Expanding the thermodynamical potential and analysis of the possible phase diagram of deconfinement in the FL model*

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Abstract: Deconfinement phase transition is studied in the FL model at finite temperature and chemical potential. At MFT approximation, phase transition can only be first order in the whole μ-T phase plane. Using a Landau expansion, we further study the phase transition order and the possible phase diagram of deconfinement. We discuss the possibilities of second order phase transitions in the FL model. From our analysis, if the cubic term in the Landau expansion could be cancelled by the higher order fluctuations, second order phase transition may occur. By an ansatz of the Landau parameters, we obtain a possible phase diagram with both the first and second order phase transitions, including the tri-critical point which is similar to that of the chiral phase transition.

Key words: deconfinement phase transition, soliton model, finite-temperature field theory

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1 Introduction

It is generally believed that at sufficiently high temperatures and densities, there is a QCD phase transition from normal nuclear matter to QGP [1, 2]. Theoretically, there are two kinds of phase transitions associated with different symmetries for two opposite quark mass limits. For $N_{\rm f} = 2+1$ massless quark flavors, the QCD lagrangian posses a chiral symmetry $SU(N_{\rm f})_{\rm R} \times SU(N_{\rm f})_{\rm L}$, which is associated with the chiral phase transition. In the heavy quark limit, QCD reduces to a pure $SU(N_{\rm c})$ gauge theory which is invariant under a global $Z(N_{\rm c})$ center symmetry. This symmetry is associated with deconfinement phase transition. The orders of these phase transitions have been studied extensively [3–5], and still remain an interesting problem [6–9]. For chiral phase transition at finite temperature in the chiral limit, the quark-antiquark condensate $\langle \bar{q}_{\rm R} q_{\rm L} \rangle$ serves as a good order parameter. The order of the phase transition depends on the quark flavors. For $N_{\rm f} = 3$ massless quark flavors it is a first order phase transition, and for $N_{\rm f} = 2$ massless quark flavors it is a second order phase transition. At finite densities, chiral phase transition has been studied by many effective models [10–12]. It is generally regarded that at high densities it is a first order phase transition. In the μ-T phase diagram of chiral phase transition, from the first order phase transition to the second order phase transition there exists a tri-critical point (TCP). For deconfinement phase transition, this does not have good order parameter, except for the infinite quark mass limit at which the Polyakov loop serves as an order parameter [13, 14]. In recent studies, the Polyakov loop has been combined into chiral models such as the Nambu-Jona-Lasinio model [15, 16] and the linear sigma model [17–19], which allows us to investigate deconfinement phase transition within the chiral models. Though the Polyakov loop is not a good order parameter, it still serves as an indicator of a rapid crossover towards deconfinement. As we know in Landau theory, for the study of phase transition and transition order, one should find a good order parameter. Once it is identified, the thermody-

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namic functions could be expanded over this order parameter and the transition order could be well studied. For deconfinement phase transition, besides the Polyakov loop, one can also search for other proper order parameters in the effective field models. In earlier studies of deconfinement, bag models have often been used to investigate the confinement mechanics and thermodynamics of deconfinement phase transition. In this paper, we use the effective bag model to study deconfinement phase transition, and mainly focus on the study of the transition order and the possible phase diagram of the deconfinement, especially the possible influence of fluctuations on the phase diagram.

The model we use here is the Friedberg-Lee (FL) soliton bag model. The FL model has been widely discussed in the past few decades [20–22]. It has been very successful in describing, phenomenologically, the static properties of hadrons and their behaviors at low energy. The model consists of quark fields interacting with a phenomenological scalar field σ. The σ field is introduced to describe the complicated nonperturbative features of QCD vacuum. It naturally gives a color confinement mechanism in QCD theory. The model has also been extended to finite temperatures and densities to study deconfinement phase transition [23–27]. Here we will try to identify the proper order parameter in this model and make an analysis of deconfinement phase transition.

The organization of this paper is as follows: in Section 2 we give a brief introduction to the FL model. The thermodynamic potential is derived and deconfinement phase transition is discussed at finite temperatures and densities at mean field theory (MFT) approximation. In Section 3, we make a Landau expansion of the thermodynamic potential. In this way the transition order is studied by analyzing the Landau coefficients. By an ansatz of Landau coefficients, we discuss the possible phase diagram of deconfinement in the FL model. The last section is the summary.

2 Thermodynamic potential and deconfinement phase transition in the FL model at MFT

We start from the Lagrangian of the FL model,

$$\mathcal{L}=\bar{\psi}(\mathrm{i}\gamma_\mu\partial^\mu-g\sigma)\psi+\frac{1}{2}(\partial_\mu\sigma)(\partial^\mu\sigma)-U(\sigma), \quad (1)$$

where

$$U(\sigma)=\frac{1}{2!}a\sigma^2+\frac{1}{3!}b\sigma^3+\frac{1}{4!}c\sigma^4+B. \quad (2)$$

ψ represents the quark field and σ denotes the phenomenological scalar field. a, b, c, g and B are the constants, which are generally fitted in with the production of the properties of hadrons appropriately at zero temperature. We shift the σ field as $\sigma\rightarrow\bar{\sigma}+\sigma'$, where $\bar{\sigma}$ and σ' are the vacuum expectation value and the fluctuation of the σ field, respectively. Then the lagrangian becomes

$$\mathcal{L}_{\mathrm{eff}}=\bar{\psi}(\mathrm{i}\gamma_\mu\partial^\mu-m_{\mathrm{q}})\psi+\frac{1}{2}(\partial_\mu\sigma')(\partial^\mu\sigma') -\frac{1}{2}m_\sigma^2\sigma'^2-U(\bar{\sigma}), \quad (3)$$

where

$$U(\bar{\sigma})=\frac{1}{2!}a\bar{\sigma}^2+\frac{1}{3!}b\bar{\sigma}^3+\frac{1}{4!}c\bar{\sigma}^4+B. \quad (4)$$

$m_{\mathrm{q}}=g\bar{\sigma}$ and $m_\sigma^2=a+b\bar{\sigma}+\frac{1}{2}c\bar{\sigma}^2$ are the effective masses of the quark and σ fields, respectively. The interactions associated with the fluctuation σ', such as σ'^3, σ'^4 and $\bar{\psi}\sigma'\psi$, are neglected in the MFT approximation.

According to finite temperature field theory, the partition function is

$$Z=\int[\mathrm{d}\bar{\psi}][\mathrm{d}\psi][\mathrm{d}\sigma']\exp\left[\int_0^\beta\mathrm{d}\tau\int\mathrm{d}^3x(\mathcal{L}_{\mathrm{eff}}+\mu\psi^\dagger\psi)\right], \quad (5)$$

where μ is the chemical potential of the quarks. Completing the integration in partition function Z, together with the thermodynamic potential $\Omega=-T\ln Z$, at mean field level, we could obtain

$$\Omega=U(\bar{\sigma})+\frac{1}{\beta}\int\frac{\mathrm{d}^3p}{(2\pi)^3}\ln(1-\mathrm{e}^{-\beta E_\sigma}) -\frac{\gamma}{\beta}\int\frac{\mathrm{d}^3p}{(2\pi)^3}\left[\ln(1+\mathrm{e}^{-\beta(E_{\mathrm{q}}-\mu)}) +\ln(1+\mathrm{e}^{-\beta(E_{\mathrm{q}}+\mu)})\right], \quad (6)$$

where β is the inverse of the temperature T and γ is a degenerate factor that $\gamma=2(\mathrm{spin})\times2(\mathrm{flavor})\times3(\mathrm{color})$. In addition, $E_\sigma=\sqrt{\vec{p}^2+m_\sigma^2}$ and $E_{\mathrm{q}}=\sqrt{\vec{p}^2+m_{\mathrm{q}}^2}$.

In our calculation, the parameters are chosen to be $a=17.7$ fm^{-2}, $b=-1457.4$ fm^{-1}, $c=20000$, $g=12.16$. The effective mass of the σ field is fixed at m_σ=550 MeV [25]. Then one could plot Ω versus $\bar{\sigma}$ for different T, as shown in Fig. 1. At zero temperature, where $\Omega=U(\bar{\sigma})$, there are two minima of the thermodynamic potential: one corresponds to the perturbative vacuum at $\bar{\sigma}=0$, and another corresponds to the physical vacuum at $\bar{\sigma}=\sigma_v$. The system is stabled at the physical vacuum at $\bar{\sigma}=\sigma_v$. It is well known that at this time the quarks are confined in a soliton bag and the system is in a hadronic phase. With the temperature increased, the physical vacuum $\bar{\sigma}=\sigma_v$ is lifted up, while the quarks are still confined until the two vacuums degenerate. At this time, the deconfinement phase transition occurs and the phase transition temperature is $T=T_c$. After that, the system is stabled at the perturbative vacuum $\bar{\sigma}=0$, where the quarks are deconfined and the system is in a deconfined phase. This is a first order phase transition.

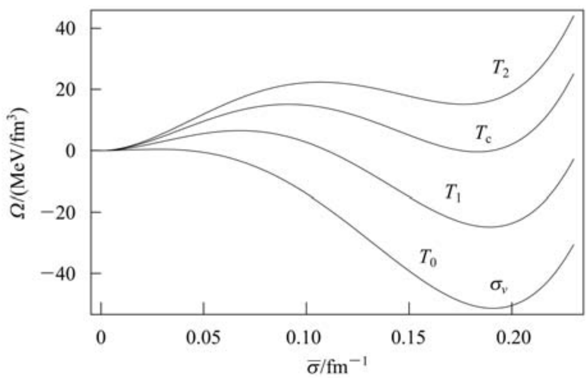

Fig. 1. The thermodynamical potentials for different temperatures and zero chemical potential: T_0=0 MeV, T_1=100 MeV, T_c=121 MeV and T_2=130 MeV.

One can also plot Ω versus $\bar{\sigma}$ at different μ for T=50 MeV, as shown in Fig. 2. The deconfinement phase transition takes place at $\mu=\mu_c$, where the two vacuums degenerate. The analysis of deconfinement phase transition at finite chemical potential is similar to that at finite temperature.

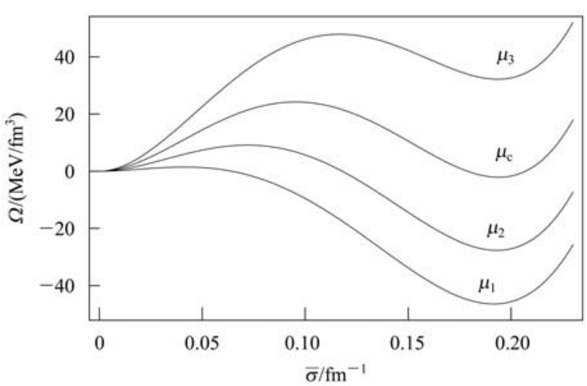

Fig. 2. The thermodynamical potentials for different chemical potentials and fixed temperature at T=50 MeV: μ_1=100 MeV, μ_2=200 MeV, μ_c=255 MeV and μ_3=300 MeV.

One can then obtain the μ-T phase diagram as shown in Fig. 3. In the whole μ-T phase plane, the transition is first order.

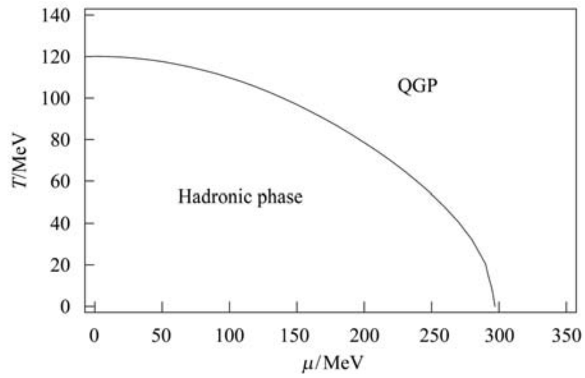

Fig. 3. The μ-T phase diagram of deconfinement at MFT in the FL model.

3 Landau expansion and the possible phase diagram of the deconfinement phase transition

In the above discussion, we know at MFT approximation in the FL model that the deconfinement phase transition is first order. One can plot the $\bar{\sigma}$ as a function of T, as shown in Fig. 4. It can be seen that at $T=T_c$, $\bar{\sigma}$ jumps from nonzero value $\bar{\sigma}=\sigma_v$ to zero value $\bar{\sigma}=0$. In the confined phase $\bar{\sigma}\neq 0$; and in the deconfined phase $\bar{\sigma}=0$. Here, $\bar{\sigma}$ can be viewed as an order parameter of the deconfinement phase transition in the FL model, so we can do a Landau expansion of Ω based on $\bar{\sigma}$ and make a thorough investigation of the phase transition order.

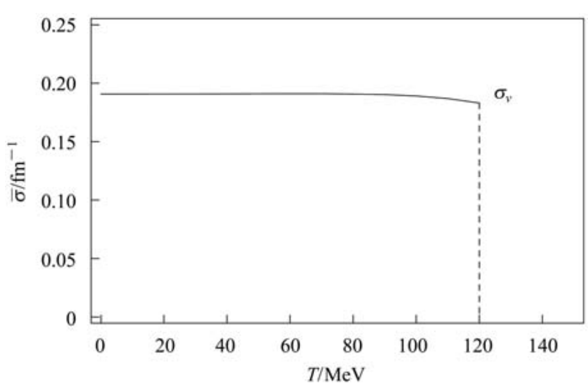

Fig. 4. $\bar{\sigma}$ as a function of T at zero chemical potential in the FL model.

At the MFT approximation, from Eq. (6), the thermodynamic potential could be power expanded by $\bar{\sigma}$ with $\bar{\sigma}^2$, $\bar{\sigma}^3$ and $\bar{\sigma}^4$. However, the analytical forms of the coefficients of the expansion are difficult to obtain. Here we will write down the effective form of the expansion as

$$\Omega=\frac{1}{2}A(T,\mu)\bar{\sigma}^2+\frac{1}{3!}B(T,\mu)\bar{\sigma}^3+\frac{1}{4!}C(T,\mu)\bar{\sigma}^4, \quad (7)$$

where $A(T,\mu)$, $B(T,\mu)$ and $C(T,\mu)$ are the effective parameters which could be determined by a numerical fitting process. This means that at certain T and μ from the configuration of the Ω versus $\bar{\sigma}$, one could fit the curve by the $\bar{\sigma}^2$, $\bar{\sigma}^3$ and $\bar{\sigma}^4$ to obtain the values of $A(T,\mu)$, $B(T,\mu)$ and $C(T,\mu)$. By Eq. (7), from Landau theory, it is clear that the cubic term $\bar{\sigma}^3$ plays a crucial role in the determination of the transition order. At MFT approximation, the fitting results indicate that $B(T,\mu)$, as a negative value, will keep decreasing with an increase of temperature and/or chemical potential. This means that this term will never be zero, and therefore the transition order of deconfinement at MFT approximation can only be first order.

Now we suppose Eq. (7) is the general form of the expansion of the thermodynamical potential by order parameter $\bar{\sigma}$ in the FL model. And we expect the corrections coming from the fluctuations to effectively modify parameters $A(T,\mu)$, $B(T,\mu)$ and $C(T,\mu)$. In principle, they could be calculated by self-consistently resuming the higher order loop diagrams led by the fluctuations of σ'. However, it is very difficult to evaluate these corrections in this way. In the following we will treat the coefficients $A(T,\mu)$, $B(T,\mu)$ and $C(T,\mu)$ as free parameters and make a general study of the phase transition order on the FL model by Landau theory.

In Landau theory, one can make a derivative of the thermodynamic potential to $\bar{\sigma}$ as

$$\frac{\mathrm{d}\Omega}{\mathrm{d}\bar{\sigma}} = A(T,\mu)\bar{\sigma} + \frac{1}{2}B(T,\mu)\bar{\sigma}^2 + \frac{1}{3!}C(T,\mu)\bar{\sigma}^3 = 0. \quad (8)$$

One can then obtain three solutions:

$$\bar{\sigma}_1 = 0, \qquad \bar{\sigma}_{2,3} = \frac{-3B \pm \sqrt{9B^2 - 24AC}}{2C}. \quad (9)$$

In our case, we assume $C > 0$, which guarantees that the vacuums are the minima. When $3B^2 \leqslant 8AC$, there is only one minimum at $\bar{\sigma} = 0$. When $3B^2 > 8AC$, there are two minima. They correspond to the perturbative vacuum at $\bar{\sigma} = 0$ and the physical vacuum at $\bar{\sigma} = \sigma_v$. When the two minima degenerate, one can obtain the condition that: $B^2 = 3AC$, when the deconfinement phase transition takes place. Thus one can draw the critical line of the deconfinement phase transition in the plane of B versus A as shown in Fig. 5. The phase plane has been divided into two parts: the left area beside the line in the plane represents the confined phase, while the right area is the deconfined phase. By analyzing the variation of the vacuum, one can find that the deconfinement phase transition can be either first or second order. If the system goes across the critical line at $B \neq 0$, the transition is first order. If the system goes across the line at $B = 0$, the transition is second order.

From the above discussion by Landau theory, we know there may be a second order phase transition in the FL model, while at the MFT level the deconfinement phase transition can only be first order. But if we consider fluctuations beyond MFT, there are maybe additional terms which cancel the cubic $\bar{\sigma}^3$ term, and the second order phase transition may be possible. That means the parameter $B(T,\mu)$ will go to zero before the transition takes place. The system will evolve from the left area to the right area across the critical line by the axis origin in Fig. 5. In our former calculation at MFT, the fluctuations of σ' in the Lagrangian have been neglected. These terms are possibly important in the cancellation of the cubic term. However, it is very difficult to calculate the thermodynamic potential including these fluctuations from the Lagrangian in the FL model. In the following, we will make an ansatz based on the form of the Landau expansion of the thermodynamic potential to mimic the deconfinement phase transition, which has both first and second order phase transition.

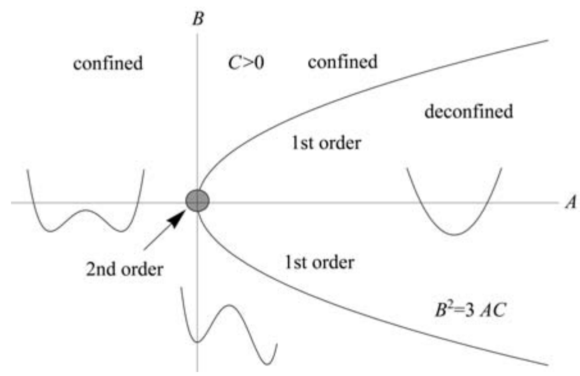

Fig. 5. Phase diagram of deconfinement on the A-B plane in the FL model.

We can devise a possible variation pattern of $A(T,\mu)$, $B(T,\mu)$ and $C(T,\mu)$. We suppose at finite temperature and zero chemical potential that the absolute value of $B(T,\mu)$ keeps decreasing and tends to zero with increasing temperature, while $A(T,\mu)$ first decreases to a negative value and then increases with increasing temperature. $C(T,\mu)$ remains positive in all the cases. By this kind of variation, from Fig. 5, one can see that the system will evolve from the confined phase to the deconfined phase across the origin of the axis, and the transition will be second order. Thus we make the following ansatz of $A(T,\mu)$, $B(T,\mu)$ and $C(T,\mu)$ as

$$A(T,\mu) = a\left[(T - T_{\mathrm{c}})\left(k_1 T - \frac{1}{T_{\mathrm{c}}}\right) + \lambda_1 \mu^2\right], \quad (10)$$

$$B(T,\mu)=b\ \exp\left[-k_2\left(\frac{T+T_c}{T_c}\right)^6+k_2+\lambda_2\mu\right], \quad (11)$$

$$C(T,\mu)=c, \quad (12)$$

where a, b and c are the parameters of the FL model which have already been given in Section 2. $k_1=4$ fm^2, $k_2=0.15$, $\lambda_1=0.5$ fm^2 and $\lambda_2=0.1$ fm are the effective parameters of the ansatz. T_c is the critical temperature of the transition at zero chemical potential, which can be seen in a later analysis. It also serves as a temperature scaling factor, whose value can be taken as T_c=180 MeV. When $T=\mu=0$, it is clear that $A(0,0)=a$, $B(0,0)=b$ and $C(0,0)=c$. One should notice that in our ansatz, with the increasing temperature, the parameter $B(T,\mu)$ will be infinitely close to zero but not zero. However, when the second order phase transition takes place, the absolute value of $B(T,\mu)$ will be sufficiently small. At zero chemical potential, from Eq. (10), one can see at $T=T_c$, $A(T_c,0)=0$. At the same time, $B(T_c,0)\approx 0$. Thus the deconfinement phase transition at zero chemical potential and finite temperature takes place at $T=T_c$=180 MeV and the transition order is second order. At zero temperature, from Eq. (11), one can see that $B(0,\mu)$ will never be zero with increasing chemical potential, which means that the transition will be first order at zero temperature and finite chemical potential.

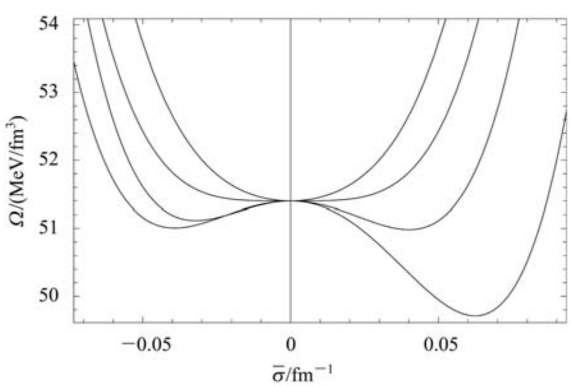

Fig. 6. The thermodynamical potentials for different temperatures and zero chemical potential. The temperatures are 100, 150, 180 and 200 MeV from bottom to top.

We can also evaluate the thermodynamic potential for different chemical potentials and temperatures. At finite temperature and zero chemical potential, the thermodynamic potential as a function of $\bar{\sigma}$ is plotted in Fig. 6. It is clear that the phase transition is second order. At zero temperature and finite chemical potential, it can be seen from Fig. 7 that the transition is first order. The deconfinement phase transition can be presented in a μ-T phase diagram, as shown in Fig. 8. From the first order phase transition to the second order phase transition there exists a TCP. The phase diagram is qualitatively consistent with that of the chiral phase transition. However, how we obtain a credible phase diagram of deconfinement through direct calculations, including the fluctuations from the Lagrangian of the FL model, deserves further investigation.

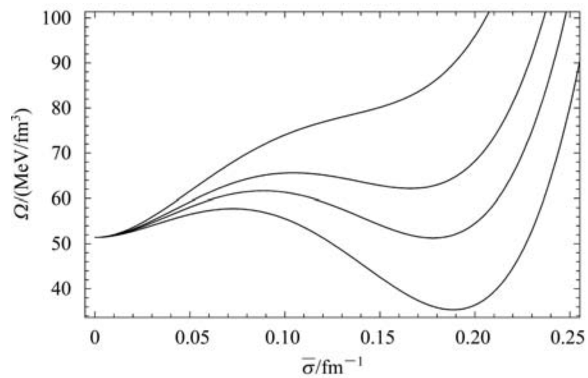

Fig. 7. The thermodynamical potentials for different chemical potentials and zero temperature. The chemical potentials are 350, 392, 420 and 470 MeV from bottom to top.

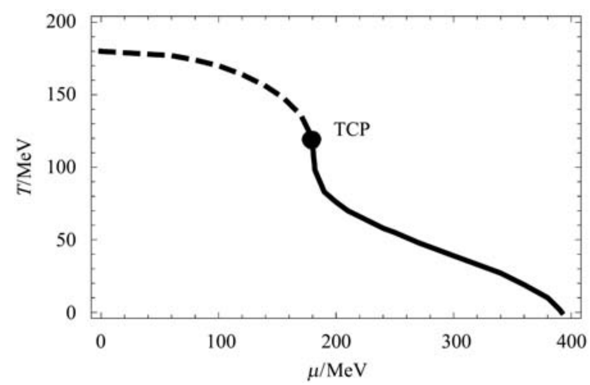

Fig. 8. μ-T phase diagram of deconfinement with TCP in the FL model.

4 Summary

In this paper we have discussed the possible phase diagram of deconfinement in the FL model. From the calculation in only the MFT approximation and without fluctuations, deconfinement phase transition can only be first order at finite temperature and chemical potential. Using the Landau expansion of the thermodynamic potential and analysis through Landau theory, we show that deconfinement phase transition can also be second order, which will not appear in the

MFT approximation but possibly will when nonlinear fluctuations are considered. Thinking of the difficulties in calculating the fluctuations, we have not done the calculation here, but made the ansatz that the Landau coefficients are certain functions of temperature and chemical potential. By this ansatz we obtain the possible μ-T phase diagram of deconfinement in the FL model, which is similar to that of the chiral phase transition. This means that deconfinement phase transition is first order at low temperature and high chemical potential, whereas it is second order at high temperature and low chemical potential. From the first order to the second order phase transition there exists a TCP.

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