Structure of ^{6}He in the frame of a cluster model*

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Abstract: A microscopic cluster model with a fully correlated Gaussian basis is developed. In the model, the stochastic variational method is used in order to calculate the ground-energy and the mean-square radius conveniently. Based on this model, the ground-energy level and radius of the neutron halo nucleus, ^{6}He, are calculated as a α+n+n three-cluster model. The results are in good agreement with the experimental data.

Key words: fully correlated Gaussian basis, stochastic variational method

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1 Introduction

Some light nuclei close to the drip line present exotic structure features called halos, because of the low nucleon separation energy and the short range of the nuclear force [1]. Among these nuclei, the two neutron halo systems, such as ^{6}He, ^{11}Li, and ^{14}Be, exhibit Borromean characteristics, i.e. these are only one bound state when the nuclei are considered as three-body systems, but no bound states in any two-body subsystems [2]. As a member of these Borromean nuclei, ^{6}He is of special interest to both experimental and theoretical physicists.

There are some questions about ^{6}He that always draw a lot of attention from experimental physisists. The first is the t+t configuration contribution to the ^{6}He ground-state wave function [3]. This was recently investigated by means of a 2n transfer reaction [3] and the result shows that the spectroscopic factor obtained for the α+2n configuration is as good as expected, but the spectroscopic factor obtained for the t+t configuration is much smaller than the theoretical predictions [3]. The second one is the correlation of two-neutron relative motion. An experiment has been performed by way of intensity interferometry [4] to investigate the spatial correlation function of the two neutrons in the halo nuclei, such as ^{6}He, ^{11}Li, and ^{14}Be. The final one is the basic properties of ^{6}He, such as r.m.s. radii, nuclear charge radius, ground energy, etc. Recently, Wang et al. proposed to determine the charge radius of a ^{6}He nucleus by using a laser spectroscopy method [5], which is independent of nuclear models.

Several theoretical models, such as the microscopic generator-coordinate method in hyperspherical formalism (HGCM) [6], the resonating-group-method (RGM) [7] and the antisymmetrized molecular dynamics (AMD) [8], have been proposed and successfully used to study the ^{6}He cluster structure. All of the microscopic models need to generate the wave-function for their systems first. Different models use different methods to generate the wave function.

In this paper, we also use a microscopic model, and the wave-function of ^{6}He is generatd by the fully correlated Gaussian basis method, which has proved to be a very useful tool not only for describing the few-body system [9, 10] but also for studying the cluster structure [7] and some other interesing physical properties, such as α-condensation [11] of nuclei. In Refs. [7] and [11], one can easily find that the wave functions of the inter-cluster motion is described with the correlated Gaussian basis but the intrinsic wave-function of the cluster is constructed by a harmonic-oscillator function. Take cluster α in Ref. [11] as an example. The intrinsic wave function of cluster α is described by the $(0s)^4$ harmonic-oscillator function with an oscillator parameter "v". If people want to

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calculate some physical quantities (energy, r.m.s. radii, etc.), they have to deal with Slater determinants [9, 11], which make the calculation tough and tedious. In order to avoid this inconvenience, we use another way to generate the intrinsic wave-function of the cluster. Since the correlated Gaussian basis represents the correlation between clusters in nuclei, we can also use it to describe the correlation between nucleons within a cluster, which means both the wave functions of inter-cluster motion and the intrinsic wave-function of the cluster are generated with the correlated Gaussian basis. Then one can find that the calculation will become easier and more convenient. To test the validity of this method, the calculated ground-energy, r.m.s. radii, and proton and neutron densities of ^{6}He are compared with the experimental data.

This paper is organized as follows. In Section 2, a brief description of the model is presented. In Section 3, our results and discussion of ^{6}He are shown. The summary is given in the last section.

2 The model

Our model is a microscopic cluster model. All nucleons in a nucleus are assumed to be arranged in clusters and treated explicitly. The wave-function is constructed with the fully correlated Gaussian basis, which represents the correlation between nucleons in clusters and correlation between clusters. What is more, it satisfies the Pauli principle exactly and has a good total momentum and parity. The nucleons are supposed to interact via an effective two-nucleon interaction, and the Schrödinger equation is solved with the variation method.

2.1 Hamiltonian of *A*-nucleon

The A-nucleon Hamiltonian, with the center-of-mass kinetic energy $\hat{T}_{\rm c.m.}$ being subtracted [12], is

$$\hat{H}=\sum_{i=1}^{A}\hat{T}_i-\hat{T}_{\rm c.m.}+\sum_{i<j}\hat{V}_{i,j}, \tag{1}$$

where $\hat{T}_i$ is the kinetic energy of the ith nucleon, $\hat{T}_{\rm c.m.}$ is the kinetic energy of center of mass, and $\hat{V}_{i,j}$ is the effective two-nucleon interaction. In this paper, we choose the Minnesota potential [13] without the L-S part but with the Coulomb potential. Since the interaction $\hat{V}_{i,j}$ remains unchanged by the displacement of the origin, the Hamiltonian is translationally invariant and depends on the internal degrees of freedom only.

2.2 Stochastic variational solution on a fully correlated Gaussian basis

The general form of a microscopic N-cluster state ψ can be expressed as

$$\psi=\mathcal{A}\{\phi_{\rm in}\eta(\vec{x})[\theta_L(\vec{x})\chi_S]_{\rm JM}\chi_{\rm TM_T}\}, \tag{2}$$

where $\vec{x}$ is a set of relative (Jacobi) coordinates of the N-cluster system. $\phi_{\rm in}$ is the intrinsic wave-function of clusters and is described as

$$\begin{aligned}\phi_{\rm in}&=\prod_{k=1}^{N}\exp\left\{-\frac{1}{2}\sum_{j>i=1}^{A_k}\beta_k(\vec{r}_i^{\,k}-\vec{r}_j^{\,k})^2\right\},\\ \beta_k&=\beta/A_k,\end{aligned} \tag{3}$$

where N is the number of clusters, β_k stands for the harmonic-oscillator parameter of the kth cluster, which is determined by RGM, $\vec{r}_i^{\,k}$ is the coordinate of the ith nucleon in the kth cluster and A_k is the nucleon number of the kth cluster.

Function η in Eq. (2) describes the relative motion of the N-cluster system,

$$\eta(\vec{x})=\exp\left\{-\frac{1}{2}\sum_{j>i=1}^{N}\alpha_{i,j}(\vec{x}_i-\vec{x}_j)^2\right\}, \tag{4}$$

where parameter $\alpha_{i,j}$ represents the correlation between the ith and jth clusters and is generated by the stochastic variational method (SVM) [7, 9]. $\chi_{{\rm SM}_S}$ and $\chi_{{\rm TM}_T}$ [9] are the spin and isospin functions, $\mathcal{A}$ is an antisymmetrizer of all of the nucleons, $\theta_{\rm L}(\vec{x})$ represents the angular part of the wave-function and is chosen as a vector-coupled product of solid spherical harmonics of the Jacobi coordinates,

$$\theta_{\rm L}(\vec{x})=[[[y_{l_1}(\vec{x}_1)y_{l_2}(\vec{x}_2)]_{L_{12}}y_{l_3}(\vec{x}_3)]_{\rm L_{123}},\cdots]_{\rm LM_L}. \tag{5}$$

The microscopic N-cluster variational wave function is taken to be

$$\psi=\sum_{k=1}^{K}C_i\psi_i, \tag{6}$$

where $\{C_i\}$ are a set of linear variational parameters. Then according to the Ritz variational method, the final A-nucleon Schrödinger-equation becomes

$$\mathcal{H}c=E\mathcal{N}c, \tag{7}$$

where $\mathcal{H}$ and $\mathcal{N}$ are respectively the matrices of the Hamiltonian and overlap

$$\begin{aligned}&\mathcal{H}_{ij}=\langle\psi_i|\,H\,|\psi_j\rangle\ \text{and}\\ &\mathcal{N}_{ij}=\langle\psi_i|\,\psi_j\rangle\ \ (i,j=1,\cdots,K).\end{aligned} \tag{8}$$

2.3 The ^{6}He nucleus

To test the validity of our method, we apply it to ^{6}He. It is known that the ground state of ^{6}He can

be well understood in terms of the three-body model (α+n+n) [7, 14, 15], although Csótó [15] suggested that t+t clusterization should also be considered, if one wants to get a more accurate separation energy of ^{6}He. In this paper, since we just want to test our method, only the first clusterization is considered.

The α+n+n three-cluster system can be arranged in two ways, which correspond to two different kinds of Jacobi coordinates (see Fig. 1). They are referred to as the T-type arrangement and the Y-type arrangement. Each arrangement is related to a model space. The T-type corresponds to the di-neutron plus α-particle description, and the Y-type to the ^{5}He plus one-neutron description. The orbit angular momentum l_i, belonging to the Jacobi coordinates x_i, is coupled to L and the spins of clusters are coupled to the total spin S. Then L and S are coupled to the total angular momentum J. However, in this paper we set all l_i to be zero just for simplicity and, for the same reason, we just deal with the Y-type arrangement.

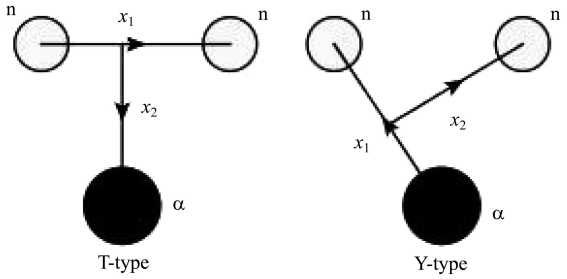

Fig. 1. Jacobi arrangements in the three-cluster model.

Because the orbit angular momentum l_i, is set to zero, Eq. (2) has a simpler expression,

$$\psi = \mathcal{A}\{\phi_{\rm in}\eta(\vec{x})\chi_{\rm SM_S}\chi_{\rm TM_T}\}. \tag{9}$$

Now the microscopic N-clusters variational wavefunction becomes

$$\psi = \sum_{k=1}^{K} C_i\psi(B_i), \tag{10}$$

with

$$\psi(B_i) = \mathcal{A}\{G(B_i,x)\chi_{\rm SM_S}\chi_{\rm TM_T}\}, \tag{11}$$

and

$$G(B_i,x) = \eta(\vec{x})\phi_{\rm in} = \exp\left\{-\frac{1}{2}\tilde{\boldsymbol{\rho}}B_i\boldsymbol{\rho}\right\}, \qquad \tilde{\boldsymbol{\rho}} = \{\rho_1,\rho_2,\rho_3,x_1,x_2\}, \tag{12}$$

where ρ_i is the Jacobi coordinate within the cluster (see Fig. 2) and B is a symmetric, positive-definite $(A-1)\times(A-1)$ matrix. From Eq. (12), one can find that the calculations in Eq. (8) become easier than in Ref. [11] because they only involve the Gaussian integration.

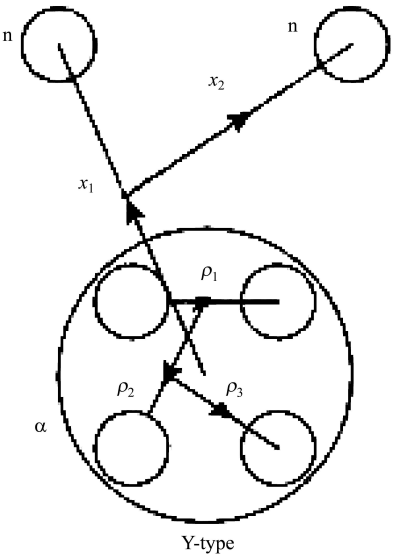

Fig. 2. Jacobi coordinates of the Y-type arrangement.

2.4 Selection of nonlinear parameters by SVM

Matrix B has two kinds of parameters: β and $\alpha_{i,j}$. As mentioned before, β is a constant, determined by RGM, and parameter $\alpha_{i,j}$ is generated by SVM [7, 9].

The typical and most economical way to select the parameters in SVM is [17]:

(1) generate several sets of (B_k^n, n=1, ⋯, N) randomly;

(2) determine the corresponding energies (E_k^1, ⋯, E_k^N) by solving the eigenvalue problem;

(3) choose the parameter B_k^n belonging to the lowest energy E_k^n as a basis parameter and add it to the previous basis states; then the parameters of the new basis become (B_1, ⋯, B_{k-1}, $B_k = B_k^n$);

(4) increase the basis dimension to k+1.

And the most naive way to generate the elements of matrix B is to choose $\alpha_{i,j}$ independently of each other randomly, but it is found that it will be slightly more advantageous to follow the procedure below [17]. Firstly, all $\alpha_{i,j}$ are generated randomly. Then the first element of α (say α_{11}) is changed K_0 times randomly. In the mean-time, other $\alpha_{i,j}$ is kept unchanged. Select the best (giving the lowest ground state energy) parameter and fix the α_{11}. The second element is fixed in the same way and this process is continued until the last $\alpha_{i,j}$. This procedure can be repeated K_1 times. In this paper, we follow this procedure to select the nonlinear parameters.

3 Results and discussion

3.1 Input parameters

There are four parameters that need to be fixed first. The first is the width parameter $\beta = m\omega/\hbar$. It is adjusted to get nearly correct values for the radii of the cluster α and to minimize its ground energy. In this paper, we choose β 0.52 fm^{-2} [7]. The second one is the exchange-mixture parameter "u" in the Minnesota potential and is chosen as 1.15 [16]. The final two parameters are $b_{\min}$ and $b_{\max}$. We set $b_{\min}$ to be 0 and $b_{\max}$ to 8. Detailed information about them can be found in Refs. [14, 17].

3.2 The structure of ^{6}He

We show the calculated ground state of ^{6}He (J=0, T=1) here. The calculated ground-energy, root-mean-square (r.m.s) radii of matter density, proton, and neutron are given in Table 1. The experimental data and the theoretical results of other models are also listed.

Table 1. Binding energy, r.m.s mass radii $r_{\rm m}$, proton radii $r_{\rm p}$ and neutron radii $r_{\rm n}$ of ^{6}He.

	E/MeV	$r_{\rm m}$/fm	$r_{\rm p}$/fm	$r_{\rm n}$/fm
AMD-m56 [8]		2.31	1.90	2.49
RGM [7]	−26.333	2.49	1.82	2.77
our model	−25.051	2.51	1.90	2.77
Exp. [18,20]	−29.271[21]	2.33–2.48	1.912(18)[5]	2.59–2.61

The extremely large radius of ^{6}He has been experimentally deduced by the reaction cross section measurement [18–20], and it is suggested that the large radius arises from the great enhancement of neutron radii caused by the neutron halo structure in ^{6}He. In Table 1, one can find that the RGM [7] reproduces the experimental neutron radius well but a little difference for proton radius. The AMD [8] model calculates the proton radius consistently with the experimental one, but there is a little discrepancy for the neutron radius. In our model, the experimental neutron and proton radii of ^{6}He are both well described.

It is not surprising that the ground energy of ^{6}He is several MeV higher than the experimental data, because our calculations are based on the simplest consideration (L-S part in Minnesota potential, orbit angular components of wave-function, and other Jacobi arrangements of clusters are ignored). Although those factors are not considered in our model, the calculated ground energy of ^{6}He is almost the same as RGM [7].

Finally, the proton and neutron densities are calculated as follows,

$$\rho(\vec{\boldsymbol{r}}) = \langle\psi| \sum_{i=1}^{A} \hat{P}_i \delta(\vec{r}_i - \vec{R} - \vec{r}) \,|\psi\rangle, \qquad (13)$$

where $\hat{P}_i$ acts in the isospin space as a proton or neutron projector, $\vec{r}_i$ denotes the ith nucleon's coordinate, and $\vec{R}$ is the central mass coordinate of ^{6}He. In Fig. 3, the neutron density has a very long tail at the large distance region compared with the proton density. This represents the neutron halo structure of ^{6}He, which is observed in many experiments. Also, the calculated density distributions are very close to the experimental data [18] and almost the same as AMD [8].

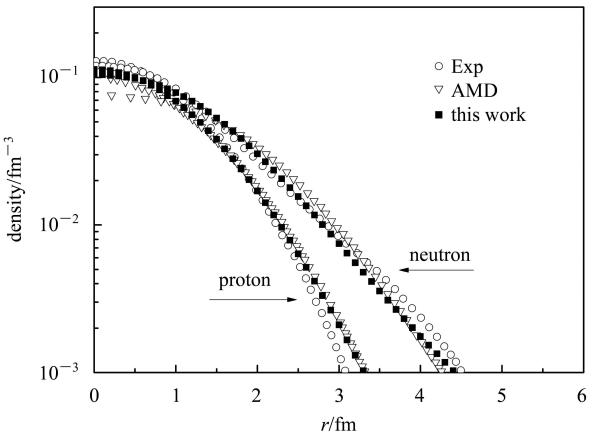

Fig. 3. The proton and neutron density distributions of ^{6}He.

4 Summery

We have used a fully correlated Gaussian basis and SVM to calculate the ground energy, r.m.s radii of matter, proton and neutron of ^{6}He. The radii of rms mass, proton and neutron distributions are in good agreement with the results of other models and experimental data, although the ground energy has a little discrepancy. We also did some calculations of proton and neutron densities, and reproduced the most important property of ^{6}He, which shows the distribution of neutrons extends far beyond protons,

as observed in many experiments. So we can conclude that our method to generate the wave-function of the inner cluster and the whole nuclear system is reasonable.

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