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Monte Carlo simulations for 20 MV X-ray spectrum reconstruction of a linear induction accelerator

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Abstract: To study the spectrum reconstruction of the 20 MV X-ray generated by the Dragon-I linear induction accelerator, the Monte Carlo method is applied to simulate the attenuations of the X-ray in the attenuators of different thicknesses and thus provide the transmission data. As is known, the spectrum estimation from transmission data is an ill-conditioned problem. The method based on iterative perturbations is employed to derive the X-ray spectra, where initial guesses are used to start the process. This algorithm takes into account not only the minimization of the differences between the measured and the calculated transmissions but also the smoothness feature of the spectrum function. In this work, various filter materials are put to use as the attenuator, and the condition for an accurate and robust solution of the X-ray spectrum calculation is demonstrated. The influences of the scattering photons within different intervals of emergence angle on the X-ray spectrum reconstruction are also analyzed.

Key words: X-ray spectrum reconstruction, Monte Carlo method, filter material, scattering photon, linear induction accelerator

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1 Introduction

The Dragon-I linear induction accelerator (LIA) is a facility which takes advantage of the electromagnetic induction principle to accelerate electrons to an energy as high as 20 MeV [1, 2]. An X-ray is generated when a high-energy electron beam is made to strike a high-Z target. There is an increasing demand to acquire a good knowledge of the X-ray spectral distribution, since it is tremendously significant for diagnostic X-ray imaging applications, such as beam-hardening correction [3], dose deposition calculation [4], duel-energy material detection [5], etc. Because the X-ray emitted in short pulses from the Dragon-I LIA is of high intensity and dose rate, it is very difficult to make direct measurement of the X-ray spectra. Various techniques have been developed for X-ray spectrum reconstruction, mainly including empirical or semiempirical models [6–8], the Compton scattering technique [9], and the attenuated transmission method (ATM) [10–15].

The ATM has attracted extensive attention since it was first proposed by Silberstein in 1932 [10]. This method is relatively simple and shows a good adaption to wide ranges of photon energy. The first step of the ATM is to obtain the transmission curve with respect to the attenuator thickness under the narrow beam condition. The transmissions of the X-ray at various depths are correlated with the energy spectrum via integral equations due to the dependence of the attenuation coefficients on the photon energy. Such a relation can be formulated as a linear equation system when the spectrum is discretized. The second step of the ATM is to reconstruct the spectra by solving the system of linear equations. As is known, the matrix of coefficients is highly ill-conditioned. It is an ineffective task to obtain the inverse of the matrix by using traditional means. Many methods have been proposed for a stable and accurate solution, such as truncated singular value decomposition [11], the Expectation-Maximization algorithm [12], as well as the BFGS quasi-Newton method [13]. Besides, Waggener et al. presented an iterative perturbation method (IPM) to minimize the differences between the measured and the calculated transmission curves for X-ray spectrum estimation [14]. Manciu et al.

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made a further improvement in the IPM by taking into account the smoothness of the megavoltage spectrum function [15].

In this work, we make a study of the spectrum reconstruction of the X-ray generated by the Dragon-I LIA based on numerical analysis of the attenuation curve. The Monte Carlo method is applied to simulate the attenuations of the X-ray in the filter materials, which enables accurate physical modeling and convenient modification of parameters. The improved IPM is used to reconstruct the X-ray spectra from the obtained transmission data. Particular analyses are made on the choice of the attenuators and the behavior of scattered radiation. The calculation results indicate that the condition for a robust and accurate solution of the spectrum reconstruction is related to the attenuation property of the filter material over the interval of the X-ray energy. The impact of different scattering photon fractions on the spectrum reconstruction is also discussed in the paper.

2 Theory and calculation for spectrum reconstruction

The X-ray generated by an accelerator has a continuous spectrum over a wide energy interval. When the polyenergetic X-ray traverses an attenuator, the transmission function can be formulated as

$$T(L)=\int_{E_{\min}}^{E_{\max}} F(E)\exp\left[-\int_L \mu_\rho(E)\cdot\rho\cdot \mathrm{d}l\right]\mathrm{d}E, \quad (1)$$

where $T(L)$ represents the transmission through the attenuator with the mass attenuation coefficient $\mu_\rho(E)$ and the material density ρ. L denotes the total path length of X-ray when passing through the attenuator. $F(E)$ represents the X-ray spectral distribution, E is the photon energy and $E_{\min}$ and $E_{\max}$ are the minimum and maximum energies, respectively.

For practical measurements, the detected transmission is affected by both the X-ray energy spectrum and the detector spectral response. The function $F(E)$ actually stands for the product of these two quantities, and the X-ray spectrum will be obtained when the detector response is factored out. In this work, the ideal model of detector that has a uniform response in energy is applied for simplicity. Under the narrow beam condition, the path length of the X-ray can be considered to be equivalent to the attenuator thickness. Then Eq. (1) associated with different attenuator thicknesses can be discretized into a system of linear equations,

$$T(x_i)=\sum_{j=1}^{n} F(E_j)\cdot\exp\left[-\mu_\rho(E_j)\cdot\rho\cdot x_i\right]\Delta E, \quad (2)$$

where $x_i(i=1,\cdots,m)$ is the thickness of the attenuator, $E_j(j=1,\cdots,n)$ is the discrete energy, $E_1=E_{\min}$, $E_n=E_{\max}$, $\Delta E=E_{\max}/(n-1)$ represents the energy bin width, and $F(E_j)$ denotes the j-th spectrum fraction.

In the improved IPM of Manciu et al. [15], each discrete spectrum value is perturbed in turn searching for the minimum value of the difference quantity given by

$$D_{\mathrm{TF}}=\frac{1}{m}\sum_{i=1}^{m}\frac{|T_{\mathrm{calc}}(x_i)-T_{\mathrm{meas}}(x_i)|}{T_{\mathrm{meas}}(x_i)} +\frac{\alpha}{n-1}\sum_{j=2}^{n}\left[F(E_j)-F(E_{j-1})\right]^2, \quad (3)$$

where α represents the importance factor of the spectrum smoothness. On the right side of Eq. (3), the first term denotes the difference quantity between the calculated and the measured transmission data ($T_{\mathrm{calc}}(x)$ and $T_{\mathrm{meas}}(x)$), and the second term is actually a penalty function related to the local gradient of the spectrum. It needs to be pointed out that $F(E)$ obtained by minimizing D_{TF} is not the exact solution but a very close one, which coincides better with the smoothness feature of a megavoltage spectrum.

At the beginning of spectrum calculation, $E_{\min}$ is set to be zero and $E_{\max}$ to be 20 MeV according to the energy of the accelerated electron beam. The number of discrete spectrum is chosen to be $n=201$, which denotes a subtle spectrum distribution with the energy bin width of $\Delta E=0.1$ MeV. μ_ρ for all materials at different photon energies discussed in this work are obtained from the table values of NIST Report No. 5652 [16] by applying the logarithmic interpolation. A set of normalized pre-spectrum $\{F_{\mathrm{initial}}(E_j)\}$ is initialized to start the process. Each spectrum fraction is perturbed in the reverse order from $j=n$ to $j=1$ by $F_\pm(E_j)=F_{\mathrm{initial}}(E_j)\pm\Delta F(E_j)$, where $\Delta F(E_j)=F_{\mathrm{initial}}(E_j)/2^k$ with $k=1,2,3,\cdots$. The difference quantities, $D_{\mathrm{TF},+}$, $D_{\mathrm{TF},-}$ and D_{TF}, are evaluated for positive, negative and no perturbations, respectively, using Eq. (3). The set of discrete spectrum corresponding to the smallest one of the three difference quantities is picked out and renormalized so as to satisfy

$$\frac{1}{2}\left[F(E_1)+F(E_n)+2\sum_{j=2}^{n-1}F(E_j)\right]\Delta E=1. \quad (4)$$

After a round of perturbations for all $F(E_j)$, the obtained set of the spectrum is used as the new pre-spectrum estimation for the next iterative calculation loop. The final X-ray spectrum is settled when the difference quantity becomes no longer smaller than that in the previous loop.

3 Results and discussions

3.1 Monte Carlo simulation model for X-ray attenuation

The Monte Carlo method, also known as random sampling technique or statistical test method, is a strong theoretical simulation method for the research of X-ray radiography [17]. In this work, we simulate the attenuated transmission measurement of the Dragon-I LIA using the Geant4 code [18]. As shown in Fig. 1, the simulation model of the experimental setup includes a 20 MeV electron beam, a 1.2 mm thick tantalum target, the attenuator and two lead collimators. The electron beam has a normalized emittance of 2000 mm·mrad, and is considered to be Gaussian distributed in profile with the FWHM of 1.5 mm. One collimator is placed in front of the attenuator, and the other is placed behind. The two collimators are both 30 cm in thickness. Each collimator has an aperture throughout the central axis, the radius of which determines the confinement of the photon emergence angle. The detector is settled close to the rear face of the second collimator, which records the photon energy flux passing through the aperture.

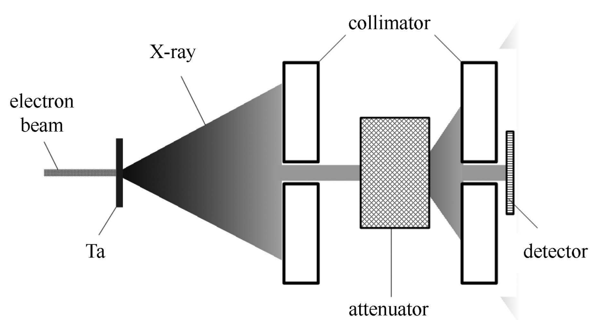

Fig. 1. Simulation model of the experimental setup for X-ray attenuation measurement.

In the following two subsections, the apertures are firstly set small enough to guarantee the narrow beam condition. Various filter materials are used to obtain transmission curves for spectrum reconstruction. Then the aperture in the second collimator is modified, which enables scattering photons within different emergence angle intervals to arrive at the detector.

3.2 Choice of the filter materials for spectrum estimation

Four materials are employed as the filter for X-ray attenuations, including carbon (C), aluminum (Al), iron (Fe) and tungsten (W). The transmission curves with respect to the mass thickness are illustrated in Fig. 2, which apply the simulation model under narrow beam condition. The numbers of transmission data (m) are 27, 25, 28 and 19 for the filter materials of C, Al, Fe and W, respectively. It can be seen that the X-ray is attenuated more rapidly in the materials of higher Z. The X-ray energy spectrum is reconstructed by the improved IPM, where both the uniform distribution (Fig. 3(a)) and the pulse function distribution (Fig. 3(b)) are used as pre-spectra. The results show that the final spectra obtained from the transmission data of C and Al are coincident well with the true spectra. But for Fe and W, the final spectra exhibit obviously larger deviations from the true spectra. Besides, the reconstructed spectra seem to not be convergent when starting with different initial guesses.

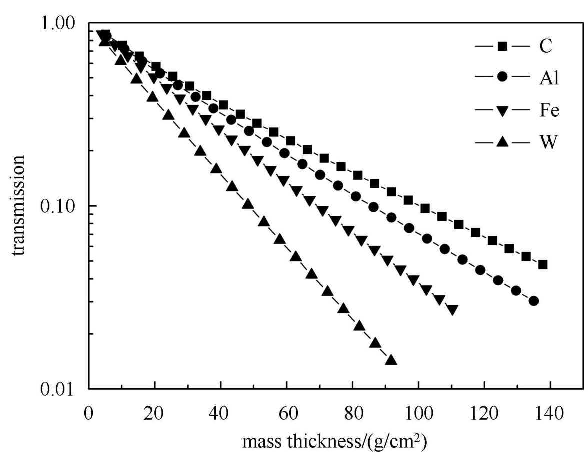

Fig. 2. The transmission curves of X-ray under the narrow beam condition.

As is known, for the system of Eq. (2), when the number of transmission data (m) associated with different attenuation lengths is equal to that of the spectrum energy bins (n), a unique physical solution of the X-ray spectrum can be solved provided that the coefficient matrix is of full rank. This condition requires that μ_ρ of the filter material be decreasing monotonically over the whole energy range being determined [19]. Practically, it means that the coefficient matrix has no proportional columns and consequently each discrete spectrum fraction cannot be replaced by others. C and Al satisfy this demand in the photon energy range between 10 keV and 20 MeV, while Fe and W fail to satisfy it since the energies corresponding to their minimum μ_ρ are less than the upper limit of the X-ray spectrum [16]. In this work, we try to make a subtle reconstruction of X-ray spectrum from quite a small set of transmission equations ($n \gg m$). The calculated results show that such a criterion for choosing a proper filter material for high-energy X-ray spectrum measurements still works here.

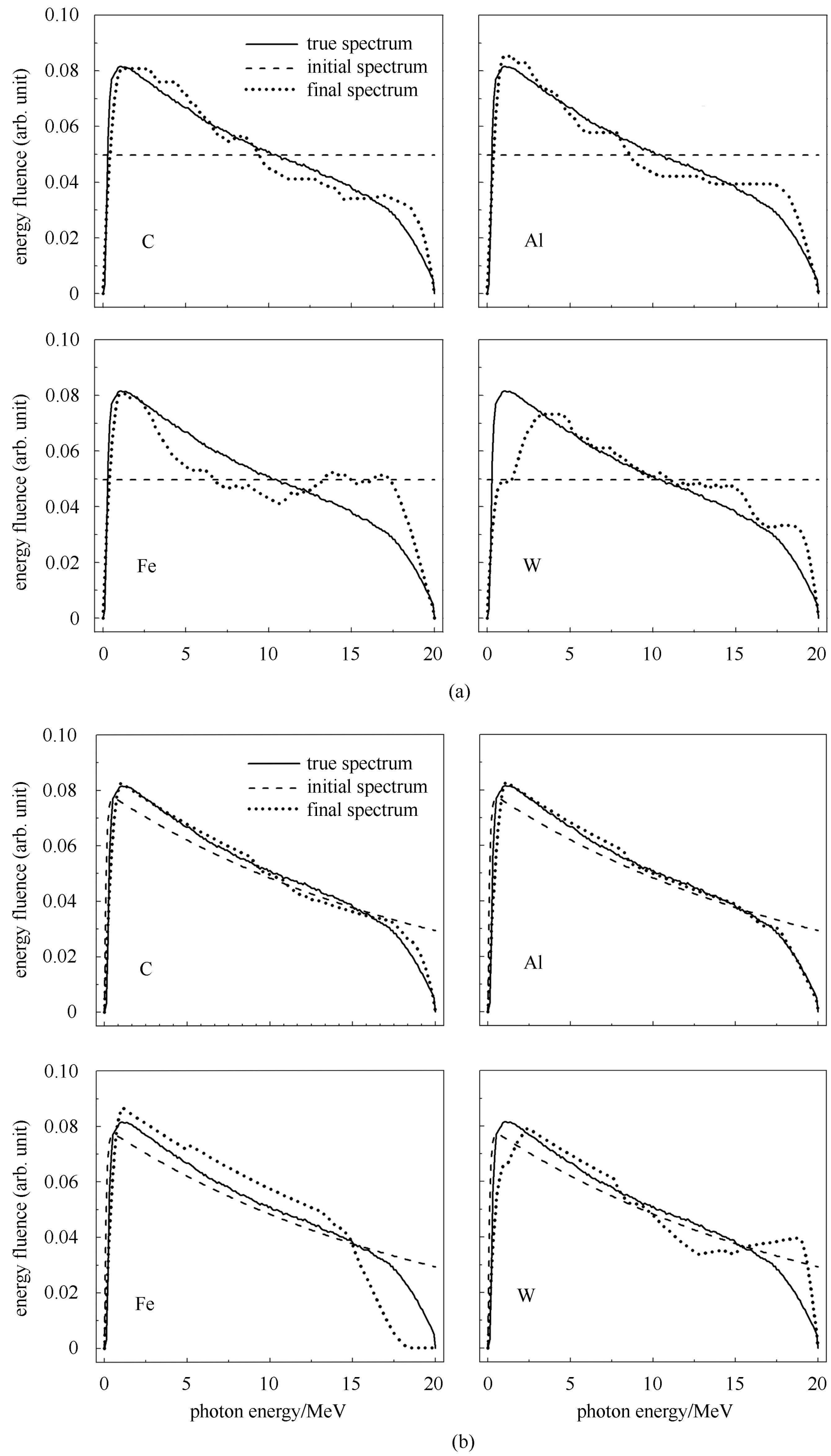

Fig. 3. Reconstruction of X-ray spectra with different filter materials. (a) Pre-spectrum of uniform distribution; (b) Pre-spectrum of pulsefunction distribution.

3.3 Impact of X-ray scattering

Practical transmission measurements are inevitably affected by X-ray scattering since actual experimental setup cannot completely shield the detector from the scattered photons. Here, we use Al as the attenuator and explore the impact of photon scattering on the X-ray spectrum reconstruction.

Firstly, we simulate the attenuation of monoenergetic photons, all of which incident at the attenuator perpendicularly. Fig. 4 shows the angular distributions of the transmitted X-ray. It is seen that the X-ray scattering is more centralized to small emergence angles for photons of higher energies. Besides, as the thickness of the attenuator increases from 1 cm (Fig. 4(a)) to 20 cm (Fig. 4(b)), the enhancement of X-ray scattering is obviously stronger for high-energy photons than the contrary.

Then the value of μ_ρ corresponding to different angle intervals of transmitted photons is calculated according to the following expression

$$\mu_\rho = -\frac{1}{d_\mathrm{m}} \ln\left(\frac{\displaystyle\int_0^{\theta_\mathrm{e}} I_\mathrm{t}(\theta')\mathrm{d}\theta'}{I_0}\right), \tag{5}$$

where I_0 is the incident photon intensity, I_t is the transmitted intensity towards a certain direction, θ_e is the maximum emergence angle of photons reaching the detector, and d_m represents the mass thickness of the attenuator. Here, we mainly focus the attention on small emergence angles. It is found that μ_ρ at low energies exhibits a good coincidence with the NIST data, while for high energies, μ_ρ is obviously smaller than the standard value, and the deviation becomes larger along with the increase of photon energy (Fig. 5). This predicates a bias of the reconstructed spectrum towards the high energy direction, which compensates for "overvaluation" of the μ_ρ in Eq. (2). The calculated spectra from transmission data which contain photon energies within different emergence angle intervals are plotted in Fig. 6. The number of transmission data remains $m = 25$ for different scattering circumstances. As the emergence angle interval is broadened, more scattered photons contribute to the detected transmission data.

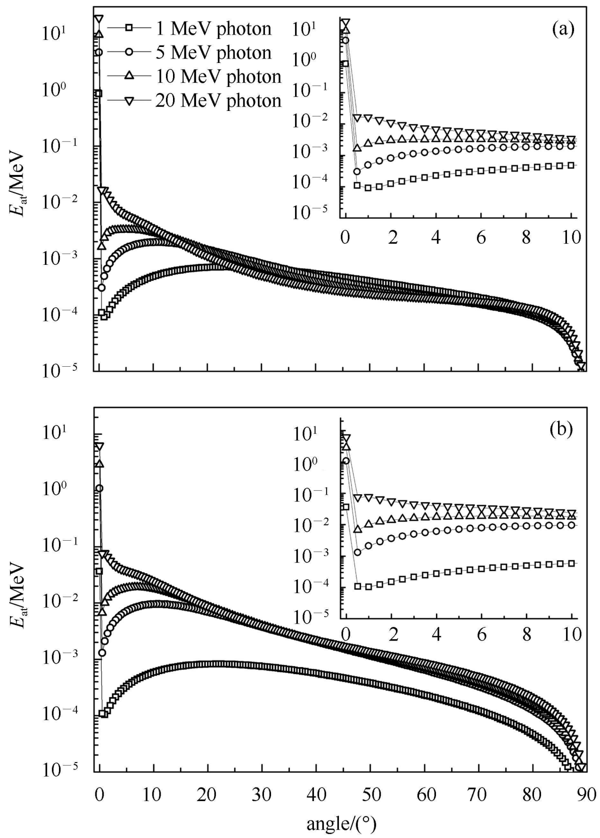

Fig. 4. Angular distribution of monoenergetic photons transmitted through aluminum. The thickness of aluminum is 1 cm for (a) and 20 cm for (b). E_at is the average transmission energy.

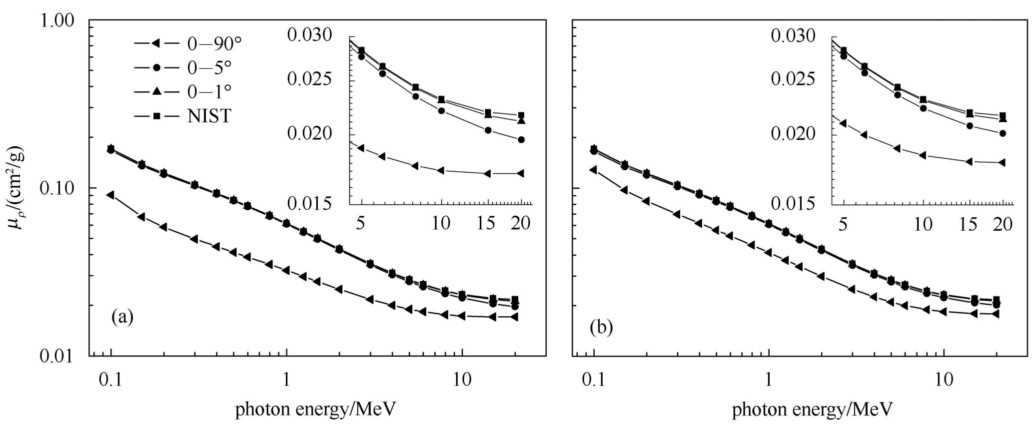

Fig. 5. Comparison of calculated mass attenuation coefficients of aluminum within various emergence angle intervals of transmitted photons. The thickness of aluminum is 1 cm for (a) and 20 cm for (b).

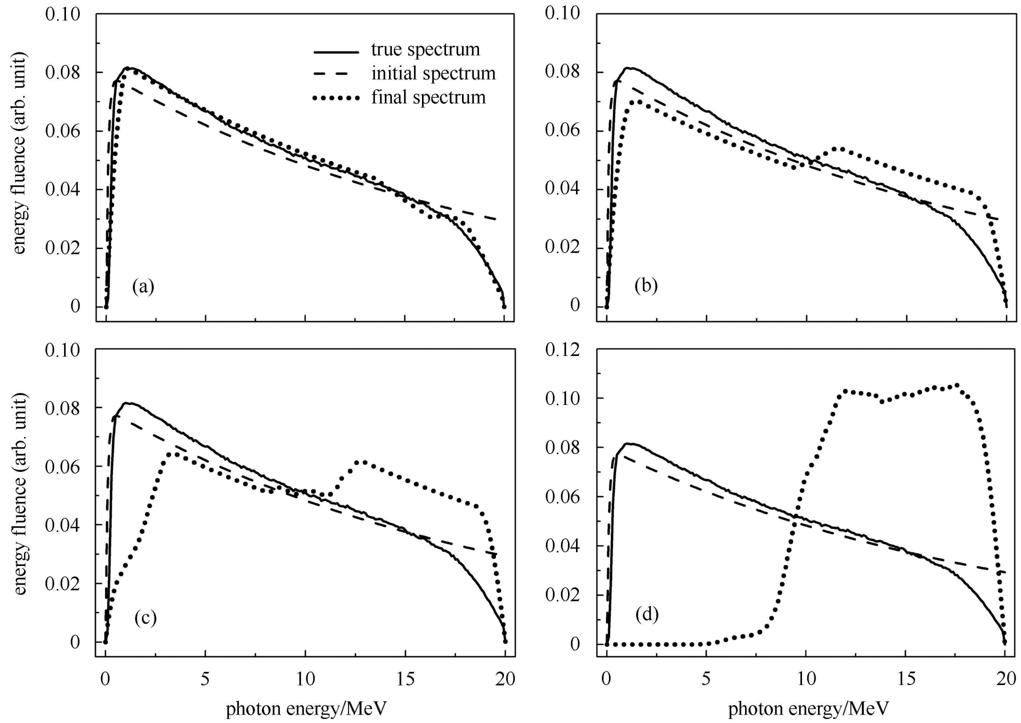

Fig. 6. Reconstructed X-ray spectra from transmission data containing photon energies within different emergence angle intervals. (a) 0–1°; (b) 0–5°; (c) 0–10°; (d) 0–90°.

Consequently, the reconstructed spectral distribution offsets more from the low-energy region to high-energy region.

4 Conclusion

In this paper, the X-ray spectrum reconstruction of Dragon-I LIA is performed based on numerical analysis of the transmission data obtained by Monte Carlo simulations. The improved IPM is applied to calculate the spectrum, which searches for a smooth function to minimize the difference between the "measured" and the calculated transmission curves. Various materials have been used as the attenuator to reconstruct the X-ray spectrum. The condition for a proper filter material in the attenuated transmission measurement is demonstrated. To achieve an accurate and robust solution of the spectrum, the filter material needs to have a mass attenuation coefficient which decreases monotonically throughout the energy range. The calculated results also show that the scattered photons will lead to a bias of the reconstructed spectrum from the low-energy region to high-energy region.

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