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Partial wave analysis of $\psi' \to \gamma\chi_{c0} \to \gamma pK^-\overline{\Lambda}$ used for searching for baryon resonance*

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Abstract Abundant ψ' events have been collected at the Beijing Electron Positron Collider-Ⅱ (BEPCⅡ) that could undoubtedly provide us with a great opportunity to study the more attractive charmonium decays. As has been noticed before, in the process of J/ψ decaying to the baryonic final states, $pK^-\overline{\Lambda}$, the evident Λ^* and N^* bands have been observed. Similarly, by using the product of χ_{cJ} from ψ' radiative decay, we may confirm this or find some extra new resonances. χ_{c0}'s data samples will be more than $\chi_{c1,2}$, taking into account the larger branching ratio of $\psi' \to \gamma\chi_{c0}$. Here, we provide explicit partial wave analysis formulae for the very interesting channel $\psi' \to \gamma\chi_{c0} \to \gamma pK^-\overline{\Lambda}$.

Key words partial wave analysis, covariant tensor, helicity amplitude, baryon resonance

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1 Introduction

As is known, the process of $\psi' \to \gamma\chi_{cJ}$ contains abundant interesting physics. Firstly, the product of χ_{cJ} in ψ' radiative decays may pronde useful information about two-gluon hadronization dynamics and glueball decays. Secondly, the radiative decays of $\psi' \to \gamma\chi_{cJ}$ are expected to be dominated by electric dipole (E1) transitions, with higher multipoles suppressed by powers of photon energy divided by quark mass [1–3], so searching for contributions of higher multipoles is promising. On the other hand, the possibility of anomalous magnetic moments of heavy quarks that are larger than those for light ones may exist [4]. Except for these things, utilizing the rich final state interaction of χ_{cJ}'s baryonic decay in searching for new baryon resonances is another meaningful topic. In the experiment at BES-III, about 10×10^9 J/ψ and 3×10^9 ψ' events can be collected per year's running according to the designed luminosity of BEPC-Ⅱ in Beijing[3)] [5]. These large data samples will provide great opportunities to perform partial wave analysis to study this topic.

Like the J/ψ case [6], one of the most interesting channels is $\chi_{cJ} \to pK^-\overline{\Lambda}$. In reality, on the experimental side, $\psi' \to \gamma\chi_{c0}$ has a larger branching ratio than the other χ_cs [7], which can provide us with a relatively larger χ_{c0} data sample at BESⅢ. Furthermore, in the real data analysis, one can isolate the χ_{c0} from χ_{c1} and χ_{c2} from the mass window cuts with little dilution.

Experimentally, in order to get more information about the resonance properties (such as J^{PC} quantum numbers, mass, width, production and decay rates, etc.), partial wave analysis (PWA) is necessary. PWA is an effective method for analyzing the experimental data of hadron spectra. There are two methods: one is based on the covariant tensor (also named Rarita Schwinger) formalism [8], and the other is based on the original helicity formalism [9]. The latter covariant helicity format was developed by Chung [10, 11]. Ref. [12] shows the connection between the covariant

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tensor formalism and the helicity one. In this short paper, we will pay more attention to the covariant tensor format, but append the helicity one for the specific process, $\psi' \to \gamma\chi_{c0} \to \gamma\Lambda(1520)\overline{\Lambda}$ with $\Lambda(1520)$ decaying to pK^-, $\overline{\Lambda}$ to $\overline{p}\pi^+$.

The paper is organized as follows. In Sec. 2, the general principle for constructing covariant tensor amplitude is introduced. In Sec. 3, we present the covariant tensor amplitudes for $\psi' \to \gamma\chi_{c0} \to \gamma pK^-\overline{\Lambda}$. In Sec. 4, the helicity formula for $\chi_{c0} \to \Lambda(1520)\overline{\Lambda} \to (pK^-)(\overline{p}\pi^+)$ is provided. Finally we present our conclusion.

2 General formalism for constructing covariant tensor amplitude

In this part, the general formulae which will be used in the following have been mentioned before in Ref. [13–15], including three parts: ψ' radiative decay to a meson, a meson decaying to two baryons and a baryon decaying to a daughter baryon and a meson. Because it can be transplanted into the case of J/ψ decay, hereafter in this paper we also refer to them as ψ.

2.1 ψ radiative decay

Denoting the ψ polarization four-vector by $\psi_\mu(m_1)$ and the polarization vector of the photon by $e_\nu(m_2)$, the general form for the decay amplitude is

$$A = \psi_\mu(m_1)e^*_\nu(m_2)A^{\mu\nu} = \psi_\mu(m_1)e^*_\nu(m_2)\sum_i \Lambda_i U_i^{\mu\nu}, \tag{1}$$

where $U_i^{\mu\nu}$ is the i-th partial wave amplitude with coupling strength determined by a complex parameter Λ_i. Because of the massless properties of the photon, there are two additional conditions: (1) the usual orthogonality condition $e_\nu q^\nu = 0$, where q is the photon momentum; (2) the gauge invariance condition (assuming the Coulomb gauge in ψ rest system) $e_\nu p^\nu_\psi = 0$, where p_ψ is the momentum of vector meson ψ. Then we yield the sum of polarization [16]

$$\sum_m e^*_\mu(m)e_\nu(m) = -g_{\mu\nu} + \frac{q_\mu K_\nu + K_\mu q_\nu}{q\cdot K} - \frac{K\cdot K}{(q\cdot K)^2}q_\mu q_\nu \equiv -g^{(\perp\perp)}_{\mu\nu} \tag{2}$$

with $K = p_\psi - q$ and $e_\nu K^\nu = 0$. To compute the differential cross section, we need an expression for $|A|^2$, the square modulus of the decay amplitude, which gives the decay probability of a certain configuration and should be independent of any particular frame.

For ψ production from e^+e^- annihilation, the electrons are highly relativistic, with the result that $J_z = \pm 1$, which is the transverse polarization. If we take the beam direction to be the z-axis, this limits m has only two values, i.e. components along x and y. Thus the radiative cross section is:

$$\begin{aligned}\frac{d\sigma}{d\Phi_n} = {} & \frac{1}{2}\sum_{m_1=1}^{2}\sum_{m_2=1}^{2}\psi_\mu(m_1)e^*_\nu(m_2)A^{\mu\nu}\times \\ & \psi^*_{\mu'}(m_1)e_{\nu'}(m_2)A^{*\mu'\nu'} = \\ & -\frac{1}{2}\sum_{m_1=1}^{2}\psi_\mu(m_1)\psi^*_{\mu'}(m_1)g^{(\perp\perp)}_{\nu\nu'}A^{\mu\nu}A^{*\mu'\nu'} = \\ & -\frac{1}{2}\sum_{\mu=1}^{2}A^{\mu\nu}g^{(\perp\perp)}_{\nu\nu'}A^{*\mu\nu'} = \\ & -\frac{1}{2}\sum_{i,j}\Lambda_i\Lambda^*_j\sum_{\mu=1}^{2}U_i^{\mu\nu}g^{(\perp\perp)}_{\nu\nu'}U_j^{*\mu\nu'} \equiv \\ & \sum_{i,j}P_{ij}\cdot F_{ij}\end{aligned} \tag{3}$$

with definition

$$P_{ij} = P^*_{ji} = \Lambda_i\Lambda^*_j, \tag{4}$$

$$F_{ij} = F^*_{ji} = -\frac{1}{2}\sum_{\mu=1}^{2}U_i^{\mu\nu}g^{(\perp\perp)}_{\nu\nu'}U_j^{*\mu\nu'}. \tag{5}$$

Note the relation

$$\sum_{m=1}^{2}\psi_\mu(m)\psi^*_{\mu'}(m) = \delta_{\mu\mu'}(\delta_{\mu1}+\delta_{\mu2}), \tag{6}$$

has been used.

The partial wave amplitude U in the covariant tensor formalism can be constructed by using pure orbital angular momentum covariant tensor $\widetilde{t}^{(L)}_{\mu_1\mu_2\cdots\mu_L}$ and covariant spin wave functions $\phi_{\mu_1\mu_2\cdots\mu_S}$ together with the metric tensor $g^{\mu\nu}$, the totally antisymmetric Levi Civita tensor $\epsilon_{\mu\nu\lambda\sigma}$ and the four-momenta of participating particles. For a process $a \to bc$, if there exists a relative orbital angular momentum L_{bc} between the particle b and c, then the pure orbital angular momentum L_{bc} state can be represented by the covariant tensor $\widetilde{t}^{(L)}_{\mu_1\mu_2\cdots\mu_L}$, which is built from the relative momentum. Here, we list the amplitude for pure S-, P-, D- and F- wave orbital angular momentum explicitly [10, 11, 13]:

$$\widetilde{t}^{(0)} = 1, \tag{7}$$

$$\widetilde{t}^{(1)}_{\mu} = \widetilde{g}_{\mu\nu}(p_a)r^{\nu}B_1(Q_{abc}) \equiv \widetilde{r}_{\mu}B_1(Q_{abc}), \tag{8}$$

$$\widetilde{t}^{(2)}_{\mu\nu} = \left[\widetilde{r}_{\mu}\widetilde{r}_{\nu} - \frac{1}{3}(\widetilde{r}\cdot\widetilde{r})(\widetilde{g}_{\mu\nu}(p_a))\right]B_2(Q_{abc}), \tag{9}$$

$$\widetilde{t}^{(3)}_{\mu\nu\lambda} = \Big[\widetilde{r}_{\mu}\widetilde{r}_{\nu}\widetilde{r}_{\lambda} - \frac{1}{5}(\widetilde{r}\cdot\widetilde{r})(\widetilde{g}_{\mu\nu}(p_a)\widetilde{r}_{\lambda} + \widetilde{g}_{\nu\lambda}(p_a)\widetilde{r}_{\mu} + \widetilde{g}_{\lambda\mu}(p_a)\widetilde{r}_{\nu})\Big]B_3(Q_{abc}), \tag{10}$$

where $r = p_b - p_c$ is the relative momentum of the two decay products in the parent particle rest frame. In the above equations,

$$\widetilde{g}_{\mu\nu}(p_a) = -g_{\mu\nu} + \frac{p_{a\mu}p_{a\nu}}{p_a^2} \tag{11}$$

is the polarization sum relation for vector meson, and

$$Q^2_{abc} = \frac{(s_a + s_b - s_c)^2}{4s_a} - s_b \tag{12}$$

with $s_a = E_a^2 - p_a^2$. $B_l(Q_{abc})$ is the Blatt Weisskopf barrier factor [13, 17],

$$B_1(Q_{abc}) = \sqrt{\frac{2}{Q^2_{abc}} + Q^2_0}, \tag{13}$$

$$B_2(Q_{abc}) = \sqrt{\frac{13}{Q^4_{abc}} + 3Q^2_{abc}Q^2_0 + 9Q^4_0}, \tag{14}$$

$$B_3(Q_{abc}) = \sqrt{\frac{277}{Q^6_{abc}} + 6Q^4_{abc}Q^2_0 + 45Q^2_{abc}Q^4_0 + 225Q^6_0}, \tag{15}$$

with $Q_0 = 0.197321/R(\mathrm{GeV}/c)$ as a hadron scale parameter, where R is the radius of the centrifugal barrier in fm.

If a is an intermediate resonance decaying into b, c, one needs to introduce into the amplitude a Breit Wigner propagator [13, 18]

$$f^{(a)}_{(bc)} = \frac{1}{m_a^2 - s_{bc} - \mathrm{i}m_a\Gamma_a}. \tag{16}$$

In the equation, $s_{bc} = (p_b + p_c)^2$ is the invariant mass-squared of b and c. m_a, Γ_a are the resonance mass and width, respectively.

Additionally, some expressions depend also on the total momentum of the ij pair, $p_{(ij)} = p_i + p_j$. When one wants to combine two angular momenta j_b and j_c into a total angular momentum j_a, if $j_a + j_b + j_c$ is an odd number, then the combination $\epsilon_{\mu\nu\lambda\sigma}p_a^{\mu}$ with p_a the momentum of the parent particle is needed in order to satisfy the requirement of parity conservation [10, 11], otherwise it is not needed.

2.2 The case of a meson decaying to two baryons

For a given hadronic decay process A→BC, in the L-S scheme at hadronic level, the initial state is described by its four-momentum p_{μ} and its spin state S_A, and the final state is described by the relative orbital angular momentum state of BC system and their spin state (S_B, S_C). The spin states (S_A, S_B, S_C) can be well represented by the relativistic Rarita Schwinger spin wave functions for particles of arbitrary half-integer spin [8]. As is well known, spin-$\frac{1}{2}$ wave function is the standard Dirac spinor $u(p,s)$ and $v(p,s)$; spin-1 wave function is the standard spin-1 polarization four-vector $\epsilon^{\mu}(p,s)$ for a particle with momentum p and spin projection s. For arbitrary spin, there have been the explicit expressions which will be introduced and used in the following. For the case of A as a meson, B as N^* with spin $n+\frac{1}{2}$ and C as $\overline{N}$ with spin $\frac{1}{2}$, the total spin of BC (S_{BC}) can be either n or $n+1$. The two S_{BC} states can be represented as [14]

$$\psi^{(n)}_{\mu_1\mu_2\cdots\mu_n} = \bar{u}_{\mu_1\mu_2\cdots\mu_n}(p_B, s_B)\gamma_5 v(p_C, s_C), \tag{17}$$

$$\Psi^{(n+1)}_{\mu_1\mu_2\cdots\mu_{n+1}} = \bar{u}_{\mu_1\mu_2\cdots\mu_n}\left(\gamma_{\mu_{n+1}} - \frac{r_{\mu_{n+1}}}{m_A + m_B + m_C}\right) \times v(p_C, s_C) + (\mu_1 \leftrightarrow \mu_{n+1}) + \cdots + (\mu_n \leftrightarrow \mu_{n+1}). \tag{18}$$

2.3 The case of one baryon decaying to a daughter baryon and a meson

For the case of A as N^* with spin $n+\frac{1}{2}$, B as N with spin $\frac{1}{2}$ and C as a meson, one needs to couple $-S_A$ and S_B first to get $S_{AB} = -S_A + S_B$ states, which are [14]

$$\phi^{(n)}_{\mu_1\mu_2\cdots\mu_n} = \bar{u}(p_B, s_B)u_{\mu_1\mu_2\cdots\mu_n}(p_A, s_A), \tag{19}$$

$$\Phi^{n+1}_{\mu_1\mu_2\cdots\mu_n} = \bar{u}(p_B, s_B)\gamma_5\widetilde{\gamma}_{\mu_{n+1}}u_{\mu_1\mu_2\cdots\mu_n}(p_A, s_A) + (\mu_1 \leftrightarrow \mu_{n+1}) + \cdots + (\mu_n \leftrightarrow \mu_{n+1}). \tag{20}$$

For Sec 2.2 and 2.3, the principle of constructing the orbital angular part is the same as Sec. 2.1. Up to now, we have introduced all principles for constructing the covariant tensor amplitude, including the orbital angular part as well as the spin part. In the concrete case, the P parity conservation may be applied, which could be expressed as

$$\eta_{\rm A}=\eta_{\rm B}\eta_{\rm C}(-1)^{L}, \tag{21}$$

where $\eta_{\rm A}$, $\eta_{\rm B}$ and $\eta_{\rm C}$ are the intrinsic parities of particles A, B, and C, respectively. From this relation, L can be only even or odd for each case, which guarantees a pure L final state.

In the following, we will present the specific process $\psi'\to\gamma\chi_{c0}\to\gamma pK^-\overline{\Lambda}$ in the framework of general covariant tensor amplitude.

3 Analysis for $\psi'\to\gamma\chi_{c0}\to\gamma pK^-\overline{\Lambda}$

Hereafter, we denote p, K^-, and $\overline{\Lambda}$ by the numbers 1, 2 and 3 for simplicity. Firstly, for $\psi'\to\gamma\chi_{c0}$, from the helicity formalism, it is easy to show that there is only one independent amplitude for ψ' radiative decay to a spin 0 meson. Hence, the amplitude is

$$U^{\mu\nu}_{\gamma\chi_{c0}}=g^{\mu\nu}f^{(\chi_{c0})}. \tag{22}$$

where $f^{(\chi_{c0})}$ means χ_{c0} has the subsequent decay. For sequential χ_{c0} decay, there may exist the following modes: $\chi_{c0}\to\Lambda_x\overline{\Lambda},\Lambda_x\to pK^-$, where Λ_x can be

$$\Lambda(1520)\frac{3}{2}^-,\ \Lambda(1600)\frac{1}{2}^+,\ \Lambda(1670)\frac{1}{2}^-,\ \Lambda(1690)\frac{3}{2}^-,$$

$$\Lambda(1800)\frac{1}{2}^-,\ \Lambda(1810)\frac{1}{2}^+,\ \Lambda(1820)\frac{5}{2}^+,\ \Lambda(1830)\frac{5}{2}^-,$$

$$\Lambda(1890)\frac{3}{2}^+,\ \Lambda(2100)\frac{7}{2}^-,\ \Lambda(2110)\frac{5}{2}^+;$$

$\chi_{c0}\to\overline{N}p,\overline{N}\to\overline{\Lambda}K^-$, where $\overline{N}$ is the anti-partner of hyperon N, and N can be

$$N(1650)\frac{1}{2}^-,\ N(1675)\frac{5}{2}^-,\ N(1700)\frac{3}{2}^-,\ N(1710)\frac{1}{2}^+$$
or
$$N(1720)\frac{3}{2}^+.$$

Another possibility, that $p\overline{\Lambda}$ may be generated from an intermediate resonance K_x is also taken into account. We would give an explicit example for how to write the amplitude for a concrete process. For Λ_x being $\Lambda(1520)\frac{3}{2}^-$, the total spin of $\Lambda(1520)$ and $\overline{\Lambda}\frac{1}{2}^-$ can be 1 or 2, corresponding to the P- wave and D-wave, respectively, because of the spin-0 property of χ_{c0}. The parity relation (21) makes P- wave impossible. Considering that this channel is recognized as a meson decaying to two fermions, according to the above description, now one can write the covariant amplitude as

$$\Phi^{(2)}_{(\Lambda(1520)3)\mu\nu}\tilde{t}^{(2)\mu\nu}_{(\Lambda(1520)3)}, \tag{23}$$

where Φ and t's meanings are implied in Eq. (9) and (20). And then considering $\Lambda(1520)\to pK^-$, the total spin of particle 1 and 2 can only be $\frac{1}{2}$, requiring D- wave following Eq. (21) and angular momentum conservation. Thus the covariant amplitude can be expressed as

$$\Phi^{(2)}_{(12)\mu\nu}\tilde{t}^{(2)\mu\nu}_{(12)}. \tag{24}$$

Here, we list all the amplitudes for the whole decay chain $\psi'\to\gamma\chi_{c0},\chi_{c0}\to\Lambda_x\overline{\Lambda},\Lambda_x\to pK^-$ up to spin-$\frac{7}{2}$ for Λ_x,

$$\Lambda_x\left(\frac{1}{2}^+\right): U^{\mu\nu}=g^{\mu\nu}\Psi^{(1)}_{(\Lambda_x3)\lambda}\tilde{t}^{(1)\lambda}_{(\Lambda_x3)}\Phi^{(1)}_{(12)\sigma}\tilde{t}^{(1)\sigma}_{(12)}f^{(\Lambda_x)}_{(12)}, \tag{25}$$

$$\Lambda_x\left(\frac{1}{2}^-\right): U^{\mu\nu}=g^{\mu\nu}\psi^{(0)}_{(\Lambda_x3)}\phi^{(0)}_{(12)}f^{(\Lambda_x)}_{(12)}, \tag{26}$$

$$\Lambda_x\left(\frac{3}{2}^+\right): U^{\mu\nu}=g^{\mu\nu}\psi^{(1)}_{(\Lambda_x3)\lambda}\tilde{t}^{(1)\lambda}_{(\Lambda_x3)}\phi^{(1)}_{(12)\sigma}\tilde{t}^{(1)\sigma}_{(12)}f^{(\Lambda_x)}_{(12)}, \tag{27}$$

$$\Lambda_x\left(\frac{3}{2}^-\right): U^{\mu\nu}=g^{\mu\nu}\Psi^{(2)}_{(\Lambda_x3)\lambda\delta}\tilde{t}^{(2)\lambda\delta}_{(\Lambda_x3)}\Phi^{(2)}_{(12)\rho\sigma}\tilde{t}^{(2)\rho\sigma}_{(12)}f^{(\Lambda_x)}_{(12)}, \tag{28}$$

$$\Lambda_x\left(\frac{5}{2}^+\right): U^{\mu\nu}=g^{\mu\nu}\Psi^{(3)}_{(\Lambda_x3)\lambda\delta\beta}\tilde{t}^{(3)\lambda\delta\beta}_{(\Lambda_x3)}\Phi^{(3)}_{(12)\rho\sigma\eta}\times \tilde{t}^{(3)\rho\sigma\eta}_{(12)}f^{(\Lambda_x)}_{(12)}, \tag{29}$$

$$\Lambda_x\left(\frac{5}{2}^-\right): U^{\mu\nu}=g^{\mu\nu}\psi^{(2)}_{(\Lambda_x3)\lambda\delta}\tilde{t}^{(2)\lambda\delta}_{(\Lambda_x3)}\phi^{(2)}_{(12)\rho\sigma}\tilde{t}^{(2)\rho\sigma}_{(12)}f^{(\Lambda_x)}_{(12)}, \tag{30}$$

$$\Lambda_x\left(\frac{7}{2}^+\right): U^{\mu\nu}=g^{\mu\nu}\psi^{(3)}_{(\Lambda_x3)\lambda\delta\beta}\tilde{t}^{(3)\lambda\delta\beta}_{(\Lambda_x3)}\phi^{(3)}_{(12)\rho\sigma\eta}\times \tilde{t}^{(3)\rho\sigma\eta}_{(12)}f^{(\Lambda_x)}_{(12)}, \tag{31}$$

$$\Lambda_x\left(\frac{7}{2}^-\right): U^{\mu\nu}=g^{\mu\nu}\Psi^{(4)}_{(\Lambda_x3)\lambda\delta\beta\xi}\tilde{t}^{(4)\lambda\delta\beta\xi}_{(\Lambda_x3)}\Phi^{(4)}_{(12)\rho\sigma\eta\zeta}\times \tilde{t}^{(4)\rho\sigma\eta\zeta}_{(12)}f^{(\Lambda_x)}_{(12)}. \tag{32}$$

Note that $\tilde{t}^{(0)}=1$ has been considered here and the $f^{(\Lambda_x)}$s differ from case to case. And $\psi^{(0)}$,$\phi^{0)}$ is expressed as [14]

$$\psi^{(0)}_{(\mathrm{BC})} = \bar{u}(p_\mathrm{B},s_\mathrm{B})\gamma_5 v(p_\mathrm{C},s_\mathrm{C}), \quad (33)$$

$$\phi^{(0)}_{(\mathrm{AB})} = \bar{u}(p_\mathrm{B},s_\mathrm{B})u(p_\mathrm{A},s_\mathrm{A}), \quad (34)$$

which can be deduced from Eq. (17) and Eq. (19).

Correspondingly, for channel $\chi_{c0} \to \overline{N}_x p, \overline{N}_x \to K^-\overline{\Lambda}$, in the same way, we can write the amplitudes up to spin-$\frac{7}{2}$ for $\overline{N}_x$ without difficulties, even though the highest spin for N_x decaying into $K^-\Lambda$ only is $\frac{5}{2}$ currently [7].

$$\overline{N}_x\left(\frac{1}{2}^+\right): U^{\mu\nu} = g^{\mu\nu}\psi^{(0)}_{(\overline{N}_x1)}\phi^{(0)}_{(\overline{N}_x1)}f^{(\overline{N}_x)}_{(23)}, \quad (35)$$

$$\overline{N}_x\left(\frac{1}{2}^-\right): U^{\mu\nu} = g^{\mu\nu}\Psi^{(1)}_{(\overline{N}_x1)\lambda}\widetilde{t}^{(1)\lambda}_{(\overline{N}_x1)}\Phi^{(1)}_{(23)\sigma}\widetilde{t}^{(1)\sigma}_{(23)}f^{(\overline{N}_x)}_{(23)}, \quad (36)$$

$$\overline{N}_x\left(\frac{3}{2}^+\right): U^{\mu\nu} = g^{\mu\nu}\Psi^{(2)}_{(\overline{N}_x1)\lambda\delta}\widetilde{t}^{(2)\lambda\delta}_{(\overline{N}_x1)}\Phi^{(2)}_{(23)\rho\sigma}\times \widetilde{t}^{(2)\rho\sigma}_{(23)}f^{(\overline{N}_x)}_{(23)}, \quad (37)$$

$$\overline{N}_x\left(\frac{3}{2}^-\right): U^{\mu\nu} = g^{\mu\nu}\psi^{(1)}_{(\overline{N}_x1)\lambda}\widetilde{t}^{(1)\lambda}_{(\overline{N}_x1)}\phi^{(1)}_{(23)\sigma}\widetilde{t}^{(1)\sigma}_{(23)}f^{(\overline{N}_x)}_{(23)}, \quad (38)$$

$$\overline{N}_x\left(\frac{5}{2}^+\right): U^{\mu\nu} = g^{\mu\nu}\psi^{(2)}_{(\overline{N}_x1)\lambda\delta}\widetilde{t}^{(2)\lambda\delta}_{(\overline{N}_x1)}\phi^{(2)}_{(23)\rho\sigma}\times \widetilde{t}^{(2)\rho\sigma}_{(23)}f^{(\overline{N}_x)}_{(23)}, \quad (39)$$

$$\overline{N}_x\left(\frac{5}{2}^-\right): U^{\mu\nu} = g^{\mu\nu}\Psi^{(3)}_{(\overline{N}_x1)\lambda\delta\beta}\widetilde{t}^{(3)\lambda\delta\beta}_{(\overline{N}_x1)}\Phi^{(3)}_{(23)\rho\sigma\eta}\times \widetilde{t}^{(3)\rho\sigma\eta}_{(23)}f^{(\overline{N}_x)}_{(23)}, \quad (40)$$

$$\overline{N}_x\left(\frac{7}{2}^+\right): U^{\mu\nu} = g^{\mu\nu}\Psi^{(4)}_{(\overline{N}_x1)\lambda\delta\beta\xi}\widetilde{t}^{(4)\lambda\delta\beta\xi}_{(\overline{N}_x1)}\Phi^{(4)}_{(23)\rho\sigma\eta\zeta}\times \widetilde{t}^{(4)\rho\sigma\eta\zeta}_{(23)}f^{(\overline{N}_x)}_{(23)}, \quad (41)$$

$$\overline{N}_x\left(\frac{7}{2}^-\right): U^{\mu\nu} = g^{\mu\nu}\psi^{(3)}_{(\overline{N}_x1)\lambda\delta\beta}\widetilde{t}^{(3)\lambda\delta\beta}_{(\overline{N}_x1)}\phi^{(3)}_{(23)\rho\sigma\eta}\times \widetilde{t}^{(3)\rho\sigma\eta}_{(23)}f^{(\overline{N}_x)}_{(23)}. \quad (42)$$

For channel $\chi_{c0} \to K_x^+K^-, K_x^+ \to p\overline{\Lambda}$, the amplitudes are listed below. Note that $J^P = 0^+,\ 1^-,\ 2^+,\ 3^-,\ 4^+\cdots$ are forbidden by the parity relation Eq. (21). In the following, the partial wave amplitude is denoted by $U^{\mu\nu}_{(LS)}$, in which L,S mean the orbital and total spin angular momentum between p and $\overline{\Lambda}$. We write the amplitudes for K_x up to spin-4,

$${K_x}^+(0^-): U^{\mu\nu} = g^{\mu\nu}\psi^{(0)}_{(K_x2)}f^{(K_x)}_{(13)}, \quad (43)$$

$${K_x}^+(1^+): U^{\mu\nu}_{(10)} = g^{\mu\nu}\psi^{(0)}_{(K_x2)}\widetilde{T}^{(1)\sigma}_{(K_x2)}\times \phi_{(13)\sigma}\epsilon_\lambda\widetilde{t}^{(1)\lambda}_{(13)}f^{(K_x)}_{(13)}, \quad (44)$$

$$U^{\mu\nu}_{(11)} = g^{\mu\nu}\epsilon^{\rho\sigma\eta\zeta}p_{K_x\rho}\epsilon_\sigma\widetilde{T}^{(1)\eta}_{(K_x2)}\times \Psi^{(1)}_{(K_x2)\eta}\widetilde{t}^{(1)\zeta}_{(13)}\phi_{(13)\zeta}f^{(K_x)}_{(13)}, \quad (45)$$

where $\epsilon^{\rho\sigma\eta\zeta}$ is the total asymmetric 4-rank tensor, and ϵ_λ and ϵ_σ denote the wave function of K_x which is the familiar polarization 4-vector of vector meson. In the following, φs imply the wave function of higher spin K_x, which are higher rank tensors. There has been a general expression for a particle of spin J, which is a rank-J tensor [10]

$$\varphi^{\alpha_1\alpha_2\cdots\alpha_J}(m) = \sum_{m_1m_2\cdots}\langle 1m_1 1m_2|2n_1\rangle\langle 2n_1 1m_3|3n_2\rangle\cdots \langle J-1n_{J-2}1m_J|Jm\rangle\varphi^{\alpha_1}(m_1)\times \varphi^{\alpha_2}(m_2)\cdots\varphi^{\alpha_J}(m_J) \quad (46)$$

with $m_i = \pm1, 0$, $(i = 1,2,\cdots J)$ and

$$\varphi^\alpha(1,-1) = \mp\frac{1}{\sqrt{2}}(0;1,\pm\mathrm{i},0),\ \varphi^\alpha(0) = (0;0,0,1). \quad (47)$$

It is interesting to note a useful relationship:

$$\varphi(-m) = (-)^m\varphi^*(m). \quad (48)$$

Next we illustrate these formulas with some examples. For $J=1$, one finds that it reduces to identities for $\varphi(1)$ and $\varphi(0)$. For $J=2$, one has [10]

$$\varphi^{\alpha\beta}(+2) = \varphi^\alpha(1)\varphi^\beta(1), \quad (49)$$

$$\varphi^{\alpha\beta}(+1) = \frac{1}{\sqrt{2}}[\varphi^\alpha(1)\varphi^\beta(0)+\varphi^\alpha(0)\varphi^\beta(1)], \quad (50)$$

$$\varphi^{\alpha\beta}(0) = \frac{1}{\sqrt{6}}\left[\varphi^\alpha(1)\varphi^\beta(-1)+\varphi^\alpha(-1)\varphi^\beta(1)+ \sqrt{\frac{2}{3}}\varphi^\alpha(0)\varphi^\beta(0)\right]. \quad (51)$$

Thus, the amplitudes for spin-2 and spin-3 of K_x are written as

$$\mathrm{K_x}^+(2^-):\ U^{\mu\nu}_{(20)} = g^{\mu\nu}\widetilde{T}^{(2)\eta\zeta}_{(\mathrm{K_x}2)}\varphi^{(2)\eta\zeta}\widetilde{t}^{(2)\rho\sigma}_{(13)}\psi^{(0)}\varphi_{\rho\sigma}f^{(\mathrm{K_x})}_{(13)} = g^{\mu\nu}P^{(2)}_{\rho\sigma\eta\zeta}\psi^{(0)}\widetilde{T}^{(2)\eta\zeta}_{(\mathrm{K_x}2)}\widetilde{t}^{(2)\rho\sigma}_{(13)}f^{(\mathrm{K_x})}_{(13)}, \tag{52}$$

$$U^{\mu\nu}_{(21)} = g^{\mu\nu}\epsilon^{\rho\sigma\eta\zeta}p_{\mathrm{K_x}\rho}\widetilde{T}^{(2)\beta\lambda}_{(\mathrm{K_x}2)}\varphi_{\beta\lambda}\widetilde{t}^{(2)\iota}_{(13)\sigma}\varphi_{\iota\eta}\Psi^{(1)}_{\zeta}f^{(\mathrm{K_x})}_{(13)} = g^{\mu\nu}P^{(2)}_{\beta\lambda\iota\eta}\epsilon^{\rho\sigma\eta\zeta}p_{\mathrm{K_x}\rho}\widetilde{T}^{(2)\beta\lambda}_{(\mathrm{K_x}2)}\widetilde{t}^{(2)\iota}_{(13)\sigma}\Psi^{(1)}_{\zeta}f^{(\mathrm{K_x})}_{(13)}, \tag{53}$$

$$\mathrm{K_x}^+(3^+):\ U^{\mu\nu}_{(30)} = g^{\mu\nu}\widetilde{T}^{(3)\lambda\delta\beta}_{(\mathrm{K_x}2)}\varphi_{\lambda\delta\beta}\psi^{(0)}\widetilde{t}^{(3)\rho\sigma\eta}_{(13)}\varphi_{\rho\sigma\eta}f^{(\mathrm{K_x})}_{(13)} = g^{\mu\nu}P^{(3)}_{\lambda\delta\beta\rho\sigma\eta}\widetilde{T}^{(3)\lambda\delta\beta}_{(\mathrm{K_x}2)}\psi^{(0)}\widetilde{t}^{(3)\rho\sigma\eta}_{(13)}f^{(\mathrm{K_x})}_{(13)}, \tag{54}$$

$$U^{\mu\nu}_{(31)} = g^{\mu\nu}\widetilde{T}^{(3)\lambda\delta\beta}_{(\mathrm{K_x}2)}\varphi_{\lambda\delta\beta}\epsilon^{\rho\sigma\eta\zeta}p_{\mathrm{K_x}\rho}\widetilde{t}^{(3)\kappa\xi}_{(13)\sigma}\Psi^{(1)}_{\eta}\varphi^{(3)}_{\zeta\kappa\xi}f^{(\mathrm{K_x})}_{(13)} = g^{\mu\nu}P^{(3)}_{\lambda\delta\beta\zeta\kappa\xi}\widetilde{T}^{(3)\lambda\delta\beta}_{(\mathrm{K_x}2)}\epsilon^{\rho\sigma\eta\zeta}p_{\mathrm{K_x}\rho}\widetilde{t}^{(3)\kappa\xi}_{(13)\sigma}\Psi^{(1)}_{\eta}f^{(\mathrm{K_x})}_{(13)}, \tag{55}$$

where the product of two φs is in reality its corresponding spin projection operator $P^{(S)}$ [13],

$$P^{(2)}_{\rho\sigma\eta\zeta}(p_{\mathrm{K_x}}) = \sum_m \varphi_{\rho\sigma}(p_{\mathrm{K_x}},m)\varphi^*_{\eta\zeta}(p_{\mathrm{K_x},m}) = \frac{1}{2}(\widetilde{g}_{\rho\eta}\widetilde{g}_{\sigma\zeta}+\widetilde{g}_{\rho\zeta}\widetilde{g}_{\sigma\eta})-\frac{1}{3}\widetilde{g}_{\rho\sigma}\widetilde{g}_{\eta\zeta}, \tag{56}$$

$$\begin{aligned}P^{(3)}_{\lambda\delta\beta\zeta\kappa\xi}(p_{\mathrm{K_x}}) = {} & \sum_m \varphi_{\lambda\delta\beta}(p_{\mathrm{K_x}},m)\varphi^{*\zeta\kappa\xi}(p_{\mathrm{K_x}},m) = \\ & \frac{1}{6}(\widetilde{g}_{\lambda\zeta}\widetilde{g}_{\delta\kappa}\widetilde{g}_{\beta\xi}+\widetilde{g}_{\lambda\zeta}\widetilde{g}_{\delta\xi}\widetilde{g}_{\beta\kappa}+\widetilde{g}_{\lambda\kappa}\widetilde{g}_{\delta\zeta}\widetilde{g}_{\beta\xi}+ \\ & \widetilde{g}_{\lambda\zeta}\widetilde{g}_{\delta\xi}\widetilde{g}_{\beta\kappa}+\widetilde{g}_{\delta\xi}\widetilde{g}_{\beta\zeta}\widetilde{g}_{\lambda\xi}+\widetilde{g}_{\delta\kappa}\widetilde{g}_{\lambda\xi}\widetilde{g}_{\beta\zeta})- \\ & \frac{1}{15}(\widetilde{g}_{\lambda\delta}\widetilde{g}_{\zeta\kappa}\widetilde{g}_{\beta\xi}+\widetilde{g}_{\lambda\delta}\widetilde{g}_{\kappa\xi}\widetilde{g}_{\beta\zeta}+ \\ & \widetilde{g}_{\lambda\delta}\widetilde{g}_{\zeta\xi}\widetilde{g}_{\beta\kappa}+\widetilde{g}_{\lambda\beta}\widetilde{g}_{\zeta\xi}\widetilde{g}_{\delta\kappa}+\widetilde{g}_{\lambda\beta}\widetilde{g}_{\zeta\kappa}\widetilde{g}_{\delta\xi}+ \\ & \widetilde{g}_{\lambda\beta}\widetilde{g}_{\kappa\xi}\widetilde{g}_{\delta\zeta}+\widetilde{g}_{\delta\beta}\widetilde{g}_{\kappa\xi}\widetilde{g}_{\lambda\kappa}+\widetilde{g}_{\delta\beta}\widetilde{g}_{\zeta\kappa}\widetilde{g}_{\lambda\xi}+ \\ & \widetilde{g}_{\delta\beta}\widetilde{g}_{\kappa\xi}\widetilde{g}_{\lambda\kappa}).\end{aligned} \tag{57}$$

So far, we have given the covariant tensor amplitudes for the process $\psi'\to\gamma\chi_{c0}\to\gamma \mathrm{pK}^-\overline{\Lambda}$, including the possible immediate resonance Λ_x to spin-$\frac{7}{2}$, $\mathrm{N_x}$ to spin-$\frac{7}{2}$ as well as $\mathrm{K_x}$ up to spin-3 being taken into account.

4 Helicity formalism

For completeness, we will give the helicity format in comparison with the tensor formula in this part. Helicity formalism has an explicit advantage that the angular dependence can be easily seen. In this section, we will give the amplitude for the decay chain $\chi_{c0}\to\Lambda(1520)\overline{\Lambda}, \Lambda(1520)\to\mathrm{pK}^-, \overline{\Lambda}\to\overline{\mathrm{p}}\pi^+$. Firstly, we want to introduce the general helicity formula expression. Consider a state with spin(parity) $= J(\eta_J)$ decaying into two states with $S(\eta_s)$ and $\sigma(\eta_\sigma)$. The decay amplitudes are given, in the rest frame of J [9][1)],

$$\mathcal{M}^{\mathrm{J}\to\mathrm{s}\sigma}_{\lambda\nu} \propto \sqrt{\frac{2J+1}{4\pi}}D^{J*}_{M\delta}(\phi,\theta,0)H^J_{\lambda\nu}, \tag{58}$$

where λ and ν are the helicities of the two final state particles s and σ with $\delta=\lambda-\nu$. The symbol M stands for the z component of the spin J in a coordinate system fixed by the production process. The helicities λ and ν are the rotational invariants by definition. The direction of the break-up momentum of the decaying particle s is given by the angles θ and ϕ in the J rest frame. Let $\hat{x},\hat{y}$ and $\hat{z}$ be the coordinate system fixed in the J rest frame. It is important to recognize, for the applications to sequential decays, the exact nature of the body-fixed (helicity) coordinate system implied by the arguments of the D function given above. Let $\hat{x}_{\mathrm{h}}$, $\hat{y}_{\mathrm{h}}$ and $\hat{z}_{\mathrm{h}}$ be the helicity coordinate system fixed by the s decay. Then by definition, $\hat{z}_{\mathrm{h}}$ describes the direction of s in the J rest frame (also termed the helicity axis), and the y axis is given by $\hat{y}_{\mathrm{h}}=\hat{z}\times\hat{z}_{\mathrm{h}}$ and $\hat{x}_{\mathrm{h}}=\hat{y}_{\mathrm{h}}\times\hat{z}_{\mathrm{h}}$. Parity conservation in the decay leads to the relationship

$$H^J_{\lambda\nu}=\eta_J\eta_s\eta_\sigma(-)^{J-s-\sigma}H^J_{-\lambda-\nu}. \tag{59}$$

Let us consider a full process A→B+C, where B and C are also the unstable particles decaying to $\mathrm{B_1+B_2}$ and $\mathrm{C_1+C_2}$, respectively. The decay amplitude is simply[2)]

$$\mathcal{M}(\lambda_{\mathrm{B_1}},\lambda_{\mathrm{B_2}},\lambda_{\mathrm{C_1}},\lambda_{\mathrm{C_2}}) = \sum_{\lambda_{\mathrm{B}},\lambda_{\mathrm{C}}}\mathcal{M}^{\mathrm{A\to B+C}}_{\lambda_{\mathrm{B}},\lambda_{\mathrm{C}}}\cdot\mathcal{M}^{\mathrm{B\to B_1+B_2}}_{\lambda_{\mathrm{B_1}},\lambda_{\mathrm{B_2}}}\cdot\mathcal{M}^{\mathrm{C\to C_1+C_2}}_{\lambda_{\mathrm{C_1}},\lambda_{\mathrm{C_2}}}, \tag{60}$$

1) An Experimenter's Guide to the Helicity Formalism, CALT-68-1148 (1984).

2) T'Jampens S, Etude de la violation de la symétrie CP dans les canaux charmonium-K*(892) par une analyse angulaire complète dépendante du temps (expérience BaBar), PhD Thesis, Univ. Paris-Sud 11, France, http://tel.archives-ouvertes.fr/tel-00002447/fr/.

with

$$\mathcal{M}^{A\to B+C}_{\lambda_B,\lambda_C} \propto \sqrt{\frac{2J_A+1}{4\pi}} D^{J_A *}_{M_A,\lambda_B-\lambda_C}(\phi_A,\theta_A,0) H^A_{\lambda_B,\lambda_C}, \tag{61a}$$

$$\mathcal{M}^{B\to B_1+B_2}_{\lambda_{B_1},\lambda_{B_2}} \propto \sqrt{\frac{2J_B+1}{4\pi}} D^{J_B *}_{\lambda_B,\lambda_{B_1}-\lambda_{B_2}}(\phi_B,\theta_B,0) H^B_{\lambda_{B_1},\lambda_{B_2}}, \tag{61b}$$

$$\mathcal{M}^{C\to C_1+C_2}_{\lambda_{C_1},\lambda_{C_2}} \propto \sqrt{\frac{2J_C+1}{4\pi}} D^{J_C *}_{-\lambda_C,\lambda_{C_1}-\lambda_{C_2}}(\phi_C,\theta_C,0) H^C_{\lambda_{C_1},\lambda_{C_2}}. \tag{61c}$$

Please note in (61c) that the first subscript of $D^{J_C *}$ is $-\lambda_C$ and NOT λ_C, although it also gives the correct result, because the quantization axis is along the direction of the momentum of particle B, so that the spin-quantization projection M_C in the particle C rest frame verifies $M_C = -\lambda_C$.

The unpolarized angular distribution is then given by averaging the initial spins and summing over the final spins:

$$\frac{d^3\Gamma}{\mathcal{N} d\Omega_A d\Omega_B d\Omega_C} \propto \frac{1}{2S_A+1} \sum_{\lambda_{B_1},\lambda_{C_1},\lambda_{B_2},\lambda_{C_2}} \times |\mathcal{M}(\lambda_{B_1},\lambda_{B_2},\lambda_{C_1},\lambda_{C_2})|^2, \tag{62}$$

where $\mathcal{N}$ is the normalization factor. Following Eq. (59), one has

$$H^{\overline{\Lambda}}_{\frac{1}{2}0} = H^{\overline{\Lambda}}_{-\frac{1}{2}0}$$

and

$$H^{\Lambda(1520)}_{\frac{1}{2}0} = -H^{\Lambda(1520)}_{-\frac{1}{2}0}.$$

Applying the above amplitude expressions, after a lengthy evaluation, one can get

$$\frac{d^3\Gamma}{\mathcal{N} d\Omega_{\chi_{c0}} d\Omega_{\Lambda(1520)} d\Omega_{\overline{\Lambda}}} \propto \Big[\frac{3}{2}\cos^2\theta_{\Lambda(1520)} - \frac{3}{2}\cos\theta_{\Lambda(1520)} + \frac{9}{2}\cos^2\theta_{\Lambda(1520)}\sin\theta_{\Lambda(1520)}\cos\phi_{\Lambda(1520)} + \frac{\sqrt{3}}{2}\cos^2\theta_{\Lambda(1520)}\cos 2\phi_{\Lambda(1520)} - \frac{\sqrt{3}}{4}\cos\theta_{\Lambda(1520)} \times \cos 2\phi_{\Lambda(1520)} - \frac{3\sqrt{3}}{4}\cos 2\phi_{\Lambda(1520)} + 1\Big] \times \left|H^{\overline{\Lambda}}_{\frac{1}{2}0}\right|^2 \left|H^{\Lambda(1520)}_{\frac{1}{2}0}\right|^2, \tag{63}$$

where the subscript $\Lambda(1520)$ denotes the angle defined in the rest frame of $\Lambda(1520)$. After integrating $\phi_{\Lambda(1520)}$s from $[0,2\pi]$, Eq. (63) becomes

$$\frac{d^3\Gamma}{\mathcal{N}' d\Omega_{\chi_{c0}} d\Omega_{\Lambda(1520)} d\Omega_{\overline{\Lambda}}} \propto \frac{3}{2}\cos^2\theta_{\Lambda(1520)} - \frac{3}{2}\cos\theta_{\Lambda(1520)} + 1, \tag{64}$$

where

$$\mathcal{N}' = \mathcal{N}\left|H^{\overline{\Lambda}}_{\frac{1}{2}0}\right|^2 \left|H^{\Lambda(1520)}_{\frac{1}{2}0}\right|^2$$

is the redefined normalization factor. For simplicity, denoting

$$\frac{d^3\Gamma}{\mathcal{N} d\Omega_{\chi_{c0}} d\Omega_{\Lambda(1520)} d\Omega_{\overline{\Lambda}}}$$

by

$$\frac{d\Gamma}{\mathcal{N}' d\Omega}.$$

Fig. 1 shows the angular distribution.

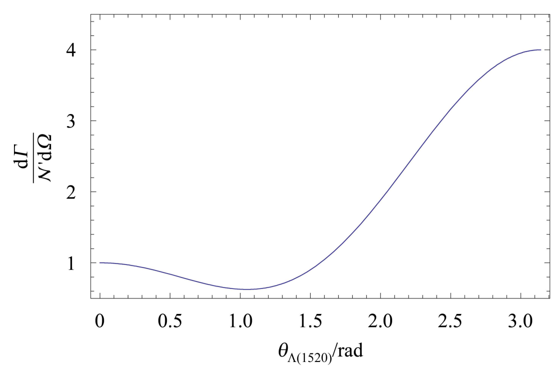

Fig. 1. The illustrative plot for the angular distribution of $\chi_{c0} \to \Lambda(1520)\overline{\Lambda}$, $\overline{\Lambda} \to \overline{p}\pi^+$, $\Lambda(1520) \to pK^-$ in helicity format.

After a back-of-the-envelope computation by using Eqs. (58)–(62), one can find the differential decay width for other Λ^* and N^* in helicity format, where Λ^* and N^* denote the excited states of baryons Λ and N.

5 Conclusion

To study the abundant hadron spectra contained in the $pK^-\overline{\Lambda}$ final states with a large data sample at BESIII, in this short paper, firstly, the relevant general tensor formalism is introduced, and then the covariant tensor amplitudes for $\psi' \to \gamma\chi_{c0} \to \gamma pK^-\overline{\Lambda}$ are given, including the possible resonance Λ_x to spin-$\frac{7}{2}$, N_x to spin-$\frac{7}{2}$ as well as K_x up to spin-3. At last, for completeness, the helicity format of differential decay width for $\chi_{c0} \to \Lambda(1520)\overline{\Lambda}, \overline{\Lambda} \to \overline{p}\pi^+, \Lambda(1520) \to pK^-$ is provided as an example, and a figure is attached. It also can be easily adapted to the case of other higher excited states of baryon Λ or N. We expect that, in future, significant physical results can be achieved through the channel $\psi' \to \gamma\chi_{c0} \to \gamma pK^-\overline{\Lambda}$ that we have proposed here.

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