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Full lattice QCD study of the κ scalar meson*

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Abstract: We studied the κ light scalar meson in 2+1 flavor full QCD with sufficiently light u and d quarks. Via lattice simulation we measured the correlators for the κ channel in the "Asqtad" improved staggered fermion formulation. After chiral extrapolation we obtained the mass of the κ meson with 826 ± 119 MeV, which is within recent experimental values of 800–900 MeV. The simulations were carried out with the MILC 2+1 flavor gauge configurations at lattice spacing $a\approx0.15$ fm.

Key words: lattice QCD, kappa meson, chiral extrapolation

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1 Introduction

Recently there has been a growing interest in the lowest scalar meson, i.e., the κ meson. The so-called κ meson ($J^P=0^+$) is a scalar meson with strangeness. In 2010, the Particle Data Group (PDG) [1] listed the meson $K_0^*(800)$ or κ with a very broad width (550 MeV). A recent analysis [2] gives its mass at about 750^{30}_{50} MeV. Moreover, the resonance of a scalar meson with $I=1/2$ is also reported to exist in the πK system with a mass m_κ of about 800 MeV [2–4]. However, the existence of the κ meson has become plausible, because some analyses of experimental data show no hints of it [5].

There have not been many lattice studies of the κ meson. Recently, lattice simulations of the mass of the κ meson have been reported mainly by three groups. Prelovsek et al. [6] have presented a rough estimation of the mass of the κ meson as 1.6 GeV by extrapolating the a_0 mass obtained from dynamical correlators with degenerate $N_f=2$ quarks on a $16^3\times32$ lattice to $(m_u+m_s)/2$. Mathur et al. [7] have studied the $u\bar{s}$ meson with the overlap fermion in the quenched approximation and obtained the mass of the $u\bar{s}$ scalar meson to be 1.41±0.12 GeV. Using the dynamical N_f=2 sea quarks and a valence strange quark, the UKQCD Collaboration [8] has studied the κ meson and estimated the κ mass to be about 1.1 GeV, which is still far from a recent experimental value of about 800 MeV.

The SCALAR Collaboration [9, 10] reported a preliminary analysis on the κ meson using a dynamical fermion for the light u/d quark and a valence approximation for the strange quark, which shows that the $I=1/2$ scalar meson has a mass of about 1.8 GeV and therefore cannot be identified with the κ meson observed in experiments. In Ref. [11], they performed a quenched QCD calculation using Wilson fermions and the plaquette gauge action. They estimated the value of the mass of the κ meson to be about 1.7 GeV, which is still about twice as large as the experimental mass $\sim$800 MeV. Hence the lattice simulations have not yet provided the final answer to the mass of the κ meson.

In principle, the extraction of the mass of the κ meson is straightforward. Following the method in the study of the light pseudoscalar decay constant f_K from the three-flavor lattice QCD in Ref. [12], considering that κ meson has both a strange s quark and a light u quark, in this work we treat the u quark as a valence approximation quark, while the strange quark mass is fixed to its physical value [12]. Then

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we perform a series of simulations with MILC gauge configurations in the presence of 2+1 flavors of Asqtad improved staggered dynamical sea quarks, generated by the MILC Collaboration [13, 14], and extrapolate the mass of the κ meson to the physical light u quark mass using simple polynomials [15].

2 Non-singlet κ correlator

We choose the point-like source and sink for the κ meson. Let **0** be the spatial point of the κ state at the source time, and $\boldsymbol{x}$ corresponding position at the sink time. To simulate the correct number of the quark species, we use the fourth-root trick [16, 17], which automatically performs the transition from four tastes to one taste per flavor for a staggered fermion at all orders. To make the 0^+ flavor non-singlet with isospin 1/2, the fermionic lattice operator at the source can be written as

$$\mathcal{O}_{\rm Source}(\mathbf{0},0) \equiv \frac{1}{\sqrt{n_{\rm r}}} \sum_{{\rm a},g} \overline{s}^{\rm a}_g(\mathbf{0},t) u^{\rm a}_g(\mathbf{0},0), \qquad (1)$$

where g is the index of the taste replica, $n_{\rm r}$ is the number of taste replicas, a is the color index, **0** is the zero vector (i.e., $\mathbf{0}=(0,0,0)$), and for notational simplicity we omit the Dirac-spinor index. At the final time slice t, the fermionic lattice operator at the sink can be written similarly as

$$\mathcal{O}_{\rm Sink}(\boldsymbol{x},t) \equiv \frac{1}{\sqrt{n_{\rm r}}} \sum_{{\rm b},g'} \overline{s}^{\rm b}_{g'}(\boldsymbol{x},t) u^{\rm b}_{g'}(\boldsymbol{x},t), \qquad (2)$$

where g' is the index of the taste replica and b is the color index. Therefore, the time slice correlator $C(t)$ for the κ meson can be evaluated by the formula

$$C(t) = \sum_{\boldsymbol{x}} \left\langle {\rm Tr}[\mathcal{O}^\dagger_{\rm Sink}(\boldsymbol{x},t)\ \mathcal{O}_{\rm Source}(\mathbf{0},0)] \right\rangle$$
$$= \frac{1}{n_{\rm r}} \sum_{\boldsymbol{x},{\rm a},{\rm b}} \sum_{g,g'} \left\langle \overline{s}^{\rm b}_{g'}(\boldsymbol{x},t) u^{\rm b}_{g'}(\boldsymbol{x},t)\ \overline{u}^{\rm a}_g(\mathbf{0},0) s^{\rm a}_g(\mathbf{0},0) \right\rangle .$$

After we perform Wick contractions of the fermion fields, and sum over the index of the taste replica [16, 18], using the light u quark Dirac operator $M_{\rm u}$ and the s quark Dirac operator $M_{\rm s}$, we obtain the time slice correlator

$$C(t) = \sum_{\boldsymbol{x}} (-)^x \left\langle {\rm Tr}[M^{-1}_{\rm u}(\boldsymbol{x},t;0,0) M^{-1^\dagger}_{\rm s}(\boldsymbol{x},t;0,0)] \right\rangle, \qquad (3)$$

where Tr is the trace over the color index, and $x = (\boldsymbol{x},t)$ is the lattice position.

The mass of the κ meson can be reliably determined on the lattice. The extraction of the mass of the κ meson is straightforward,

$$C(t) = A{\rm e}^{-m_\kappa t}, \qquad (4)$$

where we omit the unimportant contributions from the excited κ meson and the oscillating terms corresponding to a particle with opposite parity.

3 Simulations and results

We use MILC lattices with 2+1 dynamical flavors of Asqtad-improved staggered dynamical fermions, a detailed description of the simulation parameters can be found in Refs. [13, 15, 19]. We analyzed the κ correlators on the 0.15 fm MILC ensemble of 631 $16^3 \times 48$ gauge configurations with bare quark masses $am'_{\rm ud}=0.0097$ and $am'_{\rm s}=0.0484$ and bare gauge coupling $10/g^2=6.572$, which have a physical volume of approximately 2.5 fm. The mass of the dynamical s quark is close to its physical value, and the masses of the u and d quarks are degenerate.

For the light u quark Dirac operator $M_{\rm u}$ and the s quark Dirac operator $M_{\rm s}$, we have measured the point-to-point quark-line connected correlator which is described by Eq. (3). We use the conjugate gradient method (CG) to obtain the required matrix elements of the inverse fermion matrices $M_{\rm u}$ and $M_{\rm s}$. In order to improve the statistics, we computed κ correlators from eight source time slices evenly spread through the lattice (i.e., only one source time slice was chosen at a time), and averaged these correlators.

Following the method in the study of the light pseudoscalar decay constant $f_{\rm K}$ from a three-flavor lattice QCD in Ref. [12], since the κ meson has both a strange s quark and a light u quark, we should treat the u quark as a valence approximation quark, while the strange quark mass is fixed to its physical value. The physical value of the strange quark mass of the lattice ensemble used in this work is given in Refs. [15, 19], that is, $am_{\rm s} = 0.0426$, where a is the lattice spacing.

The propagators of the κ meson are calculated with the same configurations using the five u valence quarks. For this "medium" coarse ensemble we choose am_x=0.0097, 0.01067, 0.01164, 0.01261, and 0.01358, where m_x is the light valence u quark mass. In order to obtain the physical mass of the κ meson, we then perform extrapolation to the chiral (physical value of the u quark mass) limit guided by chiral perturbation theory. The propagators of the π meson are also computed with the same configurations using the same five u valence quarks.

For staggered quarks, the meson propagators have the generic single-particle form,

$$\mathcal{C}(t)=\sum_i A_i e^{-m_i t}+\sum_i A_i'(-1)^t e^{-m_i' t}+(t\to N_t-t), \tag{5}$$

where the oscillating terms correspond to a particle with opposite parity. For the κ meson correlator, we consider only one mass with each parity in the fits of Eq. (5), that is, in our concrete calculation, our operator is the state with spin-taste assignment $I\otimes I$ and its oscillating term with spin-taste assignment $\gamma_0\gamma_5\otimes\gamma_0\gamma_5$ [20]. Therefore, the κ correlator was then fitted to the following physical model,

$$C_\kappa(t)=b_\kappa e^{-m_\kappa t}+b_{K_A}(-)^t e^{-M_{K_A} t}+(t\to N_t-t), \tag{6}$$

where b_{K_A} and b_κ are two overlap factors.

This fitting model explicitly contains the κ pole, together with the corresponding negative parity state. There are four fit parameters for the meson term, but the negative parity mass was constrained by prior: the K_A. In summary, we fit the κ correlator with four parameters, which are needed to parameterize the two explicit mesons.

For u valence quark $am_x=0.0097$, the effective mass plot of the κ meson is shown in Fig. 1. The mass of the κ meson is extracted from the effective mass plot. We find that the effective mass of the κ meson suffers from large errors, especially at larger minimum time distance regions. To avoid possible large errors coming from the data at large minimum time distance $D_{\min}$, we fit the effective mass of the κ meson only in the time range $7\leqslant D_{\min}\leqslant 10$, where the effective masses are almost constant with small errors. We obtained similar results for the other u valence quarks, therefore we do not show these effective mass plots here.

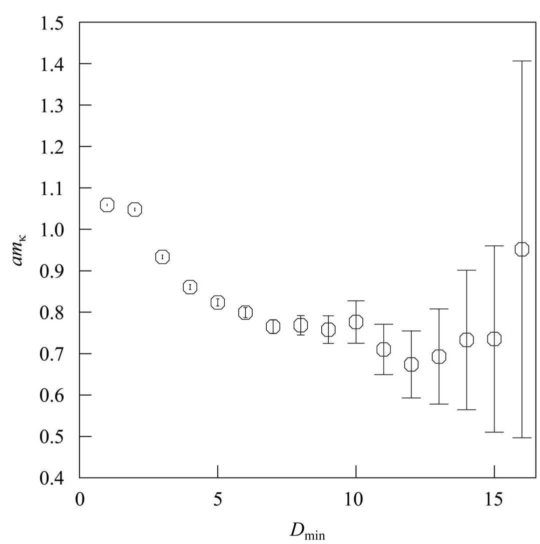

Fig. 1. The masses of the κ meson as a function of the minimum time distance included in the fit from the run with $am_x=0.0097$. The effective mass plot will be a plateau in the time range $7\leqslant D_{\min}\leqslant 10$.

In this work the κ propagators were fitted to Eq. (6) using a minimum time distance of $9a$. At this distance, the contamination from the excited states is comparable to the statistical errors, we can neglect the systematic effect due to excited states. For this calculation, our best fit gives $\chi^2/\text{dof}=5.9/12$. The fitted functional form for $am_x=0.0097$ is compared with the data in Fig. 2.

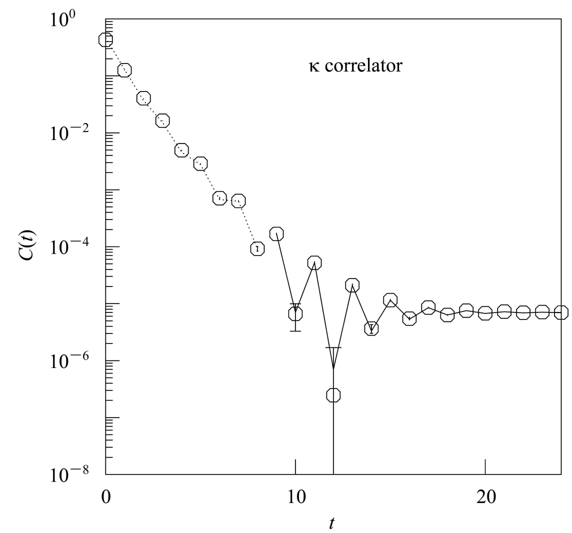

Fig. 2. The best fit to the κ correlator for $am_x=0.0097$. The fitting range is indicated by points and fitted solid lines. We shift all the points $7e-6$ in the y axis for good visualization. The y-axis is dimensionless.

The fitted masses of the κ correlators are summarized in Table 1. The errors in Column three are from both the error on the lattice spacing a and the fitted error in Column two. In order to obtain the physical mass of the κ meson, we should carry out chiral extrapolation of m_κ to the physical light u quark mass using simple polynomials [15].

As for determination of the physical quark mass m_u, we must compare the lattice meson masses with the experimental value. In this work, since the strange quark mass is fixed to its physical value, we can search the physical light quark mass m_u in the following two ways, both of which are found to give similar results: 1) According to the results in Refs. [12, 21], we can estimate the physical light quark mass m_u for the given physical value of the strange quark mass. 2) Using the pion masses on this ensemble to determine the physical light u/d quark mass. In this work, we choose the second method.

Table 1. The summary of results for the κ mass. The second and third blocks show the mass of the κ meson in the lattice unit and in GeV, respectively. Column four shows the time range for the chosen fit, Column five shows the number of degrees of freedom (dof) for the fit and Column six shows the number of configurations used in these calculations.

am_u	am_κ	m_κ/GeV	range	χ^2/dof	lat.
0.0097	0.758(33)	1.059(45)	9–24	5.9/12	631
0.01067	0.766(33)	1.083(45)	9–24	5.9/12	631
0.01164	0.775(33)	1.107(45)	9–24	6.0/12	631
0.01261	0.783(33)	1.129(45)	9–24	6.0/12	631
0.01358	0.790(33)	1.151(45)	9–24	6.3/12	631

For u valence quark $am_x = 0.0097$, the effective mass plot of the π meson is shown in Fig. 3. The mass of the π meson is extracted from the effective mass plot. We find that the effective mass of the π meson has only small errors within a broad minimum time distance region $7 \leqslant D_{\min} \leqslant 15$ and is taken to be reliable. Therefore we fit the effective mass of the π meson with the single-exponential [12] of the π propagator at $D_{\min} = 14$. We obtained similar results for the other u valence quarks, hence we do not show these effective mass plots here.

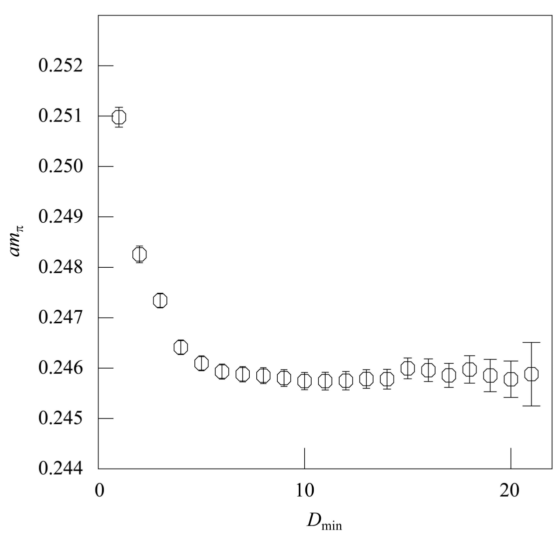

Fig. 3. The masses of the π meson as a function of the minimum time distance included in the fit from the run with $am_x = 0.0097$. The effective mass plot will be a plateau in the time range $7 \leqslant D_{\min} \leqslant 15$.

In Fig. 4, we show how the physical quark mass m_u is obtained, where m_x is the light valence u quark mass in the pion meson, and $\overline{\pi}$ is the physical pion mass in the nature, which can be found in PDG [1]. The data in Fig. 4 appear linear to the eye, hence we adopt a linear extrapolation, which has $\chi^2/\text{dof} = 0.612/5$. In fact our nonlinear fits give almost the same results. Once the physical light quark mass m_u is obtained, we then use this number as its chiral limit.

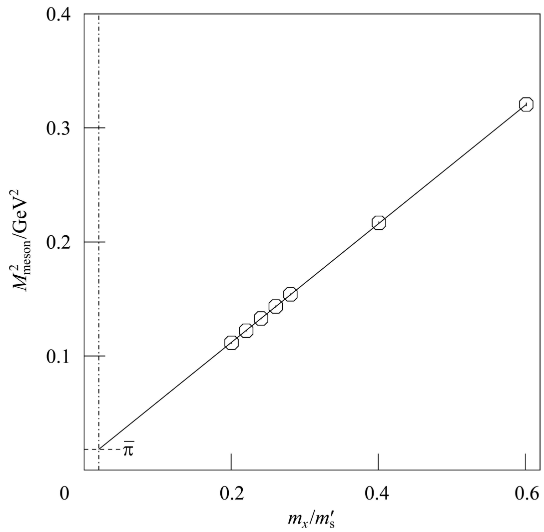

Fig. 4. The squared π meson masses as a function of the light valence quark mass am_x. The vertical dotted line shows the light quark mass m_u obtained.

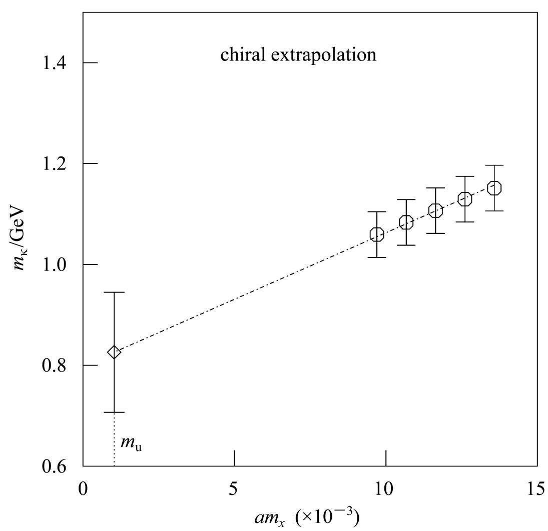

Fig. 5. The mass of the κ meson in the lattice unit as a function of the light valence u quark mass am_x. The chiral limit is obtained at the physical u quark mass m_u.

In Fig. 5, we show how physical value m_κ is extracted. The data in Fig. 5 also appear linear to the eye, therefore we choose a crude linear extrapolation, which has χ^2/dof=0.0332/3. The dashed line in Fig. 5 is the linear extrapolation of m_κ to the physical light u quark mass m_u. The chirally extrapolated mass

$m_\kappa = (826 \pm 119)$ MeV, the diamond point in Fig. 5 shows this value.

4 Summary and outlook

In this work we studied the point-to-point κ correlator for the MILC "medium" coarse ($a = 0.15$ fm) lattice ensemble in the presence of $2+1$ flavors of Asqtad which improved the staggered dynamical sea quarks, generated by the MILC Collaboration [13, 14]. We treated the light u quark as a valence approximation quark, while the strange quark mass was fixed to its physical value, and extrapolated the mass of the κ meson to the physical u quark mass using simple polynomials. We obtained the physical mass of the κ meson with 826±119MeV, which is well within the recent experimental value 800–900 MeV. Probably, it may be identified with the κ meson observed in experiments.

Of course, our preliminary results reported here definitely need improvement. In this work we performed our research on one ensemble of MILC lattice gauge configuration. That means our results are at a single lattice spacing and bare quark masses. The more physical one should be in a continuum limit (lattice spacing $a \to 0$). Hence, we need to work on different configurations.

Moreover, as discussed in Refs. [17, 20, 22], the lattice artifacts are clearly present in the κ channel in our QCD simulation on the medium coarse ($a = 0.15$ fm) lattice ensemble, we need to use the smaller lattice spacings to minimize the lattice artifacts, hence an empirical investigation of these effects is necessary. We are beginning a series of simulations at three lattice spacings $a \approx 0.12, 0.09$ and 0.06 fm lattice ensembles to investigate these effects for the physical mass of the κ meson.

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