Stochastic motion of the untrapped particle in electrostatic mode

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Abstract The stochastic energy diffusion of the untrapped particle in the electrostatic mode is investigated analytically. We find that the equilibrium electrostatic field of periodical structure plays the same role as the usual focusing magnetic field to lead the test particle to stochastic motion. The resonance overlapping criterion for the random state is given, and also the Fokker-Planck-Kolmogorov approach to diffusion is considered for our system.

Key words stochastic acceleration, electrostatic mode, diffusion

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1 Introduction

Theoretical models have been developed to investigate the transition to stochastic motion of electrons under the action of an electromagnetic wave propagating obliquely with respect to a uniform magnetic field [1–5]. In particular, the threshold value of the field amplitude has been studied. Following the Fokker-Planck-Kolmogorov (FPK) approach, the energy diffusion in the presence of a wave propagating perpendicularly to the magnetic field was studied, and the expression of the diffusion coefficient has been given analytically.

The aim of the present paper is the investigation of the energy diffusion of untrapped particle in BGK (Bernstein-Greene-Kruskal) mode moving in a perturbed monochromatic electrostatic wave. BGK mode is a kind of general electrostatic equilibrium [6], whose electrostatic field possesses spatial structure. In the case of the untrapped particle, one can build an equilibrium electrostatic field of periodical structure in space and see that the equilibrium electrostatic potential plays the same role as the magnetically confined field to lead the particle to the random state.

2 Electrostatic equilibrium

In the case of the untrapped particle, the one-dimension distribution function of the electron and the ion can be expressed as

$$f_a = V_{a0}\delta\left(v_x^2 + \frac{2q_a\phi}{m_a} - V_{a0}^2\right), \tag{1}$$

$$V_{a0}^2 \gg \left|\frac{2q_a\phi}{m_a}\right|, \tag{2}$$

where $q_a, m_a (a=i,e)$ are the charge and mass of the electron and the ion, $V_{a0}(a=i,e)$ is constant, and ϕ is the electrostatic potential and satisfies

$$\frac{\mathrm{d}^2\phi}{\mathrm{d}x^2} = -4\pi n_0 e\left(V_{i0}\left(V_{i0}^2 - \frac{2e\phi}{m_i}\right)^{-\frac{1}{2}} - V_{e0}\left(V_{e0}^2 + \frac{2q_a\phi}{m_i}\right)^{-\frac{1}{2}}\right), \tag{3}$$

where $n_0 = n_e = n_i$ is the mean density of the particle. Applying Eq. (2) to (3) has the solution

$$\phi = \phi_{\max}\sin(\varLambda_{\mathrm{D}}x), \tag{4}$$

$$\varLambda_{\mathrm{D}} = (4\pi n_0 e^2)^{-\frac{1}{2}}\left(\frac{1}{m_i V_{i0}^2} + \frac{1}{m_e V_{e0}^2}\right)^{-\frac{1}{2}}$$

can be considered as the generalized Debye length.

3 Hamiltonian of the charged particle

Suppose that there exists a perturbed monochromatic electrostatic wave $\psi\cos(kx - wt)$, then the

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Hamiltonian of a particle moving in the field has the following form

$$H=\frac{1}{2m_a}p^2+q_a\phi_{\max}\sin(\varLambda_{\rm D}x)+q_a\psi\cos(kx-wt). \quad (5)$$

Generally, Eq. (5) describes the motion of charge particle in two electrostatic waves. The interaction of the two waves can lead the particle to move stochastically, the first term on the right side of Eq. (5) is considered as the unperturbed part, and the rest as the perturbed part. However, the characteristics of the second and third terms on the right side of Eq. (5) are quite different. The second term is a small intrinsic term of our system which can be seen to play the same role as the confined magnetic field in the later analysis, while the third term represents the external perturbation whose amplitude can be very large. In the following, we want to derive the criterion for the occurrence of stochasticity, driven by only the third term. Therefore it is natural to divide H into the following form.

$$H = H_0+H_1,$$

$$H_0 = \frac{1}{2m_a}p^2+q_a\phi_{\max}\sin(\varLambda_{\rm D}x), \quad (6)$$

$$H_1 = q_a\psi\cos(kx-wt). \quad (7)$$

In the absence of the pertubation, H_0 is a periodical function of the variable x. One can transform the variables (p,x) to action-angle variables (I,θ):

$$I=\int_0^{\frac{2\pi}{\varLambda_{\rm D}}}[2m_a(H_0-q_a\phi_{\max}\sin(\varLambda_{\rm D}x))]^{\frac{1}{2}}{\rm d}x.$$

Using $H_0\gg q_a\phi_{\max}$, one gets

$$I\approx 2\pi\frac{(2m_aH_0)^{\frac{1}{2}}}{\varLambda_{\rm D}}, \quad (8)$$

$$\frac{{\rm d}\theta}{{\rm d}t}=\frac{\partial H_0}{\partial I}=\Omega. \quad (9)$$

The variable x can be expressed as where

$$x=\frac{\varLambda_{\rm D}}{2\pi}\theta+a\sin\theta, \quad (10)$$

where a is the amplitude. Substituting Eq. (10) into (7), we get

$$H_1=q_a\psi\sum_{m=-\infty}^{+\infty}J_m(ka)\cos\left[\left(m+\frac{k\varLambda_{\rm D}}{2\pi}\right)\theta-wt\right]. \quad (11)$$

Using Eqs. (11), (8), we can express H as

$$H = \frac{\varLambda_{\rm D}^2}{8m_a\pi^2}I^2+q_a\psi\sum_{m=-\infty}^{+\infty}J_m(ka)\times \cos\left[\left(m+\frac{k\varLambda_{\rm D}}{2\pi}\right)\theta-wt\right]. \quad (12)$$

4 Island overlapping criterion and FPK equation

For low amplitude ψ, the particle dynamics is well described by the Hamiltonian (12), while increasing ψ, the motion is no longer regular and the transtion to chaotic motion occurs for $\psi\geqslant\psi_{\rm c}$, where $\psi_{\rm c}$ is the threshold value. We use the Chirikov criterion [7] for the resonance overlapping to estimate $\psi_{\rm c}$. Let ΔI_n be the half-width of an island separatrix around the nth harmonic,and δI_n the distance between the nth and $n(+)$ 1st in the absence of the perturbation, then the resonance condition

$$\left(n+\frac{k\varLambda_{\rm D}}{2\pi}\right)\frac{{\rm d}\theta}{{\rm d}t}-w=0$$

defines the resonant value of the action

$$I_n=\frac{4m_a\pi^2w}{\varLambda_{\rm D}^2\left(n+\frac{k\varLambda_{\rm D}}{2\pi}\right)}.$$

The distance between the nth and n+1st harmonics is then

$$\delta I_n=\frac{4m_a\pi^2w}{\varLambda_{\rm D}^2\left(n+\frac{k\varLambda_{\rm D}}{2\pi}\right)\left(n+\frac{k\varLambda_{\rm D}}{2\pi}+1\right)}. \quad (13)$$

The expression for the separatrix width can be computed whenever the Hamiltonian (12) is well approximated by

$$H^{(n)}(I,\theta) = \frac{\varLambda_{\rm D}^2}{8m_a\pi^2}I^2+q_a\psi J_n(ka)\times \cos\left[\left(n+\frac{k\varLambda_{\rm D}}{2\pi}\right)\theta-wt\right].$$

The half-width of the separatrix is given by letting $H^{(n)}=q_a\psi J_n(ka)$, then

$$\Delta I_n=2\pi(2m_a)^{\frac{1}{2}}(2q_a\psi J_n(ka))^{\frac{1}{2}}\frac{1}{\varLambda_{\rm D}}, \quad (14)$$

and the criterion for the overlapping of the two resonances is then

$$s_n(\psi)=\frac{\Delta I_n+\Delta I_{n+1}}{\delta I_n}\geqslant 1. \quad (15)$$

Finally the threshold value of $\psi_{\rm c}$ is estimated as

$$\psi_{\rm c}^{-\frac{1}{2}} = \left(\frac{q_a}{m_a}\right)^{\frac{1}{2}}\left(n+\frac{k\varLambda_{\rm D}}{2\pi}\right)\left(n+\frac{k\varLambda_{\rm D}}{2\pi}+1\right)\times \frac{(J_n(ka))^{\frac{1}{2}}+(J_{n+1}(ka))^{\frac{1}{2}}}{\pi w}\varLambda_{\rm D}. \quad (16)$$

Applying the FPK approach [8–10], a diffusion equation for the distribution function averaged over the phases $f(I,t)$ can be written for the system described by the Hamiltonian (12)

$$\frac{\partial f}{\partial t}=\frac{1}{2}\frac{\partial}{\partial I}\left(D(I)\frac{\partial f}{\partial I}\right). \quad (17)$$

The actual expression for the diffusion coefficient $D(I)$ depends on the assumptions of the phase dynamics. If the effects of the phase correlations are taken into account $D(J)$ reads as $\frac{\ll(\Delta I)^2\gg}{t}$, where $\ll\cdots\gg$ represents the average over the initial phase.

$$D(I)\approx q_a^2\psi^2\sum_{m=0}^{+\infty}\frac{\frac{1}{\tau_c}}{\left[\left(m+\frac{k\Lambda_D}{2\pi}\right)\Omega-w\right]^2+\frac{1}{\tau_c^2}}, \quad (18)$$

where τ_c is phase correlation decay time. When $\tau_c\to 0$, $D(J)$ becomes

$$D(I)=\pi q_a^2\psi^2\sum_{m=0}^{+\infty}\delta\left(\left(m+\frac{k\Lambda_D}{2\pi}\right)\Omega-w\right). \quad (19)$$

5 Summary

In summary, the equilibrium electrostatic field is found to play the same role as the usual uniform magnetic field to lead particle moving in a monochromatic electrostatic wave to the random state.

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