Optimal design of a 7 T highly homogeneous superconducting magnet for a Penning trap*

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Abstract A Penning trap system called Lanzhou Penning Trap (LPT) is now being developed for precise mass measurements at the Institute of Modern Physics (IMP). One of the key components is a 7 T actively shielded superconducting magnet with a clear warm bore of 156 mm. The required field homogeneity is 3×10^{-7} over two 1 cubic centimeter volumes lying 220 mm apart along the magnet axis. We introduce a two-step method which combines linear programming and a nonlinear optimization algorithm for designing the multi-section superconducting magnet. This method is fast and flexible for handling arbitrary shaped homogeneous volumes and coils. With the help of this method an optimal design for the LPT superconducting magnet has been obtained.

Key words Penning trap, linear programming, magnet design, nonlinear optimization methods, pattern search.

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1 Introduction

Penning traps are devices that use both a magnetic field and an electric field to trap ions. At present, some Penning trap systems are already in use, such as HITRAP, LEBIT, MLLTRAP [1–3]. A Penning trap system called LPT (Lanzhou Penning Trap) is now developed for precise mass measurements in Institute of Modern Physics (IMP).

The uniformity, the strength and the stability of the magnetic field strongly affect the accuracy and the sensitivity of the Penning trap system. So a superconducting magnet which provides a highly homogeneous magnetic field is the key component of the penning trap. The specifications for the LPT magnet which is under design in IMP are summarized in Table 1. The central field is 7 T with a uniformity of 3×10^{-7} in the two regions of interest (ROI), lying 220 mm apart. However, due to the manufacturing and winding tolerances, it is impractical to achieve such a high homogeneity only with the main coils. So we first design the main coils with a lower homogeneous field (10^{-5}) and then the superconducting shim coils and passive shim pieces are used to reach the required homogeneity.

Table 1. Specification of the LPT magnet.

items	value
central field	7 T
homogeneity	3×10^{-7} within 1 cm^3
stray field(5×10^{-4} T line)	2 m away from the center
warm bore	ϕ156 mm
field stability	10^{-8}/h

Linear programming (LP) [4, 5] and several nonlinear optimization algorithms [6–8] have already been applied to superconducting magnet design. The linear programming method allows complete flexibility in arbitrary geometric constraints on both location and the shape of the homogeneous volume; it guaran-

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tees a globally optimal solution and has a high computation speed; but the nonlinear constrains, such as the characteristics of a superconductor (e.g. B-J characteristics), the Lorentz force, the stabilization and protection against a quench, cannot be considered. However, the nonlinear optimization algorithms, such as the genetic algorithm (GA) and the sequential quadratic programming (SQP) can introduce these nonlinear constrains. But they have not been applied very often to direct shape optimization (topology optimization), because of the severe computational costs and their difficulty in dealing with large numbers of design parameters. To take advantage of the two algorithms, we propose a two-step method which combines the linear programming and nonlinear optimization methods to design the magnet efficiently.

2 Method

As a first step Linear Programming is used to carry out the topology optimization to get the coils' initial location and shape. Then the nonlinear optimizing methods are used as the second step to further simplify the coils' shape.

2.1 First step: the linear programming method

A system of cylindrical coordinates (r,z,ϕ) is defined and a superconducting magnet that is rotationally symmetric with respected to the z axis is considered. The current density is parallel to the ϕ direction and can be specified by a function $J(r,z)$. $J(r,z)$ is allowed to be nonzero only in the specified domain called feasible coil space. For a multi-section magnet the feasible coil space can be divided into several sections $S_l(l=1,2,\cdots,L)$ as shown in Fig. 1.

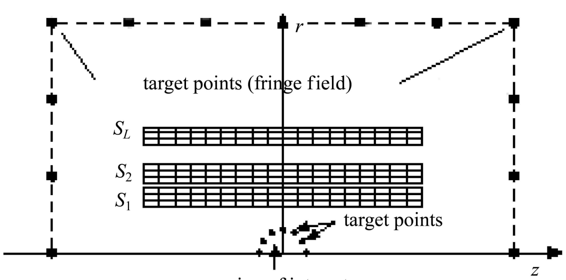

Fig. 1. The multi-sections feasible coil space with numerical grid.

With an introduction of the numerical grid (Fig. 1), the l-th feasible coil space is divided into a set of $N_l(l=1,2,\cdots,L)$ rectangular subregions $D_{l,n}(n=1,2,\cdots,N_l)$. The number of subregions determines the precision of the calculation. We typically use 1000–2000 subregions to ensure an adequate spatial resolution. The current density of each section is assumed to be uniform, i.e. J_l for l-th section. According to the Bio-Savart law, the magnetic field generated by the magnet is expressed as [9]

$$\begin{aligned} B_r(r,z) &= 2\pi\sum_{l=1}^{L}\sum_{n=1}^{N_l} J_l\int_{D_{l,n}} r'\mathrm{d}r'\mathrm{d}z' G_r(r,r',z-z'),\\ B_z(r,z) &= 2\pi\sum_{l=1}^{L}\sum_{n=1}^{N_l} J_l\int_{D_{l,n}} r'\mathrm{d}r'\mathrm{d}z' G_z(r,r',z-z'),\\ B_\phi(r,z) &= 0. \end{aligned} \tag{1}$$

The function G_z and G_r are defined by

$$\begin{aligned} G_r(r_a,r_b,z) &= \frac{\mu_0 z}{4\pi^2 r_a r_b\sqrt{(r_a+r_b)^2+z^2}}\times\\ &\quad\left[-K(k)+\frac{r_a^2+r_b^2+z^2}{(r_a-r_b)^2+z^2}E(k)\right],\\ G_z(r_a,r_b,z) &= \frac{\mu_0}{4\pi^2 r_b\sqrt{(r_a+r_b)^2+z^2}}\times\\ &\quad\left[K(k)-\frac{r_a^2-r_b^2+z^2}{(r_a-r_b)^2+z^2}E(k)\right], \end{aligned} \tag{2}$$

where $\mu_0=4\pi\times10^{-7}$ T$\cdot$m/A is the free space permeability, $K(k)$ and $E(k)$ are the complete elliptic integrals, and

$$k=\sqrt{\frac{4r_a r_b}{(r_a+r_b)^2+z^2}}. \tag{3}$$

The integral over each rectangular subregion can be calculated with a numerical integration routine. However, the field generated by each subregion with small cross section can be approximated by the field from an ideal current loop (with zero cross-sectional area). Eq. (1) then simplifies to

$$\begin{aligned} B_r(r,z) &= 2\pi\sum_{l=1}^{L}\sum_{n=1}^{N_l} J_l a_{l,n} r'_{l,n} G_r(r,r'_{l,n},z-z'_{l,n}),\\ B_z(r,z) &= 2\pi\sum_{l=1}^{L}\sum_{n=1}^{N_l} J_l a_{l,n} r'_{l,n} G_z(r,r'_{l,n},z-z'_{l,n}),\\ B_\phi(r,z) &= 0, \end{aligned} \tag{4}$$

where $a_{l,n}$ is the area of the subregion $D_{l,n}$.

An arbitrarily shaped homogeneous volume is specified by a set of M_h target points $((r_m,z_m),m=1,2,\cdots,M_h)$ (Fig. 1) on the surface of the volume. In order to control the magnet's fringe field, an additional set of M_f target points $((r_m,z_m),m=M_h+1,M_h+2,\cdots,M_h+M_f)$ are introduced (Fig. 1). We note that the current of a loop can be expressed as

$$I_{N_1+\cdots+N_{l-1}+n} = |J_l| a_{l,n}, \quad (5)$$

so the magnetic field at a target point (r_m, z_m) will be:

$$\begin{aligned} B_z(r_m,z_m) &= \sum_{l=1}^{L}\sum_{n=1}^{N_l} Z_{mn} e_l I_{N_1+\cdots+N_{l-1}+n}, \\ B_r(r_m,z_m) &= \sum_{l=1}^{L}\sum_{n=1}^{N_l} R_{mn} e_l I_{N_1+\cdots+N_{l-1}+n}, \\ Z_{mn} &= 2\pi r'_{l,n} G_z(r_m, r'_{l,n}, z_m - z'_{l,n}), \\ R_{mn} &= 2\pi r'_{l,n} G_r(r_m, r'_{l,n}, z_m - z'_{l,n}), \quad (6) \end{aligned}$$

where e_l has two values: 1 and -1, corresponding to positive and negative current direction.

For the superconducting magnet we would like to minimize the costs of the magnet, which are mostly determined by the superconductor volume. Since we assume that the current density within each section is uniform, it is possible to write the coil volume as

$$V_{\text{coils}} = 2\pi \sum_{l=1}^{L}\sum_{n=1}^{N_l} \frac{1}{J_l} r'_{l,n} I_{N_1+\cdots+N_{l-1}+n}\ . \quad (7)$$

Our goal is to find a current distribution that generates the desired field with minimum volume. It can be produced by solving the optimization problem:

$$\begin{aligned} \text{Minimize:} \quad & V_{\text{coils}} \\ \text{Subject to:} \quad & |B_z(r_{m1}, z_{m1}) - B_0| \leqslant \epsilon B_0, \quad (8) \\ & |B_z(r_{m2}, z_{m2})| \leqslant B_{z,\text{shield}}, \quad (9) \\ & |B_r(r_{m2}, z_{m2})| \leqslant B_{r,\text{shield}}, \quad (10) \\ & 0 \leqslant I_{N_1+\cdots+N_{l-1}+n} \leqslant Ic_l, \quad (11) \\ & l = 1, \cdots, L, \\ & m_1 = 1, \cdots, M_h, \\ & m_2 = M_h+1, \cdots, M_h+M_f. \end{aligned}$$

Eq. (8) are the field homogeneity constraints. Usually the homogeneous volume is centered on the z axis, where B_r can be neglected [10]. So it is sufficient to only constrain B_z. In Eq. (9) and Eq. (10) we constrain two field components of the stray field. The inequality constraint Eq. (11) is added to limit the current allowed in l-th section, which is determined by the critical current of the superconductor. This minimization problem is a standard form LP problem which can be solved with a standard LP software package, such as GLPK, PCx and Matlab. Typically, it takes less than 100 s for 1200 candidate coils in a 2.66 GHz processor with 3 GB of RAM.

2.2 Second step: rectangular coils with the nonlinear optimizing method

After the first step, the coil domains are usually non-rectangular. It is difficult to fabricate a magnet with non-rectangular coils, so we have to find a solution that can be implemented with only rectangular coils. The non-rectangular domain is then divided into a set of geometrically simple parts. These parts are replaced with rectangular regions whose shape and location parameters can be determined by using the nonlinear optimizing method. A software toolkit called DAKOTA (Design Analysis Kit for Optimization and Terascale Applications)[1)] developed by the Department of Energy's Sandia National Laboratories is used in the second step. DAKOTA provides a flexible and extensible interface between simulation codes and contains algorithms for optimization with gradient and nongradient-based methods such as MOGA (Multi-objective Genetic Algorithm), pattern search method, etc [11]. A C++ in-house code based on a nonsingular integration method has been developed for the magnetic field calculation of the multi-section magnets. It is linked with DAKOTA through the use of script languages (Bourne shell and Perl). The flowchart of the optimizing iteration is shown in Fig. 2.

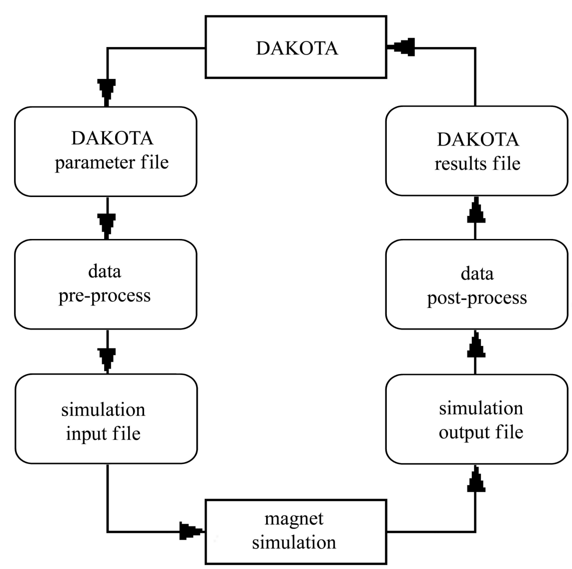

Fig. 2. Flowchart of the optimizing procedure DAKOTA coupled with the in-house code.

1) DAKOTA is a GNU General Public License (GPL) software for Large-Scale Engineering Optimization and Uncertainty Analysis. http://www.cs.sandia.gov/DAKOTA

3 Results

In order to reduce the winding volume, two types of conductors with different cross sectional areas are chosen. The multi-section feasible coil domain is preset according to the specifications of the magnet. Indeed, considering the critical current density of the superconducting wire and the thickness of the winding formers, iterative adjustment of the domain and its division is usually needed. Fig. 3 shows the final design using the LP method. This design has six primary coils and three shielding coils. The current density of the innermost section is 100 A/mm^2 which is half of the outer section. The current direction is opposite in three outermost shielding coils.

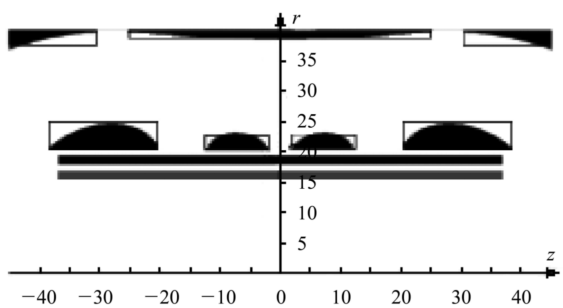

Fig. 3. The resulting coil domains after the LP optimization.

The result of the LP search consists of non-rectangular coils. In order to obtain a design with rectangular coils, we chose seven rectangular coils to replace the non-rectangular ones (Fig. 3). Considering the symmetry, there are twelve design variables (Three variables for each coil). There are two objective functions because we want to minimize both the inhomogeneity and the fringe field of the magnet. A single objective function f is set as a weighted sum of the inhomogeneous objective function and the stray field objective function:

$$f=\omega\sum_{i=1}^{M_h}(B_{z_i}-7.0)^2+\sum_{j=M_h+1}^{M_h+M_f}(|B|_j{}^2), \qquad (12)$$

where ω represents the importance of the inhomogeneous function. Using this method the multi-objective problem is reduced to a single-objective optimization.

The pattern search method has been used to get the optimal design. Fig. 4 shows a graph of the objective function value versus iteration number during the optimization. The total run lasted 360 s and completed 2800 iterations on a 1.2 GHz processor with 1 GB of RAM. Fig. 5 shows the optimal design using the pattern search method.

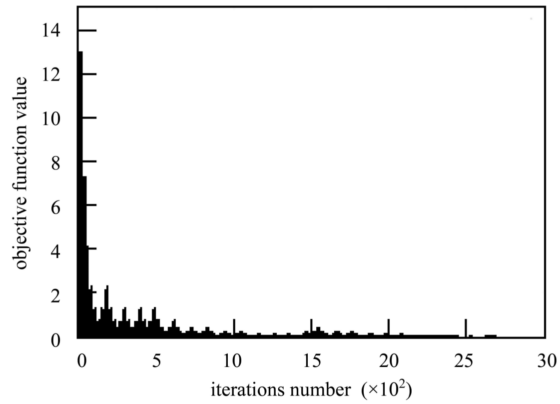

Fig. 4. Plot of the objective function values vs. iteration number.

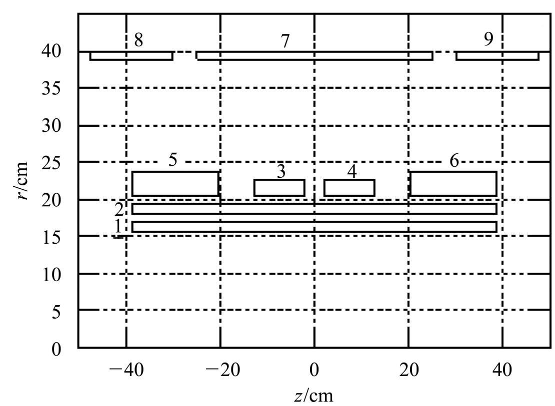

Fig. 5. Obtained optimal design using the pattern search method.

The design was also simulated with the OPERA-3D software from Cobham Technical Services. The results shows that the peak-to-peak inhomogeneity is less than 1.5×10^{-5} (Fig. 6) over a 2 cm diameter spherical volume. As shown in Fig. 7, the 5×10^{-4} T line is limited to a distance of 2 m from the magnet's center. The specifications of the main coils are shown in Table 2.

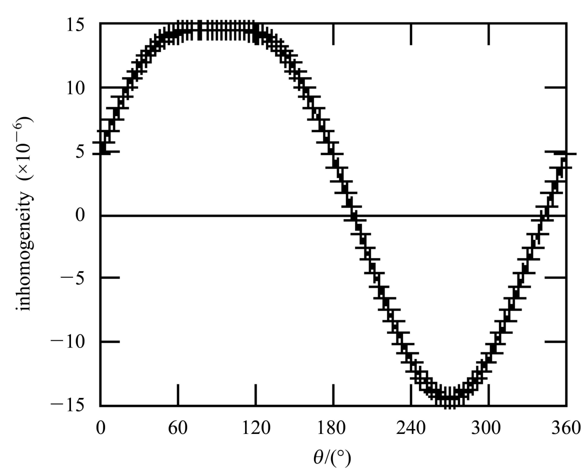

Fig. 6. The peak-to-peak inhomogeneity.

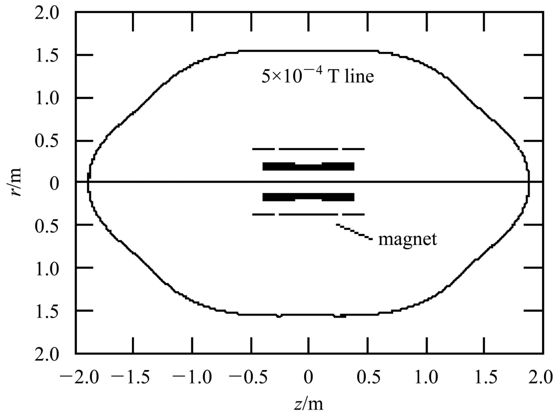

Fig. 7. Stray field distribution.

4 Conclusion

In this paper a two-step optimization method for the design of a multi-section superconducting magnet was introduced. This method uses linear programming to obtain an initial ideal coil arrangement and then adjusts this solution with the optimization software toolkit DAKOTA and the in-house magnetic field calculation code. Using the proposed method, an optimal design for LPT superconducting magnet has been obtained.

Table 2. Specification of the main coils.

parameter	value
central field	7.0001 T
homogeneity	1.5×10^{-5}
5×10^{-4} T line	
radial distance from magnet center	1.5 m
axial distance from magnet center	2.0 m
peak field	
coil 1	7.75 T
coil 2–9	6.47 T
Cu/Sc ratio	
coil 1	3.0
coil 2–9	4.33
current density($J_{\rm op}$)	
coil 1	100 A/mm^2
coil 2–6	200 A/mm^2
coil 7–9	−200 A/mm^2
current margin($\lvert J_{\rm op}/J_{\rm c}\rvert$)	
coil 1	0.5
coil 2–9	0.5

References

1 Quint W, Dilling J, Djekic S et al. Hyperfine Interactions, 2001, **132**: 457
2 Habs D, Cross M, Heinz H et al. Eur. Phys. J. A, 2005, **25**: 57
3 Ringle R, Bollen G, Lawton D et al. Eur. Phys. J. A, 2005, **25**: 59
4 XU H, Conolly S M, Scoot G C et al. IEEE Trans. Magn., 2000, **36**: 476
5 Jens H. Jensen. IEEE Trans. Magn., 2002, **38**: 3579
6 Nicholas R. IEEE Trans. Magn., 2002, **12**: 733
7 Noguchi S, Ishiyama A. IEEE Trans. Magn., 1996, **32**(4): 2655
8 Ishiyama A, Shimizu K, Sakahara A. IEEE Trans. Magn., 1994, **30**(5): 3435
9 Montgomery D B. Solenoid Magnet Design. New York: Wiley-Inter-Science,1969
10 Garrett M W. J. Appl. Phys., 1967, **38**(6): 2563
11 Josh D. Griffin, Patty D. Hough, Tammy G. Kolda et al. DAKOTA, Version 4.2 User's Manual. Sandia National Laboratories, 2008. http://www.cs.sandia. gov/DAKOTA