Low-lying states of ^{184}W and ^{184}Os nuclei *

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Abstract: The energy levels, transition energy, B(E2) values, intrinsic quadrupole moment Q_0 and potential energy surface for even-even ^{184}W and ^{184}Os nuclei were calculated using IBM-1. The predicted energy levels, transition energy, B(E2) values and intrinsic quadrupole moment Q_0 results are reasonably consistent with the experimental data. A contour plot of the potential energy surfaces shows that two interesting nuclei are deformed and have rotational characters.

Key words: IBM-1, low-lying state, potential energy surface, quadrupole moment

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1 Introduction

The quadrupole collectivity in an atomic nucleus exhibits distinct regularities, where the nuclear shape can be spherical, deformed or a shape in-between. Like other models and theories [1, 2], the interacting boson model [3] has been successful in reproducing the nuclear collective levels in terms of s and d bosons, which are essentially the collective s and d pairs of valence nucleons [4], respectively. The IBM Hamiltonian has the so-called dynamical symmetry, and the quadrupole deformation shape can be classified as spherical vibrator ($U(5)$), axially symmetric deformation ($SU(3)$), or γ-unstable deformation ($O(6)$), if the interaction strengths of the IBM Hamiltonian take specific values. The medium-to-heavy mass W and Os nuclei are located in the rear-earth mass region. Most of these nuclei are well deformed and can be populated to very high spin. The low-lying W isotopes have been studied within the framework of the interacting boson model IBM-2 [5], and their nuclear structures have been investigated using the IBM-1 model [6].

The properties of the even-even Pt and Os isotopes are investigated in the framework of the interacting boson approximation, including the neutron-proton degree of freedom. It is shown that the transition from the gamma unstable region of the heavier Pt isotopes towards the more axially symmetric deformed features of the lighter Os and Pt isotopes can be described very well by the IBA Hamiltonian [7]. The intrinsic calculation was performed in the framework of Hartree-BCS theory employing the pairing and $Q.Q$ interaction. The level energies up to I=10 in GSB and up to I=4 in the γ-band show good agreement with the experiment [8]. In this study, the calculations of the energy levels of even-even ^{184}W and ^{184}Os nuclei have been performed using the interacting boson model. The positive parity state energies, reduced probabilities of E2 transitions, B(E2) values, intrinsic quadrupole moment Q_0 and potential energy surface were calculated and compared with the experimental data.

2 The interacting boson model

The IBM has become one of the most intensively used nuclear models, due to its ability to describe the changing low-lying collective properties of nuclei across an entire major shell with a simple Hamiltonian. In the IBM the spectroscopies of low-lying collective properties of even-even nuclei were described in terms of a system of interacting s bosons (L=0) and d bosons (L=2) [9, 10]. In addition, the structure of the low-lying levels is dominated by excitations among the valence particles outside the major closed shells in this model. The number of proton bosons, N_π, and neutron bosons, N_υ, were counted from the nearest closed shell, and the total boson number $N=N_\pi+N_\upsilon$. The underlying structure of the six-dimensional unitary group $SU(6)$ of the model leads to a simple Hamiltonian capable of describ-

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ing the three dynamical symmetries. These symmetries are called $SU(5)$ vibrational [11], $SU(3)$ rotational [12] and $O(6)$ γ-unstable [13]. There are also the transitional nuclei [14], whose structures are intermediate. The IBM-1 Hamiltonian can be expressed as [13]

$$\begin{aligned} H = & \varepsilon_s s^+ s + \varepsilon_d (d^+ d) + \sum_{L=0,2,4} C_L \left[(d^+ d^+)^{(L)} . (dd)^{(L)} \right] \\ & + \frac{1}{2} \upsilon_0 \left[(d^+ d^+)_0^{(0)} s^2 + (s^+)^2 (dd)_0^{(0)} \right] \\ & + \sqrt{\frac{1}{2} \upsilon_2} \left[\left[(d^+ d^+)^{(2)} \mathrm{d}s \right]_0^{(0)} \left[s^+ d^+ (dd)^{(2)} \right]_0^{(0)} \right] \\ & + \frac{1}{2} u_0 (s^+)^{(2)} s^+ + \frac{1}{\sqrt{5}} u_2 s^+ s (d^+ d), \end{aligned} \quad (1)$$

where it can be written in general form as [15]

$$H = \varepsilon n_d + a_0 P^\dagger . P + a_1 L.L + a_2 Q.Q + a_3 T_3 . T_3 + a_4 T_4 . T_4, \quad (2)$$

where $\varepsilon = \varepsilon_d - \varepsilon_s$ is the boson energy. The parameters a_0, a_1, a_2, a_3 and a_4 designate the strength of the pairing, angular momentum, quadrupole, octupole and hexdecupole interactions between the bosons.

3 Calculated results

3.1 Energy levels

The rotational limit of the IBM-1 has been applied for the ^{184}W and ^{184}Os nuclei due to the values of the $E4_1^+/E2_1^+$ ratio ($E4_1^+/E2_1^+$=3.3 for ^{184}W and 3.2 for ^{184}Os) [16, 17]. Therefore, these nuclei have a rotational dynamical symmetry $SU(3)$ with respect to IBM-1. The calculations have been performed with no distinction made between the neutron and proton bosons. For the analysis of the excitation energies in the ^{184}W and ^{184}Os nuclei, we tried to keep the number of free parameters in the Hamiltonian to a minimum.

In the framework of IBM-1, the nuclei of ^{184}W and ^{184}Os, with Z=74 and 76, have proton boson hole numbers 4 and 3, and neutron boson hole numbers 8 and 9, respectively. The coefficient values, which have good agreement with the experimental results, are shown in Table 1. The calculated ground band g-, β- and γ- bands and the experimental data of the low lying states are plotted in Fig. 1(a, b) for the ^{184}W and ^{184}Os nuclei. There is good agreement from the comparison of the IBM-1 calculations (open circle) with the experimental data (solid circle) [18, 19], but this is deviated in the high spin (energies) of the experimental data. Furthermore, the IBM-1 model is successful in predicting the β_{2^-} and γ_{2^-} bands for the ^{184}W nucleus, as shown in Tables 2 and 3, respectively.

In addition, the IBM-1 transition energy calculations are plotted in Fig. 2(a, b) as a function of angular momentum for nuclei interest. The comparison between IBM-1 calculations (open circle) and the experimental data (solid circle) [18, 19] shows a good agreement between them in the low energy.

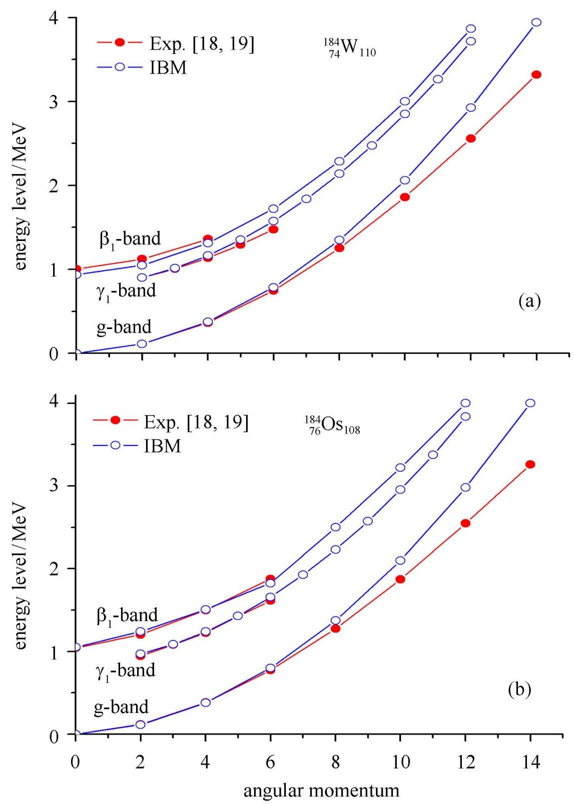

Fig. 1. (color online) The energy levels as a function of angular momentum.

3.2 The B(E2)value

The reduced matrix elements of the E2 operator (T^{E2}) have the form [11]

$$T^{\mathrm{E2}} = \alpha 2 [d^\dagger s + s^\dagger d]^{(2)} + \beta 2 [d^\dagger d]^{(2)}, \quad (3)$$

where ($s^\dagger$, $d^\dagger$) and (s, d) are the creation and annihilation operators for the s and d bosons, respectively, while $\alpha 2$ and $\beta 2$ are two parameters. The B(E2) values are then given by

$$B(\mathrm{E2}, J_\mathrm{i} \to J_\mathrm{f}) = \frac{1}{2J_\mathrm{i}+1} |\langle J_\mathrm{f} \| T^{\mathrm{E2}} \| J_\mathrm{i} \rangle|^2. \quad (4)$$

For the calculations of the absolute B(E2) values, two parameters, $\alpha 2$ and $\beta 2$ of Eq. (3), were adjusted according to the experimental $B(\mathrm{E2};\, 2_1^+ \to 0_1^+)$. Table 4 shows the values of the $\alpha 2$ and $\beta 2$ parameters, which were obtained in the present calculations. The calculated results of the reduced probability transitions, the B(E2) values, and the experimental data [19] are plotted in Fig. 3 for the ^{184}W and ^{184}Os nuclei. Fig. 3 shows a good agreement between the IBM-1 calculations (open circle) and experimental data (solid circle).

Table 1. The adopted values for the parameters used in the IBM-1 calculations. All parameters are given in MeV, except N and CHI (CHI is a constant that is dependent on the dynamical symmetry).

nucleus	N	ε	a_0	a_1	a_2	a_3	a_4	CHI
^{184}W	12	0.0	0.0140	0.0146	−0.0115	0.0	0.0	−1.333
^{184}Os	12	0.0	0.0160	0.0146	−0.0125	0.0	0.0	−1.333

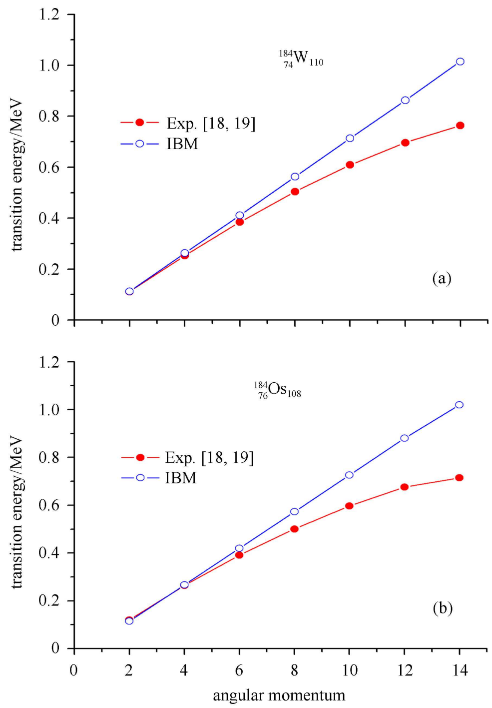

Fig. 2. (color online) The transition energy as a function of angular momentum.

Table 2. The β_2-bands for the ^{184}W nucleus (in MeV).

J^π	IBM-1	Exp. [18, 19]
0^+	1.517	1.322
2^+	1.629	1.431
4^+	1.819	
6^+	2.230	

Table 3. The γ_2-bands for the ^{184}W nucleus (in MeV).

J^π	IBM-1	Exp. [18, 19]
2^+	1.684	1.386
3^+	1.796	1.523
4^+	1.890	
5^+	2.005	
6^+	2.301	

Table 4. The coefficients of T^{E2} (in eb).

nucleus	$\alpha 2$	$\beta 2$
^{184}W	0.10801	−0.14391
^{184}Os	0.10501	−0.11650

Also, we can see that the calculated values deviate significantly from the experimental data, even showing an opposite trend of changing with increasing spin in ^{184}W because the IBM-1 calculations failed in the high spin energy levels. The B(E2) value decreases when the energy level is higher than the experimental data.

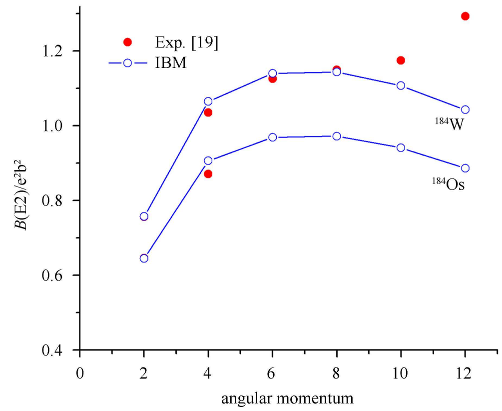

Fig. 3. (color online) The comparison between the IBM-1 calculations and experimental data for even-even ^{184}W and ^{184}Os nuclei.

3.3 The quadrupole moments

The intrinsic quadrupole moments of the nuclei can be derived from the transition rate $B(\mathrm{E2}, J\rightarrow J-2)$ values according to Eq. (5) [20].

$$B(\mathrm{E2})=\frac{15}{32\pi}\frac{(J-1)}{(2J-1)}\frac{J}{(2J+1)}e^2Q_0^2(J\rightarrow J-2). \quad (5)$$

Table 5 presents the calculation of the intrinsic quadrupole moment Q_0 within the framework of IBM-1 for the even-even ^{184}W and ^{184}Os nuclei. The presented results for Q_0 are consistent with the expectations and from phenomenological systematics, and are compared with previous experimental results [21].

Table 5. The intrinsic quadrupole moment Q_0 (in b) for the ground state band.

nucleus	IBM-1	Exp. [21]
^{184}W	6.160	6.168
^{184}Os	5.700	5.694

3.4 Potential energy surface

In recent years, the potential energy surface (PES) by the Skyrme mean field method was mapped onto the PES of the IBM Hamiltonian [22–25]. The expectation value of the IBM-1 Hamiltonian with the coherent state ($|N,\beta,\gamma\rangle$) is used to create the IBM energy surface [15].

The state is a product of the boson creation operators ($b_c^\dagger$), with

$$|N,\beta,\gamma\rangle = \frac{1}{\sqrt{N!}}(b_c^\dagger)^N|0\rangle, \quad (6)$$

$$b_c^\dagger = (1+\beta^2)^{-1_2}\{s^\dagger+\beta[\cos\gamma(d_0^\dagger) + \sqrt{1/2}\sin\gamma(d_2^\dagger+d_{-2}^\dagger)]\}. \quad (7)$$

The energy surface, as a function of β and γ, has been given by [3]

$$E(N,\beta,\gamma) = N\varepsilon_d\beta^2/(1+\beta^2)+N(N-1)/(1+\beta^2)^2(\alpha_1\beta^4 +\alpha_2\beta^3\cos3\gamma+\alpha_3\beta^2+\alpha_4), \quad (8)$$

where the α_i's are related to the coefficients C_L, ν_2, ν_0, u_2 and u_0 of Eq. (1).

The calculated potential energy surfaces for the even-even ^{184}W and ^{184}Os nuclei are presented in Fig. 4. This shows that these two nuclei are deformed and have rotational-like characters. The prolate deformation is more deep than oblate in these nuclei.

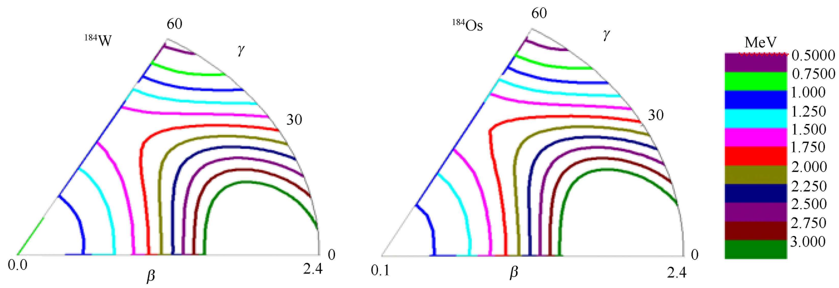

Fig. 4. (color online) The potential energy surfaces for even-even ^{184}W and ^{184}Os nuclei.

4 Summary

The interacting boson model (IBM-1) was used to calculate the energy levels (positive parity), the reduced probability of E2 transitions, the intrinsic quadrupole moment Q_0, and the potential energy surface for ^{184}W and ^{184}Os nuclei. The predicted low-lying levels (energies, spins and parities), the reduced probability of E2 transitions and the intrinsic quadrupole moments were reasonably consistent with the experimental results. The potential energy surfaces for ^{184}W and ^{184}Os nuclei show that these two nuclei are deformed and have transitional dynamical symmetry $SU(3)$–$O(6)$ characters.

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