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Application of the transfer matrix for tuning the CSNS-DTL

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Abstract: In the construction of a drift tube LINAC (DTL), many factors caused during the fabrication and assembly of the structure cells cause the electric field distribution not as the same as the design curve. To solve this problem, the traditional way is to solve the equation of Slater's perturbation theorem to obtain the electrical field variation due to local frequency shift. However, that is very difficult under complicated conditions. Since the field perturbation equation is similar to the particle's transverse motion equation, which can be simply solved by using the transfer matrix method, we thus propose to apply a transfer matrix method in tuning the DTL. We demonstrate the availability and advantages of this method with 3D microwave code simulation and the LabVIEW calculation program. After two iterations, the initial error of the electric field of 19.5% has been improved greatly down to 1.3%– −4.5%. This indicates that the transfer matrix method is very useful and convenient for the simplification of tuning procedures.

Key words: CSNS DTL, transfer matrix, electrical field perturbation, LabVIEW

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1 Introduction

The DTL of China Spallation Neutron Source (CSNS) consists of seven tanks and aims to accelerate the H^- ion beam to 132 MeV.

E_{0D} stands for the design axial electrical field distribution in Tank-1 of the CSNS DTL, which is linearly ramped from cell 1 to cell 23, and then keeps constant until the exit end of the last cell, shown as Fig. 1 [1]. However, errors caused in the machining and assembly of the structural cells cause the field distribution E_{0M} to be not the same as E_{0D}. To solve this problem, the traditional way is to solve the equation of Slater's perturbation theorem to obtain the electrical field variation due to local frequency shift and then to adjust the length of slug tuners according to local frequency shift to achieve the design field. However, this is very difficult under some complicated conditions. Since the field perturbation equation is similar to the particle's transverse motion equation, which can be simply solved by using the transfer matrix method, thus in this paper we propose to apply the transfer matrix method for tuning the CSNS DTL too and demonstrate its availability and advantages using 3D microwave simulation and the LabVIEW calculation program.

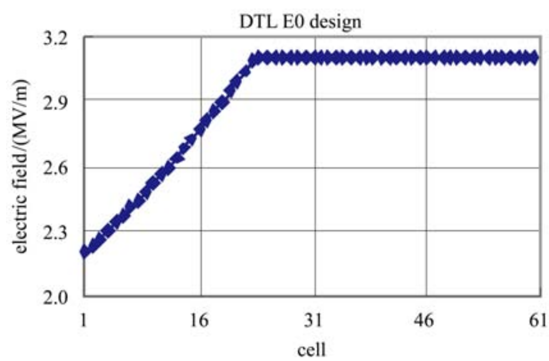

Fig. 1. The axial electrical field distribution designed in Tank-1 of CSNS DTL.

2 The particle transport equation and transfer matrix method

2.1 Slater's perturbation theorem

Slater's perturbation theorem has always been used to tune the longitudinal electric field distribution of a DTL. Supposing f_0 is the DTL's resonant

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frequency, for a small perturbation δf, the local frequency shift is [2]:

$$f(z)=f_0+\delta f(z). \tag{1}$$

When $\delta f \ll f_0$, the perturbation has a negligible effect on the cavity's resonant frequency. However, it can induce the longitudinal electrical field distribution E_z:

$$\frac{\mathrm{d}^2E_z}{\mathrm{d}z^2}-\frac{4\pi^2\delta f}{c^2}[2f_0-\delta f]E_z=0, \tag{2}$$

$$\frac{\mathrm{d}}{\mathrm{d}z}\left[\frac{\delta E_z}{E_0}\right]=0(z=0 \text{ and } z=L). \tag{3}$$

Here, δE_z is the electrical field variation.

With Eq. (2) and the boundary condition (3), we can calculate the deviation of the electrical field distribution.

2.2 Particle transport equation

Slater's equation is very difficult to calculate under some complicated perturbations. However, it is similar to the particle transverse movement equation in a quadrupole channel, which is [3]:

$$\frac{\mathrm{d}^2u}{\mathrm{d}z^2}+\Omega_U(z)u=0. \tag{4}$$

Here, we define $u=(\beta_{\mathrm{s}}\gamma_{\mathrm{s}})^{1/2}U$, where β_{s} and γ_{s} are the synchronous velocity and relativistic factor, respectively, U is the transverse direction, either x or y, and the coefficient $\Omega_U(z)$ is a piecewise constant in the transport channel. The transfer matrix method has always been applied to solve this kind of equation. For example, a particle's transverse movement of a transport channel with N elements can be calculated in the following transfer matrix method:

$$\begin{pmatrix} x \\ x' \end{pmatrix}=M\begin{pmatrix} x_0 \\ x'_0 \end{pmatrix}=\prod_{j=N}^{j=1}M_j\cdot\begin{pmatrix} x_0 \\ x'_0 \end{pmatrix}. \tag{5}$$

Here x_0 and x'_0 are the initial displacement and divergence while x and x' are the final values, and M_j is the transfer matrix in the j-th section. It should be noticed that (2) and (4) are very similar, thus we consider applying this transfer matrix method in calculating the electrical field perturbation.

2.3 Transfer matrix for the electrical field perturbation

According to the discussion above, Slater's equation should have a similar transfer matrix as the particle transport equation, just like [4]:

$$\begin{pmatrix} E \\ E' \end{pmatrix}=M(z)\begin{pmatrix} E_0 \\ E'_0 \end{pmatrix}=\prod_{j=N}^{j=1}M_j(L_j)\begin{pmatrix} E_0 \\ E'_0 \end{pmatrix}. \tag{6}$$

Here, E_0 and E'_0 are the initial distribution and divergence of the electrical field, respectively, while E and E' are the final values, and $M_j(L_j)$ is the transfer matrix in the j-th section. The transfer matrix $M(z)$ has three different expressions in different sections:

$$M(z)=\begin{bmatrix} \cos(kz) & \frac{1}{k}\sin(kz) \\ -k\sin(kz) & \cos(kz) \end{bmatrix}$$

$$(\delta f<0)\text{Focusing section} \tag{7}$$

$$M(z)=\begin{bmatrix} \mathrm{ch}(kz) & \frac{1}{k}\mathrm{sh}(kz) \\ k\mathrm{sh}(kz) & \mathrm{ch}(kz) \end{bmatrix}$$

$$(\delta f>0)\text{ Defocusing section} \tag{8}$$

$$M(z)=\begin{bmatrix} 1 & z \\ 0 & 1 \end{bmatrix}\quad(\delta f=0)\text{ Drifting section} \tag{9}$$

$$\text{Parameter } k=\left|\frac{4\pi^2\delta f}{c^2}[2f_0-\delta f]\right|^{1/2}. \tag{10}$$

3 The transfer matrix for the CSNS DTL using the LabVIEW program

Tank-1 of the CSNS-DTL contains three short module tanks, the first of which is under construction, mainly consisting of 28 drift tubes with stems, 4 slug tuners, as well as 14 post couplers. According to the tuning requirements, the deviation of measured electrical field distribution E_{0M} should be less than 1% from E_{0D} by adjusting the slug tuners and post coupler angles. The main procedure is to modify the penetrating length of the 4 slug tuners according to the transfer matrix solution and then rotate the coupler angles when the field deviation is no more than 5%. In the CSNS DTL, each slug tuner has a diameter of 150 mm and a maximum inner depth of 100 mm. All of them can alter the cavity's frequency of ±1 MHz [5].

The rough tuning procedure of slug tuners consists of three steps. The first step is to measure the filed distribution E_{0M} with a bead-pull system; the second step is, due to the deviation between the E_{0M} and E_{0D}, to calculate the field modification applying the transfer matrix method; and the third step is to adjust the length of tuners according to the relationship between the parameter k and the tuners' length. Since the first module tank is still under fabrication, we take the simulation of the electric field with 3D microwave code to replace the measurement and then use the LabVIEW program to calculate the tuners' length by applying the transfer matrix method.

3.1 Simulation of the electrical field

At first, with the four slug tuners' penetrating depths being all 50 mm, and the post couplers being inserted in the same length and with the same tip angle, the simulation of the peak field distribution E_{peak} is expressed in Fig. 2.

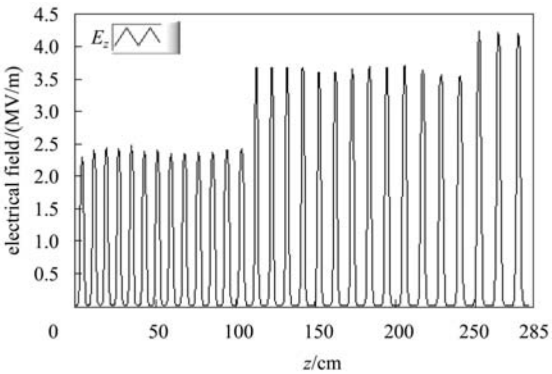

Fig. 2. The simulation of peak field for the first module tank

The relation between the peak field E_z and the averaged value E_0, L_{T} is the length of the Tank-1:

$$\int_0^{L_{\text{T}}} E_z^2 \text{d}z = L_{\text{T}} E_0^2. \tag{11}$$

We can calculate the normalization value of this averaged field $E_{0\text{S}}$ in case of invariability of the cavity's energy, drawn in Fig. 3. Compared with $E_{0\text{D}}$ preliminarily, the maximum deviation of the field is about 19.5%.

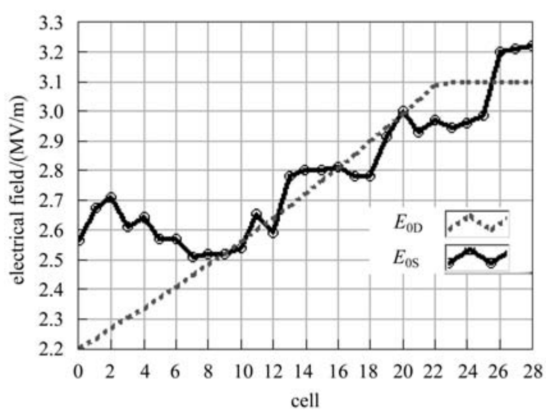

Fig. 3. The simulation and design of the averaged field distribution for the first module tank.

3.2 Calculating the field perturbation with the transfer matrix

These four slug tuners are located at Z=0.33 m, 1.0 m, 1.66 m and 2.33 m. With them, the first module tank is divided into 5 drifting sections and 4 perturbation sections, the length of which is 0.15 m, it is the same case for the diameter of the slug tuner.

The essential part of the transfer matrix method is to account for the local frequency shift according to the linear fitting tilts of $E_{0\text{S}}$ and $E_{0\text{D}}$. Supposing the tilts and electrical distributions are modified in the perturbation sections, we can take the data in the middle of fore-and-after drifting sections as the entry-and-exit values of the transfer matrix.

Take the second tuner as an example. The perturbation section is from Z=0.9237 to 1.0737 meters, and the simulated field of cell-9 (E_1=2.8718 MeV/m) and cell-16 are taken as the transfer matrix's entry and exit electrical field, respectively. In the $E_{0\text{D}}$ curves, the tilts of cell 9 and 16 both are 0.4265, which means that the tilt in the second perturbation section should keep constant, whereas the tilt changes from 0.177 to 0.317 in $E_{0\text{S}}$. So the main target of the second tuner's tuning process is to calculate the frequency shift due to the tilt's change applied with transfer the matrix method, and then account for the tuners' length to adjust with the LabVIEW program.

3.3 Accounting for the adjusting length with the LabVIEW program

The second perturbation section is a defocusing section, which changes the tilt from 0.177 to 0.317 in simulation. The electrical field perturbation equation is:

$$\begin{pmatrix} E_2 \\ E_2' \end{pmatrix} = M_2(z) \begin{pmatrix} E_1 \\ E_1' \end{pmatrix}. \tag{12}$$

Here, E_1 and E_1' represent the initial distribution and divergence of the second perturbation section, while E_2 and E_2' are the final values. Since the second transfer matrix $M_2(z)$ is the defocusing section,

$$M_2(z) = \begin{bmatrix} \text{ch}(k_2 z) & \dfrac{1}{k_2}\text{sh}(k_2 z) \\ k_2 \text{sh}(k_2 z) & \text{ch}(k_2 z) \end{bmatrix}. \tag{13}$$

With the values of Z, E_1, E_1' and E_2' mentioned before, and

$$E_2' = k_2 \text{sh}(k_2 z) E_1 + \text{ch}(k_2 z) E_1'. \tag{14}$$

It is easy to solve the value of parameter k_2 in the LabVIEW program, mainly consisting of three loops corresponding to three sections with different transfer matrixes.

Here, the resonant frequency f_0 is 324 MHz and due to the relation of parameter k and frequency shift

δf in Eq. (10), we can get:

$$\delta f=\frac{-6\times10^6\times[-54\pi^2+(-625k^2\pi^2+2916\pi^4)^{1/2}]}{\pi^2}. \tag{15}$$

In each perturbation section, the slug tuner's effect on the local frequency (MHz/mm):

$$\delta f_0=(1/(50\times4))\times(2.8285/0.15). \tag{16}$$

Therefore, in the 0.15 m perturbation section, as k changes by 0.01, the local frequency shifts by about 351.8 Hz; while as the tuner's penetrating length alters by about 0.1 mm, the local frequency shifts by about 1 kHz.

In the tuning result, the second tuner should be pulled out by about 12 mm, as shown in Fig. 4. In the same way, we can solve the other three tuner's frequency shifts and lengths to adjust.

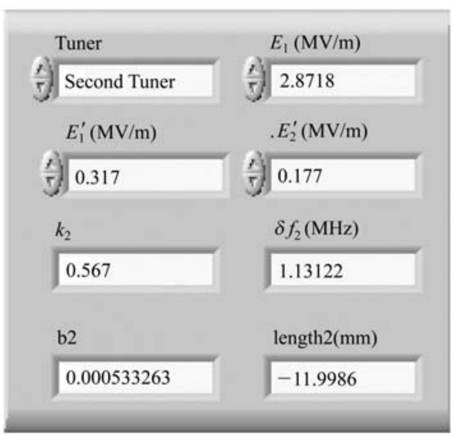

Fig. 4. The tuning results of the second tuner.

4 Checking results with simulation

Four tuners are adjusted for the first time according to the program results. Then we can simulate it with the 3D microwave code under the same condition to check the first tuning effect. In Fig. 5, E_{1S} and E_{2S} express the simulation field distribution before and after tuning. Comparing them shows that the field's deviation makes a great deal progress with the maximum deviation dropping to 7%, which demonstrates that the transfer matrix method is viable in tuning the DTL's electrical field perturbation.

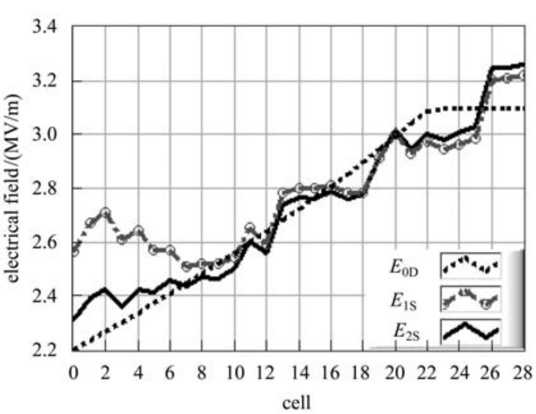

Fig. 5. The design and simulation field distributions before and after first tuning.

5 Iterative repetition

However, there are some assumptions in the tuning program, which should be regulated in experiment, such as the tuners' effects on the local frequency shift which are not completely the same in different lengths. It is because of this that it is difficult to achieve the best results only for one tuning procedure, but we could repeat the tuning processes–iterative repetition to avoid this problem.

After the second tuning, the max deviation gets down to 1.3%– −4.5%, except cell 27–29 affected by the face angles' sudden change, which can be solved with more tuners in the whole Tank-I, shown in Fig. 6.

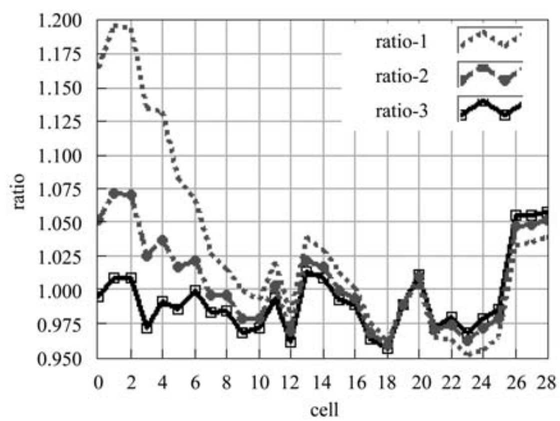

Fig. 6. The ratios of three simulation fields to E_{0D} Ratio-1: before tuning, Ratio-2: after the first. tuning, Ratio-3: after the second tuning.

On the whole, two tuning processes can fulfill the requirement provided the deviation should be less than ±5% in rough tuning with slug tuners. After rough tunings, the tabs on post couplers could be used to tune and achieve more defined results.

6 Conclusions

In the DTL's manufacture, we realize that many factors cause differences between the measured field distributions and the designed ones, which is difficult to solve using the traditional Slater solution under some complicated conditions. However, due to the similar equation to the particle's transverse movement, we apply the transfer matrix method in the tuning procedure for the CSNS DTL. By using a calculation program, simulation and iteration repetition, it has demonstrated that this transfer matrix method really could have a good result and simplify the tuning procedure. However, the iterations and accuracy will have little difference in experiment and in simulation, so an in-depth study should be promoted with an experimental focus on the modification of the assumption parameters and less iteration.

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