Velocity bunching in travelling wave accelerator with low acceleration $\operatorname{gradient}^*$

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Abstract: We present the analytical and simulated results of our study of the influence of the acceleration gradient in the velocity bunching process, which is a bunch compression scheme that uses a travelling wave accelerating structure as a compressor. Our study shows that the bunch compression application with low acceleration gradient is more tolerant to phase jitter and more successful in obtaining a compressed electron beam with symmetrical longitudinal distribution and low energy spread. We also present a transverse emittance compensation scheme to compensate the emittance growth caused by the increase of the space charge force in the compressing process, which is easy to adjust for different compression factors.

Key words: travelling wave accelerating structure, velocity bunching, acceleration gradient, emittance compensation

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1 Introduction

In recent years, the demand for the application of high brightness—low emittance, high current, with subpicosecond pulse length—electron beams has increased dramatically. In the fourth generation synchrotron light source community, high-brightness beams are needed for application in short wavelength free electron lasers (FEL), as well as in the inverse-Compton-scattering (ICS) generation of short X-ray pulses. Short electron bunches are also required in the study of novel accelerating techniques, such as plasma-based accelerators and the generation of coherent THz radiation.

Short bunches are commonly obtained by magnetic compression. In this scheme, the bunch is compressed when drifting through a series of dipoles arranged in a chicane configuration, which can introduce an energydependent path length. Therefore, an electron bunch with a proper time-energy correlation can be shortened in the chicane. The time-energy correlation along the bunch can be tuned by means of an accelerating section upstream from the chicane. Although great progress has been made in this field, magnetic compression may introduce momentum spread and transverse emittance dilution due to the bunch self-interaction via coherent synchrotron radiation [1]. To obtain a smaller and more symmetrical electron beam, a linear energy-time correlation is required along the bunch, which can be realized by an accelerating structure at a higher harmonic [2], with respect to the main accelerating linac RF.

Velocity bunching relies on the phase slippage between the electrons and the rf wave that occurs during the acceleration of nonultrarelativistic electrons. This was experimentally observed in a photocathode rf gun [3] and it has been proposed to be integrated into the velocity bunching scheme in the next photoinjector designs using a dedicated rf structure downstream of the rf electron source [4]. Previous experimental works have shown the compression ability of the velocity bunching method [5, 6]. Furthermore, the emittance growth in the compressing process was completely compensated by long solenoids [7] when the compression factor is 3.

This paper mainly focuses on the beam bunching in the travelling wave structure with low acceleration gradient (4 MV/m) instead of the high acceleration gradient (normally 20 MV/m) that was used in previous work. A brief analysis of the velocity bunching mechanism is presented in the Section 2. In Section 3, the analytical and simulated results of bunch compression within high and low acceleration gradient accelerators are described. The conclusion finds that a travelling wave accelerating structure with a low acceleration gradient is more tolerant to phase jitter and it is easier to obtain a compressed electron beam with symmetrical longitudinal distribution and low energy spread in the velocity bunching process. A transverse emittance compensation

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scheme is shown in Section 4, which is easy to be adjusted for different compression factors. Section 5 presents a summary of this paper.

2 Velocity bunching mechanism

In the velocity bunching process, the longitudinal phase space rotation is based on a correlated timevelocity chirp in the electron bunch, so that electrons on the tail of the bunch are faster than electrons in the bunch head. This rotation occurs inside a travelling rf wave of a long multicell rf structure, which applies an off crest energy chirp to the injected beam as well as accelerating it. This is possible if the injected beam moves slightly slower than the phase of the rf wave. So that when injected at the zero crossing field phase it slips back to phases where the field is accelerating, but is simultaneously chirped and compressed.

An electron in an rf travelling wave accelerating structure experiences the longitudinal electric field:

$$E_z = E_0 \sin \phi, \tag{1}$$

where E_0 is the peak field, $\phi = kz - \omega t + \phi_0$ is the phase of the electron with respect to the wave and ϕ_0 is the injection phase of the electron with respect to the rf wave. The evolution of ϕ can be expressed as a function of z:

$$\frac{\mathrm{d}\phi}{\mathrm{d}z} = k - \omega \frac{\mathrm{d}t}{\mathrm{d}z} = k - \frac{\omega}{\beta c} = k \left(1 - \frac{\gamma}{\sqrt{\gamma^2 - 1}}\right). \tag{2}$$

The energy gradient can be written as[8]:

$$\frac{\mathrm{d}\gamma}{\mathrm{d}z} = \alpha k \sin\phi, \qquad (3)$$

where $\alpha \equiv eE_0/mc^2k$ is defined as dimensionless vector potential amplitude of the wave. Eqs. (2) and (3) with the initial conditions $\gamma_{z=0} = \gamma_0$ and $\phi_{z=0} = \phi_0$ describe the longitudinal motion of an electron in the rf structure. Using a separation of variables approach, one can get

$$\alpha \cos\phi + \gamma - \sqrt{\gamma^2 - 1} = C,\tag{4}$$

the ϕ can be expressed as a function of γ :

$$\phi(\gamma) = \arccos\left(\frac{C - \gamma + \sqrt{\gamma^2 - 1}}{\alpha}\right),\tag{5}$$

where the constant C is set by the initial conditions of the problem: $C = \alpha \cos \phi_0 + \gamma_0 - \sqrt{\gamma_0^2 - 1}$. The final phase of electron at the exit of the accelerator is

$$\phi_{\rm e}(\gamma_{\rm e}) = \arccos\left(\frac{C - \gamma_{\rm e} + \sqrt{\gamma_{\rm e}^2 - 1}}{\alpha}\right). \tag{6}$$

If the $\gamma_{\rm e}$ is high enough, $\sqrt{\gamma_{\rm e}^2 - 1} - \gamma_{\rm e} \approx 0$. With the approximation $\gamma_0 - \sqrt{\gamma_0^2 - 1} \approx 1/(2\gamma_0)$. Then, the final phase becomes:

$$\phi_{\rm e} = \arccos(\cos\phi_0 + 1/(2\alpha\gamma_0)). \tag{7}$$

Expanding Eq. (7) to the first order in the initial energy spread and initial phase spread gives

$$\Delta\phi_{\rm e} = \frac{\sin\phi_0}{\sin\phi_{\rm e}} \Delta\phi_0 + \frac{1}{2\alpha\gamma_0^2\sin\phi_{\rm e}} \Delta\gamma_0.$$
(8)

Hence, depending upon the incoming energy and phase extents (initial bunch length), the phase of injection in the rf structure ϕ_0 can be tuned to minimize the phase extent of extraction (final bunch length).

3 Bunch compression in low and high gradient accelerators

When the acceleration gradient is higher, the imposed velocity difference within the bunch is more sensitive to the injection phase ϕ_0 . Because the bunch compression is achieved by the velocity difference within the bunch, the compression factor is more sensitive to the injection phase. By using Eq. (8) with initial conditions, one can calculate the compressed bunch length evolutions as a function of the injection phase when the peak acceleration gradients are $E_0=4$ MV/m and $E_0=20$ MV/m. The calculated results are similar to the simulated phase scanning results (by using the code ASTRA [9]), as shown in Fig. 1.



Fig. 1. The rms bunch length as a function of relative phase (There is about 28.7° phase shift between the two results. In the figure, we do a translation for contrast).

The simulation shows a coincident result to the analytical conclusion. This indicates that higher precision phase jitter control and power supply for the solenoids to compensate the transverse emittance growth (the emittance compensation scheme will be presented in next section) are needed in experimental practice when the acceleration gradient is high. A lower acceleration gradient in the velocity bunching process has been confirmed to be more tolerant to phase jitter and power precision. Furthermore, with a lower gradient, the compressed electron beam tends to develop more symmetrical longitudinal distribution. Analytical and simulated results will be presented in the next paragraph.

The velocity bunching process can be shown as in Fig. 2. To compress the electron bunch, the injection phase should be set at a phase that is tens degrees off the crest. The velocity of the initial beam is smaller than the phase velocity of the rf wave $(\beta_{\phi} = 1)$, there is phase slippage between the initial and compressed beam. For a higher gradient accelerator, the phase slippage is smaller, but to obtain the same compression factor as in the low gradient accelerator, the injection phase should be closer to the crest. When the injection phase is farther from the crest, one can get a compressed beam with a single spike and most of electrons are in the spike, but the compression factor is too large. Consequently, emittance compensation is difficult at present. In this paper, the accelerators are normal S-band (2856 MHz), $2/3\pi$ mode, 3 m long travelling wave accelerators. One can also can set an appropriate injection phase to make the phase slippage in the range of linearity within a short accelerator. Then, the electron bunch is imposed on a more linear negative energy chirp (the speed of electrons in the head is slower than the speed of electrons in the tail) and the electron bunch can be compressed enormously in a drift space with appropriate length, which has been presented in Ref. [6] and named by the authors as "ballistic bunching".



Fig. 2. Velocity bunching process.

Eq. (1) can be expanded as:

$$E_z = E_0 \left(\phi - \frac{\phi^3}{3!} + \frac{\phi^5}{5!} - \frac{\phi^7}{7!} + \frac{\phi^9}{9!} \cdots \right). \tag{9}$$

If there are no higher order terms in the equation, then the longitudinal distribution state of the initial beam can be preserved during the velocity bunching process when the initial energy spread is very small, or it can be treated as linear. Unfortunately, there are always higher order terms in the equation. So, reducing the E_0 may be an available method to prevent longitudinal distortion. By using Eq. (2) and (3), the numerical calculation results show the rationality of the analysis. However, the space charge force and magnetic force are not considered. In the following content, the simulation results will be presented.

Longitudinal distributions after compression by different E_0 should be compared. The current profiles of the initial bunch and the compressed bunches (with almost identical bunch length) are shown in Fig. 3. From the above figure, it appears that a travelling wave accelerating structure with low acceleration gradient is more successful to obtain compressed electron with symmetrical longitudinal distribution. Fig. 4 shows the bunch



Fig. 3. (color online) The current profiles of the initial bunch (blue line) and the compressed bunches (black line: compressed bunch in 4 MV/m gradient accelerator; red line: compressed bunch in 20 MV/m gradient accelerator).



Fig. 4. RMS bunch length evolution along the longitudinal position.

length evolutions along the beam direction when the acceleration gradients are $E_0=4$ MV/m and $E_0=20$ MV/m.

As we know, to avoid longitudinal distortion in the magnetic compression, an X-band accelerator system is normally used to linearize the energy chirp of the electron beam [2], which costs almost one million dollars. Consequently, a detailed theoretical and experimental study of this problem in the velocity bunching process is worth doing.

When the E_0 is higher, the energy spread in the bunch is greater, because the bunch compression is achieved by the velocity difference within the bunch. Fig. 5 shows a contrast of the energy spread evolutions when the gradients are $E_0=4$ MV/m and $E_0=20$ MV/m.



Fig. 5. RMS energy spread evolutions at different acceleration gradients.

4 Emittance compensation

In this section, an emittance compensation scheme will be suggested during the velocity bunching process.



Fig. 6. The electric field of accelerators along the beam line.

The electric and magnetic fields along the beam line are shown in Fig. 6 and Fig. 7. The peak acceleration gradients of the photocathode rf gun, accelerator 1 and accelerator 2 are 80 MV/m, 4 MV/m and 20 MV/m, respectively. Four solenoids are used for emittance compensation, B_0 is located at 10 cm downstream from the photocathode, and the other three are located around the accelerators.



Fig. 7. The magnetic field of solenoids along the beam line.

To prevent irreversible emittance growth during bunch compression, it is essential to preserve the laminarity of the electron beam with an envelope propagated as close as possible to a Brillouin-like flow. This is represented by an invariant envelope [10] as generalized to the context of beam compression and thus increasing I during acceleration. Mismatches between the space charge force and the external focusing gradient produce slice envelope oscillations that cause normalized emittance oscillations. It has been shown that, in order to keep such oscillations under control during the velocity bunching. the beam has to be injected into the accelerator with a laminar envelope waist ($\sigma'=0$) and the envelope has to be matched to the accelerating and focusing gradients so that it can stay close to an equilibrium mode [10]. Long solenoids around the accelerator are used to provide the required focus.

The transverse emittance and beam size evolutions along the beam direction are shown in Fig. 8. For electron bunches with different compression factors at different injection phases, one only needs to scan the strength of the solenoids to compensate the emittance growth, as shown in Fig. 8.

Figure 9 shows the current profiles for different compressed beam at different injection phases. We should point out that the magnetic forces of long solenoids (around the accelerator) can affect the compression, which can be found by contrasting the results in Fig. 9 and the phase scanning results at $E_0=4$ MV/m without the long solenoids in Fig. 1. From this contrast, one can also find that the maximum compression factor is lower and the compression factor is more tolerant to the phase jitter when the magnetic forces of long solenoids are considered in the simulation. The primary reason for this



Fig. 8. (color online) The transverse emittance and beam size evolutions along the beam direction at different injection phases, the optimal magnetic strengths of solenoids are also shown in the figure.



Fig. 9. (color online) The current profiles for different compressed beams at different injection phases.

must be the rearrangement of electrons in the bunch caused by the magnetic forces. However, a strict analysis of this problem cannot be provided by the authors at present.

When the E_0 is higher, the average energy of electrons is higher, and then the influence of the magnetic forces of long solenoids is lower and the maximum compression factor can be higher. But, the higher energy of electrons means that the compensation of the transverse emittance is more difficult, and the problem would be worse when the compression factor is expected to be great (because of the high current I, see the analysis in the third paragraph of this section). Consequently, the influential Ref. [7] simply presents the transverse emittance compensation result when the compression factor is 3 (20 MV/m accelerator was used to compress the electron bunch in the reference). If a 4 MV/m accelerator is used in the bunching process, the normalized emittance oscillations can be easier kept close to an equilibrium mode because of the lower energy. In our scheme, the maximum compression factor can be achieved at about 4.5 while the transverse emittance is compensated completely, and the next accelerator 2 is used to preserve it irreversibly.

5 Summary and conclusion

In our analyses, we contrast the ASTRA simulated results of velocity bunching process in travelling wave structure with low (4 MV/m) and high (20 MV/m) acceleration gradients. Our results show that the bunch compression application with low acceleration gradient is more tolerant to phase jitter and should be useful for obtaining better performance beams with symmetrical longitudinal distribution and low energy spread. Furthermore, a successful improvement of transverse emittance during compression is possible with optimized long solenoids, which is easily satisfied for different compressing factors.

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