Uncertainties induced by statistical fluctuations in fast mono-energetic neutron yield measurements

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Abstract: A plastic scintillation detector was used to measure the yield of deuterium-deuterium (DD) neutrons or deuterium-tritium (DT) neutrons. Collisions of fast neutrons with hydrogen nucleus in a scintillator generated recoil protons, the energies of which were fully deposited in the scintillator. The statistical fluctuation of the protons' number and that of the protons' total energy were two sources of measurement uncertainty. Based on DT neutrons, this paper represents the algorithms of computing the probability density functions of the two sources. Uncertainties of the measurement induced by statistical fluctuations were finally computed by constructing the probability density functions of the proton number and that of the neutron number.

Key words: probability density function, statistic fluctuation, uncertainty

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1 Introduction

In inertial confinement fusion (ICF) experiments at the Shenguang III laser prototype facility [1], incident laser beams deposited a portion of their energy on a deuterium-tritium (DT) or deuterium-deuterium (DD) filled target, and then the target imploded to produce DT neutrons or DD neutrons. Neutron yield measurements gave a sensitive and direct sign about whether thermonuclear fusion has taken place [2]. Using a plastic scintillation detector to measure the fusion neutron yield was one of the mature techniques [3]. It was needed to evaluate the uncertainty of a result in the process of measuring the DT neutron or DD neutron yield by the scintillation detector. In all the uncertainty elements, there are two elements induced by proton number statistical and protons' energy statistical fluctuation. These two elements of uncertainty became the main components of total uncertainty when the incident neutron number was small. While computing these two elements, it was required to use the probability density functions (PDFs) of the proton number and protons' energy.

Taking DT neutrons as an example, we concretely analyzed and represented the calculation method and the results of the two PDFs. Based on the energy sum of protons deposited in the scintillator, the two uncertainty components have been deduced. Because the recoil proton energy spectrums were always rectangular, the method used in this paper was also applicable to the related calculation of mono-energy DD neutrons.

2 Principle of the measurement DT neutron yield

The fusion neutron yield measurement system was mainly composed of a scintillation detector, a DC high-voltage power supply, an attenuator and a high-speed digital oscilloscope. The scintillation detector was made mostly of a plastic scintillator, a photomultiplier tube (PMT) and their corresponding electronic part.

When laser beams shot a target, a synchro device sent a trigger signal with a few nanoseconds width to the digital oscilloscope, and the oscilloscope recorded the signals. Meanwhile, the fusion target produced DT neutrons, and they collided with the plastic scintillator of the detector to produce recoil protons. The protons deposited their kinetic energy in the scintillator yielding characteristic fluorescence. The fluorescence was converted to an electrical pulse through PMT, the electrical pulse was fed into the oscilloscope with or without an attenuator. The neutron signal recorded in the oscilloscope was integrally calculated and processed, after which a fusion neutron yield was obtained. The integral area of the neutron signal was directly proportional to the sum of the protons' energy. Statistical fluctuation existed in the sum of the protons' energy deposited in the scintillator. When the proton number was small, there would be a high probability that the integral area of the neutron signal was

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not directly proportional to the proton number.

When a DT-filled target capsule imploded, 14 MeV DT neutrons were generated with an isotropic distribution. The neutrons incidented on the scintillator were a part of the total neutron yield. The scintillator consisted mainly of carbon atoms and hydrogen atoms. The high-energy DT neutron interaction with carbon nuclei or hydrogen nuclei was mainly through elastic scattering collisions. The carbon nuclei obtained a much smaller energy from the collision than the protons did. When calculating the fluorescence yield in the scintillator, it was reasonable to only consider the energy of recoil protons.

3 Two types of probability distributions

The fact that a proton can be produced from a neutron colliding with a hydrogen nucleus can be seen as a Bernoulli problem [4]. If the probability of neutron-to-proton conversion was ε, then the probability of a neutron not converting into a proton was $1-\varepsilon$. The probability that n neutrons incident on the scintillator and produce k protons was equal to

$$p(n,k)=\frac{n!\varepsilon^k(1-\varepsilon)^{n-k}}{k!(n-k)!}, \tag{1}$$

where $k=0,\ 1,\ 2,\ \cdots,\ n$. We used detector #9 that had a cylinder scintillator with a size of 5.71 cm thick $\times$7.62 cm diameter to measure neutron yield; the scintillator type was BC422 [5]. The ε of detector #9 was about 0.188.

Because a proton was a charged particle, and its range in the scintillator was much shorter compared with the dimensions of the scintillator, it was reasonable to assume that the proton deposited all its energy in the scintillator and the energy was finally converted into photon energy. The recoil proton spectrum induced by mono-energy neutron was rectangular. In other words, there was an equal probability for the proton to have any energy between zero and the maximum value, and the latter was equal to mono-energy DT neutron energy [4, 6]. The PDF of the proton energy was

$$f(x)=1/14 \quad 0\ \text{MeV}\leqslant x\leqslant 14\ \text{MeV}. \tag{2}$$

The total energy Z of two protons was the sum of the two protons energy deposited in the scintillator. That was $Z=X+Y$, where X was the energy of one proton, and Y the energy of the other. The PDFs of X and Y can be represented by Eq. (2) and Eq. (3).

$$f(y)=1/14 \quad 0\ \text{MeV}\leqslant y\leqslant 14\ \text{MeV}. \tag{3}$$

The PDF of random vector (X,Y) was $f(x,y) = f(x)\times f(y)=1/14^2$, where $f(x,y)$ was obtained by multiplying $f(x)$ and $f(y)$, and its physical meaning was that the two protons had no correlation. The cumulative probability distribution of the total energy Z was $F_Z(z)=P(Z\leqslant z)=\iint\limits_{x+y\leqslant z} g(x,y)\mathrm{d}x\mathrm{d}y$ [7]. As a result, the PDF of Z was

$$f_z(z)=\mathrm{d}F_z/\mathrm{d}z=\begin{cases} z/14^2 & \text{if } 0\leqslant z<14, \\ (28-z)/14^2 & \text{if } 14\leqslant z\leqslant 28. \end{cases} \tag{4}$$

From Eq. (4), we knew that the PDF of the energy sum of the two protons was triangular. At energy 14 MeV, which was the mathematical expectation value of the two protons' energy, the probability reached the maximum value.

The larger the proton number was, the more steps were needed to calculate their PDFs, and the more time was taken. It was subject to make mistakes when calculating by hand, so we selected the symbolic calculation software Mathematica [8] for computing integrals and derivatives. Many codes were written to compute the PDFs of the total energy of 2, 3,$\cdots$, 56 protons.

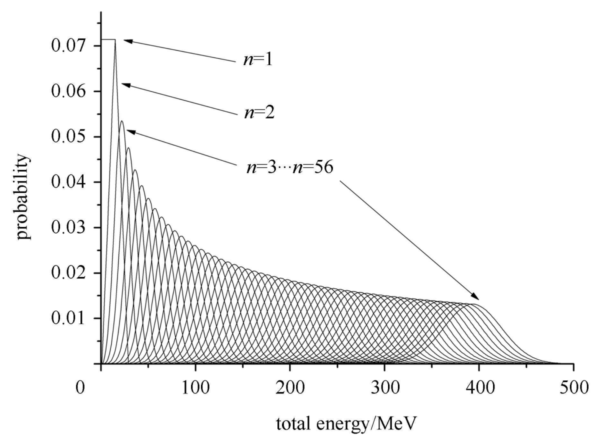

Fig. 1. PDFs of 1–56 protons' energy.

Figure 1 showed the computing results by Mathematica, and the probabilities of the energy between 500 MeV and 784 MeV were not plotted, because they were close to zero. When the proton number was bigger than 56, the normal (Gaussian) distribution functions could be taken as the PDFs of the total protons' energy. According to Eq. (2), it was easy to get the average energy of one proton $u_1=7$ MeV, and the variance $v_1=49/3$ (MeV)^2. The mean of m protons' energy was $u=m\times u_1=7n$ MeV. The variance of m protons was $v=m\times v_1=49m/3$ (MeV)^2. The PDF of $m(m\geqslant 56)$ protons' energy could be given by the normal distribution function:

$$g(m,z)=\sqrt{\frac{3}{98m\pi}}\exp\left(\frac{-3(z-7m)^2}{98m}\right). \tag{5}$$

In Fig. 2, the Gaussian distribution (indicated with a line) was compared with the PDF of 56 protons' energy, which was computed by the software Mathematica (indicated with dots). The two results were almost identical, so we used the normal distributions for large proton numbers.

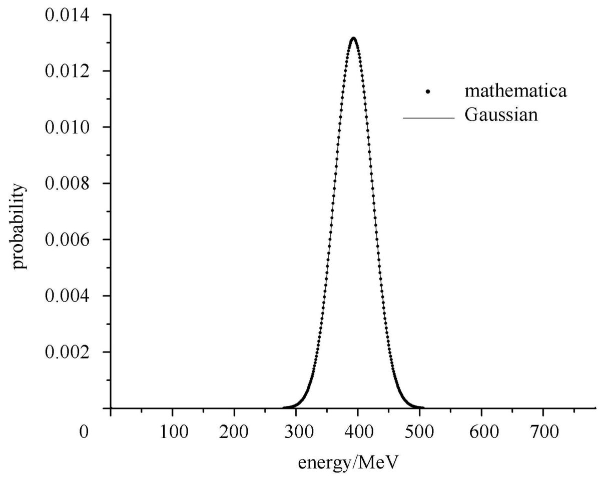

Fig. 2. PDF of 56 protons' energy.

4 Definition of calibration factor *f* and its measurement

In the calibration experiments of the detector, a calibration factor was defined by

$$f=\frac{\text{integral area of neutron pulses}}{\text{total protons' energy}}. \tag{6}$$

In March 2012, we calibrated the detector #9 at the Dense Plasma Focus facility [9]. The sum of the integral area of 8 shots neutron signal was $\sum\limits_{i=1}^{8}A_i$. The neutron yields of 8 shots were deduced by the Ag activation monitor. The sum of neutrons passed through the scintillator of the detector #9 was $\sum\limits_{i=1}^{8}N_i\Omega$, where Ω was the ratio of the neutrons crossed from the scintillator to the total neutron yield. The mean energy of a proton deposited in the scintillator was 7 MeV, thus

$$f=\frac{\sum\limits_{i=1}^{8}A_i}{\varepsilon\times 7\times\sum\limits_{i=1}^{8}N_i\Omega}=0.00873\,\text{ns}\cdot\text{V/MeV}. \tag{7}$$

5 Experimental results

At the Shenguang III laser prototype facility, the DT neutron signal with an integral area of 51.15 ns·V was measured by the detector #9, and the corresponding energy of protons was 5859.1 MeV (i.e. 51.15 ns·V/f). Assuming that the energy of each proton was 14 MeV and each proton deposited full energy in the plastic scintillator, the number of recoiled protons was at least 419 because 5859.1 MeV/14 MeV was equal to 418.5. It was deduced that the PDF of the proton number, which corresponds to 5859.1 MeV protons' energy, could be given by:

$$G(m,5859.1)=\frac{g(m,5859.1)}{\sum\limits_{m=419}^{\infty}g(m,5859.1)}, \tag{8}$$

where k=419, 420, $\cdots$, ∞. In fact, number 1790 was big enough to take place of ∞. $\sum\limits_{m=1791}^{\infty}g(m,5859.1)$ was so small compared with $\sum\limits_{m=419}^{1790}g(m,5859.1)$ that it could be neglected. Through calculation with Eq. (5) and Eq. (8), we knew that the mathematical expectation value of the proton number was 837. The probability that the proton number lay between 821 and 853 (i.e. 837±16) was 68.3%, and the relative uncertainty was 1.91%.

Since 837 protons were produced in the scintillator, the number of neutrons crossed the scintillator was 837 at least. Giving k the value of 837 in the Eq. (1), it was deduced that the PDF of the neutron number, which corresponded to 837 protons, could be given by:

$$P(n,837)=\frac{p(n,837)}{\sum\limits_{n=837}^{\infty}p(n,837)}, \tag{9}$$

where $n=837$, 838, $\cdots$, ∞. In fact, the number 6900 was large enough to take place of ∞. $\sum\limits_{n=6901}^{\infty}p(n,837)$ was so small compared with $\sum\limits_{n=837}^{\infty}p(n,837)$ that it could be neglected. Through calculation with Eq. (1) and Eq. (8), we knew that the mathematical expectation value of the neutron number was 4452. The probability that the neutron number lay between 4314 and 4590 ((i.e. 4452±162) was 68.3%, and the relative uncertainty was 3.10%.

Considering the uncertainty induced by both the PDF of the proton number and that of the neutron number, the probability that the neutron yield lay between 4290 and 4614 (i.e. 4452±138) was 68.3%, and the relative uncertainty was 3.64%. The 3.64% was the square root of the square sum of 1.91% and 3.10% [10]. i.e., $\sqrt{1.91^2+3.10^2}\,\%=3.64\%$. Three experiment results and three interpolation results were listed in Table 1. The proton number uncertainties in the second column and the neutron number uncertainties in the third column were induced by the statistic fluctuations of protons' energy and that of proton number, respectively. The combined uncertainties of neutron number in the fourth column were induced by the previous two statistic fluctuations.

6 Conclusions

Through analysis of one DT-target implosion experiment and the related computation, we obtained the sum

Table 1. Results of proton number, neutron number and their uncertainties.

sum of protons' energy/MeV	proton number and uncertainty(%)	neutron number and uncertainty(%)	neutron yield and combined uncertainty(%)
5859.1[a]	837±16(1.91%)	4452±138(3.10%)	4452±162(3.64%)
6603.7[a]	943±17(1.80%)	5015±146(2.91%)	5015±172(3.42%)
8539.1[b]	1220±19(1.56%)	6489±167(2.57%)	6489±195(3.01%)
10478.1[b]	1497±21(1.40%)	7963±185(2.32%)	7963±216(2.71%)
12417.1[b]	1774±23(1.30%)	9436±201(2.13%)	9436±235(2.49%)
14364.2[a]	2052±25(1.22%)	10915±216(1.98%)	10915±254(2.33%)

a The energy was from experiment. b The energy was interpolation value.

of the protons' energy deposited in the scintillator and the corresponding PDF of the proton number and that of the neutron number. As a result, the uncertainty of the neutron yield was also obtained. The method was also applicable to the mono-energy neutrons (the 2.45 MeV DD neutrons, for example) of other energy in the scintillator.

As Table 1 demonstrated, the more the sum of protons' energy was, the larger the numbers of protons or neutrons and their uncertainties were, and the less their relative uncertainties were. In order to improve the measurement accuracy and reduce the relative uncertainties of the number of protons and that of neutrons, the scintillation detector should be arranged as close as possible to the DT target. As a result, the more neutrons incidented on the scintillator were, the more recoil protons were produced, and the more energy of the protons was deposited in the scintillator.

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