)], \tag{50}$$

where $m_{\rm q}(Q^2)=m_{\bar{\rm q}}(Q^2)$ for all quarks and due to the equality of the s and $\bar{\rm s}$ quarks masses, the two eigenvalues of the strange distributions (in the last bracket) are equal.

In the modified minimal subtraction ($\overline{\rm MS}$) scheme, the renormalization group equation for the running mass of quarks has the following form [34, 35]:

$$\left[Q^2\frac{\partial}{\partial Q^2}-\beta(\alpha_{\rm s})\frac{\partial}{\partial\alpha_{\rm s}}+\left(\frac{1}{2}+\gamma_m(\alpha_{\rm s})\right)m\frac{\partial}{\partial m}\right]$$
$$\times R(Q^2/\mu^2,\alpha_{\rm s},m/Q)=0. \tag{51}$$

The running mass equation $m(Q^2)$ (in analogy with the running coupling constant) is governed by:

$$Q^2\frac{\partial m}{\partial Q^2}=-\gamma_m(\alpha_{\rm s})m(Q^2), \tag{52}$$

and finally its solution is:

$$m(Q^2)=m(\mu^2)\exp\left[-\int_{\mu^2}^{Q^2}\frac{{\rm d}Q^2}{Q^2}\gamma_m(\alpha_{\rm s}(Q^2))\right]. \tag{53}$$

The numerical values for the light quark masses, which are denoted here by $m(\mu^2)$, are those which were indicated just below Eq. (7).

We assume the following function for the $E_{\rm q}$s in Eq. (50):

$$E_{\rm q}=A_{\rm q}(m_{\rm q})\ x^{B_{\rm q}(m_{\rm q})}\ (1-x)^{C_{\rm q}(m_{\rm q})}. \tag{54}$$

Eq. (47) shows that $E(x,Q^2)$ should be equal to 1 at $Q^2=Q_0^2$, and hence in Eq. (50) $E_{\rm u}=E_{\rm d}=E_{\rm s}\to1$ when $Q^2\to Q_0^2$. However, the natures of the two sides of Eq. (50) are actually different. The left hand side of Eq. (50) can be calculated, based on the Valon framework [15], while its right hand side comes from χQM [17, 23]. As a result we consider an additional coefficient to fill this gap between the two models and write $E_{\rm u}=E_{\rm d}=E_{\rm s}=N$. Thereupon in Eq. (54), when $Q^2\to Q_0^2$ we have:

$$\text{as } Q^2\to Q_0^2:$$
$$\lim A_{\rm u}=\lim A_{\rm d}=\lim A_{\rm s}=N,$$
$$\lim B_{\rm u}=\lim B_{\rm d}=\lim B_{\rm s}=0,$$
$$\lim C_{\rm u}=\lim C_{\rm d}=\lim C_{\rm s}=0, \tag{55}$$

where N can be determined from Eq. (50) when its left hand side is calculated at $Q^2=Q_0^2$ using a regression method [36–38].

Also, we know that quarks at high energy scales can be considered massless [35]. According to Eq. (54), this condition implies the following limits:

$$\text{as } Q^2\to\infty:$$
$$\lim A_{\rm u}=\lim A_{\rm d}=\lim A_{\rm s},$$
$$\lim B_{\rm u}=\lim B_{\rm d}=\lim B_{\rm s},$$
$$\lim C_{\rm u}=\lim C_{\rm d}=\lim C_{\rm s}. \tag{56}$$

One of the simple functions which satisfies these conditions can be indicated by:

$$A_{\rm q}(m_{\rm q}) = N\left(\frac{m_{\rm q}(Q^2)}{m_{\rm q}(\mu^2)}\right)^{-A},$$
$$\begin{pmatrix}B_{\rm q}(m_{\rm q})\\ C_{\rm q}(m_{\rm q})\end{pmatrix} = \begin{pmatrix}B\\ C\end{pmatrix}\left[-\lg\left(\frac{m_{\rm q}(Q^2)}{m_{\rm q}(\mu^2)}\right)\right]\left(\frac{\lg[m_{\rm q}(Q^2)]}{\lg[m_{\rm q}(Q'^2)]}\right), \tag{57}$$

here Q'^2 refers to the limit of large energy value in which the quarks can be considered massless.

Using Eq. (50) at $Q^2=Q_0^2$ will tend the value of N to 2.785. To find A, B and C, we substitute the evolved $\delta\bar{u}$, $\delta\bar{d}$ and $\delta\bar{s}$ distributions into the first equation of the DGLAP equations:

$$\frac{\rm d}{{\rm d}\lg Q^2}\delta q(x,Q^2) = \frac{\alpha_{\rm s}}{2\pi}\int_x^1\frac{{\rm d}y}{y}\left[\delta q(y,Q^2)\delta P_{qq}\left(\frac{x}{y}\right)\right.$$
$$\left.+\delta g(y,Q^2)\delta P_{\rm qg}\left(\frac{x}{y}\right)\right]. \tag{58}$$

(Note that, according to Ref. [21] we can calculate the 'frozen' gluon distribution, $\delta g(x)$, from χQM and evolve it using $\delta M_{\rm g}(n,Q^2)$). Considering Eq. (58), we get three equations for $\delta\bar{u}$, $\delta\bar{d}$ and $\delta\bar{s}$, which should be solved numerically for A, B and C. Consequently the results for the parameters in Eq.(57) are:

$$A=0.797,\ B=-0.502,\ C=-1.306. \tag{59}$$

Substituting Eqs. (57) and (59) into Eq. (54), we can obtain the numerical value for the evolution functions, $E_{\rm q}$s, at any given values of Q^2 and x. The results are shown in Fig. 7.

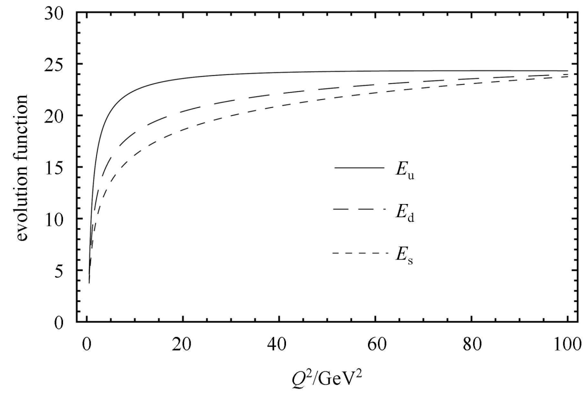

Fig. 7. The evolution functions in Eq. (50) at x=0.3.

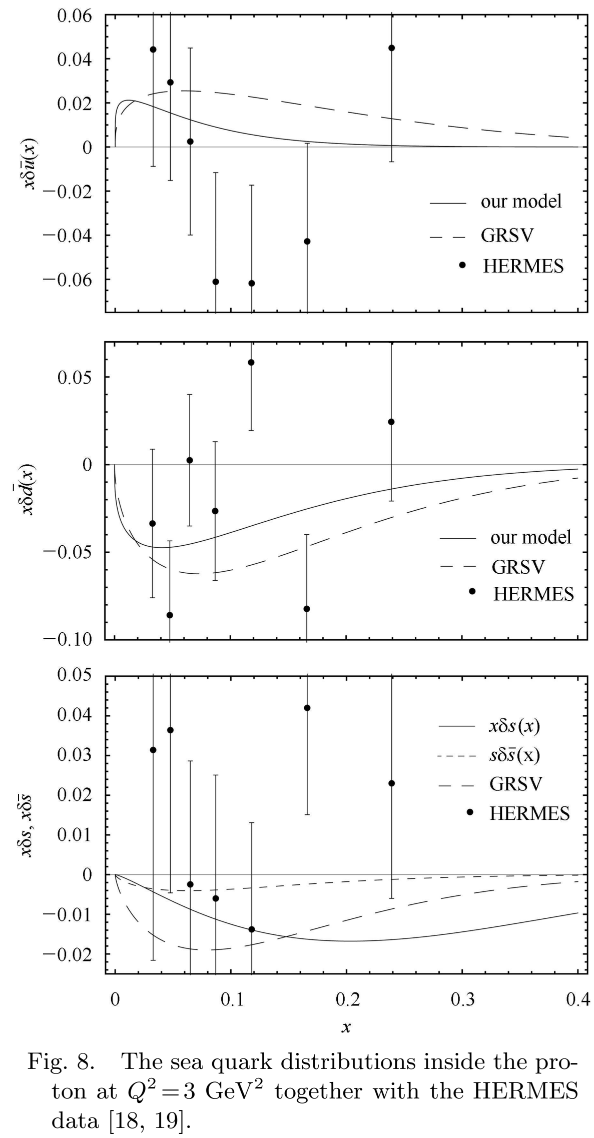

Fig. 8. The sea quark distributions inside the proton at $Q^2=3$ GeV^2 together with the HERMES data [18, 19].

Using these functions we can evolve the sea quark densities to higher values of energy scales. The results are depicted in Fig. 8 at Q^2=3 GeV^2.

7 Conclusions

The polarized distribution function of mesons cannot be supposed to vanish trivially, as otherwise all sea quark distributions in Eq. (14) would be zero. In addition, there exist a variety of reliable studies that have calculated the PPDF of mesons [3–5]. We have determined these polarized parton densities by calculating the ratios of the polarized valence densities inside the meson using Monte Carlo algorithms and the expansion of the PPDFs in terms of quark masses (see Eq. (25)). In cases where the polarized valence density of the pion is given (using any model), this method can offer the corresponding functions for kaon and eta.

The orbital angular momentum was used to calculate the meson spins. These equations are written and solved for the proton [6] to justify the spin crisis, and we solved them for mesons to calculate the parton orbital angular momentum of mesons and justify the zero spin of mesons. These calculations can be considered as additional evidence for the existence of non-zero polarized valence distributions for mesons. The agreement with the result in Ref. [17] confirms the validity of our calculations.

Due to the fact that the mass of the quarks can be responsible for chiral symmetry breaking, we employed the mass dependence of the proton quark densities, using the running mass equation [25], to reveal their asymmetry in a clearer way. The functional form of the eigenvalues of the evolution operator could be extracted, given appropriate boundary conditions for its parameters. By numerical solution of the DGLAP evolution equations, the numerical values of the required parameters in Eq. (57) were obtained. At high enough energies, where the quarks become massless, these eigenvalues tend to each other and the degenerate formulation can be used.

For further research, the bare quark distributions inside the proton can be obtained theoretically rather than phenomenologically, based on the solution of the Dirac equation under a specified potential. The asymmetry of polarized light quark distributions can also be investigated, considering the charge asymmetry of parton densities, and this is something we hope to work on in future.

Appendix A

The coefficients in the function $ax^b(1-x)^c$ have a typical expansion as follows:

$$(a,\ b \text{ and } c)=\sum_{i=0}^{3} R_i\ \alpha_{\rm s}^i. \tag{A1}$$

Their numerical values for polarized valence, gluon and singlet sector of pion, kaon and eta are calculated based on the valon model. They are tabulated below (Table A1). The $\alpha_{\rm s}$ in Eq. (A1) denotes the running coupling constant at NLO approximation. We have taken $\Lambda_{\overline{\rm MS}}$=0.200 GeV in all parts of these calculations.

Table A1.

		R_0	R_1	R_2	R_3
	a	0.292	1.592	−1.399	0.561
δq_v^π	b	0.610	2.160	−1.884	0.797
	c	2.628	−4.435	4.940	−2.215
	a	0.229	3.784	−2.899	0.983
$\delta\Sigma^\pi$	b	0.0132	3.520	−3.039	1.221
	c	2.404	−3.509	3.664	−1.632
	a	−0.0540	4.832	−7.796	5.515
δg^π	b	−0.598	3.220	−4.905	4.505
	c	5.135	−12.79	14.84	−6.364
	a	0.269	1.564	−1.302	0.522
δq_v^K	b	0.586	2.098	−1.840	0.778
	c	2.783	−4.496	5.026	−2.256
	a	0.229	3.725	−2.907	0.993
$\delta\Sigma^K$	b	0.00802	3.455	−3.019	1.212
	c	2.433	−3.547	3.703	−1.648
	a	−0.0423	4.688	−7.443	5.212
δg^K	b	−0.592	3.129	−4.692	4.298
	c	5.186	−12.89	14.98	−6.436
	a	0.245	1.527	−1.193	0.478
δq_v^η	b	0.553	2.008	−1.777	0.751
	c	2.994	−4.590	5.152	−2.314
	a	0.179	3.559	−2.377	0.749
$\delta\Sigma^\eta$	b	−0.00478	3.358	−2.988	1.195
	c	2.759	−3.589	3.763	−1.683
	a	−0.0954	4.478	−7.264	5.417
δg^η	b	−0.582	2.933	−4.177	3.836
	c	5.657	−13.55	15.81	−6.762

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