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Calculation of equation of state of QCD at zero temperature and finite chemical potential*

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Abstract In this paper we calculate the equation of state (EOS) of QCD at zero temperature and finite chemical potential by using several models of quark propagators including the Dyson-Schwinger equations (DSEs) model, the hard-dense-loop (HDL) approximation and the quasi-particle model. The results are analyzed and compared with the known results in the literature.

Key words equation of state (EOS), DSEs, HDL, QCD, density

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1 Introduction

As is well known, the study of partition function plays a key role in the equilibrium statistical field theory. The thermal properties of the system, and hence the equation of state (EOS), are completely determined by the partition function. The calculation of the partition function of quantum chromodynamics (QCD) is a contemporary focus [1–7]. In addition, it is well known that in astrophysics the study of the neutron star depends crucially on the assumed EOS [8, 9]. The study of EOS of QCD is thus of extreme importance.

Recently, we proposed a new method for calculating the partition function, and hence the EOS of QCD at finite quark chemical potential μ [6]. We find that the pressure density of QCD at finite chemical potential and zero temperature can be expressed as :

$$\mathcal{P}(\mu) = \mathcal{P}(\mu)|_{\mu=0} + \int_0^{\mu} \mathrm{d}\mu' \rho(\mu') = \mathcal{P}(\mu)|_{\mu=0} - N_{\mathrm{c}} N_{\mathrm{f}} Z_2 \int_0^{\mu} \mathrm{d}\mu' \int \frac{\mathrm{d}^4 p}{(2\pi)^4} \mathrm{tr}\left\{G[\mu'](p)\gamma_4\right\}, \quad (1)$$

where $\mathcal{P}(\mu)$ and $\rho(\mu)$ are the pressure density and quark number density at finite μ, respectively. N_{c} and N_{f} denote the number of colors and of flavors. Here $G[\mu](p)$ is the renormalized quark propagator at finite μ and Z_2 is the wave-function renormalization constant for the quark field. The trace operation is over Dirac indices. From Eq. (1) it can be seen that the pressure density is the sum of two terms: the first term is only a μ-independent constant and we will ignore it in the following because it is not interesting for our purpose; the second term, which contains all the nontrivial μ-dependence, is totally determined by $G[\mu](p)$. This means once the full quark propagator at finite μ is known, one can rigorously obtain the EOS of QCD from Eq. (1). Unfortunately the full quark propagator, especially the one at finite μ, is far from being solved. Therefore, when one actually applies Eq. (1) to calculate the EOS, one has to resort to various QCD models. In this paper we will use this formula to calculate the EOS of QCD in three different models include the Dyson-Schwinger equations (DSEs) model, the hard-dense-loop approximation (HDL) and the quasi-particle model.

2 Dyson-Schwinger equations

Over the past few years, considerable progress has been made in the framework of the DSEs approach [10, 11]. If one adopts the rainbow approximation of DSEs and ignores the μ dependence of the dressed gluon propagator, it can be shown that the dressed

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quark propagator at finite μ is obtained from the one at $\mu=0$ by the following substitution [12, 13]:

$$G^{-1}[\mu](p)=G^{-1}(\tilde{p})=\mathrm{i}\gamma\cdot\tilde{p}A(\tilde{p}^2)+B(\tilde{p}^2), \quad (2)$$

where $\tilde{p}=(\vec{p},p_4+\mathrm{i}\mu)$ and $G^{-1}(p)=\mathrm{i}\gamma\cdot pA(p^2)+B(p^2)$ is the inverse dressed quark propagator at $\mu=0$.

In order to obtain $G[\mu](p)$ using Eq. (2), one needs to specify the form of the dressed quark propagator at $\mu=0$. In this work, we adopt the following meromorphic form of the dressed quark propagator proposed by Ref. [13]:

$$G(p)=\sum_{j=1}^{n_P}\left(\frac{r_j}{\mathrm{i}\not{p}+a_j+\mathrm{i}b_j}+\frac{r_j}{\mathrm{i}\not{p}+a_j-\mathrm{i}b_j}\right). \quad (3)$$

In the numerical calculation in this work, we use three sets of parameters given in Ref. [13], which represent three forms of the propagator: three real poles (3R), two pairs of complex conjugate poles (2CC), and one real pole and one pair of complex conjugate poles (1R1CC). These parameters are listed in Table 1.

With this form of dressed quark propagator one obtains the following expression of the quark number density

$$\rho(\mu)=\frac{2N_{\mathrm{c}}N_{\mathrm{f}}}{3\pi^2}\sum_{j=1}^{n_P}r_j\theta(\mu-|a_j|)\left(\mu^2-\frac{a_j^2b_j^2}{\mu^2}-a_j^2+b_j^2\right)^{\frac{3}{2}}, \quad (4)$$

Because the expression of $\rho(\mu)$ contains step functions, when μ is smaller than a critical value $\mu_0=\min\{|a_j|\}$, the quark number density vanishes identically. This result agrees qualitatively with the general conclusion of Ref. [14].

Substituting Eq. (4) into Eq. (1) one can immediately obtain the pressure density in DSEs approach. The μ dependent part reads

$$\mathcal{P}(\mu)=\int_0^{\mu}\mathrm{d}\mu'\rho(\mu')=\frac{2N_{\mathrm{c}}N_{\mathrm{f}}}{3\pi^2}\sum_{j=1}^{n_P}r_j\ \theta(\mu-|a_j|)I, \quad (5)$$

where $I(\mu;a_j,b_j)$ is:

$$I(\mu;a_j,b_j)\ \equiv\ \frac{3(a_j^4+b_j^4-6a_j^2b_j^2)}{16}\ln\frac{\sqrt{\mu^2+b_j^2}+\sqrt{\mu^2-a_j^2}}{\sqrt{\mu^2+b_j^2}-\sqrt{\mu^2-a_j^2}}+\frac{3(a_j^2-b_j^2)|a_jb_j|}{2}\arctan\sqrt{\frac{b_j^2(\mu^2-a_j^2)}{a_j^2(\mu^2+b_j^2)}}+$$
$$\left[\frac{\mu^2}{4}-\frac{5}{8}(a_j^2-b_j^2)\right]\sqrt{(\mu^2-a_j^2)(\mu^2+b_j^2)}+\frac{a_j^2b_j^2}{2\mu^2}\sqrt{(\mu^2-a_j^2)(\mu^2+b_j^2)}. \quad (6)$$

Table 1. The parameters for the quark propagator from Ref. [13].

Parameterization	r_1	a_1/GeV	b_1/GeV	r_2	a_2/GeV	b_2/GeV	r_3	a_3/GeV
2CC	0.360	0.351	0.08	0.140	−0.899	0.463	–	–
1R1CC	0.354	0.377	–	0.146	−0.91	0.45	–	–
3R	0.365	0.341	–	1.2	−1.31	–	−1.06	−1.40

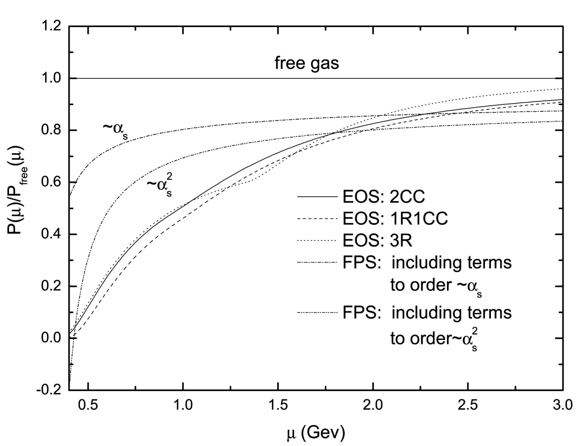

Fig. 1. The pressure density relative to the free quark gas pressure $\mathcal{P}_{\mathrm{free}}=N_{\mathrm{c}}N_{\mathrm{f}}\mu^4/(12\pi^2)$.

The numerical result is shown in Fig. 1. In Fig. 1 we give a comparison between the DSEs result and the perturbative QCD result given in Ref. [15] (named as FPS in Fig. 1). It is to be noted that in a large region of μ (< 1.7 GeV) the DSEs result is smaller than the perturbative QCD result. This fact may be important for the study of neutron stars.

3 Hard-dense-loop approximation

As is well known, the hard thermal/dense loop approximation (HTL/HDL) is considered to be a good approximation for quark-gluon plasma at high temperature/density, where quark is deconfined and chiral symmetry is restored [17]. The quark propagator under HTL/HDL approximation in QCD can be written as [17]:

$$G_{\mathrm{H}}(p)=\frac{-1}{D_+(p)}\frac{\gamma_4+\mathrm{i}\hat{p}\cdot\vec{\gamma}}{2}+\frac{-1}{D_-(p)}\frac{\gamma_4-\mathrm{i}\hat{p}\cdot\vec{\gamma}}{2}, \quad (7)$$

where $\hat{p}=\vec{p}/|\vec{p}|$. The form of the functions $D_{\pm}(p)$ is

$$D_{\pm}(p) = -\mathrm{i}p_4+\mu\pm|\vec{p}|+\frac{m_{\mathrm{q}}^2}{|\vec{p}|}\left[Q_0\left(\frac{\mathrm{i}p_4-\mu}{|\vec{p}|}\right)\mp Q_1\left(\frac{\mathrm{i}p_4-\mu}{|\vec{p}|}\right)\right], \quad (8)$$

where $m_{\mathrm{q}} \equiv g\mu/(\sqrt{6}\pi)$ is the quark thermal mass with g being the strong coupling constant, Q_0 and Q_1 are Legendre functions of the second kind. Therefore, based on HDL quark propagator and Eq. (1) one can obtain the EOS under HDL approximation. To do this, first we note the result of the previous section that under some critical chemical potential μ_0 the quark number density vanishes identically. Based on a general argument, Ref. [14] find that $\mu_0 \sim 307$ MeV and our calculation shows that $\mu_0 = 351$ MeV (for 2CC case), which is determined by the poles of quark propagator at zero μ. Second, it is generally believed that HDL approximation is valid for high chemical potential μ. Here we assume that the HDL quark propagator (7) is applicable in the range $\mu > \mu_0 = 351$ MeV. Because for $\mu < \mu_0$ the quark number density $\rho(\mu)$ vanishes identically, and also because the HDL quark propagator (7) is only applicable in the range $\mu>\mu_0$, when one calculates the pressure density under HDL approximation by means of Eq. (1), one should take the lower limit of μ' integration to be $\mu'=\mu_0$. Thus one can obtain the pressure density under HDL approximation:

$$\mathcal{P}(\mu)=-N_{\mathrm{c}}N_{\mathrm{f}}\int_{\mu_0}^{\mu}\mathrm{d}\mu'\int\frac{\mathrm{d}^4p}{(2\pi)^4}\mathrm{tr}\left\{G_{\mathrm{H}}(p)\gamma_4\right\}.$$

With HDL quark propagator (7) one can also find the quark number density as follows:

$$\rho(\mu) = 2N_{\mathrm{c}}N_{\mathrm{f}}\int\frac{\mathrm{d}^3\vec{p}}{(2\pi)^3}\left[\theta(\mu-\omega_+)Z_+-\theta(\mu-\omega_-)Z_-\right]+ N_{\mathrm{c}}N_{\mathrm{f}}\int\frac{\mathrm{d}^3\vec{p}}{(2\pi)^3}\int_{-|\vec{p}|}^{|\vec{p}|}\frac{\mathrm{d}\omega}{2\pi}\left[\rho_+(\omega)+\rho_-(\omega)\right]\times \left[1-2\theta(\omega-\mu)\right], \quad (9)$$

where $\rho_{\pm}$ is the spectral density of $1/D_{\pm}$ and $Z_{\pm}$ is defined as

$$Z_{\pm}(|\vec{p}|)=\frac{\omega_{\pm}-|\vec{p}|^2}{2m_{\mathrm{q}}^2} \quad (10)$$

with $\omega_{\pm}$ $(>|\vec{p}|)$ being the solutions of the following equations

$$\frac{|\vec{p}|(\omega_+-|\vec{p}|)}{m_{\mathrm{q}}^2}-1=\frac{1}{2}\left(1-\frac{\omega_+}{|\vec{p}|}\right)\ln\frac{\omega_++|\vec{p}|}{\omega_+-|\vec{p}|}, \quad (11)$$

$$\frac{|\vec{p}|(\omega_-+|\vec{p}|)}{m_{\mathrm{q}}^2}+1=\frac{1}{2}\left(1+\frac{\omega_-}{|\vec{p}|}\right)\ln\frac{\omega_-+|\vec{p}|}{\omega_--|\vec{p}|}. \quad (12)$$

The numerical results of $\rho(\mu)$ and $\mathcal{P}(\mu)$ are shown in Fig. 2 and Fig. 3, respectively. A comparison between the HDL result and FPS result is also shown in Fig. 4. From Fig. 4 it can be seen that the HDL pressure density is much larger than the FPS one, and as μ tends to infinity, the HDL pressure density tends to the free quark gas result from above, whereas the FPS one tends to the free quark gas result from below. In our opinions such a behavior is understandable. In QGP phase the interaction is still strong and QGP behaves as a perfect fluid [18]. Consequently the pressure should be stronger than a weakly coupled quark gas.

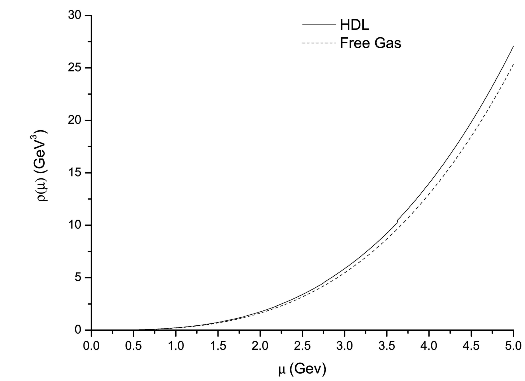

Fig. 2. The quark number density under HDL approximation.

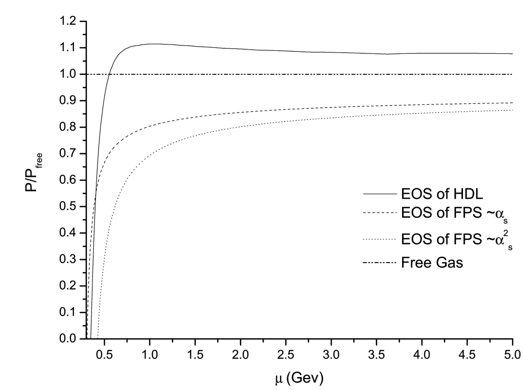

Fig. 3. The pressure density under HDL approximation.

4 Quasi-particle model

In the quasi-particle model, it was assumed that a system of interacting particles can be effectively described as an ideal gas of noninteracting quasi-particles with a temperature and density dependent mass. So the quark propagator in the quasi-particle model should have the form of a free quark propagator with such an effective mass:

$$G^{-1}[\mu](p) = \mathrm{i}\gamma \cdot \tilde{p} + m(\mu). \tag{13}$$

Here we choose the effective mass to be [19]:

$$m^2(\mu) = \frac{(N_c^2 - 1)\mu^2}{8N_c\pi^2} g^2(\mu), \tag{14}$$

$$g^2(\mu) = \frac{16\pi^2}{9\ln[\lambda(\mu + T_s)/(T_c\pi)]^2}, \tag{15}$$

where the parameters $\lambda = 6.6$, $T_c = 170$ MeV and $T_s = -0.78T_c$. Substituting such a propagator into Eq. (1), one would find the EOS for the quasi-particle model. The numerical result and a comparison with the perturbative result [16] is shown in Fig. 4. From Fig. 4 one finds that the pressure in the quasi-particle model just lies between the leading order perturbative QCD result and the next-to-leading order one. Therefore at large chemical potential the quasi-particle model should be regarded as a perturbative model.

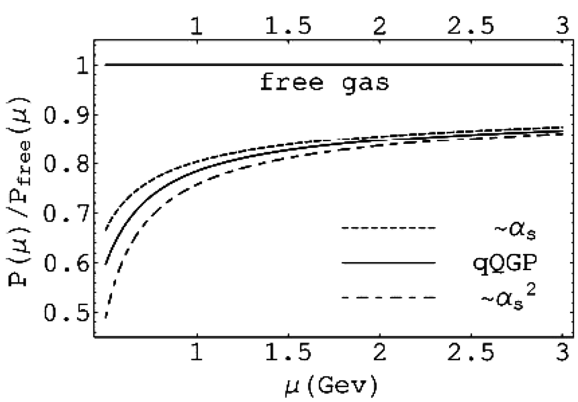

Fig. 4. The pressure density in quasi-particle model.

5 Summary

In this paper we calculate the EOS of QCD at zero temperature and finite chemical potential using three different models: the DSEs model, the HDL approximation and the quasi-particle model.

From DSEs approach we find that the quark number density vanishes when μ is smaller than a critical value μ_0 which is determined by the pole position of the quark propagator. In addition, the pressure obtained from DSEs approach is smaller than the perturbative QCD result in a large region of μ. This fact may be important for the study of neutron stars.

Under HDL approximation, we calculate the EOS and find that the HDL pressure density is much larger than the FPS one (the perturbative QCD result) [16]. It is also found that as μ tends to infinity, the HDL pressure density tends to the free quark gas result from above, whereas the FPS one tends to the free quark gas result from below.

In the quasi-particle model, it is found that the obtained pressure density just lies between the leading order perturbative QCD result and the next-to-leading order one. This indicates that at high chemical potential the quasi-particle model is a perturbative model.

The determination of the EOS of QCD is a long-standing problem in strong interaction physics. Lattice QCD calculations and phenomenological models try to pin down a usable EOS since two decades ago. We expect that the results obtained in this work can be useful for the study of EOS of QCD.

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