A new method to generate relativistic comb bunches with tunable subpicosecond spacing*

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Abstract: We propose and analyze a scheme to produce comb bunches, i.e. a bunch consisting of micro-bunch trains, with tunable subpicosecond spacing. In the scheme, the electron beam is first deflected by a deflecting cavity which introduces a longitudinal-dependent linear transverse kick to the particles. After passing through a drift space, the transverse beam size is linearly coupled to the longitudinal position of the particle inside the beam, and a mask is placed there to tailor the beam, then the mask distribution is imprinted on the beam's longitudinal distribution. A quadrupole magnet and another deflecting cavity are used in the beam line to compensate the transverse angle due to the first deflecting cavity. Analysis shows that the number, length, and spacing of the trains can be controlled through the parameters of the deflecting cavity and the mask. Such electron bunch trains can be applied to an infrared free electron laser, a plasma-wakefield accelerator and a supper-radiance THz source.

Key words: bunch trains, mask, deflecting cavity

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1 Introduction

There is an increasing demand for electron containing micro-bunch trains with tunable subpicosecond spacing for various applications. For example, such an electron beam is useful for producing high-brightness electron superradiance when the radiation frequency or the sub-harmonic of the radiation is matched to the bunch modulated frequency [1–3]. In a novel beam driven acceleration mechanism, e.g. based on plasma or dielectric wakefield acceleration [4, 5], the linearly ramped comb beam with microbunches separated by the desired excitation frequency is used to significantly increase the transformer ratio thereby enabling high energy accelerators driven by medium-to-low energy accelerator drivers.

A number of methods have been proposed to generate such bunch trains. For example, the electron beam can be modulated by one or two laser beams in a wiggler or undualtor with the inverse free electron laser effect to produce equidistance trains of micro size micro-bunches [6]. In this scheme, about 50% of the incoming charge remains in an un-modulated background. Illuminating the cathode of a photocathode RF gun with comb-like trains of UV lasers can also be used [7, 8], but this method is most suitable for long spacing between microbunches. When the spacing decreases, the space charge effect which is prominent at low energy tends to wash out charge modulation. Tailoring the relativistic electron with an interceptive mask located in a dispersion section or a transverse-longitudinal emittance exchanger is also proposed and demonstrated. Tunable subpicosecond spacing bunch trains can be generated with various beam and mask parameters [9, 10].

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In this paper, a scheme to generate intense tunable spacing microbunch trains is proposed and analyzed. The proposed scheme is similar to that of tailoring the beam with a mask located in a dispersion section except that here we replace the dispersion section with two deflecting cavities and a quadrupole to generate the transverse-longitudinal correlated beam distribution.

2 The scheme of the proposal

The scheme for generating subpicosecond spacing bunch trains is shown in Fig. 1. It consists of two deflecting cavities, a quadrupole magnet and a mask. The beam is deflected by the first cavity at the zero RF phase to induce the longitudinal correlated deflecting angle, and after a drift line, the beam transverse beam size in the x direction is dominated by the longitudinal correlated deflecting angle, and the mask is placed in this region. The mask spoils the particle's emittance that stick its solid part. These particles are subsequently lost along the beam transport line, then the shadow of the mask is imprinted onto the beam's transverse distribution and converted into the longitudinal pattern because of the transverse longitudinal correlation. The quadpole and the second deflecting cavity are used to correct the longitudinal correlated divergence angel to control the increase in the beam's emittance. In this scheme, the tailored beam parameters can be easily controlled by varying the parameters of the deflecting cavities and mask. Furthermore, bunch spacing can be adjusted by using a compressor located downstream of the beam line with a longitudinal correlated energy spread.

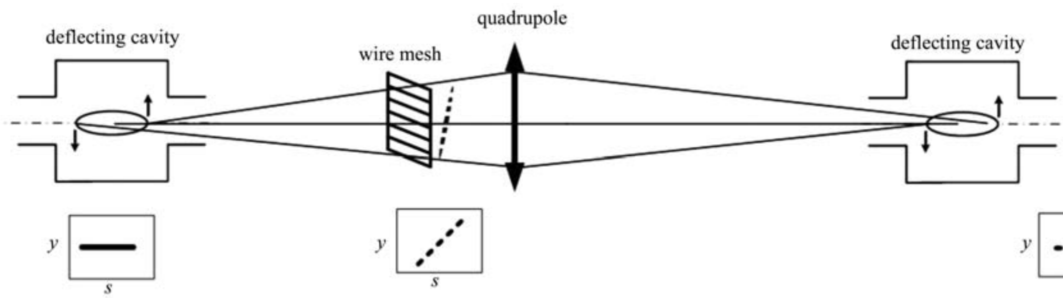

Fig. 1. Simplified schematic of bunch train generation.

3 Analysis and numerical results

We assume an initial Gaussian beam transverse and longitudinal distribution with an rms beam size σ_y and σ_z. The initial beam distribution in (y, z) space at the mask position without deflecting can then be written as

$$f(y,s)=\frac{N_0}{2\pi\sigma_y\sigma_z}\mathrm{e}^{-y^2/2\sigma_y^2}\mathrm{e}^{-s^2/2\sigma_s^2},$$

where N_0 is the number of electrons in the beam. With the deflecting cavity on, the particles are deflected with a longitudinal position correlated angle $y'=y'_0+su_1$, where

$$u_1=\frac{keV_{\mathrm{def}}}{W},$$

W is the particle kinetic energy, k is the wave number of the work frequency of the deflecting cavity, and V_{def} is the deflecting voltage. Then the particle's transverse position y after a drift l is related to the initial particle's coordinates by the equation $y=y_0+ly'_0+lu_1s$, where $y_0+ly'_0$ is the initial particle's position without deflecting and it is due to the beam's transverse emittance. The bunch distribution at the mask position with the deflecting cavity on becomes

$$f(y,s)=\frac{N_0}{2\pi\sigma_y\sigma_s}\mathrm{e}^{-(y-lu_1s)^2/2\sigma_y^2}\mathrm{e}^{-s^2/2\sigma_s^2}. \qquad (1)$$

Assuming the i-th wire of the mask is located at y_{wi} with the width d_i (the beam will be lost from y_{wi}–$d_i/2$ to $y_{wi}+d_i/2$). The modulating function of the mask can be written as follows,

$$T(y)=\begin{cases}0, y_{wi}-d_i/2<y<y_{wi}+d_i/2\\ 1\end{cases}. \qquad (2)$$

And then with the mask modulation, the bunch distribution after the mask is

$$f(y,s)=\frac{N_0}{2\pi\sigma_y\sigma_s}\mathrm{e}^{-(y-lu_1s)^2/2\sigma_y^2}\mathrm{e}^{-s^2/2\sigma_s^2}T(y). \qquad (3)$$

Integration of this formula over y gives the beam density N as a function of s, $N(s)=\int f(y,s)\mathrm{d}y$.

First, we consider the modulated result of the i-th wire, the beam density can be written as

$$N(s) = \frac{N_0}{2\pi\sigma_y\sigma_s}\mathrm{e}^{-s^2/2\sigma_s^2}\left(\int_{-\infty}^{y_{wi}-d_i/2}+\int_{y_{wi}+d_i/2}^{\infty}\right)\mathrm{e}^{-(y-lu_1s)^2/2\sigma_y^2}\mathrm{d}y$$

$$= \frac{N_0}{2\pi\sigma_y\sigma_s}\mathrm{e}^{-s^2/2\sigma_s^2}\left(\int_{-\infty}^{\infty}-\int_{y_{wi}-d_i/2}^{y_{wi}+d_i/2}\right)\mathrm{e}^{-(y-lu_1s)^2/2\sigma_y^2}\mathrm{d}y. \quad (4)$$

With the substitution $z=(y-lu_1s)/\sqrt{2}\sigma_y$:

$$N(s) = \frac{N_0}{\sqrt{2\pi}\sigma_s}\mathrm{e}^{-s^2/2\sigma_s^2}\left(\sqrt{\pi}-\int_{(y_{wi}-d_i/2-lu_1s)/\sqrt{2}\sigma_y}^{(y_{wi}+d_i/2-lu_1s)/\sqrt{2}\sigma_y}\mathrm{e}^{-z^2}\mathrm{d}z\right)$$

$$= \frac{N_0}{\sqrt{2\pi}\sigma_s}\mathrm{e}^{-s^2/2\sigma_s^2}\left(1-\frac{1}{2}\left(\mathrm{erf}\left(\frac{(y_{wi}+d_i/2-lu_1s)}{\sqrt{2}\sigma_y}\right)-\mathrm{erf}\left(\frac{(y_{wi}-d_i/2-lu_1s)}{\sqrt{2}\sigma_y}\right)\right)\right). \quad (5)$$

The error function $\mathrm{erf}(x)$ is defined as

$$\mathrm{erf}(x)=\frac{2}{\sqrt{\pi}}\int_0^x \mathrm{e}^{-x'^2}\mathrm{d}x'. \quad (6)$$

The density distribution can be separated into two parts, one is the initial distribution and the other is the mask's modulation assuming that the electron hitting the solid parts of the mask is lost with a probability 1. For other mask wires, the 'shadow' can be calculated by subtracting the corresponding number of terms similar to the second part:

$$N(s)=\frac{N_0}{\sqrt{2\pi}\sigma_s}\mathrm{e}^{-s^2/2\sigma_s^2}\left(1-\frac{1}{2}\sum_i\left(\mathrm{erf}\left(\frac{(y_{wi}+d_i/2-lu_1s)}{\sqrt{2}\sigma_y}\right)-\mathrm{erf}\left(\frac{(y_{wi}-d_i/2-lu_1s)}{\sqrt{2}\sigma_y}\right)\right)\right). \quad (7)$$

The sum over i extends over all the wires covered by the incoming bunch. For the case of a mask with equidistant wires the spacing can be written for example as $y_{wi}=i+D$, $i=...-1,0,+1,...$ and $d_i=d$, where D is the mask's period and d is the solid part width, respectively.

For the ideal case that the beam transverse size due to its emittance is much smaller than that due to the deflection and also much smaller than the width of wire, i.e., $\sigma_y=(\beta\varepsilon_{\mathrm{N}}/\gamma)^{1/2}\ll lu_1\sigma_s$, $\sigma_y\ll d$, the particle's transverse position in the y direction at mask is determined by s, $y\approx lu_1s$. This means that the beamlet with the same deflection will be either scattered or not dependable upon whether it hits a wire or not. Therefore, the pattern imprinted on the beam by the mask will be sharp. In the opposite case, there will always be some beamlet electrons scattered or some not, and therefore the mask pattern will be partially washed out.

For the ideal case, the longitudinal density distribution after the mask can be written as:

$$N(s)=\frac{N_0}{\sqrt{2\pi}\sigma_s}\mathrm{e}^{-s^2/2\sigma_s^2}T(lu_1s). \quad (8)$$

The distance between the microbunches can be written as

$$\Delta s_i=D_i/lu_1. \quad (9)$$

The width of the microbunch is equal to:

$$\Delta s_{\mathrm{microbunch}}=(D_i-d_i)/lu_1. \quad (10)$$

The center of the microbunch corresponding to the beam's center is:

$$s_{\mathrm{center\_}i}=y_i/lu_1. \quad (11)$$

Obviously the divergence angle of the beam through the mask in the y direction, $y'=y'_0+su_1$, is correlated with the longitudinal position s and will cause an increase of project emittance. In order to compensate this angle, a quadrupole and another deflecting cavity are used in the beam line. This quadrupole is placed near the mask and the distance to the first and the second deflecting cavity is l_{a} and l_{b}, respectively. Then at the second deflecting cavity exit, the position and divergence angle can be written as

$$y = \left(1-\frac{l_{\mathrm{a}}}{f}\right)y_0+\left(l_{\mathrm{a}}+l_{\mathrm{b}}-\frac{l_{\mathrm{a}}l_{\mathrm{b}}}{f}\right)y'_0 +\left(l_{\mathrm{a}}+l_{\mathrm{b}}-\frac{l_{\mathrm{a}}l_{\mathrm{b}}}{f}\right)u_2s,$$

$$y' = -\frac{1}{f}y_0+\left(1-\frac{l_{\mathrm{b}}}{f}\right)y'_0+\left(u_1+u_2-\frac{l_{\mathrm{b}}u_2}{f}\right)s,$$

where

$$u_1=\frac{keV_{\mathrm{def1}}}{W},\quad u_2=\frac{keV_{\mathrm{def2}}}{W},$$

and f is the quadrupole focus length. Set $u_1 = u_2$ and $f = 2l_a = 2l_b$, we can find that $y = -y_0$ and $y' = -\frac{1}{2l_a}y_0 - y'_0$. The longitudinal correlated transverse distribution and the divergence angle are compensated well with them.

The charge distribution of equations is plotted in Fig. 2 for various σ_{y0}. In this calculation, the rms bunch length, mask period, and the solid part width of the mask are assumed to be 0.3 mm (corresponding to 1 ps), 2 mm and 1 mm, and the voltage of the deflecting cavity is adjusted to ensure that the beam transverse size is 5 mm rms at the mask position. The microbunch period and width are 400 fs and 200 fs, respectively, which are predicted from Eqs. (9) and (10). The calculation confirms that when $\sigma_{y0} \ll d/2$, the current profile follows the mask shape and is crisp. And as σ_{y0} increases and approaches $d/2$, charge appears at all positions, and the shadow becomes less crisp and the charge distribution becomes continuous.

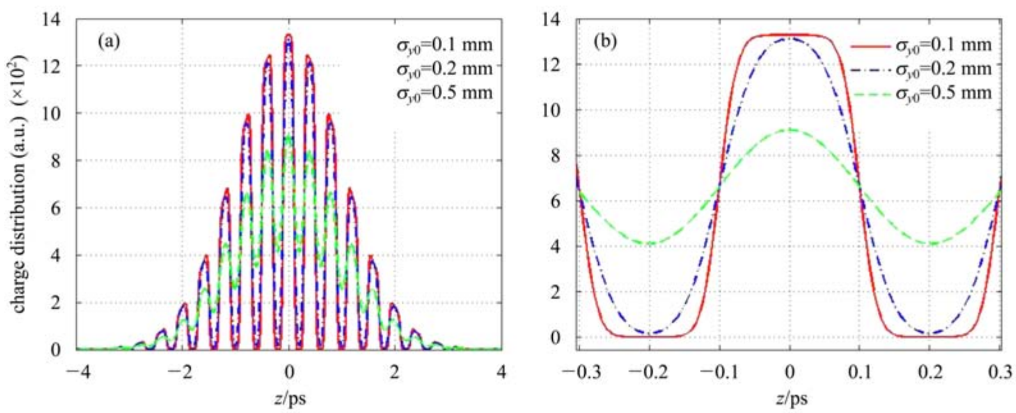

Fig. 2. The longitudinal charge distribution for various initial transverse beam sizes. (a) The total charge distribution and (b) A close-up of the charge distribution for -0.3 ps$< z <$0.3 ps.

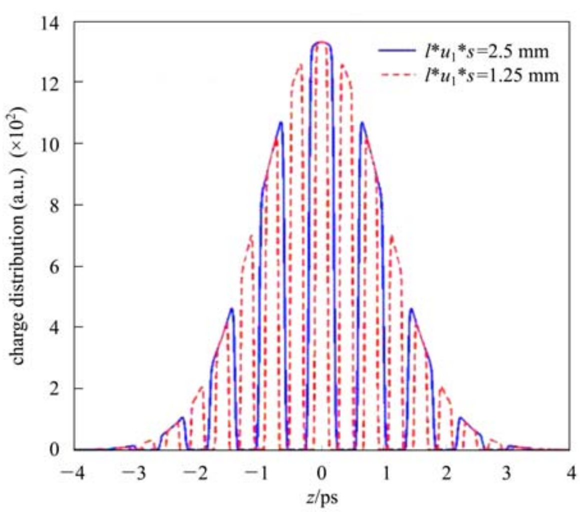

Fig. 3. Bunch charge distribution with different bunch spacing generated by adjusting the deflecting voltage. The initial rms bunch length is 1 ps and the mask period is 1 mm. The bunch spacing is 400 fs and 800 fs, respectively.

The bunch spacing between the microbunches and microbunch width can be adjusted through changing the mask parameters or the deflecting voltage. Figs. 3 and 4 show examples of generating bunch trains with different bunch spacing and width by adjusting the mask parameter and deflecting voltage, respectively. The ratio d/D is set to 1/2 in these calcula-

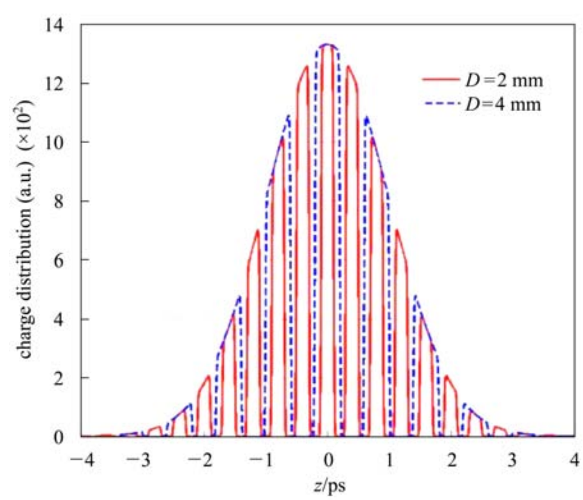

Fig. 4. Bunch charge distribution with different bunch spacings generated by adjusting the mask period. The initial rms bunch length is 1 ps and the transverse beam size at mask position with the deflecting cavity on is 5 mm. The bunch spacing is 400 fs and 800 fs corresponding to the mask period of 2 mm and 4 mm, respectively.

tions and it results in the microbunch width which is half the bunch spacing. The ratio can be optimized during the mask design to generate microbunches with different bunch widths with the same bunch spacing.

4 Summary

In summary, we present a general concept for potentially generating relativistic sub-picosecond electron bunch trains with adjustable period and bunch widths. The spacing and the number of microbunches can be adjusted through the mask design and deflecting voltage setting. The scheme can be used for a frequency tunable narrowband THz radiation source, and can also be applied to the plasma wakefield acceleration to increase the transformer ratio.

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