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# Possible heavy molecular states composed of a pair of excited charm-strange mesons*

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**Abstract:** The $P$-wave charm-strange mesons $\mathrm{D_{s0}}(2317)$ and $\mathrm{D_{s1}}(2460)$ lie below the DK and $\mathrm{D^*K}$ threshold respectively. They are extremely narrow because their strong decays violate the isospin symmetry. We study the possible heavy molecular states composed of a pair of excited charm strange mesons. As a byproduct, we also present the numerical results for the bottonium-like analogue.

**Key words:** molecular state, $P$-wave charmed-strange mesons

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## 1 Introduction

In the past seven years, the experimentally observed new charmonium or charmonium-like states include X(3872), Y(3940), Y(4260), Z(3930), X(3940), Y(4325), Y(4360), Y(4660), $\mathrm{Z^+}(4430)$, $\mathrm{Z^+}(4050)$, $\mathrm{Z^+}(4250)$ and Y(4140) etc [1–11]. It's difficult to accommodate all these states especially those charged ones in the conventional quark model. Many of these new states lie close to the threshold of two charmed mesons. A natural speculation is that some of them may be the molecular states composed of two charmed mesons [12–19].

In the framework of the meson exchange model, we have investigated the possible loosely bound molecular states composed of a pair of the ground state $S$-wave heavy mesons and a pair of $S$-wave and $P$-wave heavy mesons in Refs. [15–19]. In this work we go one step further and study the possible molecular system composed of a pair of $P$-wave heavy mesons in the $(0^+,\ 1^+)$ doublet according to the classification of the heavy quark symmetry.

The non-strange $P$-wave $(0^+,\ 1^+)$ heavy mesons are very broad with a width around several hundred MeV [20]. Instead of forming a stable molecular state, the system composed of a pair of non-strange $P$-wave heavy meson decays rapidly. Experimental identification of such a molecular state will be very difficult. The attractive interaction between the meson pairs may lead to a possible threshold enhancement in the production cross section.

In contrast, the $P$-wave charm-strange mesons $\mathrm{D_{s0}}(2317)$ and $\mathrm{D_{s1}}(2460)$ lie below the DK and $\mathrm{D^*K}$ threshold respectively. They are extremely narrow because their strong decays violate the isospin symmetry. The future experimental observation of the possible heavy molecular states composed of a pair of excited charm strange mesons may be feasible if they really exist. We study the charmonium-like system composed of a pair of excited charm strange mesons in this work.

This paper is organized as follows. We review the formalism in Section 2 and present the results in Section 3–5. The last section is the discussion and the conclusion.

## 2 Formalism

We list the flavor wave functions of the possible molecular states composed of the $P$-wave $(0^+,\ 1^+)$ heavy doublet in Tables 1–2. $\mathrm{D_0^*}$ denotes

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($D_0^{*0}$, $D_0^{*+}$, $D_{s0}^{*+}$) while $D_1$ denotes ($D_1^0$, $D_1^+$, $D_{s0}^+$). The neutral $D_0^*$-$\bar{D}_1$ system with the parameter $c=\pm1$ corresponds to the positive and negative charge parity respectively.

Table 1. The flavor wave function of the $D_0^*$-$\bar{D}_0^*$ and $D_1$-$\bar{D}_1$ system.

| $D_0^*$-$\bar{D}_0^*$ | | $D_1$-$\bar{D}_1$ | |
|---|---|---|---|
| state | wave function | state | wave function |
| $\Phi^+$ | $D_0^{*+}\bar{D}_0^{*0}$ | $\Phi^{**+}$ | $D_1^+\bar{D}_1^0$ |
| $\Phi^-$ | $D_0^{*-}D_0^{*0}$ | $\Phi^{**-}$ | $D_1^-D_1^0$ |
| $\Phi^0$ | $\frac{1}{\sqrt{2}}(D_0^{*0}\bar{D}_0^{*0}-D_0^{*+}D_0^{*-})$ | $\Phi^{**0}$ | $\frac{1}{\sqrt{2}}(D_1^0\bar{D}_1^0-D_1^+D_1^-)$ |
| $\Phi_8^0$ | $\frac{1}{\sqrt{2}}(D_0^{*0}\bar{D}_0^{*0}+D_0^{*+}D_0^{*-})$ | $\Phi_8^{**0}$ | $\frac{1}{\sqrt{2}}(D_1^0\bar{D}_1^0+D_1^+D_1^-)$ |
| $\Phi_s^+$ | $D_{s0}^+\bar{D}_0^*$ | $\Phi_s^{**+}$ | $D_{s1}^+\bar{D}_1^0$ |
| $\Phi_s^-$ | $D_{s0}^{*-}D_0^{*0}$ | $\Phi_s^{**-}$ | $D_{s1}^-D_1^0$ |
| $\Phi_s^0$ | $D_{s0}^{*+}D_0^{*-}$ | $\Phi_s^{**0}$ | $D_{s1}^+D_1^-$ |
| $\bar{\Phi}_s^0$ | $D_{s0}^{*-}D_0^{*+}$ | $\bar{\Phi}_s^{**0}$ | $D_{s1}^-D_1^+$ |
| $\Phi_{s1}^0$ | $D_{s0}^{*+}D_{s0}^{*-}$ | $\Phi_{s1}^{**0}$ | $D_{s1}^+D_{s1}^-$ |

Table 2. The flavor wave function of the $D_0^*$-$\bar{D}_1$ system. The parameter $c=\pm1$ for the $D_0^*$-$\bar{D}_1$ system with positive and negative charge parity respectively.

| $D_0^*$-$\bar{D}_1$ | |
|---|---|
| state | wave function |
| $\Phi^{*+}/\widehat{\Phi}^{*+}$ | $\frac{1}{\sqrt{2}}(D_0^{*+}\bar{D}_1^0+cD_1^+\bar{D}_0^{*0})$ |
| $\Phi^{*-}/\widehat{\Phi}^{*-}$ | $\frac{1}{\sqrt{2}}(D_0^{*0}D_1^-+cD_1^0D_0^{*-})$ |
| $\Phi^{*0}/\widehat{\Phi}^{*0}$ | $\frac{1}{2}[(D_0^{*0}\bar{D}_1^0+cD_1^0\bar{D}_0^{*0})-(D_0^{*+}D_1^-+cD_1^+D_0^{*-})]$ |
| $\Phi_8^{*0}/\widehat{\Phi}_8^{*0}$ | $\frac{1}{2}[(D_0^{*0}\bar{D}_1^0+cD_1^0\bar{D}_0^{*0})+(D_0^{*+}D_1^-+cD_1^+D_0^{*-})]$ |
| $\Phi_s^{*+}/\widehat{\Phi}_s^{*+}$ | $\frac{1}{\sqrt{2}}(D_{s0}^{*+}\bar{D}_1^0+cD_{s1}^+\bar{D}_0^{*0})$ |
| $\Phi_s^{*-}/\widehat{\Phi}_s^{*-}$ | $\frac{1}{\sqrt{2}}(D_0^{*0}D_{s1}^-+cD_1^0D_{s0}^{*-})$ |
| $\Phi_s^{*0}/\widehat{\Phi}_s^{*0}$ | $\frac{1}{\sqrt{2}}(D_{s0}^{*+}D_1^-+cD_{s1}^+D_0^{*-})$ |
| $\bar{\Phi}_s^{*0}/\widehat{\bar{\Phi}}_s^{*0}$ | $\frac{1}{\sqrt{2}}(D_0^{*+}D_{s1}^-+cD_1^+D_{s0}^{*-})$ |
| $\Phi_{s1}^{*0}/\widehat{\Phi}_{s1}^{*0}$ | $\frac{1}{\sqrt{2}}(D_{s0}^{*+}D_{s1}^-+cD_{s1}^+D_{s0}^{*-})$ |

### 2.1 Effective lagrangian

With the help of the heavy quark symmetry and chiral symmetry, the strong interaction between the $P$-wave ($0^+$, $1^+$) heavy doublet reads

$$\begin{aligned}\mathcal{L} &= \mathrm{i}g'\mathrm{Tr}[S_\mathrm{b}\gamma_\mu\gamma_5 A^\mu_\mathrm{ba}\bar{S}_\mathrm{a}]+\mathrm{i}\beta'\mathrm{Tr}[S_\mathrm{b}v^\mu(V_\mu-\rho_\mu)_\mathrm{ba}\bar{S}_\mathrm{a}]\\ &\quad +\mathrm{i}\lambda'\mathrm{Tr}[S_\mathrm{b}\sigma^{\mu\nu}F_{\mu\nu}(\rho)_\mathrm{ba}\bar{S}_\mathrm{a}]+g'_\sigma\mathrm{Tr}[S_\mathrm{a}\sigma\bar{S}_\mathrm{a}], \end{aligned} \tag{1}$$

where $S$ represents the ($0^+$, $1^+$) doublet. Its matrix representation is

$$S = \frac{1}{2}(1+\not{v})[D_1^\mu\gamma_\mu\gamma_5-D_0^*], \tag{2}$$

$$\bar{S} = \gamma^0 S^\dagger\gamma^0. \tag{3}$$

At the leading order, the axial vector field reads

$$A^\mu_\mathrm{ab}=\frac{1}{2}(\xi^\dagger\partial^\mu\xi-\xi\partial^\mu\xi^\dagger)_\mathrm{ab}=\frac{\mathrm{i}}{f_\pi}\partial^\mu\mathcal{P}_\mathrm{ab}+\dots, \tag{4}$$

where

$$\mathcal{P}=\begin{pmatrix}\frac{\pi^0}{\sqrt{2}}+\frac{\eta}{\sqrt{6}} & \pi^+ & K^+\\ \pi^- & -\frac{\pi^0}{\sqrt{2}}+\frac{\eta}{\sqrt{6}} & K^0\\ K^- & \bar{K}^0 & -\frac{2\eta}{\sqrt{6}}\end{pmatrix}. \tag{5}$$

$\rho^\mu_\mathrm{ab}$ and $F^{\mu\nu}(\rho)_\mathrm{ab}$ represent the vector meson field and its strength tensor

$$\rho^\mu_\mathrm{ab} = \frac{\mathrm{i}g_v}{\sqrt{2}}\mathcal{V}^\mu_\mathrm{ab}, \tag{6}$$

$$\begin{aligned}F^{\mu\nu}(\rho)_\mathrm{ab} &= \partial^\mu\rho^\nu_\mathrm{ab}-\partial^\nu\rho^\mu_\mathrm{ab}+[\rho^\mu_\mathrm{ab},\rho^\nu_\mathrm{ab}]\\ &= \frac{\mathrm{i}g_v}{\sqrt{2}}(\partial^\mu\mathcal{V}^\nu-\partial^\nu\mathcal{V}^\mu)_\mathrm{ab}+\dots,\end{aligned} \tag{7}$$

where $g_v=m_\rho/f_\pi$ with $m_\rho=0.77$ GeV and $f_\pi=0.132$ GeV. $\mathcal{V}$ is the nonet vector meson matrices

$$\mathcal{V}=\begin{pmatrix}\frac{\rho^0}{\sqrt{2}}+\frac{\omega}{\sqrt{2}} & \rho^+ & K^{*+}\\ \rho^- & -\frac{\rho^0}{\sqrt{2}}+\frac{\omega}{\sqrt{2}} & K^{*0}\\ K^{*-} & \bar{K}^{*0} & \phi.\end{pmatrix}. \tag{8}$$

Similarly the scalar field $\sigma$ represents the scalar nonet. All the coupling constants $g',\beta',\lambda'$ and $g'_\sigma$ are real.

In our calculation we only need the effective lagrangian at the tree level

$$\mathcal{L}_{D_0^*D_0^*\mathcal{V}} = \sqrt{2}g_v\beta' v^\mu(\mathcal{V}_\mu)_{\rm ba}D_{0\rm b}^*D_{0\rm a}^{*\dagger},$$

$$\mathcal{L}_{D_1D_1\mathcal{V}} = -\sqrt{2}g_v\beta' v^\mu(\mathcal{V}_\mu)_{\rm ba}(D_{1\rm b}\cdot D_{1\rm a}^\dagger) +2\sqrt{2}{\rm i}g_v\lambda'(\partial_\mu\mathcal{V}_\nu-\partial_\nu\mathcal{V}_\mu)_{\rm ba}D_{1\rm b}^\mu D_{1\rm a}^{\nu\dagger},$$

$$\mathcal{L}_{D_0^*D_1\mathcal{V}} = -\sqrt{2}g_v\lambda'(\partial_\mu\mathcal{V}_\nu-\partial_\nu\mathcal{V}_\mu)_{\rm ba}\epsilon^{\alpha\mu\nu\beta}v_\beta \times(D_{1\rm b\alpha}D_{0\rm a}^{*\dagger}+D_{0\rm b}^*D_{1\rm a\alpha}^\dagger),$$

$$\mathcal{L}_{D_1D_1\mathcal{P}} = \frac{2{\rm i}g'}{f_\pi}\partial_\mu\mathcal{P}_{\rm ba}D_{1\rm b\alpha}D_{1\rm a\beta}^\dagger\epsilon^{\alpha\mu\beta\nu}v_\nu,$$

$$\mathcal{L}_{D_0^*D_1\mathcal{P}} = -\frac{2g'}{f_\pi}\partial_\mu\mathcal{P}_{\rm ba}(D_{1\rm b}^\mu D_{0\rm a}^{*\dagger}+D_{0\rm b}^*D_{1\rm a}^{\mu\dagger}),$$

$$\mathcal{L}_{D_0^*D_0^*\sigma} = 2g'_\sigma D_{0\rm a}^*D_{0\rm a}^{*\dagger}\sigma,$$

$$\mathcal{L}_{D_1D_1\sigma} = -2g'_\sigma(D_{1\rm a}\cdot D_{1\rm a}^\dagger)\sigma.$$

None of the coupling constants $g'$, $\lambda'$, $g'_\sigma$ are known precisely although there exists some crude theoretical estimation [21]. We allow the parameters involved in this work to vary around the values extracted from the QCD sum rule approach (QSR).

### 2.2 Derivation of the effective potential

We follow Refs. [15, 16] to derive the effective potential of the heavy molecular system. Interested readers may consult Refs. [15, 16] for details. As usual, the monopole type form factor (FF) is introduced at every interaction vertex in order to account for the non-point-like structure effect of each interacting particle and cure the singularity of the effective potential.

$$F(q)=\frac{\Lambda^2-m^2}{\Lambda^2-q^2}. \tag{9}$$

$\Lambda$ is the phenomenological cutoff parameter. Generally $\Lambda$ is expected to be larger than the exchanged meson mass and lies around 1–3 GeV.

The effective potential in the coordinate space reads

$$\mathcal{V}(r)=\frac{1}{(2\pi)^3}\int{\rm d}\boldsymbol{q}\mathcal{V}(\boldsymbol{q})F(q)^2{\rm e}^{-{\rm i}\boldsymbol{q}\cdot\boldsymbol{r}}, \tag{10}$$

$$\frac{1}{\boldsymbol{q}^2+m^2}\longrightarrow Y(\Lambda,m,r), \tag{11}$$

$$\frac{\boldsymbol{q}^2}{\boldsymbol{q}^2+m^2}\longrightarrow Z(\Lambda,m,r), \tag{12}$$

$$Y(\Lambda,m,r) = \frac{1}{4\pi r}({\rm e}^{-mr}-{\rm e}^{-\Lambda r})-\frac{\xi^2}{8\pi\Lambda}{\rm e}^{-\Lambda r}, \tag{13}$$

$$Z(\Lambda,m,r) = -\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right)Y(\Lambda,m,r) = -\frac{{\rm e}^{-mr}m^2}{4\pi r}-\frac{{\rm e}^{-\Lambda r}\xi^2}{4\pi r}+\frac{{\rm e}^{-\Lambda r}\xi^2\Lambda}{8\pi} +\frac{{\rm e}^{-\Lambda r}\Lambda^2}{4\pi r}, \tag{14}$$

with $\xi=\sqrt{\Lambda^2-m^2}$.

We collect the meson masses in Table 3.

Table 3. The meson masses [20].

| meson | mass/GeV | mason | mass/GeV |
|---|---|---|---|
| $D_0^{*0}$ | 2.4 | $D_0^{*\pm}$ | 2.4 |
| $D_1^0$ | 2.42 | $D_1^\pm$ | 2.42 |
| $\rho^0$ | 0.77 | $\rho^\pm$ | 0.77 |
| $\phi$ | 1.020 | $\pi^0$ | 0.135 |
| $\eta$ | 0.548 | $\sigma$ | 0.66 |
| meson | mass/GeV | | |
| $D_{s0}^{*\pm}$ | 2.317 | | |
| $D_{s1}^\pm$ | 2.46 | | |
| $\omega$ | 0.782 | | |
| $\pi^\pm$ | 0.140 | | |
| $f_0(980)$ | 0.98 | | |

## 3 The $D_0^*$-$\bar{D}_0^*$ case

In the $D_0^*$-$\bar{D}_0^*$ case, the pseudoscalar meson exchange is forbidden by parity and angular momentum conservation. $\Phi_s^\pm$ and $\Phi_s^0$ ($\bar{\Phi}_s^0$) states don't exist. For $\Phi^\pm$, $\Phi^0$, $\Phi_8^0$, $\Phi_{s1}^0$, the effective potential reads

$$V(r)_{\rm Total}^{\Phi^\pm,0} = -\frac{1}{4}g_v^2\beta'^2[-Y(\Lambda,m_\rho,r) +Y(\Lambda,m_\omega,r)]-g_\sigma'^2Y(\Lambda,m_\sigma,r), \tag{15}$$

$$V(r)_{\rm Total}^{\Phi_8^0} = -\frac{1}{4}g_v^2\beta'^2[3Y(\Lambda,m_\rho,r) +Y(\Lambda,m_\omega,r)]-g_\sigma'^2Y(\Lambda,m_\sigma,r), \tag{16}$$

$$V(r)_{\rm Total}^{\Phi_{s1}^0} = -\frac{1}{2}g_v^2\beta'^2Y(\Lambda,m_\phi,r)-g_\sigma'^2Y(\Lambda,m_{f_0},r), \tag{17}$$

We use the MATSLISE package to solve the Schrödinger equation with the effective potentials. We collect the variation of the binding energy $E$ (in unit of MeV) and the root-mean-square radius $r$ (in unit of fm) with the cutoff and the coupling constants in Table 4.

As the coupling constants increase, the attraction becomes stronger. The cutoff parameter reflects the non-point-like structure of the interacting hadrons at each vertex. Its value is the hadronic size. In this work we assume the "reasonable" cutoff should be larger than the exchanged light meson mass and be around 1–3 GeV.

Simply for comparison, we also collect the numerical results for the other possible molecular states in the same multiplet although their experimental observation may be difficult because of the broad width of the non-strange $(0^+,\ 1^+)$ charmed mesons.

Table 4. The variation of the binding energy $E$ (in unit of MeV) and the root-mean-square radius $r_{\rm rms}$ (in unit of fm) with the cutoff and the coupling constants for the $D_0^*$-$\bar{D}_0^*$ system.

| states | $\Lambda$ | $E$ | $r_{\rm rms}$ |
|---|---|---|---|
| $\beta'=0.84$, $g'_\sigma=0.761$ | | | |
| $\Phi$ | – | – | – |
| $\Phi_8$ | 1.6 | −8.1 | 1.28 |
| | 1.7 | −14.4 | 1.01 |
| | 1.8 | −21.9 | 0.86 |
| | 1.9 | −30.3 | 0.76 |
| $\Phi_{s1}$ | – | – | – |
| $\beta'=0.98$, $g'_\sigma=0.761$ | | | |
| $\Phi$ | – | – | – |
| $\Phi_8$ | 1.4 | −9.2 | 1.23 |
| | 1.45 | −14.0 | 1.04 |
| | 1.5 | −19.6 | 0.92 |
| | 1.55 | −25.8 | 0.83 |
| $\Phi_{s1}$ | – | – | – |
| $\beta'=1.12$, $g'_\sigma=0.761$ | | | |
| $\Phi$ | – | – | – |
| $\Phi_8$ | 1.25 | −6.9 | 1.39 |
| | 1.3 | −12.9 | 1.09 |
| | 1.35 | −20.4 | 0.92 |
| | 1.4 | −29.2 | 0.81 |
| $\Phi_{s1}$ | 3.0 | −17.8 | 0.87 |
| | 3.2 | −25.6 | 0.75 |
| | 3.4 | −34.1 | 0.66 |

## 4 The $D_1$-$\bar{D}_1$ case

The effective potential of the $\Phi^{**\pm}$, $\Phi^{**0}$, $\Phi_8^{**0}$, $\Phi_s^{**\pm}$, $\Phi_s^{**0}$, $\Phi_{s1}^{**0}$ systems reads

$$\begin{aligned}
&V(r)^{\Phi^{**\pm,0}[J]}_{\rm Total}=\\
&-\ \frac{1}{4}g_v^2\beta'^2\mathcal{C}(J)[-Y(\Lambda,m_\rho,r)+Y(\Lambda,m_\omega,r)]\\
&-\ \lambda'^2g_v^2\mathcal{B}(J)[-Z(\Lambda,m_\rho,r)+Z(\Lambda,m_\omega,r)]\\
&+\ \frac{g'^2}{2f_\pi^2}\mathcal{A}(J)[-Z(\Lambda,m_\pi,r)+\frac{1}{3}Z(\Lambda,m_\eta,r)]\\
&-\ g_\sigma'^2\mathcal{C}(J)Y(\Lambda,m_\sigma,r),
\end{aligned}\tag{18}$$

$$\begin{aligned}
&V(r)^{\Phi_8^{**0}[J]}_{\rm Total}=\\
&-\ \frac{1}{4}g_v^2\beta'^2\mathcal{C}(J)[3Y(\Lambda,m_\rho,r)+Y(\Lambda,m_\omega,r)]\\
&-\ \lambda'^2g_v^2\mathcal{B}(J)[3Z(\Lambda,m_\rho,r)+Z(\Lambda,m_\omega,r)]\\
&+\ \frac{g'^2}{2f_\pi^2}\mathcal{A}(J)[3Z(\Lambda,m_\pi,r)+\frac{1}{3}Z(\Lambda,m_\eta,r)]\\
&-\ g_\sigma'^2\mathcal{C}(J)Y(\Lambda,m_\sigma,r),
\end{aligned}\tag{19}$$

$$V(r)^{\Phi_s^{**\pm}[J]}_{\rm Total}=-\frac{g'^2}{3f_\pi^2}\mathcal{A}(J)Z(\Lambda,m_\eta,r),\tag{20}$$

$$V(r)^{\Phi_s^{**0}/\bar{\Phi}_s^{**0}[J]}_{\rm Total}=-\frac{g'^2}{3f_\pi^2}\mathcal{A}(J)Z(\Lambda,m_\eta,r),\tag{21}$$

$$\begin{aligned}
V(r)^{\Phi_{s1}^{**0}[J]}_{\rm Total}=&-\frac{1}{2}g_v^2\beta'^2\mathcal{C}(J)Y(\Lambda,m_\phi,r)\\
&-2\lambda'^2g_v^2\mathcal{B}(J)Z(\Lambda,m_\phi,r)\\
&+\frac{2g'^2}{3f_\pi^2}\mathcal{A}(J)Z(\Lambda,m_\eta,r)\\
&-g_\sigma'^2\mathcal{C}(J)Y(\Lambda,m_{f_0},r),
\end{aligned}\tag{22}$$

where $\mathcal{A}(J)$, $\mathcal{B}(J)$ and $\mathcal{C}(J)$ denote

$$\begin{aligned}
\mathcal{A}(J)\equiv&\sum_{\lambda_1\lambda_2\lambda_3\lambda_4}\langle 1\lambda_1;1\lambda_2|J,m\rangle\langle 1\lambda_3;1\lambda_4|J,m\rangle\\
&\times\frac{1}{\vec{q}^2}[\vec{\epsilon}_1^{\lambda1}\cdot(\vec{q}\times\vec{\epsilon}_3^{\lambda3*})\vec{\epsilon}_2^{\lambda2}\cdot(\vec{q}\times\vec{\epsilon}_4^{\lambda4*})],
\end{aligned}\tag{23}$$

$$\begin{aligned}
\mathcal{B}(J)\equiv&\sum_{\lambda_1\lambda_2\lambda_3\lambda_4}\langle 1\lambda_1;1\lambda_2|J,m\rangle\langle 1\lambda_3;1\lambda_4|J,m\rangle\\
&\times\frac{1}{\vec{q}^2}[(\vec{\epsilon}_1^{\lambda1}\cdot\vec{q})(\vec{\epsilon}_2^{\lambda2}\cdot\vec{q})(\vec{\epsilon}_3^{\lambda3*}\cdot\vec{\epsilon}_4^{\lambda4*})+(\text{c.t.s})],
\end{aligned}\tag{24}$$

$$\begin{aligned}
\mathcal{C}(J)\equiv&\sum_{\lambda_1\lambda_2\lambda_3\lambda_4}\langle 1\lambda_1;1\lambda_2|J,m\rangle\langle 1\lambda_3;1\lambda_4|J,m\rangle\\
&\times[(\vec{\epsilon}_1^{\lambda1}\cdot\vec{\epsilon}_3^{\lambda3*})(\vec{\epsilon}_2^{\lambda2}\cdot\vec{\epsilon}_4^{\lambda4*})],
\end{aligned}\tag{25}$$

$\vec{\epsilon}_1,\vec{\epsilon}_2,\vec{\epsilon}_3,\vec{\epsilon}_4$ are the polarizations of the initial and final states. c.t.s denotes

$$\begin{aligned}
\text{c.t.s}=&(\vec{\epsilon}_3^{\lambda3*}\cdot\vec{q})(\vec{\epsilon}_4^{\lambda4*}\cdot\vec{q})(\vec{\epsilon}_1^{\lambda1}\cdot\vec{\epsilon}_2^{\lambda2})\\
&-(\vec{\epsilon}_1^{\lambda1}\cdot\vec{q})(\vec{\epsilon}_4^{\lambda4*}\cdot\vec{q})(\vec{\epsilon}_2^{\lambda2}\cdot\vec{\epsilon}_3^{\lambda3*})\\
&-(\vec{\epsilon}_2^{\lambda2}\cdot\vec{q})(\vec{\epsilon}_3^{\lambda3*}\cdot\vec{q})(\vec{\epsilon}_1^{\lambda1}\cdot\vec{\epsilon}_4^{\lambda4*}),
\end{aligned}$$

The values of $\mathcal{A}(J)$, $\mathcal{B}(J)$ and $\mathcal{C}(J)$ with different quantum numbers are listed in Table 5.

Table 5. The values of $\mathcal{A}(J)$, $\mathcal{B}(J)$ and $\mathcal{C}(J)$ with different quantum numbers.

| $J$ | $\mathcal{A}(J)$ | $\mathcal{B}(J)$ | $\mathcal{C}(J)$ |
|---|---|---|---|
| 0 | $\frac{2}{3}$ | $\frac{4}{3}$ | 1 |
| 1 | $\frac{1}{3}$ | $\frac{2}{3}$ | 1 |
| 2 | $-\frac{1}{3}$ | $-\frac{2}{3}$ | 1 |

Table 6. The variation of the binding energy $E$ (in unit of MeV) and the root-mean-square radius $r_{\rm rms}$ (in unit of fm) with the cutoff and the coupling constant for the $D_1$-$\bar{D}_1$ system when only the pseudoscalar meson exchange is considered. Here, we scan the cutoff range $\Lambda \leqslant 3.2$ GeV.

| $g'=0.80, \beta'=0, \lambda'=0, g'_\sigma=0$ | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| state | $J^P=0^+$ | | | $J^P=1^+$ | | | $J^P=2^+$ | | |
| | $\Lambda$ | $E$ | $r_{\rm rms}$ | $\Lambda$ | $E$ | $r_{\rm rms}$ | $\Lambda$ | $E$ | $r_{\rm rms}$ |
| $\Phi^{**}$ | 1.9 | −5.4 | 1.33 | – | – | – | – | – | – |
| | 2.0 | −10.6 | 0.98 | – | – | – | – | – | – |
| | 2.1 | −17.9 | 0.78 | – | – | – | – | – | – |
| | 2.2 | −27.9 | 0.65 | – | – | – | – | – | – |
| $\Phi_8^{**}$ | – | – | – | – | – | – | 1.1 | −4.5 | 1.49 |
| | – | – | – | – | – | – | 1.2 | −13.6 | 0.93 |
| | – | – | – | – | – | – | 1.3 | −28.9 | 0.69 |
| $\Phi_s^{**}$ | 2.7 | −6.58 | 1.13 | – | – | – | – | – | – |
| | 2.8 | −18.5 | 0.70 | – | – | – | – | – | – |
| | 2.9 | −36.4 | 0.52 | – | – | – | – | – | – |
| $\Phi_{s1}^{**}$ | – | – | – | – | – | – | 2.7 | −7.8 | 1.03 |
| | – | – | – | – | – | – | 2.8 | −20.5 | 0.66 |
| | – | – | – | – | – | – | 2.9 | −39.2 | 0.50 |

| $g'=1.06, \beta'=0, \lambda'=0, g'_\sigma=0$ | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| state | $J^P=0^+$ | | | $J^P=1^+$ | | | $J^P=2^+$ | | |
| | $\Lambda$ | $E$ | $r_{\rm rms}$ | $\Lambda$ | $E$ | $r_{\rm rms}$ | $\Lambda$ | $E$ | $r_{\rm rms}$ |
| $\Phi^{**}$ | 1.1 | −4.0 | 1.58 | 2.1 | −3.4 | 1.62 | – | – | – |
| | 1.2 | −9.9 | 1.08 | 2.2 | −7.4 | 1.14 | – | – | – |
| | 1.3 | −19.0 | 0.82 | 2.3 | −13.3 | 0.88 | – | – | – |
| | 1.4 | −31.9 | 0.67 | 2.4 | −21.3 | 0.72 | – | – | – |
| $\Phi_8^{**}$ | – | – | – | – | – | – | 0.75 | −3.6 | 1.72 |
| | – | – | – | – | – | – | 0.8 | −8.0 | 1.23 |
| | – | – | – | – | – | – | 0.9 | −23.6 | 0.81 |
| $\Phi_s^{**}$ | 1.9 | −4.3 | 1.39 | 3.0 | −11.7 | 0.86 | – | – | – |
| | 2.0 | −18.9 | 0.71 | 3.1 | −25.6 | 0.60 | – | – | – |
| | 2.1 | −43.4 | 0.50 | 3.2 | −44.8 | 0.47 | – | – | – |
| $\Phi_{s1}^{**}$ | – | – | – | – | – | – | 1.9 | −5.1 | 1.28 |
| | – | – | – | – | – | – | 2.0 | −20.5 | 0.68 |
| | – | – | – | – | – | – | 2.1 | −45.7 | 0.49 |

| $g'=1.32, \beta'=0, \lambda'=0, g'_\sigma=0$ | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| state | $J^P=0^+$ | | | $J^P=1^+$ | | | $J^P=2^+$ | | |
| | $\Lambda$ | $E$ | $r_{\rm rms}$ | $\Lambda$ | $E$ | $r_{\rm rms}$ | $\Lambda$ | $E$ | $r_{\rm rms}$ |
| $\Phi^{**}$ | 0.75 | −3.0 | 1.86 | 1.35 | −2.7 | 1.85 | – | – | – |
| | 0.85 | −10.3 | 1.11 | 1.50 | −10.0 | 1.04 | – | – | – |
| | 0.95 | −22.8 | 0.82 | 1.65 | −23.1 | 0.73 | – | – | – |
| | 1.05 | −41.3 | 0.65 | 1.75 | −35.9 | 0.61 | – | – | – |
| $\Phi_8^{**}$ | – | – | – | – | – | – | 0.65 | −12.0 | 1.11 |
| | – | – | – | – | – | – | 0.70 | −22.0 | 0.88 |
| | – | – | – | – | – | – | 0.75 | −35.9 | 0.73 |
| $\Phi_s^{**}$ | 1.55 | −8.4 | 1.03 | 2.25 | −9.7 | 0.95 | – | – | – |
| | 1.60 | −18.5 | 0.73 | 2.30 | −16.9 | 0.74 | – | – | – |
| | 1.65 | −32.4 | 0.58 | 2.40 | −37.1 | 0.53 | – | – | – |
| $\Phi_{s1}^{**}$ | – | – | – | – | – | – | 1.50 | −2.6 | 1.78 |
| | – | – | – | – | – | – | 1.55 | −9.3 | 0.98 |
| | – | – | – | – | – | – | 1.60 | −19.8 | 0.71 |

Table 7. The variation of the binding energy $E$ (in unit of MeV) and the root-mean-square radius $r_{\rm rms}$ (in unit of fm) with the cutoff and the coupling constants for the $D_1$-$\bar{D}_1$ system. Here, we scan the cutoff range $\Lambda \leqslant 3.2$ GeV.

| $g'=0.80$, $\beta'=0.84$, $\lambda'=0.42$, $g'_\sigma=0.761$ | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| state | $J^P=0^+$ | | | $J^P=1^+$ | | | $J^P=2^+$ | | |
| | $\Lambda$ | $E$ | $r_{\rm rms}$ | $\Lambda$ | $E$ | $r_{\rm rms}$ | $\Lambda$ | $E$ | $r_{\rm rms}$ |
| $\Phi^{**}$ | 1.9 | −7.1 | 1.18 | – | – | – | – | – | – |
| | 2.0 | −13.0 | 0.91 | – | – | – | – | – | – |
| | 2.1 | −21.0 | 0.74 | – | – | – | – | – | – |
| | 2.2 | −31.7 | 0.62 | – | – | – | – | – | – |
| $\Phi_8^{**}$ | – | – | – | – | – | – | 1.0 | −5.7 | 1.41 |
| | – | – | – | – | – | – | 1.05 | −11.6 | 1.06 |
| | – | – | – | – | – | – | 1.1 | −19.5 | 0.87 |
| $\Phi_s^{**}$ | 2.7 | −6.58 | 1.13 | – | – | – | – | – | – |
| | 2.8 | −18.5 | 0.70 | – | – | – | – | – | – |
| | 2.9 | −36.4 | 0.52 | – | – | – | – | – | – |
| $\Phi_{s1}^{**}$ | – | – | – | – | – | – | 2.6 | −3.2 | 1.70 |
| | – | – | – | – | – | – | 2.9 | −10.0 | 1.03 |
| | – | – | – | – | – | – | 3.2 | −18.8 | 0.79 |
| | – | – | – | – | – | – | 3.5 | −28.6 | 0.66 |

| $g'=1.06$, $\beta'=0.98$, $\lambda'=0.49$, $g'_\sigma=0.761$ | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| state | $J^P=0^+$ | | | $J^P=1^+$ | | | $J^P=2^+$ | | |
| | $\Lambda$ | $E$ | $r_{\rm rms}$ | $\Lambda$ | $E$ | $r_{\rm rms}$ | $\Lambda$ | $E$ | $r_{\rm rms}$ |
| $\Phi^{**}$ | 1.1 | −4.1 | 1.57 | 2.1 | −5.1 | 1.35 | – | – | – |
| | 1.2 | −10.1 | 1.07 | 2.2 | −9.9 | 1.01 | – | – | – |
| | 1.3 | −19.3 | 0.82 | 2.3 | −16.6 | 0.81 | – | – | – |
| | 1.4 | −32.4 | 0.66 | 2.4 | −25.5 | 0.67 | – | – | – |
| $\Phi_8^{**}$ | – | – | – | – | – | – | 0.8 | −8.4 | 1.21 |
| | – | – | – | – | – | – | 0.85 | −16.9 | 0.93 |
| | – | – | – | – | – | – | 0.9 | −29.7 | 0.76 |
| $\Phi_s^{**}$ | 1.9 | −4.3 | 1.39 | 2.9 | −3.1 | 1.61 | – | – | – |
| | 1.95 | −10.4 | 0.92 | 3.0 | −11.7 | 0.86 | – | – | – |
| | 2.0 | −18.9 | 0.71 | 3.1 | −25.6 | 0.60 | – | – | – |
| | 2.05 | −29.9 | 0.58 | 3.15 | −34.5 | 0.53 | – | – | – |
| $\Phi_{s1}^{**}$ | – | – | – | – | – | – | 1.75 | −3.6 | 1.60 |
| | – | – | – | – | – | – | 1.8 | −6.9 | 1.20 |
| | – | – | – | – | – | – | 1.9 | −16.1 | 0.83 |
| | – | – | – | – | – | – | 2.0 | −28.1 | 0.66 |

| $g'=1.32$, $\beta'=1.12$, $\lambda'=0.56$, $g'_\sigma=0.761$ | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| state | $J^P=0^+$ | | | $J^P=1^+$ | | | $J^P=2^+$ | | |
| | $\Lambda$ | $E$ | $r_{\rm rms}$ | $\Lambda$ | $E$ | $r_{\rm rms}$ | $\Lambda$ | $E$ | $r_{\rm rms}$ |
| $\Phi^{**}$ | 0.8 | −6.1 | 1.38 | 1.4 | −4.9 | 1.42 | – | – | – |
| | 0.85 | −10.3 | 1.12 | 1.5 | −10.6 | 1.02 | – | – | – |
| | 0.9 | −15.7 | 0.95 | 1.6 | −19.0 | 0.80 | – | – | – |
| | 0.95 | −22.5 | 0.82 | 1.7 | −30.5 | 0.66 | – | – | – |
| $\Phi_8^{**}$ | – | – | – | – | – | – | 0.8 | −55.1 | 0.62 |
| | – | – | – | – | – | – | 0.9 | −118.0 | 0.48 |
| | – | – | – | – | – | – | 1.0 | −213.9 | 0.39 |
| $\Phi_s^{**}$ | 1.55 | −8.4 | 1.03 | 2.2 | −4.4 | 1.42 | – | – | – |
| | 1.60 | −18.7 | 0.73 | 2.25 | −9.7 | 0.95 | – | – | – |
| | 1.65 | −32.4 | 0.58 | 2.3 | −16.9 | 0.74 | – | – | – |
| $\Phi_{s1}^{**}$ | – | – | – | – | – | – | 1.45 | −5.4 | 1.32 |
| | – | – | – | – | – | – | 1.50 | −12.5 | 0.92 |
| | – | – | – | – | – | – | 1.55 | −22.0 | 0.73 |
| | – | – | – | – | – | – | 1.60 | −33.7 | 0.62 |

Table 8. The variation of the binding energy $E$ (in unit of MeV) and the root-mean-square radius $r_{\rm rms}$ (in unit of fm) with the cutoff and the coupling constant for the $D_0^*$-$\bar{D}_1$ system when only the pseudoscalar meson exchange is considered.

| $g'=0.80, \beta'=0, \lambda'=0, g'_\sigma=0$ | | | | | | |
|---|---|---|---|---|---|---|
| states | $c=+1$ | | | $c=-1$ | | |
| | $\Lambda$ | $E$ | $r_{\rm rms}$ | $\Lambda$ | $E$ | $r_{\rm rms}$ |
| $\Phi^*$ | – | – | – | – | – | – |
| $\Phi^*_8$ | 1.1 | −4.4 | 1.51 | – | – | – |
| | 1.2 | −13.4 | 0.94 | – | – | – |
| | 1.3 | −28.6 | 0.69 | – | – | – |
| | 1.4 | −51.3 | 0.55 | – | – | – |
| $\Phi^*_{s1}$ | 2.7 | −5.2 | 1.27 | – | – | – |
| | 2.8 | −16.0 | 0.75 | | | |
| | 2.85 | −23.6 | 0.63 | | | |
| | 2.9 | −32.6 | 0.55 | | | |

| $g'=1.06, \beta'=0, \lambda'=0, g'_\sigma=0$ | | | | | | |
|---|---|---|---|---|---|---|
| states | $c=+1$ | | | $c=-1$ | | |
| | $\Lambda$ | $E$ | $r_{\rm rms}$ | $\Lambda$ | $E$ | $r_{\rm rms}$ |
| $\Phi^*$ | – | – | – | 2.2 | −7.1 | 1.17 |
| | – | – | – | 2.3 | −12.8 | 0.90 |
| | – | – | – | 2.4 | −20.7 | 0.73 |
| | – | – | – | 2.5 | −30.9 | 0.61 |
| $\Phi^*_8$ | 0.8 | −8.0 | 1.24 | – | – | – |
| | 0.85 | −14.5 | 0.98 | – | – | – |
| | 0.9 | −23.5 | 0.81 | – | – | – |
| | 0.95 | −35.3 | 0.69 | – | – | – |
| $\Phi^*_s$ | – | – | – | 3 | −9.0 | 0.98 |
| | – | – | – | 3.05 | −14.5 | 0.78 |
| | – | – | – | 3.1 | −21.4 | 0.66 |
| | – | – | – | 3.15 | −29.6 | 0.57 |
| $\Phi^*_{s1}$ | 1.9 | −4.2 | 1.43 | – | – | – |
| | 1.95 | −10.1 | 0.95 | – | – | – |
| | 2.0 | −18.3 | 0.73 | – | – | – |
| | 2.05 | −29.0 | 0.60 | – | – | – |

| $g'=1.32, \beta'=0, \lambda'=0, g'_\sigma=0$ | | | | | | |
|---|---|---|---|---|---|---|
| states | $c=+1$ | | | $c=-1$ | | |
| | $\Lambda$ | $E$ | $r_{\rm rms}$ | $\Lambda$ | $E$ | $r_{\rm rms}$ |
| $\Phi^*$ | – | – | – | 1.5 | −9.7 | 1.06 |
| | – | – | – | 1.6 | −17.7 | 0.82 |
| | – | – | – | 1.7 | −28.6 | 0.67 |
| $\Phi^*_8$ | 0.8 | −54.2 | 0.63 | – | – | – |
| | 0.825 | −65.4 | 0.58 | – | – | – |
| | 0.85 | −77.9 | 0.55 | – | – | – |
| | 0.875 | −92.0 | 0.51 | – | – | – |
| $\Phi^*_s$ | – | – | – | 2.25 | −7.9 | 1.05 |
| | – | – | – | 2.3 | −14.5 | 0.80 |
| | – | – | – | 2.35 | −23.0 | 0.65 |
| | – | – | – | 2.4 | −33.5 | 0.55 |
| $\Phi^*_{s1}$ | 1.55 | −8.9 | 1.02 | – | – | – |
| | 1.6 | −19.1 | 0.73 | – | – | – |
| | 1.65 | −32.9 | 0.59 | – | – | – |
| | 1.7 | −50.5 | 0.49 | – | – | – |

Table 9. The variation of the binding energy $E$ (in unit of MeV) and the root-mean-square radius $r_{\rm rms}$ (in unit of fm) with the cutoff and the coupling constants for the $D_0^*$-$\bar{D}_1$ system.

| $g'=0.80$, $\beta'=0.84$, $\lambda'=0.42$, $g'_\sigma=0.761$ | | | | | | |
|---|---|---|---|---|---|---|
| states | $c=+1$ | | | $c=-1$ | | |
| | $\Lambda$ | $E$ | $r_{\rm rms}$ | $\Lambda$ | $E$ | $r_{\rm rms}$ |
| $\Phi^*$ | – | – | – | – | – | – |
| $\Phi_8^*$ | 1.0 | −5.6 | 1.42 | – | – | – |
| | 1.05 | −11.5 | 1.07 | – | – | – |
| | 1.1 | −19.3 | 0.88 | – | – | – |
| | 1.15 | −28.8 | 0.76 | – | – | – |
| $\Phi_{s1}^*$ | 2.7 | −4.0 | 1.56 | – | – | – |
| | 3.0 | −10.8 | 1.01 | | | |
| | 3.3 | −19.1 | 0.79 | | | |
| | 3.6 | −28.3 | 0.67 | | | |

| $g'=1.06, \beta'=0.98, \lambda'=0.49, g'_\sigma=0.761$ | | | | | | |
|---|---|---|---|---|---|---|
| states | $c=+1$ | | | $c=-1$ | | |
| | $\Lambda$ | $E$ | $r_{\rm rms}$ | $\Lambda$ | $E$ | $r_{\rm rms}$ |
| $\Phi^*$ | – | – | – | 2.1 | −4.9 | 1.39 |
| | – | – | – | 2.2 | −9.6 | 1.03 |
| | – | – | – | 2.3 | −16.1 | 0.82 |
| | – | – | – | 2.4 | −24.8 | 0.68 |
| $\Phi_8^*$ | 0.8 | −8.3 | 1.22 | – | – | – |
| | 0.85 | −16.7 | 0.93 | – | – | – |
| | 0.9 | −29.5 | 0.76 | – | – | – |
| | 0.95 | −46.7 | 0.64 | – | – | – |
| $\Phi_s^*$ | – | – | – | 2.95 | −4.7 | 1.33 |
| | – | – | – | 3.05 | −14.5 | 0.78 |
| | – | – | – | 3.1 | −21.4 | 0.66 |
| | – | – | – | 3.15 | −29.6 | 0.57 |
| $\Phi_{s1}^*$ | 1.8 | −6.1 | 1.29 | – | – | – |
| | 1.9 | −14.6 | 0.88 | – | – | – |
| | 2.0 | −26.0 | 0.69 | – | – | – |
| | 2.1 | −40.0 | 0.58 | – | – | – |

| $g'=1.32$, $\beta'=1.12$, $\lambda'=0.56$, $g'_\sigma=0.761$ | | | | | | |
|---|---|---|---|---|---|---|
| states | $c=+1$ | | | $c=-1$ | | |
| | $\Lambda$ | $E$ | $r_{\rm rms}$ | $\Lambda$ | $E$ | $r_{\rm rms}$ |
| $\Phi^*$ | – | – | – | 1.5 | −10.4 | 1.03 |
| | – | – | – | 1.6 | −18.7 | 0.81 |
| | – | – | – | 1.7 | −30.0 | 0.66 |
| $\Phi_8^*$ | 0.8 | −55.0 | 0.63 | – | – | – |
| | 0.825 | −67.5 | 0.58 | – | – | – |
| | 0.85 | −82.1 | 0.54 | – | – | – |
| | 0.875 | −98.8 | 0.51 | – | – | – |
| $\Phi_s^*$ | – | – | – | 2.25 | −7.9 | 1.05 |
| | – | – | – | 2.3 | −14.5 | 0.80 |
| | – | – | – | 2.35 | −23.0 | 0.65 |
| | – | – | – | 2.4 | −33.5 | 0.55 |
| $\Phi_{s1}^*$ | 1.45 | −5.1 | 1.37 | – | – | – |
| | 1.5 | −11.9 | 0.95 | – | – | – |
| | 1.55 | −21.1 | 0.75 | – | – | – |
| | 1.6 | −32.5 | 0.63 | – | – | – |

We collect the variation of the binding energy $E$ and the root-mean-square radius $r$ with the cutoff and the coupling constants in Tables 6–7. With the pseudoscalar meson exchange force alone and $g' = 0.80$, there exists an isovector $\Phi^{**}$ with $J^P = 0^+$, a $\Phi_8^{**}$ state with $J^P = 2^+$, an isoscalar $\Phi_s^{**}$ state with $J^P = 0^+$ and an isoscalar $\Phi_{s1}^{**}$ state with $J^P = 2^+$. Increasing $g'$ to 1.06, we can find the bound state solution for $\Phi^{**}$ with $J^P = 1^+$ and $\Phi_s^{**}$ with $J^P = 1^+$ besides the above-mentioned bound states. With $g' = 1.32$, the above bound states still exist. We notice that corresponding cutoff $\Lambda$ becomes smaller with the larger $g'$.

Including all the exchange meson contributions, we list the numerical results in Table 7. From Tables 6 and 7 we note that the pseudoscalar meson exchange potential is dominant in the total effective potential. Thus, it is reasonable to consider pseudoscalar meson exchange potential only when studying whether there exists a bound state solution for the $D_1$-$\bar{D}_1$ case. Meanwhile, for the $B_1$-$\bar{B}_1$ system, we list the results in Table A2.

## 5 The $D_0^*$-$\bar{D}_1$ case

In the $D_0^*$-$\bar{D}_1$ case, there are both direct and crossed scattering channels in the derivation of the effective potential in the momentum space. In the crossed channel, the mass difference $q_0$ between the initial and final states (i.e., $D_0^*$ and $D_1$) should be kept. We introduce

$$\mu_{\rm m} = \sqrt{m^2 - q_0^2}, \qquad \alpha = \sqrt{\Lambda^2 - q_0^2},$$

where the subscript "m" denotes the exchanged meson. Accordingly,

$$F(q) = \frac{\Lambda^2 - m^2}{\Lambda^2 - q^2} = \frac{\alpha^2 - \mu^2}{\alpha^2 + \boldsymbol{q}^2}.$$

After the Fourier transformation

$$\frac{1}{q^2 - m^2} = \frac{1}{q_0^2 - \boldsymbol{q}^2 - m^2} \longrightarrow -Y(\alpha,\mu,r), \quad (26)$$

$$\frac{\boldsymbol{q}^2}{q^2 - m^2} = \frac{\boldsymbol{q}^2}{q_0^2 - \boldsymbol{q}^2 - m^2} \longrightarrow -Z(\alpha,\mu,r), \quad (27)$$

the effective potential for the $\Phi^{*\pm}$, $\widehat{\Phi}^{*\pm}$, $\Phi^{*0}$, $\widehat{\Phi}^{*0}$, $\Phi_8^{*0}$, $\widehat{\Phi}_8^{*0}$, $\Phi_s^{*\pm}$, $\widehat{\Phi}_s^{*\pm}$, $\Phi_s^{*0}$, $\widehat{\Phi}_s^{*0}$, $\Phi_{s1}^{*0}$, $\widehat{\Phi}_{s1}^{*0}$ systems read as

$$\begin{aligned} V(r)_{\rm Total}^{\Phi^{*\pm,0}/\widehat{\Phi}^{*\pm,0}} &= -\frac{1}{4}g_v^2\beta'^2[-Y(\Lambda,m_\rho,r)+Y(\Lambda,m_\omega,r)] + c\Bigg\{\frac{2}{3}g_v^2\lambda'^2[-Z(\alpha,\mu_\rho,r) \\ &\quad +Z(\alpha,\mu_\omega,r)] - \frac{1}{6}\frac{g'^2}{f_\pi^2}\left[-Z(\alpha,\mu_\pi,r)+\frac{1}{3}Z(\alpha,\mu_\eta,r)\right]\Bigg\} - g_\sigma'^2 Y(\Lambda,m_\sigma,r), \end{aligned} \quad (28)$$

$$\begin{aligned} V(r)_{\rm Total}^{\Phi_8^{*0}/\widehat{\Phi}_8^{*0}} &= \frac{1}{4}g_v^2\beta'^2[3Y(\Lambda,m_\rho,r)+Y(\Lambda,m_\omega,r)] + c\Bigg\{\frac{2}{3}g_v^2\lambda'^2[3Z(\alpha,\mu_\rho,r) \\ &\quad +Z(\alpha,\mu_\omega,r)] - \frac{1}{6}\frac{g'^2}{f_\pi^2}\left[3Z(\alpha,\mu_\pi,r)+\frac{1}{3}Z(\alpha,\mu_\eta,r)\right]\Bigg\} - g_\sigma'^2 Y(\Lambda,m_\sigma,r), \end{aligned} \quad (29)$$

$$V(r)_{\rm Total}^{\Phi_s^{*\pm}/\widehat{\Phi}_s^{*\pm}} = c\frac{1}{9}\frac{g'^2}{f_\pi^2}Z(\alpha,\mu_\eta,r), \quad (30)$$

$$V(r)_{\rm Total}^{\Phi_s^{*0},\bar{\Phi}_s^{*0}/\widehat{\Phi}_s^{*0},\widehat{\bar{\Phi}}_s^{*0}} = c\frac{1}{9}\frac{g'^2}{f_\pi^2}Z(\alpha,\mu_\eta,r), \quad (31)$$

$$V(r)_{\rm Total}^{\Phi_{s1}^{*0}/\widehat{\Phi}_{s1}^{*0}} = -\frac{1}{2}g_v^2\beta'^2 Y(\Lambda,m_\phi,r) + c\left\{\frac{4}{3}g_v^2\lambda'^2 Z(\alpha,\mu_\phi,r) - \frac{2}{9}\frac{g'^2}{f_\pi^2}Z(\alpha,\mu_\eta,r)\right\} - g_\sigma'^2 Y(\Lambda,m_{f_0},r). \quad (32)$$

The $D_0^*$-$\bar{D}_1$ system is very similar to the D-$\bar{D}^*$ case and is particularly interesting since X(3872) is often speculated to be a D-$\bar{D}^*$ molecular candidate. The only difference is that both components in the $D_0^*$-$\bar{D}_1$ system are extremely narrow $P$-wave states. We first focus on the pseudoscalar meson exchange, which is repulsive for the $\Phi_{s1}^{*0}$ state with negative charge parity. The $J^{PC} = 1^{++}$ $\Phi_{s1}^{*0}$ state appears as shown in Table 8–9. By comparing the result listed in Table 8 and that in Table 9, one notices that the pseudoscalar meson exchange is dominant in the $D_0^*$-$\bar{D}_1$ system, which shows that it is reasonable to consider the pseudoscalar meson exchange potential only when we investigate whether there exists the bound state solution for the $D_0^*$-$\bar{D}_1$ system. The result for the $B_0^*$-$\bar{B}_1$ system corresponds to the pseudoscalar meson exchange only.

## 6 Discussion and conclusion

Both $D_{s0}(2317)$ and $D_{s1}(2460)$ lie below the DK and $D^*$K threshold respectively. They are extremely narrow. The possible molecular states composed of the $D_{s0}(2317)$ and $D_{s1}(2460)$ may be observable experimentally if they really exist. In this work we have studied such systems carefully. As a byproduct, we collect the numerical results for the bottomonium-like analogue in the appendix.

One should be cautious that our numerical results are quite sensitive to the values of the hadronic coupling constants. The values are larger than (or around the upper bound of) those derived from the crude estimate with the light cone QCD sum rule approach [21]. Future lattice QCD simulations may help extract these coupling constants more precisely. Since the hadronic coupling constants are not known well, we allow them to vary. As shown in the numerical result, the binding energy is also sensitive to the value of the cutoff introduced in the form factor. Thus, further study and improvement of the potential model are still desirable.

Here, we need to emphasize that a monopole form factor is introduced in the numerical calculation of this work. In fact, there are many types of form factor, such as the dipole form factor. When taking the other type of the form factor, the qualitative conclusion keeps the same as that obtained in this work. Both the form factor and the cutoff are necessary and important for the hadronic system since the components are not point-like particles. They are hadrons with internal structure. When dealing with the loosely bound heavy molecular states, only the relatively soft degree of freedom is expected to play the dominant role. The exchanged soft mesons should not "see" the quark/gluon structure of the heavy meson. That's the physical meaning of the form factor and the cutoff.

So long as these couplings are big enough, there may even appear deeply bound states including radial and orbital excitations. However they are no more the "conventional" molecular states, which are loosely bound with a typical binding energy around several to several tens MeV and a radius around 1.5–3 fm. Therefore we do not list numerical results for the deeply bound cases in this work.

From our calculation there may exist two loosely bound $0^{++}$ charmonium-like states, the first of which is composed of the $D_{s0}$ $\bar{D}_{s0}$ pair and lies around 4.61 GeV. The other one is around 4.9 GeV and composed of the $D_{s1}$ and $\bar{D}_{s1}$ pair. There exists the $2^{++}$ $D_{s1}$ $\bar{D}_{s1}$ state, which lies around 4.9 GeV. The $1^{++}$ state around 4.75 GeV is composed of the $D_{s0}$ and $\bar{D}_{s1}$ pair. This state is very interesting because of its similarity to X(3872).

The dominant decay modes of the above states are the open-charm modes $D_s^{(*)}\bar{D}_s^{(*)}$. The other characteristic decay modes are the hidden-charm modes $J/\psi\phi$, $\eta_c\eta'$, $\eta_c f_0(980)$, $\chi_{cJ}\eta'$, $\chi_{cJ}f_0(980)$, $\psi'\phi$, $\psi''\phi$, $\eta_c(2S)\eta'$, etc. for the possible $C$=+ molecular states. One may easily exhaust the possible final states according to the $C/P$ parity and angular momentum conservation and kinematical considerations. These states may be significantly narrower than the conventional charmonium around the same mass region because of their molecular nature. However, their widths should be larger than those of X(3872) due to much larger phase space and more decay modes.

These states might be produced from B or $B_s$ decays if kinematically allowed. Those states with $J^{PC} = 0^{++}$, $2^{++}$ may be produced from the two photon fusion process at the $e^+e^-$ collider at B factories. The other possible facilities to look for them are RHIC, Tevatron and LHCb. Investigations of these states may help us understand the puzzling X(3872) state.

*The authors would like to thank Wei-Zhen Deng for useful discussions.*

## Appendix A

### Possible molecular states composed of a pair of excited bottom-strange mesons

We collect the numerical results for the bottomonium-like system composed of a pair of excited bottom-strange mesons in the appendix. Since neither $B_{s0}$ nor $B_{s1}$ is observed experimentally, we follow Ref. [22] and use the mass values $m_{B_0}(J^P = 0^+) = 5.627$ GeV, $m_{B_1}(J^P = 1^+) = 5.674$ GeV, $m_{B_{s0}}(J^P = 0^+) = 5.718$ GeV and $m_{B_{s1}}(J^P = 1^+) = 5.765$ GeV.

Table A1. The variation of the binding energy $E$ (in unit of MeV) and the root-mean-square radius $r_{\rm rms}$ (in unit of fm) with the cutoff and the coupling constants for the $B_0^*$-$\bar{B}_0^*$ system.

| states | $\Lambda$ | $E$ | $r_{\rm rms}$ |
|---|---|---|---|
| $\beta'=0.84$, $g'_\sigma=0.761$ | | | |
| $\Phi$ | – | – | – |
| $\Phi_8$ | 1.15 | −3.0 | 1.41 |
| | 1.2 | −6.5 | 1.04 |
| | 1.25 | −11.1 | 0.85 |
| | 1.35 | −23.4 | 0.66 |
| $\Phi_{s1}$ | 2.2 | −5.4 | 1.00 |
| | 2.3 | −8.3 | 0.84 |
| | 2.4 | −11.6 | 0.73 |
| $\beta'=0.98$, $g'_\sigma=0.761$ | | | |
| states | $\Lambda$ | $E$ | $r_{\rm rms}$ |
| $\Phi^{\pm,0}$ | – | – | – |
| $\Phi_8^0$ | 1.11 | −6.0 | 1.08 |
| | 1.15 | −10.8 | 0.87 |
| | 1.19 | −16.8 | 0.75 |
| | 1.23 | −23.9 | 0.66 |
| $\Phi_{s1}^0$ | 1.9 | −5.9 | 0.98 |
| | 2.0 | −10.5 | 0.78 |
| | 2.1 | −16.2 | 0.66 |
| | 2.2 | −22.8 | 0.58 |
| $\beta'=1.12$, $g'_\sigma=0.761$ | | | |
| states | $\Lambda$ | $E$ | $r_{\rm rms}$ |
| $\Phi^{\pm,0}$ | – | – | – |
| $\Phi_8^0$ | 1.05 | −5.6 | 1.12 |
| | 1.1 | −13.3 | 0.82 |
| | 1.15 | −24.0 | 0.67 |
| | 1.2 | −37.3 | 0.58 |
| $\Phi_{s1}^0$ | 1.7 | −5.0 | 1.06 |
| | 1.75 | −7.9 | 0.88 |
| | 1.8 | −11.3 | 0.77 |
| | 1.85 | −15.2 | 0.69 |

Table A2. The variation of the binding energy $E$ (in unit of MeV) and the root-mean-square radius $r_{\rm rms}$ (in unit of fm) with the cutoff and the coupling constant for the $B_1$-$\bar{B}_1$ system when only the pseudoscalar meson exchange is considered. The $B_1$-$\bar{B}_1$ system is easier to form a bound state than the $D_1$-$\bar{D}_1$ case. In this table, we only give the result for the $B_1$-$\bar{B}_1$ system with the typical coupling constant $g'=0.80$, 1.06. We scan the cutoff range $\Lambda \leqslant 3.1$ GeV.

| $g'=0.80$, $\beta'=0$, $\lambda'=0$, $g'_\sigma=0$ | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| state | $J^P=0^+$ | | | $J^P=1^+$ | | | $J^P=2^+$ | | |
| | $\Lambda$ | $E$ | $r_{\rm rms}$ | $\Lambda$ | $E$ | $r_{\rm rms}$ | $\Lambda$ | $E$ | $r_{\rm rms}$ |
| $\Phi^{**}$ | 1.0 | −6.4 | 0.94 | 1.6 | −1.7 | 1.52 | – | – | – |
| | 1.1 | −12.2 | 0.73 | 1.8 | −6.8 | 0.84 | – | – | – |
| | 1.2 | −20.3 | 0.60 | 2.0 | −16.5 | 0.58 | – | – | – |
| | 1.3 | −31.2 | 0.50 | 2.2 | −32.2 | 0.44 | – | – | – |
| $\Phi_8^{**}$ | – | – | – | – | – | – | 0.8 | −13.9 | 0.74 |
| | – | – | – | – | – | – | 0.85 | −20.8 | 0.64 |
| | – | – | – | – | – | – | 0.9 | −29.7 | 0.56 |
| $\Phi_s^{**}$ | 1.7 | −6.2 | 0.81 | 2.45 | −3.4 | 1.03 | – | – | – |
| | 1.8 | −17.4 | 0.53 | 2.55 | −9.3 | 0.65 | – | – | – |
| | 1.9 | −34.5 | 0.40 | 2.65 | −18.2 | 0.49 | – | – | – |
| $\Phi_{s1}^{**}$ | – | – | – | – | – | – | 1.65 | −3.0 | 1.10 |
| | – | – | – | – | – | – | 1.7 | −6.7 | 0.78 |
| | – | – | – | – | – | – | 1.8 | −18.3 | 0.51 |

(Table A2 Continued)

| $g'=1.06$, $\beta'=0$, $\lambda'=0$, $g'_\sigma=0$ | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| state | $J^P=0^+$ | | | $J^P=1^+$ | | | $J^P=2^+$ | | |
| | $\Lambda$ | $E$ | $r_{\rm rms}$ | $\Lambda$ | $E$ | $r_{\rm rms}$ | $\Lambda$ | $E$ | $r_{\rm rms}$ |
| $\Phi^{**}$ | 0.8 | −18.2 | 0.69 | 1.1 | −6.1 | 0.95 | – | – | – |
| | 0.85 | −24.8 | 0.61 | 1.2 | −11.2 | 0.74 | – | – | – |
| | 0.9 | −32.8 | 0.55 | 1.3 | −18.4 | 0.61 | – | – | – |
| $\Phi^{**}_8$ | – | – | – | – | – | – | 0.8 | −70.2 | 0.44 |
| | – | – | – | – | – | – | 0.825 | −81.8 | 0.41 |
| | – | – | – | – | – | – | 0.85 | −94.7 | 0.39 |
| $\Phi^{**}_{\rm s}$ | 1.3 | −3.8 | 1.02 | 1.8 | −5.0 | 0.88 | – | – | – |
| | 1.35 | −9.5 | 0.70 | 1.9 | −14.6 | 0.56 | – | – | – |
| | 1.4 | −17.6 | 0.55 | 2.0 | −29.3 | 0.42 | – | – | – |
| $\Phi^{**}_{\rm s1}$ | – | – | – | – | – | – | 1.3 | −4.1 | 0.98 |
| | – | – | – | – | – | – | 1.35 | −10.0 | 0.68 |
| | – | – | – | – | – | – | 1.4 | −18.3 | 0.54 |

Table A3. The variation of the binding energy $E$ (in unit of MeV) and the root-mean-square radius $r_{\rm rms}$ (in unit of fm) with the cutoff and the coupling constant for the $\rm B^*_0$-$\rm \bar{B}_1$ system when only the pseudoscalar meson exchange is considered.

| $g'=0.80$, $\beta'=0$, $\lambda'=0$, $g'_\sigma=0$ | | | | | | |
|---|---|---|---|---|---|---|
| states | $c=+1$ | | | $c=-1$ | | |
| | $\Lambda$ | $E$ | $r_{\rm rms}$ | $\Lambda$ | $E$ | $r_{\rm rms}$ |
| $\Phi^*$ | – | – | – | 1.6 | −1.8 | 1.50 |
| | – | – | – | 1.8 | −6.9 | 0.84 |
| | – | – | – | 2.0 | −16.5 | 0.58 |
| | – | – | – | 2.2 | −32.2 | 0.44 |
| $\Phi^*_8$ | 0.8 | −14.3 | 0.74 | – | – | – |
| | 0.825 | −17.6 | 0.68 | – | – | – |
| | 0.875 | −25.6 | 0.59 | – | – | – |
| | 0.9 | −30.3 | 0.56 | – | – | – |
| $\Phi^*_{\rm s}$ | – | – | – | 2.45 | −3.5 | 1.02 |
| | – | – | – | 2.55 | −9.5 | 0.65 |
| | – | – | – | 2.65 | −18.4 | 0.49 |
| | – | – | – | 2.75 | −30.4 | 0.40 |
| $\Phi^*_{\rm s1}$ | 1.65 | −3.0 | 1.12 | – | – | – |
| | 1.75 | −11.7 | 0.62 | – | – | – |
| | 1.8 | −18.1 | 0.52 | – | – | – |
| | 1.85 | −26.0 | 0.45 | – | – | – |

(Table A3 Continued)

| states | $c=+1$ | | | $c=-1$ | | |
|---|---|---|---|---|---|---|
| | $g'=1.06, \beta'=0, \lambda'=0, g'_\sigma=0$ | | | | | |
| | $\Lambda$ | $E$ | $r_{\rm rms}$ | $\Lambda$ | $E$ | $r_{\rm rms}$ |
| $\Phi^*$ | – | – | – | 1.05 | −4.4 | 1.09 |
| | – | – | – | 1.25 | −14.8 | 0.66 |
| | – | – | – | 1.35 | −23.2 | 0.56 |
| | – | – | – | 1.45 | −34.2 | 0.48 |
| $\Phi^*_8$ | 0.8 | −71.4 | 0.44 | – | – | – |
| | 0.825 | −83.0 | 0.41 | – | – | – |
| | 0.85 | −95.9 | 0.39 | – | – | – |
| | 0.875 | −110.1 | 0.37 | – | – | – |
| $\Phi^*_s$ | – | – | – | 1.75 | −2.3 | 1.26 |
| | – | – | – | 1.85 | −9.6 | 0.67 |
| | – | – | – | 1.95 | −21.8 | 0.48 |
| | – | – | – | 2.05 | −39.3 | 0.38 |
| $\Phi^*_{s1}$ | 1.3 | −4.1 | 0.98 | – | – | – |
| | 1.35 | −9.9 | 0.68 | – | – | – |
| | 1.4 | −18.2 | 0.54 | – | – | – |
| | 1.45 | −29.1 | 0.45 | – | – | – |

| states | $c=+1$ | | | $c=-1$ | | |
|---|---|---|---|---|---|---|
| | $g'=1.32, \beta'=0, \lambda'=0, g'_\sigma=0$ | | | | | |
| | $\Lambda$ | $E$ | $r_{\rm rms}$ | $\Lambda$ | $E$ | $r_{\rm rms}$ |
| $\Phi^*$ | – | – | – | 0.8 | −6.9 | 0.96 |
| | – | – | – | 0.9 | −14.2 | 0.73 |
| | – | – | – | 1.0 | −24.9 | 0.58 |
| | – | – | – | 1.1 | −39.5 | 0.49 |
| $\Phi^*_8$ | 0.8 | −177.2 | 0.33 | – | – | – |
| | 0.81 | −187.1 | 0.32 | – | – | – |
| | 0.82 | −197.3 | 0.31 | – | – | – |
| | 0.83 | −208.1 | 0.31 | – | – | – |
| $\Phi^*_s$ | – | – | – | 1.4 | -4.6 | 0.94 |
| | – | – | – | 1.5 | −9.7 | 0.68 |
| | – | – | – | 1.6 | −25.8 | 0.46 |
| | – | – | – | 1.65 | −36.8 | 0.40 |
| $\Phi^*_{s1}$ | 1.1 | −3.1 | 1.13 | – | – | – |
| | 1.15 | −10.0 | 0.70 | – | – | – |
| | 1.2 | −20.9 | 0.53 | – | – | – |
| | 1.25 | −35.8 | 0.43 | – | – | – |

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