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Constraints from Type Ia supernovae on the Λ-CDM model in Randers-Finsler space[*]

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Abstract: Gravitational field equations in Randers-Finsler space of approximate Berwald type are investigated. A modified Friedmann equation and a new luminosity distance-redshift relation is proposed. A best-fit to the Type Ia supernovae (SNe) observations yields that the Ω_Λ in the Λ-CDM model is suppressed to almost zero. This fact indicates that the astronomical observations on the Type Ia SNe can be described well without invoking any form of dark energy. The best-fit age of the universe is given. It is in agreement with the age of our galaxy.

Key words: dark energy, Finsler geometry, Type Ia supernovae, luminosity distance-redshift relation

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1   Introduction

The phenomenon that our universe is expanding was first observed by E. Hubble [1] in 1929, soon after the birth of general relativity. Since the 1990s, rapid progress on the observations of the Type Ia SNe have been made [2–5]. In order to explain the observations, dark energy with the property of negative pressure must be involved in the framework of the standard cosmological model. Multiple kinds of models have been proposed in the past decade [6]. Some of them suggested an evolving canonical scalar field with a potential [7–9]. Others tried to modify general relativity to make up for the gap between the theory and the observations [10–12]. One of the most famous candidates of dark energy is the cosmological constant Λ. However, it requires "fine tuning" in an early epoch of the universe [13].

Finsler geometry, which includes the Riemann geometry as its special case, supplies a new systematic approach to the problems mentioned above. Gravity in a Finsler space has been studied for a long time [14–16]. A theory of the gauge transformations in the context of Finsler space has been discussed

by G. Asanov [17–19] and S. Ikeda [20, 21], and its applications to general relativity has been suggested by R. Beil [22, 23]. The corresponding gravitational field equation was derived from the Riemannian osculating metric in Ref. [17]. Considering the consistency with the Bianchi identity in Finsler geometry and the Einstein's general covariance principle, we gave a new gravitational field equation in a Berwald-Finsler space [24]. In the framework of Finsler geometry, a modified Newtonian gravity has been proposed, which agrees quite well with the asymptotically flat rotation curves of spiral galaxies without invoking dark matter [25].

In this paper, we test possible constraints from the Type Ia SNe on cosmology in Finsler geometry. Based on preceding work [24, 26], we present a new relation between the luminosity distance and redshift. A best-fit to the Type Ia SNe data indicates that our universe is matter-dominated in Finsler space. A reasonable age of our universe is proposed, which is consistent with that obtained from radioactive dating of isotopes in stars and from white dwarfs in our galaxy.

The paper is organized as follows. In Section 2, a brief introduction of Finsler geometry is presented.

In Section 3, we discuss the gravitational field equation in a Randers-Finsler space of approximate Berwald type. A new luminosity-redshift relation is set up in Section 4. It is one of the keys to understanding the Type Ia SNe observations without invoking the dark energy hypothesis. The numerical study is carried out in Section 5. In Section 6, we give concluding remarks and the comparison bewtween our age prediction and that of other models.

2   Finsler geometry

Let us first introduce the basic concepts and notions in Finsler geometry. We use $T_x M$ to denote the tangent space at a point x on a manifold M, i.e., $x \in M$. TM is the tangent bundle of M. Each element of TM is described by (x, y), where $x \in M$ and $y \in T_x M$. The natural projection $\pi : TM \to M$ is defined as $\pi(x, y) \equiv x$.

Finsler geometry has its genesis in integrals of the form

$$\int_a^b F(x^1, \cdots, x^n; y^1, \cdots, y^n) \, dt, \qquad (1)$$

where x^i stands for the position and $y^i \equiv dx^i/dt$ for the velocity. The integrand F is called a Finsler structure.

A Finsler structure of M [27]

$$F : TM \to [0, \infty)$$

is a function with the following properties.
(i) Regularity: F is C^∞ on the entire slit tangent bundle $TM \backslash 0$.
(ii) Positive homogeneity: $F(x, \lambda y) = \lambda F(x, y)$ for all $\lambda > 0$.
(iii) Strong convexity: the $n \times n$ Hessian matrix

$$g_{\mu\nu} \equiv \frac{\partial}{\partial y^\mu} \frac{\partial}{\partial y^\nu} \left(\frac{1}{2} F^2 \right) \qquad (2)$$

is positive-definite at every point of $TM \backslash 0$. (Metrics used in physics are not always positive-definite. Thus, the condition (iii) should be loosened for applications in physics.) And the lowering and raising of indices in this paper are carried out by the fundamental tensor $g_{\mu\nu}$ defined above and its inverse $g^{\mu\nu}$. The Cartan tensor $A_{\lambda\mu\nu}$, which is defined as

$$A_{\lambda\mu\nu} \equiv \frac{F}{4} \frac{\partial}{\partial y^\lambda} \frac{\partial}{\partial y^\mu} \frac{\partial}{\partial y^\nu} (F^2) , \qquad (3)$$

is regarded as a measurement of deviation from the Riemannian manifold.

According to Chern's work [28], each Finsler manifold admits a unique linear connection, called the Chern connection. It is torsion-free and almost

metric-compatible. The connection coefficients take a form as

$$\Gamma_{\mu\nu}^\alpha = \gamma_{\mu\nu}^\alpha - g^{\alpha\lambda} \left(A_{\lambda\mu\beta} \frac{N_\nu^\beta}{F} - A_{\mu\nu\beta} \frac{N_\lambda^\beta}{F} + A_{\nu\lambda\beta} \frac{N_\mu^\beta}{F} \right), \qquad (4)$$

where $\gamma_{\mu\nu}^\alpha$ are the formal Christoffel symbols of the second kind

$$\gamma_{\mu\nu}^\alpha \equiv \frac{1}{2} g^{\alpha s} \left(\frac{\partial g_{s\mu}}{\partial x^\nu} - \frac{\partial g_{\mu\nu}}{\partial x^s} + \frac{\partial g_{\nu s}}{\partial x^\mu} \right). \qquad (5)$$

N_ν^μ is defined as

$$N_\nu^\mu \equiv \gamma_{\nu\alpha}^\mu y^\alpha - \frac{A_{\nu\lambda}^\mu}{F} \gamma_{\alpha\beta}^\lambda y^\alpha y^\beta. \qquad (6)$$

The curvature tensor of a Finsler space is given as [27]

$$R_{\kappa}^{\ \lambda}{}_{\mu\nu} = \frac{\delta \Gamma_{\kappa\nu}^\lambda}{\delta x^\mu} - \frac{\delta \Gamma_{\kappa\mu}^\lambda}{\delta x^\nu} + \Gamma_{\alpha\mu}^\lambda \Gamma_{\kappa\nu}^\alpha - \Gamma_{\alpha\nu}^\lambda \Gamma_{\kappa\mu}^\alpha, \qquad (7)$$

where

$$\frac{\delta}{\delta x^\mu} = \frac{\partial}{\partial x^\mu} - N_\mu^\nu \frac{\partial}{\partial y^\mu}. \qquad (8)$$

The Ricci scalar is defined as

$$Ric \equiv g^{\mu\nu} R_{\mu\nu} , \qquad (9)$$

where

$$R_\nu^\mu \equiv \frac{y^\lambda}{F} R_{\lambda}^{\ \mu}{}_{\nu\kappa} \frac{y^\kappa}{F} = \frac{y^\lambda}{F} \left(\frac{\delta}{\delta x^\nu} \frac{N_\lambda^\mu}{F} - \frac{\delta}{\delta x^\lambda} \frac{N_\nu^\mu}{F} \right). \qquad (10)$$

The Ricci tensor $Ric_{\mu\nu}$, first introduced by Akbar-Zadeh [29], is

$$Ric_{\mu\nu} \equiv \frac{\partial^2 \left(\frac{1}{2} F^2 Ric \right)}{\partial y^\mu \partial y^\nu} = \left[\frac{1}{2} F^2 Ric \right]_{y^\mu y^\nu}. \qquad (11)$$

A Randers space is a specific type of Finsler space, whose Finsler structure takes the form [30]

$$F(x, y) \equiv \alpha(x, y) + \beta(x, y), \qquad (12)$$

where

$$\alpha(x, y) \equiv \sqrt{\tilde{a}_{\mu\nu}(x) y^\mu y^\nu}, \qquad (13)$$

$$\beta(x, y) \equiv \tilde{b}_\mu(x) y^\mu. \qquad (14)$$

The $\tilde{a}_{\mu\nu}$ are the components of a Riemannian metric and the $\tilde{b}_\mu$ are those of a 1-form. Lower-case Greek indices run from "0" to "3". Specifically, the lowering and raising of indices for the terms with a tilde are carried out by $\tilde{a}_{\mu\nu}$ and its inverse $\tilde{a}^{\mu\nu}$ instead of the fundamental tensor.

A Finsler structure F is said to be of Berwald type if the Chern connection coefficients $\Gamma_{\mu\nu}^\alpha$ in natural coordinates have no y dependence. A Randers

space is said to be of Berwald type if and only if [31]

$$\tilde{b}_{\mu|\nu} \equiv \frac{\partial \tilde{b}_\mu}{\partial x^\nu} - \tilde{b}_\kappa \tilde{\gamma}^\kappa_{\mu\nu} = 0, \qquad (15)$$

where $\tilde{\gamma}^\kappa_{\mu\nu}$ are the Christoffel symbols of a Riemannian metric $\tilde{\alpha}$. After some tedious calculations, one obtains that

$$\Gamma^\kappa_{\mu\nu} = \tilde{\gamma}^\kappa_{\mu\nu}. \qquad (16)$$

So the curvature tensor of a Randers space of Berwald type is given as

$$R_{\kappa \ \mu\nu}^{\lambda} = \frac{\partial \tilde{\gamma}^\lambda_{\kappa\nu}}{\partial x^\mu} - \frac{\partial \tilde{\gamma}^\lambda_{\kappa\mu}}{\partial x^\nu} + \tilde{\gamma}^\lambda_{\alpha\mu} \tilde{\gamma}^\alpha_{\kappa\nu} - \tilde{\gamma}^\lambda_{\alpha\nu} \tilde{\gamma}^\alpha_{\kappa\mu} , \qquad (17)$$

and the corresponding Ricci tensor $Ric_{\mu\nu}$ is

$$Ric_{\mu\nu} = \frac{1}{2}(R_{\mu \ \alpha\nu}^{\alpha} + R_{\nu \ \alpha\mu}^{\alpha}). \qquad (18)$$

3   The Friedmann model in Randers-Finslerian space

In order to investigate the Friedmann-Robertson-Walker (FRW) cosmology, we take $\tilde{\alpha}_{\mu\nu}$ to be the form

$$\tilde{a}_{\mu\nu} = \mathrm{diag}\left(1, -\frac{R^2(t)}{1-kr^2}, -R^2(t)r^2, -R^2(t)r^2\sin^2\theta\right), \qquad (19)$$

where $k = 0, +1, -1$ stands for a flat, closed or open universe, respectively. With the condition (15) in mind and assuming that the space of our universe is almost homogeneous and isotropic, we take

$$\tilde{b}_\mu = (\tilde{b}_0, 0, 0, 0), \qquad (20)$$

where $\tilde{b}_0$ is a small constant. This renders the Randers space which we investigate to be an approximate Berwald one. It is known that all the tangent spaces of a Berwald space are linearly isometric to a common Minkowski one, the mathematical setting in which Einstein's theory of special relativity is most conveniently formulated.

Using the identities (2), (17) and (18), one may directly calculate the Ricci tensor in the Randers space of approximate Berwald type. Non-zero components are listed below:

$$Ric_{00} = -3\frac{\ddot{R}}{R}\tilde{a}_{00},$$

$$Ric_{ij} = -\left(\frac{\ddot{R}}{R} + 2\frac{\dot{R}^2}{R^2} + \frac{2k}{R^2}\right)\tilde{a}_{ij} . \qquad (21)$$

The trace of the Ricci tensor $Ric_{\mu\nu}$ gives the scalar curvature $S \equiv g^{\mu\nu}Ric_{\mu\nu}$,

$$S = -6\frac{\alpha}{F}\left(\frac{\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} + \frac{k}{R^2}\right)$$

$$-3\frac{\ddot{R}}{R}\frac{\alpha^2}{F^2}\left(\frac{\beta}{F}\tilde{a}_{00}\frac{y^0}{\alpha}\frac{y^0}{\alpha} - 2\tilde{a}_{00}\frac{y^0}{\alpha}\tilde{b}^0\right)$$

$$-\left(\frac{\ddot{R}}{R} + 2\frac{\dot{R}^2}{R^2} + \frac{2k}{R^2}\right)\frac{\alpha^2}{F^2}\left(\frac{\beta}{F}\tilde{a}_{ij}\frac{y^i}{\alpha}\frac{y^j}{\alpha}\right). \quad (22)$$

A new gravitational field equation in the Berwald-Finsler space is given as [24]

$$\left[Ric_{\mu\nu} - \frac{1}{2}g_{\mu\nu}S\right] + \left\{\frac{1}{2}B_{\alpha \ \mu\nu}^{\alpha} + B_{\mu \ \nu\alpha}^{\alpha}\right\} = 8\pi G T_{\mu\nu}, \qquad (23)$$

where

$$B_{\mu\nu\alpha\beta} = -A_{\mu\nu\lambda}R_{\theta \ \alpha\beta}^{\lambda}y^\theta/F. \qquad (24)$$

T_ν^μ is the energy-momenta tensor as $T_\nu^\mu = \mathrm{diag}(\rho, -p, -p, -p)$, where $p \equiv p(x)$ and $\rho \equiv \rho(x)$ are, respectively, the pressure and the energy density of the cosmic fluid.

In a Randers space of approximate Berwald type, the gravitational field equation (23) reduces to

$$\left[Ric_{\mu\nu} - \frac{1}{2}g_{\mu\nu}S\right] = 8\pi G T_{\mu\nu}, \qquad (25)$$

because the terms $B_{\alpha \ \mu\nu}^{\alpha}$ and $B_{\mu \ \nu\alpha}^{\alpha}$ are zero. $Ric_{\mu\nu}$ and S are given by the identities (21) and (22).

For the sake of simplicity, we introduce two parameters A and B as

$$A \equiv \frac{\alpha}{F}\left(\frac{\beta}{F}\tilde{a}_{00}\frac{y^0}{\alpha}\frac{y^0}{\alpha} - 2\tilde{a}_{00}\frac{y^0}{\alpha}\tilde{b}^0\right) \qquad (26)$$

and

$$B \equiv \frac{\alpha}{F}\left(\frac{\beta}{F}\tilde{a}_{ij}\frac{y^i}{\alpha}\frac{y^j}{\alpha}\right) . \qquad (27)$$

The 0-0 component of the field equation (25) gives the modified Friedmann equation

$$\frac{\alpha}{F}\left(\frac{\dot{R}^2}{R^2} + \frac{k}{R^2}\right) - \frac{1}{2}\frac{\alpha}{F}\frac{\ddot{R}}{R}A$$

$$+\frac{1}{6}\frac{\alpha}{F}\left(\frac{\ddot{R}}{R} + 2\frac{\dot{R}^2}{R^2} + 2\frac{k}{R^2}\right)B = \frac{8\pi G}{3}\rho \qquad (28)$$

and the i-j component of (25) gives

$$3\frac{\alpha}{F}\left(2\frac{\ddot{R}}{R} + \frac{\dot{R}}{R^2} + \frac{k}{R^2}\right) + \frac{9}{2}\frac{\ddot{R}}{R}\frac{\alpha}{F}A$$

$$+\frac{1}{2}\frac{\alpha}{F}\left(\frac{\ddot{R}}{R} + 2\frac{\dot{R}}{R^2} + 2\frac{k}{R^2}\right)B = 8\pi G(-3p). \quad (29)$$

Subtracting the above two equations, we have

$$\frac{\alpha}{F}\frac{\ddot{R}}{R}(1+A)=-\frac{4\pi G}{3}(\rho+3p) . \quad (30)$$

The 0-0 component of the field equation (28) can be rewritten into the form

$$\frac{\alpha}{F}\left(\frac{\dot{R}^2}{R^2}+\frac{k}{R^2}\right)\left(1+\frac{B}{3}\right)(1+A)$$

$$=\frac{8\pi G}{3}\rho(1+A)+\frac{4\pi G}{3}(\rho+3p)\left(-\frac{A}{2}+\frac{B}{6}\right). \quad (31)$$

Assuming that A and B are both time-independent, implementation of time derivative $\dfrac{\mathrm{d}}{\mathrm{d}t}$ on both sides of the equation (31) and using (30) again leads us to

$$-\frac{\dot{R}}{R}\left[\left(1+\frac{B}{3}\right)(\rho+3p)+\rho\left(2+\frac{3A}{2}+\frac{B}{6}\right)\right.$$

$$\left.+p\left(-\frac{3A}{2}+\frac{B}{2}\right)\right]=\dot{\rho}\left(1+\frac{3A}{4}+\frac{B}{12}\right)$$

$$+\dot{p}\left(-\frac{3A}{4}+\frac{B}{4}\right). \quad (32)$$

With the equations of state $p_i = w_i\rho_i$ of each individual component i (where the constant $w_i = 0, -1, -1/3$ corresponds to matter, vacuum and "curvature" respectively), the equation (32) can be solved,

$$\rho_i \propto R^{-\frac{3(1+w_i)+\frac{3}{2}(1-w_i)A+\frac{1}{2}(1+3w_i)B}{1+\frac{3}{4}(1-w_i)A+\frac{1}{12}(1+3w_i)B}} . \quad (33)$$

4   A new luminosity-redshift relation

We adopt the conventional definitions

$$H(a)\equiv\frac{\dot{R}}{R}, \qquad \rho_{\text{crit0}}\equiv\frac{3H_0^2}{8\pi G} , \quad (34)$$

$$\Omega_{i0}\equiv\frac{\rho_{i0}}{\rho_{\text{crit0}}}=\left(\frac{8\pi G}{3H_0^2}\right)\rho_{i0} , \quad (35)$$

and

$$\rho_k\equiv-\frac{3k}{8\pi GR_0^2a^{-2}}, \quad \rho_{\text{vac}}=\rho_\Lambda\equiv\frac{\Lambda}{8\pi G} . \quad (36)$$

Combining the equations (31) and (33), one obtains

$$H(a)=H_0\left[\sum_{i(k)}\Omega_{i0}f_i(w_i,A,B)a^{-n_i(w_i,A,B)}\right]^{1/2} , \quad (37)$$

where

$$f_i(w_i,A,B)=\frac{1+\frac{3}{4}(1-w_i)A+\frac{1}{12}(1+3w_i)B}{\left(1+\frac{B}{3}\right)(1+A)}, \quad (38)$$

$$n_i(w_i,A,B)$$

$$=-\frac{3(1+w_i)+\frac{3}{2}(1-w_i)A+\frac{1}{2}(1+3w_i)B}{1+\frac{3}{4}(1-w_i)A+\frac{1}{12}(1+3w_i)B}. \quad (39)$$

Here $\Omega_k=1-\Omega_M-\Omega_\Lambda$. The notation $\sum_{i(k)}$ denotes that the sum includes Ω_k. As an approximation, we do not take the radiation term into account due to its little influence on the predictions of the Λ-CDM model when using the Type Ia SNe data alone.

The luminosity distance d_{L} as a function of the redshift z of a supernova is [32]

$$d_{\text{L}}(z)=\frac{(1+z)}{H_0\sqrt{|\Omega_{k0}|}}\text{sinn}\left[H_0\sqrt{|\Omega_{k0}|}\int_{1/(1+z)}^1\frac{\mathrm{d}a}{a^2H(a)}\right],$$
$$(40)$$

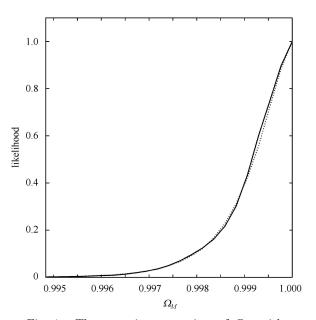where "sinn" stands for "sin" (if $k>0$), "1" (if $k=0$) or "sinh" (if $k < 0$) under certain circumstances.

Fig. 1.   The posterior constraints of Ω_M with $A=-3$ and $B=-1$ using all data. The solid line indicates the fully marginalized posterior of Ω_M. The dotted line shows the mean likelihood of the samples. The fact that the two match well implies that the mean likelihood is well constrained and our result is justifiable. We take the Hubble constant to be $H_0 = 70.5$ km $\cdot$ s^{-1} $\cdot$ Mpc^{-1} instead of a base variable parameter, because the Type Ia supernova data alone cannot put a decent constraint on it. The center value lies at $\Omega_M = 0.9997$. The best-fit likelihood $\mathcal{L}$ for a total number of 307 data points is $-\ln\mathcal{L} = 182.1819$. Corresponding to a high best-fit Ω_M, a low best-fit $\Omega_\Lambda = 0.0003$ should be anticipated, which indicates a matter-dominated universe.

Substituting the equation (37) into (40), we get a luminosity distance-redshift relation that looks like

$$d_{\mathrm{L}}(z) = d_{\mathrm{L}}(z; \Omega_M, \Omega_\Lambda, A, B). \tag{41}$$

5   Numerical study

The distance modulus μ is related to the luminosity distance via

$$\mu \equiv m - M = 5\log_{10}[d_{\mathrm{L}}(\mathrm{Mpc})] + 25, \tag{42}$$

where m is the apparent magnitude of the source and M its absolute magnitude. And it is μ and z that the Supernova Project measured. A total uncertainty of μ (denoted by "σ_μ") and the corresponding redshift z were presented in the Ref. [5].

The χ^2 statistic in our fit is

$$\chi^2_{\mathrm{SN}}(\Omega_M, \Omega_\Lambda) \equiv \sum_{i=1}^{307} \frac{[\mu_{\mathrm{obs}}(z_i) - \mu_{\mathrm{th}}(z_i; \Omega_M, \Omega_\Lambda)]^2}{\sigma_\mu(z_i)^2}, \tag{43}$$

Fig. 2. Best-fit 68% and 95% confidence regions from the marginalized posterior distributions in the Ω_Λ-Ω_M plane for the Finsler cosmological model with $A = -3$ and $B = -1$. The inner contour denotes the 68% confidence limit and the outer one denotes the 95% one. The cross "+" at the upper left corner denotes the best-fit values of $(\Omega_\Lambda, \Omega_M) = (0.0003,\ 0.9997)$ in the modified Friedmann model. Compared with the $(\Omega_\Lambda, \Omega_M) = (0.72,\ 0.28)$ in the reference [34], our result indicates that in a Randers-Finsler universe, no dark energy, and only matter components, exist.

where $\mu_{\mathrm{th}}(z_i)$ is obtained by Eq. (42), while $\mu_{\mathrm{obs}}(z_i)$ and $\sigma_\mu(z_i)$ come from the observations. We employ the Markov Chain Monte-Carlo (MCMC) techniques [33] to explore the parameter space. The likelihood function looks like $\mathcal{L} \propto \mathrm{e}^{-\chi^2_{\mathrm{SN}}(\Omega_M, \Omega_\Lambda)/2}$. For simplicity, we take $A = -3$ and $B = -1$, leaving Ω_Λ(or Ω_M) to be the only free parameter in our model with the constraint $\Omega_M + \Omega_\Lambda = 1$ of a $k = 0$ flat universe.

The marginalized posterior and the mean likelihood distributions of the density parameter Ω_M are shown in Fig. 1. The two contours in Fig. 2 and Fig. 3 respectively line out the 68% and 95% confidence regions of the marginalized distribution in the Ω_Λ-Ω_M and Age-Ω_M planes. Best-fit values of $\Omega_M = 0.9997^{+0.0003}_{-0.0009}$ and $\Omega_\Lambda = 0.0003^{+0.0001}_{-0.0003}$ are obtained
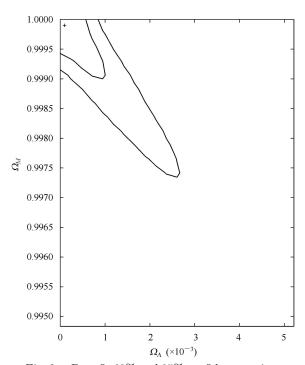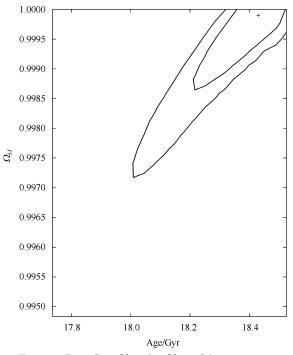

Fig. 3. Best-fit 68% and 95% confidence regions from the marginalized posterior distributions in the Age-Ω_M plane for the Finsler cosmological model with $A = -3$ and $B = -1$. The inner contour denotes the 68% confidence limit and the outer one denotes the 95% one. The cross "+" at the upper right corner in the above figure denotes that the best-fit value of the age of the universe in our modified Friedmann model is 18.298 Gyr. This prediction is consistent with the 10 to 20 Gyr estimate obtained from radioactive dating of isotopes in stars [35, 36] and the 6.5 to 10 Gyr minimum age given by the white dwarfs in our Galactic disk [37, 38]. Thus, the Type Ia SNe data could be well explained by a Randers-Finsler space-time without invoking any form of dark energy.

with $-\ln\mathcal{L} = 182.1819$ for a total number of 307 data points. The almost vanishing Ω_Λ indicates that, in our model, there is no need to invoke the Ω_Λ term in the Einstein field equation to account for the supernova observations. The best-fit age of the universe is $18.298^{+0.102}_{-0.013}$ Gyr.

6   Conclusion

In this paper, we have initiated an exploration on the possibility of a modified Friedmann model in a Randers-Finsler space of approximate Berwald type as an alternative to the dark energy hypothesis. Wondering whether the space-time of our universe is a Randers-Finslerian manifold instead of a Riemannian one, we have rewritten the Einstein field equation in such a space and the new Friedmann equation

was also given. A best-fit to the Type Ia SNe data suppresses the effective density parameter Ω_Λ in the Λ-CDM model to almost zero. This fact demonstrates that a Randers-Finsler geometrical explanation of the "accelerated" expanding universe without invoking dark energy is possible.

Moreover, the best-fit age of the universe in our model is consistent with the 10 to 20 Gyr estimate obtained from radioactive dating of isotopes in stars [35, 36] and the 6.5 to 10 Gyr minimum age given by the white dwarfs in our Galactic disk [34, 37, 38].

Lastly, one should note that the parameters A and B are actually time dependent. Taking them to be constant parameters is merely a zeroth-order approximation. To study the time-dependent behavior of these parameters, one should consider the high-order terms of A and B. Relevant research is being undertaken.

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