

Systematic studies on α -decay half-lives for super heavy nuclei^{*}

ZHANG Di-Da(张地大)^{1,2;1)} CHEN Bao-Qiu(陈宝秋)² MA Zhong-Yu(马中玉)²

¹ School of Nuclear Engineering and Technology, East China Institute of Technology, Fuzhou, Jiangxi 344000, China

² China Institute of Atomic Energy, P.O. Box 275(18), Beijing 102413, China

Abstract Radioactive decay of super heavy nuclei via the emission of α -particles has been studied theoretically in the preformed cluster model (PCM). The nucleus-nucleus (NN) potential is obtained by double folding the density distributions of the α -particle and the daughter nucleus with a realistic effective interaction. The M3Y effective interaction, supplemented by a zero-range pseudo-potential for exchange term, is used to calculate the NN potential. The α decay half-lives for 317 nuclei at $Z=102$ – 120 are performed in the PCM framework with the theoretical Q values extracted from the Möller-Nix-Kratz and Liran-Marinov-Zeldes mass tables and are compared with the experimental data. The calculated results are also compared with those obtained by using Q values from the Muntian-Hofmann-Patyk-Sobiczewski and Myers-Swiatecki mass estimates.

Key words preformed cluster model (PCM), α -decay half lives, super heavy nuclei, mass tables

PACS 23.60.+e, 27.90.+b

1 Introduction

More than 40 years ago, theoretical study predicted that there was the possible existence of a stable island of super-heavy elements (SHE) in the periodic table [1]. An exciting challenge in the study of SHE is the quest for a stable island, where the next magic number beyond $Z=82$ and $N=126$ may be located in the island. In order to do so, a continuing effort has been devoted to the investigation of SHE both in experiments and theoretical studies.

The synthesis of SHE has been apparently advanced in recent years [2, 3] using both cold (e.g. Zn on Pb) and warm (e.g. Ca on U, Pu and Cm) fusion reactions at GSI, Dubna and Riken. On the other hand, much theoretical work on SHE is to explore the properties of these nuclei and the next possible magic numbers of both proton and neutron. The macro-microscopic calculations predicted spherical shell closures at $Z=114$ and $N=184$ [4]. In the relativistic mean field (RMF) calculations, Rutz et al. [5] predicted $Z=120$ and $N=172$ as the next magic shells in the spherical RMF theory, meanwhile Pa-

tra et al. [6] using the axially deformed RMF theory predicted $Z=120$ and $N=184$ as the next possible magic numbers for SHE. In the Skyrme Hartree-Fock calculations with parameter sets of SKI3 and SKI4 the most pronounced shell effects at $Z=120$ and $N=184$ [7] are obtained, while in the Hartree-Fock-Bogoliubov calculations with the finite range Gogny force one predicted $Z=120, 126$ and $N=172, 184$ as the possible proton shell closures [7]. Of late, Tapas et al. [8] applied the new effective field theory to analyze the isotopic and isotonic chains of SHE and search the next shell closures. They predicted $Z=120, N=172$ and $Z=120, N=258$ as possible spherical doubly magic super heavy nuclei. As mentioned above, most theoretical studies predict $Z=120$ as a possible magic number of proton in SHE. To investigate if the so-called stable island could really exist around the above Z and N values, one could explore whether these nuclei have long half-lives of α decay using a quantum tunneling model.

It is known that theoretical explanations for α radioactive decay in terms of a quantum mechanical barrier penetration are a great success. Previously,

Received 28 July 2009

^{*} Supported by National Natural Science Foundation of China (10875150, 10775183, 10535010) and Major State Basic Research Development Programme in China (2007CB815000)

1) E-mail: zhangdidazhangdida@126.com

©2009 Chinese Physical Society and the Institute of High Energy Physics of the Chinese Academy of Sciences and the Institute of Modern Physics of the Chinese Academy of Sciences and IOP Publishing Ltd

such researches were mainly concentrated on two theoretical models, one is the preformed cluster model (PCM) [9, 10], and the other is the super asymmetric fission model (SAFM) [11, 12]. Basu [13] introduced double folding (DF) potential with M3Y and density dependent M3Y (DDM3Y) to the PCM and SAFM approaches to investigate the α decay half-lives for SHE. The nucleus-nucleus (NN) potentials between the α -particle and the daughter nuclei are obtained in the DF model [14]. The DF potential obtained using the M3Y effective interaction supplemented by a zero-range pseudo-potential for the single nucleon exchange is more appropriate because of its microscopic nature. The DF potential with M3Y and DDM3Y has been extended to investigate the decay half-lives for SHE [15–20] in the PCM and SAFM approach recently. Although the two approaches with the experimental decay energies can produce the α decay half lives in good agreement with the experimental data, both can lead to large deviations from the experimental data while using theoretical Q values instead. Therefore a good mass formula is a crucial factor for the theoretical predictions of the α decay half lives. The main purpose of this work is to test various mass formulae and systematically study the α decay half lives for SHE. We adopt Q values extracted from Möller-Nix-Kratz (MNK) [21] and Liran-Marinov-Zeldes (LMZ) [22] mass tables, which are calculated in a finite-range droplet model and semiempirical shell-model mass equation respectively. The α decay half lives for 317 nuclei ($Z = 102$ – 120) are performed in the PCM framework and compared with the existing experimental data. The present results are also compared with those obtained from the other model [20] using Q values from Muntian-Hofmann-Patyk-Sobiczewski (M) [23] and Myers-Swiatecki (MS) [24–26] mass tables.

The paper is arranged as follows. The formulae of the PCM are presented in Section 2, the calculated results and discussions are given in Section 3, and Section 4 gives a brief summary of this work.

2 Preformed cluster model and double folded potential

Buck et al. [9, 10] proposed a preformed cluster model for α -decay. The ground state of the parent nucleus in this cluster model is assumed to be an α -particle orbiting the daughter nucleus. The orbit is denoted by a large value of the global quantum number $G = 2n + L$, where n is the node number of radial motion and L is the angular momentum. In the

PCM, the cluster is assumed to be formed before it penetrates the barrier and its preformation factor is included in the calculations.

The total interaction potential between the α particle and the daughter nucleus is equal to the sum of the nuclear potential, the Coulomb potential, and the centrifugal barrier,

$$V(R) = V_N(R) + V_C(R) + \frac{\hbar^2 L(L+1)}{2\mu R^2}, \quad (1)$$

where R is the distance between the α particle and the daughter nucleus, and $\mu = \frac{M_\alpha M_d}{M}$ is the reduced mass. M_α, M_d and M are the masses of the α -particle, the daughter nucleus, and the parent nucleus, respectively. L is the angular momentum, a Langer modified centrifugal barrier is used with $L(L+1)$ replaced by $\left(L + \frac{1}{2}\right)^2$.

The nuclear potential $V_N(R)$ has been given in various forms. In the present work, the nuclear potential is obtained in a double folding model as [14, 27]

$$V_N(R) = \lambda \int \int \rho_\alpha(\mathbf{r}_1) \rho_d(\mathbf{r}_2) v(\varepsilon, \mathbf{s}) d\mathbf{r}_1 d\mathbf{r}_2, \quad (2)$$

where λ is the renormalization factor, ρ_α and ρ_d are the density distribution functions of the α particle and the daughter nucleus, respectively. $\mathbf{s} = \mathbf{R} + \mathbf{r}_2 - \mathbf{r}_1$ is the relative vector between interacting nucleon pair. The density distribution function for the α -particle has the Gaussian form [14, 27]:

$$\rho_\alpha(r) = 0.4229 \exp(-0.7024r^2). \quad (3)$$

The density distribution of the daughter nucleus can be described by the spherically symmetric Fermi function [28–30]:

$$\rho_d(r) = \frac{\rho_0}{1 + \exp((r-c)/a)}. \quad (4)$$

The equivalent sharp radius r_ρ , the half density radius c and the diffuseness a for the Fermi density distribution are given by [31, 32]:

$$c = r_\rho / (1 - \pi^2 a^2 / 3r_\rho^2), \quad r_\rho = 1.13 A_d^{1/3}, \quad a = 0.54 \text{ fm}. \quad (5)$$

The value of the central density ρ_0 is fixed by equating the volume integral of the density distribution function to the mass number A_d of the daughter nucleus.

The nucleon-nucleon effective interaction between the α -particle and the daughter nucleus, is given by the M3Y and has following form [14, 27]:

$$v(\varepsilon, \mathbf{s}) = 7999 \frac{e^{-4s}}{4s} - 2134 \frac{e^{-2.5s}}{2.5s} - 276(1 - 0.005\varepsilon)\delta(\mathbf{s}), \quad (6)$$

where the last term is the zero range pseudo-potential representing the single-nucleon exchange term and ε

is the energy per nucleon.

The Coulomb potential between the daughter nucleus and α -particle is taken as [28, 30, 33]:

$$V_C(R) = \begin{cases} \frac{Z_\alpha Z_d e^2}{R} & R > R_c \\ \frac{Z_\alpha Z_d e^2}{2R_c} \left[3 - \left(\frac{R}{R_c} \right)^2 \right] & R \leq R_c, \end{cases} \quad (7)$$

where $R_c = 1.2A_d^{1/3}$ and A_d is the mass number of the daughter nucleus [28, 30, 33]. Z_α and Z_d are atomic numbers of the α particle and the daughter nucleus, respectively. The renormalization factor λ in the nuclear potential is determined by imposing the Bohr-Sommerfeld quantization condition:

$$\int_{R_1}^{R_2} dr \sqrt{\frac{2\mu}{\hbar^2} [Q - V(r)]} = (2n+1) \frac{\pi}{2} = (G-L+1) \frac{\pi}{2}. \quad (8)$$

The global quantum number G is given by, [34]

$$G = \begin{cases} 20 & (N > 126) \\ 18 & (82 < N \leq 126) \\ 16 & (N \leq 82). \end{cases} \quad (9)$$

The α decay width Γ_α is given by the following formula, [10, 35]

$$\Gamma_\alpha = PF \frac{\hbar^2}{4\mu} \exp\left[-2 \int_{R_2}^{R_3} k(R) dR\right], \quad (10)$$

where P is the preformation factor, the factor F is determined as:

$$F \int_{R_1}^{R_2} \frac{dR}{2k(R)} = 1. \quad (11)$$

And the wave number is given by:

$$k(r) = \sqrt{\frac{2\mu}{\hbar^2} |Q - V(R)|}, \quad (12)$$

where μ is the reduced mass and Q is the release energy of the α decay.

$$Q = (M - M_\alpha - M_d)C^2. \quad (13)$$

R_i are the classical turning points. The second and third turning points of the action integral are evaluated numerically by using the following equations:

$$V(R_2) = Q = V(R_3). \quad (14)$$

The α decay half-life is then related to the width by [35]

$$T_{1/2} = \hbar \ln 2 / \Gamma_\alpha. \quad (15)$$

The preformation factor P of the α cluster is chosen to be 1 for even-even nuclei, 0.6 for odd- A nuclei, and

$P=0.35$ for odd-odd nuclei.

3 Results and discussion

We evaluate the second and third turning points by solving Eq. (14) along with the nuclear potential and the Coulomb potential given by Eq. (2) and Eq. (7) respectively, and R_1 equals zero. We consider that all the decay processes are zero angular momentum transfer, spherical charge distribution for Coulomb potential and M3Y effective interaction for the nucleon-nucleon interaction. The theoretical α decay energies used in our present work are taken from two different mass tables, viz. $Q_{\text{th}}^{\text{MKNK}}$ [21] and $Q_{\text{th}}^{\text{LMZ}}$ [22]. In our calculations the values of the renormalization factor λ used in calculating the nuclear potentials are varying from 0.51 to 0.58, and it decreases as the proton numbers increase. The theoretical alpha decay half lives obtained with $Q_{\text{th}}^{\text{MKNK}}$ and $Q_{\text{th}}^{\text{LMZ}}$ in the area of $Z=102-120$ are shown in Fig. 1 and Fig. 2. The results obtained with the two mass formulae show quite different trends: the values of $T_{1/2}^{\text{LMZ}}$ vary regularly in our calculated regions, the half lives increase monotonously as a function of the neutron numbers for all isotope chains due to the value of $Q_{\text{th}}^{\text{LMZ}}$ decreasing monotonously. On the contrary, the values of $T_{1/2}^{\text{MKNK}}$ behave structured.

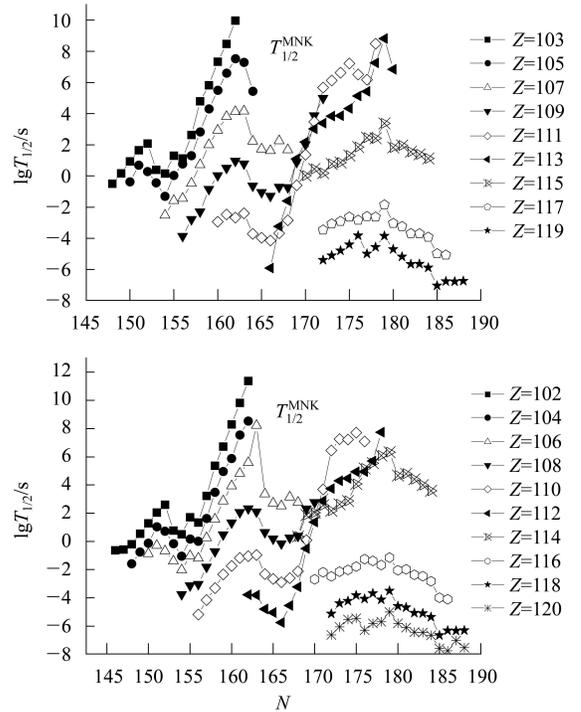


Fig. 1. Theoretical α -decay half-lives of nuclei calculated in the PCM with $Q_{\text{th}}^{\text{MKNK}}$.

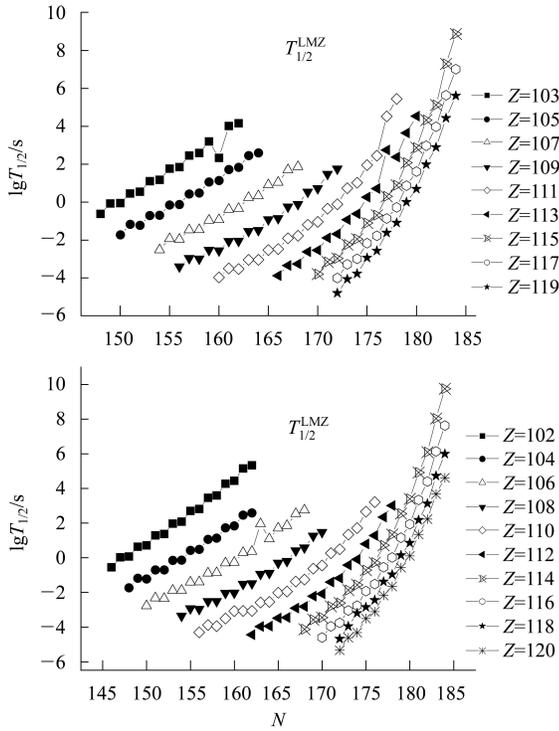


Fig. 2. Theoretical α -decay half-lives of nuclei calculated in the PCM with Q_{th}^{LMZ} .

To study the prediction power of the theoretical Q values used in the present work, we compared our results with the experimental data [3, 36–44]. Fig. 3 shows the comparison between the experimental data and the theoretical α decay half lives evaluated with Q_{th}^{MNK} and Q_{th}^{LMZ} . In general the theoretical values $T_{1/2}^{LMZ}$ can reproduce the experimental data reasonably. In particular, both of the two theoretical estimations can reproduce the experimental data very well when $Z=116$. In most cases the theoretical values $T_{1/2}^{MNK}$ overestimated the experimental data. In some cases, for example, when $Z=113$ and 114, the theoretical results deviate from the experimental data up to two or three orders of magnitude.

We also compare our results calculated by using Q_{th}^{MNK} [21] with other theoretical evaluations by Chowdhury, etc. [20] for the even Z nuclei in the area of $Z=102$ – 120 . They calculated the α -decay half lives of super heavy nuclei in the SAFM with DDM3Y interaction and Q_{th}^M [23] and Q_{th}^{MS} [24–26]. The comparison between our results and Chowdhury's is shown in Fig. 4. From Fig. 4 we can see that for each isotope chain the α -decay half lives $T_{1/2}^{MNK}$ varying with respect to the neutron number have a very similar

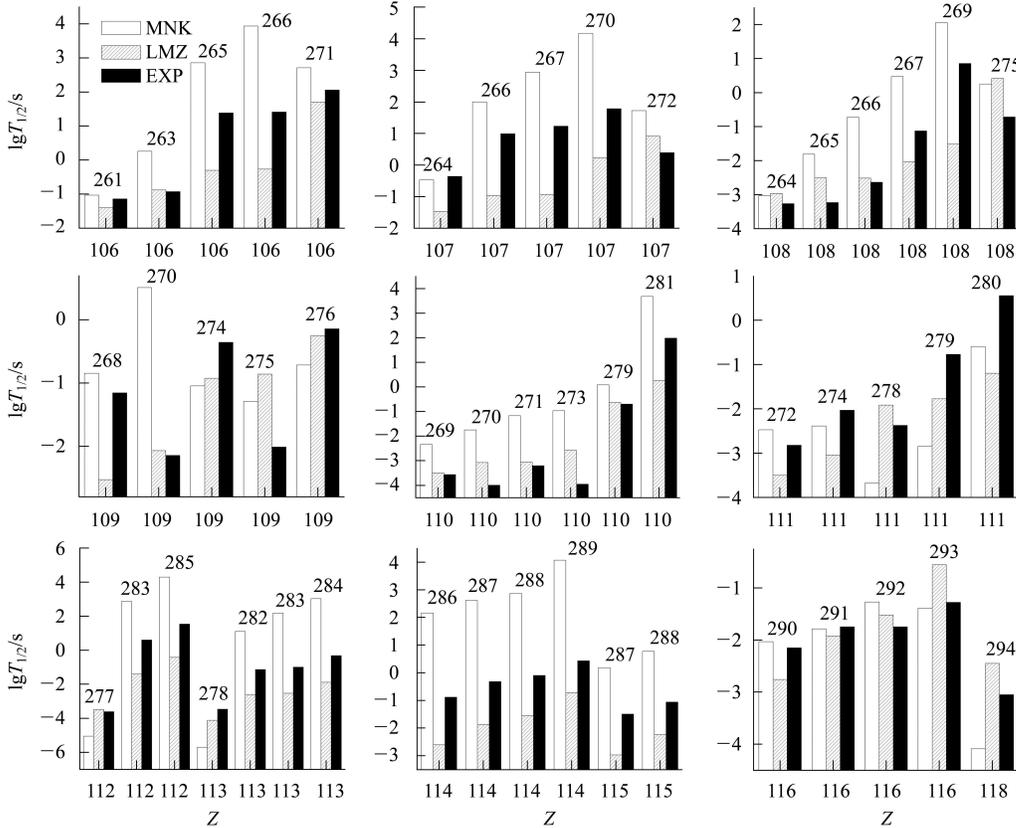


Fig. 3. Comparison of theoretical and experimental α decay half-lives $T_{1/2}(s)$ for 50 nuclei. The hollow columns and the diagonal filled line columns show the calculated half lives in the PCM with M3Y effective interaction, and Q_{th}^{MNK} or Q_{th}^{LMZ} respectively. The filled columns show the experimental data.

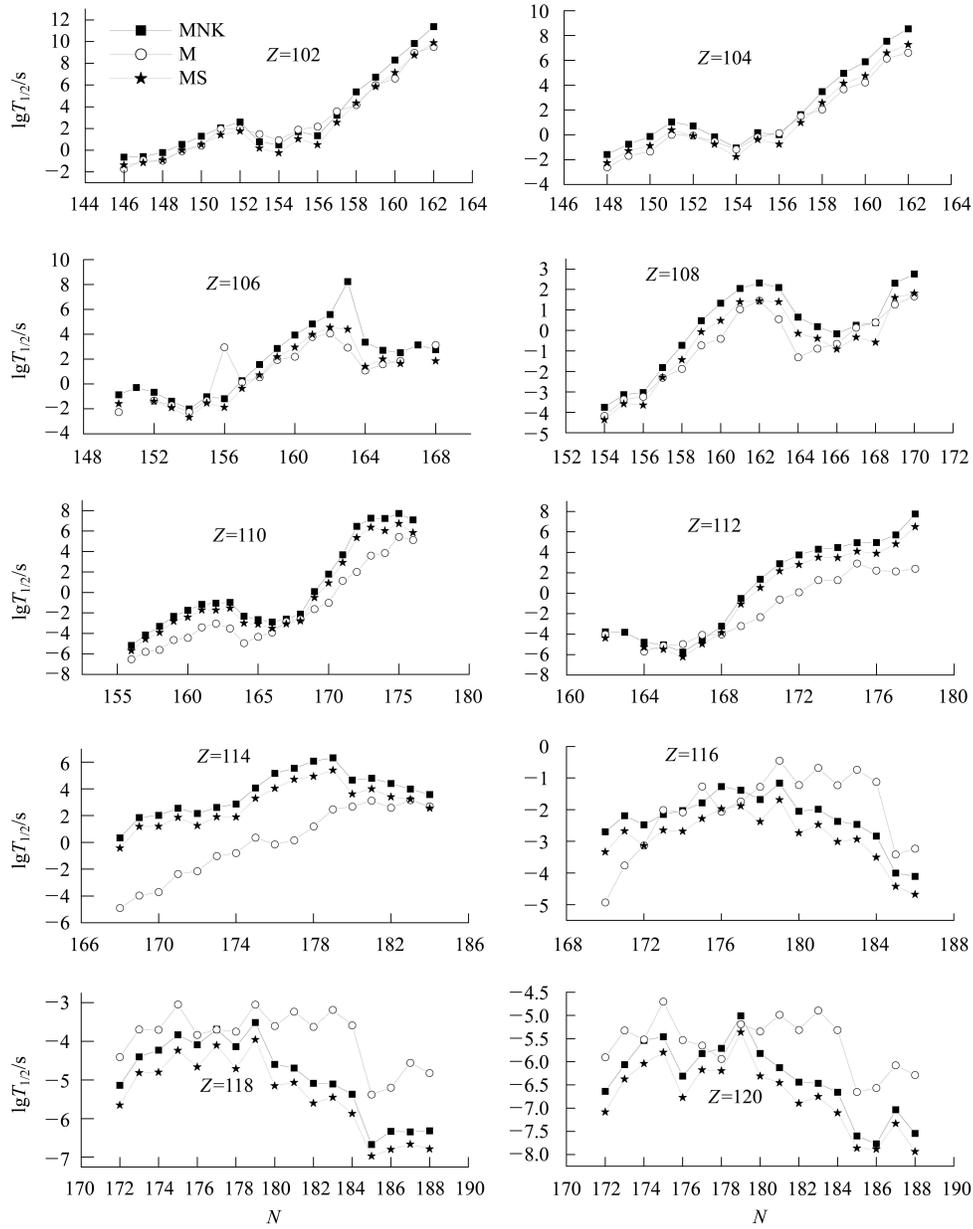


Fig. 4. Comparison of $T_{1/2}^{\text{MNK}}$ with $T_{1/2}^{\text{M}}$ and $T_{1/2}^{\text{MS}}$. The rectangles are the α -decay half-lives calculated in PCM with M3Y interaction and $Q_{\text{th}}^{\text{MNK}}$ in the present work. The circles and asterisks are evaluated in SAFM framework with DDM3Y interaction using Q_{th}^{M} and $Q_{\text{th}}^{\text{MS}}$, respectively.

trend as $T_{1/2}^{\text{MS}}$, but in most cases the former ones are slightly larger than the latter ones due to the fact that the decay energies $Q_{\text{th}}^{\text{MNK}}$ are smaller than $Q_{\text{th}}^{\text{MS}}$. For nuclei with $Z=102$ and 104 , the values of $T_{1/2}^{\text{M}}$ are very close to those of $T_{1/2}^{\text{MNK}}$ and $T_{1/2}^{\text{MS}}$ though they are different. While the proton number Z increases, the value of $T_{1/2}^{\text{M}}$ becomes more and more obviously deviated from the other two.

The present results of the α decay half lives do not indicate a pronounced island of increased stability around $Z=114$ or $Z=120$. In general the α decay half lives decrease with increasing Z up to $Z=120$.

This observation is consistent with that observed in Ref. [20].

4 Summary

We have performed systematic evaluations of the α -decay half lives for 317 nuclei in the area of $Z=102$ – 120 in the PCM framework with the M3Y interaction and theoretical decay energies $Q_{\text{th}}^{\text{MNK}}$ and $Q_{\text{th}}^{\text{LMZ}}$. The comparisons of our results with the experimental data show that the theoretical half lives can reproduce the experimental data reasonably in most

cases. We also compared our results obtained by using $Q_{\text{th}}^{\text{MKN}}$ with those of Ref. [20] in the SAFM framework with DDM3Y interaction, and Q_{th}^{M} or $Q_{\text{th}}^{\text{MS}}$. In general good agreement is found. In addition, our re-

sults do not indicate that there is a long life island at $Z=114$ or 120 . The present data will be useful for further studies of the α -decay half-lives of SHE.

References

- 1 Gareev F A, Kalinkin B N, Sobiczewski A. Phys. Lett., 1966, **22**: 500
- 2 Hofmann S, Muenzenberg G. Rev. Mod. Phys., 2000, **72**: 733; Hofmann S et al. Eur. Phys. J. A, 2007, **32**: 251; Armbruster P. Eur. Phys. J. A, 2000, **7**: 7
- 3 Oganessian Y T et al. Nature (London), 1999, **400**: 242; Oganessian Y T et al. J. Phys. G: nucl. Part. Phys., 2007, **34**: R165
- 4 Möller P, Nix R. Nucl. Phys. A, 1992, **549**: 64
- 5 Rutz K, Bender M, Greiner G. Phys. Rev. C, 1997, **56**: 238
- 6 Patra S K, Gupta R K, Greiner G, Mod. Phys. Lett. A, 1997, **12**: 1727; Patra S K, WU C L, Praharaaj C R et al. Nucl. Phys. A, 1999, **651**: 117
- 7 Bender M, Rutz K, Reinhard P G et al. Phys. Rev. C, 1999, **60**: 034304; Kruppa A T, Bender M, Nazarewicz W et al. Phys. Rev. C, 2000, **61**: 034313
- 8 Tapas S, Patra S K, Sharma B K et al. Phys. Rev. C, 2004, **69**: 044315
- 9 Buck B, Merchant A C, Perez S M. Phys. Rev. Lett., 1990, **65**: 2975; J. Phys. G: Nucl. Part. Phys., 1991, **17**: 1223; 1992, **18**: 143
- 10 Buck B, Merchant A C, Perez S M. Phys. Rev. C, 1992, **45**: 2247; Buck B, Merchant A C, Perez S M. At. Data Nucl. Data Table, 1993, **54**: 53
- 11 Poenaru D N, Greiner W, Depta K et al. At. Data Nucl. Data Tables, 1986, **34**: 423
- 12 Poenaru D N, Schnabel D, Greiner W, et al. At. Data Nucl. Data Tables, 1991, **48**: 231
- 13 Basu D N. Phys. Rev. C, 2002, **66**: 027601; Phys. Lett. B, 2003, **566**: 90
- 14 Satchler G R, Love W G. Phys. Rep., 1979, **55**: 183
- 15 XU C, REN Z Z. Nucl. Phys. A, 2005, **753**: 174; XU C, REN Z Z. Phys. Rev. C, 2006, **73**: 041301
- 16 Gambhir Y K, Bhagwat A, Gupta M. Phys Rev. C, 2005, **71**: 037301
- 17 Peter Mohr. Phys. Rev. C, 2006, **73**: 031301(R)
- 18 Chowdhury P R, Samanta C, Basu D N. Phys. Rev. C, 2006, **73**: 014612
- 19 Samanta C, Chowdhury P R, Basu D N. Nucl. Phys. A, 2007, **789**: 142
- 20 Chowdhury P R, Samanta C, Basu D N. At. Data. Nucl. Data Tables, 2008, **94**: 781
- 21 Möeller P, Nix J R, Kratz K L. At. Data Nucl. Data Tables, 1997, **66**: 131
- 22 Liran S, Marinov A, Zeldes N. nucl-th/0102055
- 23 Myers W D, Swiatecki W J. Nucl. Phys. A, 1996, **601**: 141
- 24 Muntian I, Patyk Z, Sobiczewski A. Acta Phys. Pol. B, 2001, **32**: 691
- 25 Muntian I, Hofmann S, Patyk Z et al. Acta Phys. Pol. B, 2003, **34**: 2073
- 26 Muntian I, Patyk Z, Sobiczewski A. Phys. At. Nucl., 2003, **66**: 1015
- 27 Kobos A M, Brown B A, Hodgson P E et al. Nucl. Phys. A, 1982, **384**: 65
- 28 Walecka J D. Theoretical Nuclear Physics and Subnuclear Physics. Oxford: Oxford Univ. Press, 1995
- 29 Hahn B, Ravenhall D G, Hofstadter R. Phys. Rev., 1956, **101**: 1131
- 30 Bohr A, Mottelson B R. Nuclear Structure, Vol. 1. Singapore: World Scientific, 1998
- 31 Srivastava D K, Basu D N, Ganguly N K. Phys. Lett. B, 1983, **124**: 6
- 32 Srivastava D K, Ganguly N K, Hodgson P E. Phys. Lett. B, 1974, **51**: 439
- 33 Preston M A, Bhaduri R K. Structure of the Nucleus. Addison-Wesley, Reading, MA, 1975
- 34 Buck B, Johnston J C, Merchant A C et al. Phys. Rev. C, 1996, **53**: 2841
- 35 Gurvitz S A, Kaelbermann G. Phys. Rev. Lett., 1987, **59**: 262
- 36 Hofmann S, Ninov V, Heßberger F P et al. Z. Phys. A, 1995, **350**: 277
- 37 Hofmann S. Rep. Prog. Phys., 1998, **61**: 639
- 38 Hofmann S, Ninov V, Heßberger F P et al. Z. Phys. A, 1996, **354**: 229
- 39 Düllmann Ch E, Brüche W, Dressler R et al. Nature, 2002, **418**: 859
- 40 Hofmann S, Ninov V, Heßberger F P et al. Z. Phys. A, 1995, **350**: 281
- 41 Morita K et al. J. Phys. Soc. Jpn., 2004, **73**: 2593
- 42 Wilk P A, Gregorich K E, Türler A et al. Phys. Rev. Lett., 2000, **85**: 2697
- 43 Audi G, Wapstra A H. Nucl. Phys. A, 2003, **729**: 3
- 44 Hofmann S, Heßberger F P, Ackermann D et al. Eur. Phys. J. A, 2001, **10**: 5