Dynamical CP violation of the generalized Yang-Mills model

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Abstract: Starting from the generalized Yang-Mills model which contains, besides the vector part V_μ, also a scalar part S and a pseudoscalar part P. It is shown, in terms of the Nambu-Jona-Lasinio (NJL) mechanism, that CP violation can be realized dynamically. The combination of the generalized Yang-Mills model and the NJL mechanism provides a new way to explain CP violation.

Key words: CP violation, Yang-Mills theory, dynamical symmetry breaking

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1 Introduction

Since the violation of parity symmetry proposed by Lee and Yang [1] was confirmed, the study of symmetry and symmetry breaking has played a central role in particle physics. It was argued that the elementary electric dipole moments would vanish due to the combined charge conjugation and parity symmetry, i.e., CP symmetry. However, it was then pointed out by Ramsey and independently by Jackson and collaborators [2] that T invariance was also an assumption and needed to be checked experimentally. Since then the search for CP violation has been vigorously pursued. In 1964, CP symmetry was eventually found to be violated in the kaon system by Val Fitch, James Cronin, and collaborators [3].

Within the framework of the Standard Model, there are two sources of CP violation. One is the CKM model [4, 5], where the source of CP violation comes from the phase [5] δ in the CKM mixing matrix for quarks. All the laboratory experimental results [6] related to CP violation and mixing phenomena are consistent with the CKM model up to now. The Standard Model has another source of CP violation in addition to one that appears in the CKM matrix. This source of CP violation arises in the strong interaction sector of the theory from the term $\theta(\alpha_s/8\pi)G\tilde{G}$, which is of topological origin.

CP violation is central to understand the phenomena in cosmology as well as in particle physics. In 1967, Andre Sakharovit pointed out that CP violation plays an important role in generating the baryon asymmetry in the universe. However, the CP violation in the Standard Model is not sufficient to generate the desired amount of baryon asymmetry and one needs a source of CP violation above and beyond what is present in the Standard Model. Further, new sources of CP violation beyond the standard model could also show up in particles production at the Large Hadron Collider, and in the new generation of experiments underway on neutrino physics.

In addition to the baryon asymmetry in the universe there are other avenues which may reveal the existence of a new source of the CP violation that exists in the Standard Model. The electric dipole moments (EDMs) of elementary particles and atoms are prime candidates for these. The largest values of EDMs in the framework of the Standard Model are very small. In the past decades a significant body of work on CP violation beyond the standard CKM model has appeared. It encompasses spontaneous CP violation models [7–10], left-right symmetric models [11] and so on.

At present, the Standard Model of particle physics stands triumphant, and all the data obtained from many experiments in particle physics are in agree-

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ment with this model. Higgs fields and the original Higgs mechanism [12] have appeared in the literature of gauge theories and unified theories in two distinct guises: one is to break the gauge symmetry and the other is to provide a mass for the physical particles. Nevertheless, there are still some open problems in it. A very important problem is that so far there is not any experimental evidence to support the existence of elementary Higgs particles. There are some suggestions to avoid the difficulties for introducing the Higgs fields and Higgs mechanism. Some authors have suggested that the Higgs fields are bound state of fermion—anti-fermion pairs [13–17].

To overcome the difficulties of the usual gauge theories and Higgs mechanism is the motivation for much effort for a long time. In our previous work [18,19], a maximally generalized Yang-Mills model (MGYMM) which contains, besides the vector part V_μ, also an axial-vector part A_μ, a scalar part S, a pseudoscalar part P and a tensor part $T_{\mu\nu}$ is constructed and the dynamical breaking of gauge symmetry in the model is also discussed. It is shown, in terms of the Nambu-Jona-Lasinio mechanism (NJL) [20], that the gauge symmetry breaking can be realized dynamically in the maximally generalized Yang-Mills model. The combination of the maximally generalized Yang-Mills model and the NJL mechanism provides a way to overcome the difficulties related to the Higgs fields and the Higgs mechanism in the usual spontaneous symmetry breaking theory. At the same time this combination can also provide a new mechanism of CP violation.

The main objective of the present paper is to construct a new CP violation model by using the generalized Yang-Mills model (GYMM) which contains, besides the vector part V_μ, also a scalar part S and a pseudoscalar part P. In this model the symmetry of CP will be broken down dynamically.

2 Maximally generalized Yang-Mills model

In this section, we will review the model which we proposed in Ref.[18]. In the usual Yang-Mills theory gauge invariance is assured through the demand that vector gauge transform as $\gamma_\mu V_\mu \to U(\gamma_\mu V_\mu)U^{-1} - (\gamma_\mu \partial_\mu U)U^{-1}$. Recently, some authors [21–23] have again studied the Yang-Mills theory and constructed generalized Yang-Mills theories (GYMT) in which pseudoscalar boson fields [21], axial-vector fields [22] or scalar fields [23] are considered to be also acceptable as gauge fields. Following these discussions, we have constructed a maximally generalized Yang-Mills model (MGYMM) [18].

The main idea of MGYMM is as follows: Consider a Lagrangian which is invariant under a Lie group with N generators. Corresponding to each generator of the Lie group there is one gauge field, it does not matter whether vector field or other fields. One can choose the first $N_{\rm V}$ to be associated with an equal number of vector gauge fields and the last N' to be associated with an equal number of the other fields. Naturally $N_{\rm V}+N'=N$. By taking each of the generators and multiplying it by one of its associated gauge fields and summing them together, we construct a maximally generalized Dirac covariant derivative D as

$$D=\gamma_\mu \partial_\mu - {\rm i}\gamma_\mu V_\mu + \Phi, \tag{1}$$

with $\Phi = S + {\rm i}\gamma_5 P - {\rm i}\gamma_\mu A_\mu \gamma_5 + \sigma_{\mu\nu} T_{\mu\nu}$ being the generic gauge field in which $S = gS^cT^c$ is a scalar field, $P = gP^bT^b$ a pseudoscalar field, $V_\mu = gV_\mu^aT^a$ a vector field, $A_\mu = gA_\mu^dT^d$ an axial-vector field and $T_{\mu\nu} = gT_{\mu\nu}^eT^e$ a tensor field, the superscript a varies from 1 to $N_{\rm V}$, b varies from $N_{\rm V}+1$ to $N_{\rm V}+N_{\rm P}$, c varies from $N_{\rm V}+N_{\rm P}+1$ to $N_{\rm V}+N_{\rm P}+N_{\rm S}$, d varies from $N_{\rm V}+N_{\rm P}+N_{\rm S}+1$ to $N_{\rm V}+N_{\rm P}+N_{\rm S}+N_{\rm A}$ and the superscript e varies from $N_{\rm V}+N_{\rm P}+N_{\rm S}+N_{\rm A}+1$ to $N_{\rm V}+N_{\rm P}+N_{\rm S}+N_{\rm A}+N_{\rm T}$.

By defining the transformation for the gauge fields as

$$-{\rm i}\gamma_\mu V_\mu+\Phi \to U(-{\rm i}\gamma_\mu V_\mu+\Phi)U^{-1}-(\gamma_\mu\partial_\mu U)U^{-1}, \tag{2}$$

we can obtain that $D\to UDU^{-1}$. Then we can build up the Lagrangian which contains only the matter fields and convariant derivatives, and possesses both the Lorentz and gauge invariance

$$L=-\bar{\Psi}D\Psi+\frac{1}{2g^2}\tilde{\rm Tr}\left(\frac{1}{8}({\rm Tr}D^2)^2-\frac{1}{2}{\rm Tr}D^4\right), \tag{3}$$

where the trace with the tilde is over the gauge (or the Lie group) matrices and the one without the tilde is over matrices of the spinorial representation of the Lorentz group. The expansion of Eq. (3) will be

$$L = -\bar{\Psi}(\gamma_\mu\partial_\mu - {\rm i}\gamma_\mu V_\mu+\Phi)\Psi-\frac{1}{2g^2}\tilde{\rm Tr}(\partial_\mu V_\nu-\partial_\nu V_\mu - {\rm i}[V_\mu,V_\nu])^2-\frac{1}{4g^2}\tilde{\rm Tr}\left[{\rm Tr}(\gamma_\mu\partial_\mu\Phi-{\rm i}\{\gamma_\mu V_\mu,\Phi\})^2\right]. \tag{4}$$

As in the usual Yang-Mills theories, when D acts on the matter fields Ψ, its gauge fields are going to be multiplied by constants (the charges) $Q_{\rm V}$, $Q_{\rm S}$, $Q_{\rm P}$, $Q_{\rm A}$ and $Q_{\rm T}$ with the result $D\Psi=(\gamma_\mu\partial_\mu-{\rm i}Q_{\rm V}\gamma_\mu V_\mu+Q_{\rm S}S+{\rm i}Q_{\rm P}\gamma_5 P-{\rm i}Q_{\rm A}\gamma_\mu A_\mu\gamma_5+Q_{\rm T}\sigma_{\mu\nu}T_{\mu\nu})\Psi$. From the

Standard Model we can conclude that $Q_{\rm V}=1$. Following this consideration, we can finally obtain the Lagrangian of the maximally generalized Yang-Mills model

$$L = -\bar{\Psi}(\gamma_\mu\partial_\mu - {\rm i}\gamma_\mu V_\mu + Q_{\rm S}S + {\rm i}Q_{\rm P}\gamma_5 P - {\rm i}Q_{\rm A}\gamma_\mu A_\mu\gamma_5 + Q_{\rm T}\sigma_{\mu\nu}T_{\mu\nu})\Psi - \frac{1}{2g^2}\tilde{\rm T}{\rm r}(\partial_\mu V_\nu - \partial_\nu V_\mu - {\rm i}[V_\mu,V_\nu])^2 - \frac{1}{4g^2}\tilde{\rm T}{\rm r}\,[{\rm Tr}(\gamma_\mu\partial_\mu\Phi - {\rm i}\{\gamma_\mu V_\mu,\Phi\})^2]\,. \quad (5)$$

When taking $\Phi={\rm i}\gamma_5 P$ or $\Phi=-{\rm i}\gamma_\mu A_\mu\gamma_5$, one will obtain the theories which have been given in Ref. [21] and Ref. [22].

3 Generalized Yang-Mills model and its dynamical CP violation

In this section, we will investigate the dynamical CP violation of the generalized Yang-Mills model with $\Phi=S+{\rm i}\gamma_5 P$. Taking $\Phi=S+{\rm i}\gamma_5 P$ in the covariant derivative Eq. (1), then Eq. (5) changes to be

$$L = -\bar{\Psi}\gamma_\mu(\partial_\mu - {\rm i}V_\mu)\Psi - \bar{\Psi}(Q_{\rm S}S + {\rm i}Q_{\rm P}\gamma_5 P)\Psi - \frac{1}{2g^2}\tilde{\rm T}{\rm r}(\partial_\mu V_\nu - \partial_\nu V_\mu - {\rm i}[V_\mu,V_\nu])^2 - \frac{1}{4g^2}\tilde{\rm T}{\rm r}\{{\rm Tr}[\gamma_\mu\partial_\mu(S+{\rm i}\gamma_5 P) - {\rm i}\{\gamma_\mu V_\mu,(S+{\rm i}\gamma_5 P)\}]^2\}. \quad (6)$$

From Eq. (6) one can find that the Lagrangian is invariant under CP and T. For convenience, here and after we will neglect the interaction terms between the scalar field and the pseudoscalar field. The relevance of the Lagrangian density (6) then consists simply of

$$L = -\bar{\Psi}\gamma_\mu(\partial_\mu - {\rm i}V_\mu)\Psi - \bar{\Psi}(Q_{\rm S}S + {\rm i}Q_{\rm P}\gamma_5 P)\Psi - \frac{1}{2g^2}\tilde{\rm T}{\rm r}(\partial_\mu V_\nu - \partial_\nu V_\mu - {\rm i}[V_\mu,V_\nu])^2 - \frac{1}{g^2}\tilde{\rm T}{\rm r}[(\partial_\mu S - {\rm i}\{V_\mu,S\})^2 + (\partial_\mu P - {\rm i}[V_\mu,P])^2]. \quad (7)$$

The first term on the right side is the same as the usual matter term of a gauge theory, the second is the same as the Yukawa term, the third is the kinetic energy term of the vector gauge fields in the usual Yang-Mills theory and the fourth is similar to the gauge-invariant kinetic energy of scalar and pseudoscalar bosons in the non-Abelian adjoint representation. And one can also easily find that in this model it does not include the Higgs potential V which is necessary in the spontaneously broken gauge theories. In the model presented here, if the symmetry breaking is achieved spontaneously, then the Higgs potential V also has to be introduced explicity.

Here, if we regard the scalar fields in the present model not as the Higgs fields related to the spontaneous symmetry breaking, then we do not need to introduce the Higgs potential V to the model. And in the conditions that the Higgs potential V disappear in the MGYMM, we can regard the scalar fields as the physical vacuum no longer as the Higgs fields as some authors have done [24–26]. We will show that by using the NJL mechanism the symmetry breaking can be realized dynamically.

The equation of motion, from the Lagrangian (7), is given by

$$\gamma_\mu(\partial_\mu - {\rm i}gV^a_\mu T^a)\Psi + (G_{\rm S}S^cT^c + {\rm i}G_{\rm P}\gamma_5 P^bT^b)\Psi = 0, \quad (8)$$

$$(\partial^2_\mu - g^2 d_{\rm S}V^a_\mu V^a_\mu)S^c - G_{\rm S}\bar{\Psi}T^c\Psi = 0, \quad (9)$$

$$(\partial^2_\mu - g^2 f_{\rm P}V^a_\mu V^a_\mu)P^b - {\rm i}G_{\rm P}\bar{\Psi}\gamma_5 T^b\Psi = 0, \quad (10)$$

$$(\partial_\mu F^a_{\mu\nu} + gf^{abc}V^b_\mu F^c_{\mu\nu}) + [g^2 d_{\rm S}(S^c)^2 + g^2 f_{\rm P}(P^b)^2]V^a_\nu - {\rm i}g\bar{\Psi}\gamma_\nu T^a\Psi = 0, \quad (11)$$

in which, $G_{\rm S}=gQ_{\rm S}$, $G_{\rm P}=gQ_{\rm P}$, $F^a_{\mu\nu}=\partial_\mu V^a_\nu - \partial_\nu V^a_\mu + gf^{abc}V^b_\mu V^c_\nu$ and $d_{\rm S}=d^{abc}d^{abc}$, $f_{\rm P}=f^{abc}f^{abc}$ (in $d_{\rm S}$ and $f_{\rm P}$, where a varies from 1 to $N_{\rm V}$, b varies from $N_{\rm V}+1$ to $N_{\rm V}+N_{\rm P}$, and c varies from $N_{\rm V}+N_{\rm P}+1$ to $N_{\rm V}+N_{\rm P}+N_{\rm S}$). Multiplying the left and right-hand side of Eq. (11) by V^a_ν, we obtain

$$\{(\partial_\mu F^a_{\mu\nu} + gf^{abc}V^b_\mu F^c_{\mu\nu}) + [g^2 d_{\rm S}(S^c)^2 + g^2 f_{\rm P}(P^b)^2]V^a_\nu - {\rm i}g\bar{\Psi}\gamma_\nu T^a\Psi\}V^a_\nu = 0. \quad (12)$$

After taking vacuum expectation value of Eq. (12), to the lowest-order approximation in $\hbar$, we obtain [24–26]

$$f_{\rm V}\langle V^a_\mu V^a_\mu\rangle = d_{\rm S}\langle(S^c)^2\rangle + f_{\rm P}\langle(P^b)^2\rangle, \quad (13)$$

in which $f_{\rm V}=f^{abc}f^{abc}$ (a, b, c vary from 1 to $N_{\rm V}$). We can see that in the ground state, Eq. (13) gives an important relation about the vector gauge bosons, the pseudoscalar fields and the scalar fields. As is well known, if the vacuum (the ground state of the model) expectation value of the scalar fields (or the pseudoscalar fields) is non-vanishing, the gauge symmetry will be broken down. Here one can choose the scalar bosons S^{c_1} and P^{b_1} to be associated with the unit generator $T^{c_1}=T^{b_1}=1/\sqrt{2N_{\rm d}}$. times the $N_{\rm d}\times N_{\rm d}$ unit matrix ($N_{\rm d}$: the dimensions of the fundamental representation), and if there is no unit generator in the Lie group one can introduce a unit one to it. We denote the vacuum expectation of the scalar bosons

$$\langle S^c\rangle = \langle S^{c_1}\rangle \neq 0, \quad \langle P^b\rangle = \langle P^{b_1}\rangle \neq 0\,. \quad (14)$$

The nonzero expectation value of P^b implies that the vacuum state is not an eigenstate of CP. In order to exhibit more clearly the CP-violating character of the solution, we may perform a unitary transformation under which P^b is unchanged, but

$$\Psi \rightarrow \mathrm{e}^{-\mathrm{i}\frac{1}{2}\gamma_5\alpha}\Psi. \tag{15}$$

Therefore, the quadratic expressions

$$\bar{\Psi}\Psi \rightarrow \bar{\Psi}\mathrm{e}^{-\mathrm{i}\gamma_5\alpha}\Psi = \bar{\Psi}(\cos\alpha - \mathrm{i}\gamma_5\sin\alpha)\Psi, \tag{16}$$

$$\mathrm{i}\bar{\Psi}\gamma_5\Psi \rightarrow \mathrm{i}\bar{\Psi}\gamma_5\mathrm{e}^{-\mathrm{i}\gamma_5\alpha}\Psi = \bar{\Psi}(\sin\alpha + \mathrm{i}\gamma_5\cos\alpha)\Psi. \tag{17}$$

Hence, by choosing

$$\tan\alpha = \frac{G_{\mathrm{P}}\langle P^b\rangle T^b}{G_{\mathrm{S}}\langle S^c\rangle T^c}, \tag{18}$$

we have

$$\bar{\Psi}(G_{\mathrm{S}}\langle S^c\rangle T^c + \mathrm{i}G_{\mathrm{P}}\gamma_5\langle P^b\rangle T^b)\Psi \rightarrow \bar{\Psi}M_{\Psi}\Psi, \tag{19}$$

with the fermion mass

$$M_{\Psi} = \left[\frac{1}{2N_{\mathrm{d}}}(G_{\mathrm{s}}\langle S^{c_1}\rangle)^2 + \frac{1}{2N_{\mathrm{d}}}(G_{\mathrm{P}}\langle P^{b_1}\rangle)^2\right]^{\frac{1}{2}}. \tag{20}$$

Then after taking the vacuum expectation values of Eq. (9) and Eq. (10) to the lowest-order approximation in $\hbar$, we obtain the self-consistency equations as

$$M_{\mathrm{S}}^2\langle S^{c_1}\rangle = -G_{\mathrm{S}}\sqrt{\frac{1}{2N_{\mathrm{d}}}}\langle\bar{\Psi}\Psi\rangle = \mathrm{i}G_{\mathrm{S}}\sqrt{\frac{1}{2N_{\mathrm{d}}}}\mathrm{Tr}S_{\mathrm{F}}(0), \tag{21}$$

$$M_{\mathrm{P}}^2\langle P^{b_1}\rangle = -\mathrm{i}G_{\mathrm{P}}\sqrt{\frac{1}{2N_{\mathrm{d}}}}\langle\bar{\Psi}\gamma_5\Psi\rangle = -G_{\mathrm{P}}\sqrt{\frac{1}{2N_{\mathrm{d}}}}\mathrm{Tr}[\gamma_5 S_{\mathrm{F}}(0)], \tag{22}$$

in which $M_{\mathrm{S}}^2 = g^2 d_{\mathrm{S}}\langle V_\mu^a V_\mu^a\rangle$, $M_{\mathrm{P}}^2 = g^2 f_{\mathrm{P}}\langle V_\mu^a V_\mu^a\rangle$. Comparing Eq. (21) with Eq. (22), we have

$$\langle S^{c_1}\rangle = -\frac{\mathrm{i}f_{\mathrm{P}}G_{\mathrm{S}}\mathrm{Tr}S_{\mathrm{F}}(0)}{d_{\mathrm{S}}G_{\mathrm{P}}\mathrm{Tr}[\gamma_5 S_{\mathrm{F}}(0)]}\langle P^{b_1}\rangle = -\frac{\mathrm{i}f_{\mathrm{P}}G_{\mathrm{S}}\mathrm{Tr}\displaystyle\int\frac{\mathrm{d}^4p}{(2\pi)^4}\frac{1}{-\mathrm{i}\gamma_\mu p_\mu - (G_{\mathrm{S}}\langle S^{c_1}\rangle T^{c_1} + \mathrm{i}G_{\mathrm{P}}\gamma_5\langle P^{b_1}\rangle T^{b_1})}}{d_{\mathrm{S}}G_{\mathrm{P}}\mathrm{Tr}\displaystyle\int\frac{\mathrm{d}^4p}{(2\pi)^4}\frac{\gamma_5}{-\mathrm{i}\gamma_\mu p_\mu - (G_{\mathrm{S}}\langle S^{c_1}\rangle T^{c_1} + \mathrm{i}G_{\mathrm{P}}\gamma_5\langle P^{b_1}\rangle T^{b_1})}}\langle P^{b_1}\rangle = -\frac{f_{\mathrm{P}}G_{\mathrm{S}}}{d_{\mathrm{S}}G_{\mathrm{P}}\tan\alpha}\langle P^{b_1}\rangle. \tag{23}$$

From Eq. (23) and Eq. (20), we obtain

$$M_{\Psi} = \sqrt{\frac{f_{\mathrm{P}}^2G_{\mathrm{S}}^4 + d_{\mathrm{S}}^2G_{\mathrm{P}}^4\tan^2\alpha}{2N_{\mathrm{d}}}}\frac{\langle P^{b_1}\rangle}{d_{\mathrm{S}}G_{\mathrm{P}}\tan\alpha}. \tag{24}$$

Substituting Eq. (13) and Eq. (23) into Eq. (22), we can rewrite the self-consistency equation (22) as

$$(g^2f_{\mathrm{P}}^3G_{\mathrm{S}}^2 + g^2d_{\mathrm{S}}f_{\mathrm{P}}^2G_{\mathrm{P}}^2\tan^2\alpha)\langle P^{b_1}\rangle^3 = -\mathrm{i}f_{\mathrm{V}}d_{\mathrm{S}}G_{\mathrm{P}}^3\tan^2\alpha\sqrt{\frac{1}{2N_{\mathrm{d}}}}\langle\bar{\Psi}\gamma_5\Psi\rangle. \tag{25}$$

In this self-consistency equation, with an invariant momentum cut-off at $p^2 = \Lambda^2$, in the momentum integral, $\langle\bar{\Psi}\gamma_5\Psi\rangle$ will be finite quantities as follows:

$$\langle\bar{\Psi}\gamma_5\Psi\rangle = -\mathrm{i}\mathrm{Tr}[\gamma_5S_{\mathrm{F}}(0)] = \mathrm{Tr}\int\frac{\mathrm{d}^4p}{(2\pi)^4}\frac{-\mathrm{i}\gamma_5}{-\mathrm{i}\gamma_\mu p_\mu - (G_{\mathrm{S}}\langle S^{c_1}\rangle T^{c_1} + \mathrm{i}G_{\mathrm{P}}\gamma_5\langle P^{b_1}\rangle T^{b_1})}$$
$$= \mathrm{Tr}\int\frac{\mathrm{d}^4p}{(2\pi)^4}\frac{-\mathrm{i}\gamma_5[\mathrm{i}\gamma_\mu p_\mu - (G_{\mathrm{S}}\langle S^{c_1}\rangle T^{c_1} + \mathrm{i}G_{\mathrm{P}}\gamma_5\langle P^{b_1}\rangle T^{b_1})]}{p^2 + M_\Psi^2} = \frac{\mathrm{i}}{2\pi^2}M_\Psi\sin\alpha\left[M_\Psi^2\ln\left(\frac{\Lambda^2}{M_\Psi^2} + 1\right) - \Lambda^2\right]. \tag{26}$$

Substituting (26) into (25), we have

$$\langle P^{b_1}\rangle^2 = \zeta\left[\frac{(f_{\mathrm{P}}^2G_{\mathrm{S}}^4 + d_{\mathrm{S}}^2G_{\mathrm{P}}^4\tan^2\alpha)\langle P^{b_1}\rangle^2}{2N_{\mathrm{d}}d_{\mathrm{S}}^2G_{\mathrm{P}}^2\tan^2\alpha}\ln\left(\frac{2N_{\mathrm{d}}d_{\mathrm{S}}^2G_{\mathrm{P}}^2\Lambda^2\tan^2\alpha}{(f_{\mathrm{P}}^2G_{\mathrm{S}}^4 + d_{\mathrm{S}}^2G_{\mathrm{P}}^4\tan^2\alpha)\langle P^{b_1}\rangle^2} + 1\right) - \Lambda^2\right], \tag{27}$$

in which

$$\zeta = \frac{f_{\mathrm{V}}G_{\mathrm{P}}^2\tan\alpha\sin\alpha\sqrt{f_{\mathrm{P}}^2G_{\mathrm{S}}^4 + d_{\mathrm{S}}^2G_{\mathrm{P}}^4\tan^2\alpha}}{4\pi^2N_{\mathrm{d}}(g^2f_{\mathrm{P}}^3G_{\mathrm{S}}^2 + g^2d_{\mathrm{S}}f_{\mathrm{P}}^2G_{\mathrm{P}}^2\tan^2\alpha)}. \tag{28}$$

From Eq. (27) and Eq. (23), one can finally obtain that the non-vanishing vacuum expectation values of the scalar field S and the pseudoscalar field P are completely determined by the self-energy of the fermions. Note that Eq. (27) and Eq. (23) determine the magnitude and the direction of the vacuum state $C_0 = (\langle S^c\rangle, \langle P^b\rangle)$. Evidently, the gauge bosons acquired masses, due to the non-vanishing vacuum expectation values of the scalar field S and the pseudoscalar field P which are determined by the self-energy of the fermions. So the CP and T symmetry is broken dynamically; but the product CPT symmetry remains intact. The amplitude of the CP violation [27] is

$$A_{-}=\frac{G_{\mathrm{P}}^{2}\sin\alpha\cos\alpha}{2N_{\mathrm{d}}(k^{2}+M_{\mathrm{P}}^{2})},\tag{29}$$

where k denotes the 4-momentum transfer.

In the literature of usual spontaneous CP violation models there exist some unnatural aspects: the need of spontaneous symmetry breakdown, and thus the necessity of an ad-hoc adjunction of a scalar multiplet of Higgs fields. The dynamical CP violation of GYMM is an attempt in this area. It is shown, by using the NJL mechanism, that the gauge symmetry and the CP symmetry can be broken down dynamically. Through the discussion of the present paper, it presents a basic mechanism about dynamical CP violation.

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