Single-particle resonance states of ^{122}Zr in relativistic mean-field theory combined with real stabilization method*

MEI Hua(梅花)[1] CHEN Hong(陈洪)[1;1)] YAO Jiang-Ming(尧江明)[2;2)]

1 (School of Physical Science and Technology, Southwest University, Chongqing 400715, China)
2 (School of Physics, Peking University, Beijing 100871, China)

Abstract Single-particle resonance states of ^{122}Zr are studied in the real stabilization method within the framework of relativistic mean field theory. Two efficient methods are adopted to extract the resonance energy and width of ^{122}Zr. The results are compared with those obtained from the analytic continuation in the coupling approach and scattering phase-shift methods.

Key words resonance state, real stabilization method, resonance energy and width, density of states

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1 Introduction

The development of radioactive ion beam facilities have made it possible to study exotic nuclei. Most of these nuclei are loosely bound and often exhibit resonances with a pronounced single-particle character since the Fermi surface approaches the continuum. Since resonances have a great effect on the formation of halo and giant halo[1], much effort has been devoted to the study of resonance state in nuclei. So far, several methods have been used to investigate the property of resonance, including R-matrix[2], K-matrix[3], S-matrix[4] and several bound-state-like methods, such as Real Stabilization Method (RSM)[5], Analytical Continuation in the Coupling Constant (ACCC)[6] approach and Complex Scaling Method (CSM)[7].

In the past decades, relativistic mean-field (RMF) theory has been successfully applied to study the properties of nuclei at and far from the line of β-stability. Combined with RMF theory, there are several approaches being developed to deal with resonance states, including RMF-SPS[8], RMF-ACCC[9] and RMF-RSM[10]. Furthermore, different physics quantities, including phase shift[11, 12] and density of states[13], are proposed to extract the resonance energy and width.

In this paper, we will study the single-particle resonance states in ^{122}Zr with the newly developed RMF-RSM approach. Two efficient methods will be adopted to extract the corresponding resonance energy and width.

2 The relativistic mean-field model

The starting point of the RMF theory with meson-exchange providing nucleon-nucleon interaction is the standard effective Lagrangian density constructed with the nucleon field (ψ), two isoscalar meson fields (σ and ω_μ), the isovector meson field ($\boldsymbol{\rho}_\mu$) and the photon field (A_μ)[14, 15],

$$\mathcal{L} = \bar{\psi}\left[\mathrm{i}\gamma^\mu\partial_\mu - M - g_\sigma\sigma - g_\omega\gamma^\mu\omega_\mu - g_\rho\gamma^\mu\boldsymbol{\tau}\cdot\boldsymbol{\rho}_\mu - e\gamma^\mu A_\mu\frac{1-\tau_3}{2}\right]\psi + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{1}{2}m_\sigma^2\sigma^2 - \frac{1}{3}g_2\sigma^3 - \frac{1}{4}g_3\sigma^4 -$$

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1) E-mail: chenh@swu.edu.cn

2) E-mail: jmyao@pku.edu.cn

$$\frac{1}{4}\Omega_{\mu\nu}\Omega^{\mu\nu}+\frac{1}{2}m_\omega^2\omega_\mu\omega^\mu+\frac{1}{4}c_3(\omega_\mu\omega^\mu)^2-\frac{1}{4}\boldsymbol{R}_{\mu\nu}\cdot\boldsymbol{R}^{\mu\nu}+\frac{1}{2}m_\rho^2\boldsymbol{\rho}_\mu\cdot\boldsymbol{\rho}^\mu-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}\ , \quad (1)$$

where M and $m_{\rm i}(g_{\rm i})$ ($\rm i=\sigma,\omega_\mu,\boldsymbol{\rho}_\mu$) are the masses (coupling constants) of the nucleon and the mesons respectively. The field tensors of the vector mesons and the electromagnetic field are defined as,

$$\Omega^{\mu\nu}=\partial^\mu\omega^\nu-\partial^\nu\omega^\mu, \quad (2a)$$

$$\boldsymbol{R}^{\mu\nu}=\partial^\mu\boldsymbol{\rho}^\nu-\partial^\nu\boldsymbol{\rho}^\mu, \quad (2b)$$

$$F^{\mu\nu}=\partial^\mu A^\nu-\partial^\nu A^\mu. \quad (2c)$$

From Eq. (1), one can obtain the Dirac equation for single-particle orbits and its wavefunctions $\psi_{\rm i}$ and classical time-independent inhomogeneous Klein-Gordon equations for the meson fields and photon field via variational principle. With the restriction of spherical symmetry, the coupled Dirac equation and Klein-Gordon equations are solved iteratively in r-space with box size $R_{\max}$. More details about the numerical techniques can be found in Ref. [15].

3 The real stabilization method

For a bound state, the eigenenergy will not change with the box size $R_{\max}$. For those states with energy greater than zero, there are also some states stable against $R_{\max}$. Such stable states correspond to resonances. It is the basic idea of real stabilization method and will be used to find out the single-particle resonance states in ^{122}Zr. Furthermore, in this paper, we will introduce two methods proposed by Maier et al.[12] and Mandelshtam et al.[13] to extract the corresponding resonance energies and widths. In the following, these two methods will be called Maier method and Mandelshtam method respectively for brevity.

In Maier method, the resonance energy E_γ is determined by the condition $\partial E^2/\partial R_{\max}^2=0$, at which, the box size is labeled as $\bar{R}_{\max}$. Assuming that the phase shift from the potential scattering $\eta_{\rm l,pot}(E)$ varies slowly with respect to the box size, i.e., $\partial\eta_{\rm l,pot}(E)\partial R_{\max}\sim 0$, the resonance width Γ is simply given by,

$$\Gamma=\frac{2\sqrt{E_\gamma^2+2E_\gamma M}}{-(E_\gamma+M)\bar{R}_{\max}-(E_\gamma^2+2E_\gamma M)[\partial E/\partial R_{\max}|_{\bar{R}_{\max}}]^{-1}}. \quad (3)$$

In Mandelshtam method, the resonance parameters could be calculated by computing the density of states $\rho_{\rm R}(E)$ from the stabilization diagram of the eigenenergies $E_{\rm R}$ vs $R_{\max}$. The density of states is composed of contributions from the localized space and its orthogonal space,

$$\rho_{\rm R}(E)=\rho_{\rm R}^{\rm Q}(E)+\rho_{\rm R}^{\rm P}(E)\ . \quad (4)$$

$\rho_{\rm R}^{\rm Q}(E)$ is the expected resonant part which is stable against $R_{\max}$ for $R_{\max}$ outside the Q region, and which, for the case of an isolated resonance, is expected to be[16]

$$\rho^{\rm Q}(E)\simeq\pi^{-1}\frac{\Gamma/2}{(E_\gamma-E)^2+\Gamma^2/4}\ . \quad (5)$$

Assuming that the resonant part of $\rho_{\rm R}(E)$ stabilized at relatively small $R_{\max}$, where $\rho_{\rm R}^{\rm P}(E)$ is negligible, the density of state $\rho_{\rm R}(E)$ approximately equals to $\rho_{\rm R}^{\rm Q}(E)$ and determined by[13],

$$\rho_{\rm R}(E)\simeq\rho_{\rm R}^{\rm Q}(E)=\frac{1}{\Delta R}\sum_j\left|\frac{{\rm d}E_{\rm j}(R')}{{\rm d}R'}\right|^{-1}_{E_{\rm j}(R')=E}\ , \quad (6)$$

where

$$E_{\rm j}(R')=E,\quad R_{\max}-\Delta R/2<R'<R_{\max}+\Delta R/2. \quad (7)$$

The index j sums the derivatives of the $E_{\rm j}$ vs $R_{\max}$ curves at the intersections of the curves with the constant E line, if they lie in the ΔR region over which we have averaged.

4 Numerical calculations and results

The calculation of single-particle resonance states in ^{122}Zr within RMF-RSM approach is composed of two steps. First, we perform RMF calculation for the single-particle energy levels in ^{122}Zr with PK1 effective interactions[17] by changing the box size $R_{\max}$ from 6 fm to 50 fm. Then, we extract the resonance parameters with both Maier method and Mandelshtam method.

Figure 1 shows four neutron single-particle energy levels with positive energies, which are stable against $R_{\max}$. For the state $\nu h9/2$, the lowest state falls down quickly at 10 fm and then stabilizes until $R_{\max}=27$ fm. It implies that there is a narrow $\nu h9/2$ resonance state with resonance energy $E_\gamma=2.43$ MeV. The same behavior happens for $\nu i13/2$ state with $E_\gamma=5.91$ MeV. However, for $\nu f5/2$ and $\nu i11/2$ states, the stabilization is not obvious which implies two wide resonance states.

In order to extract the resonance parameters with Mandelshtam method, the averaged densities of the four resonance states are plotted in Fig. 2. According to Eq. (5), the peak of $\rho_{\rm R}(E)$ corresponds to the resonance energy E_γ, at which, $\rho_{\rm R}(E)$ is inversely proportional to the resonance width Γ. Fig. 2 shows that $\nu h9/2$ state has a narrow resonance peaked at $E_\gamma = 2.43$ MeV and $\nu i11/2$ state has a wide resonance peaked at $E_\gamma = 10.6$ MeV.

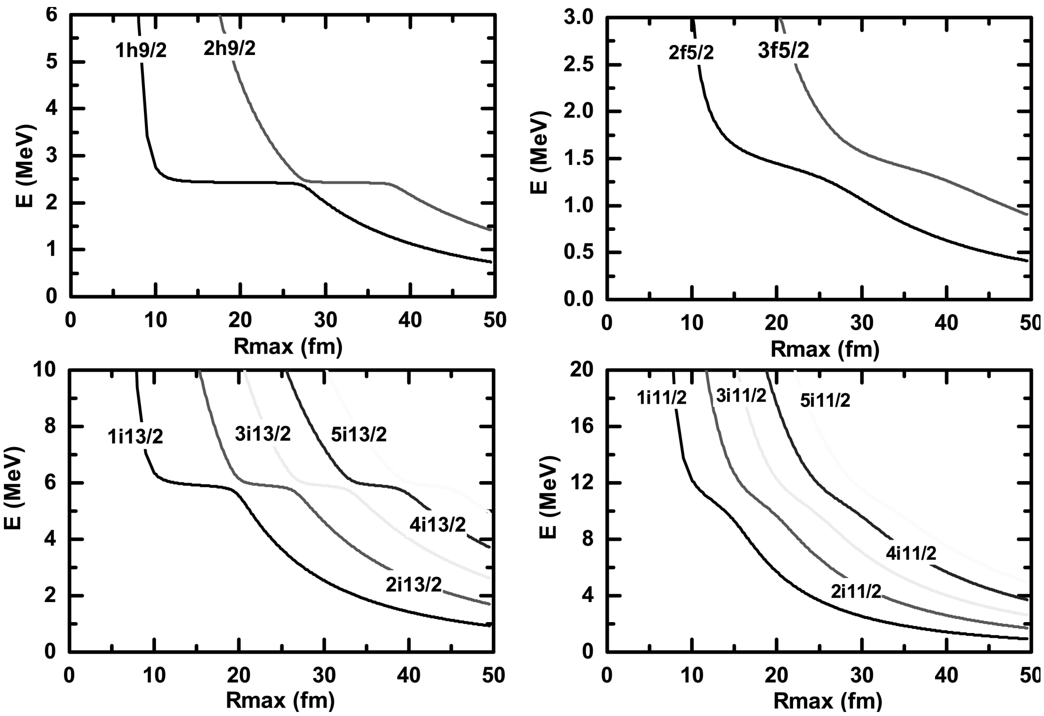

Fig. 1. Eigenenergies of $\nu h9/2$, $\nu f5/2$, $\nu i13/2$ and $\nu i11/2$ states as functions of the box size $R_{\max}$ in ^{122}Zr.

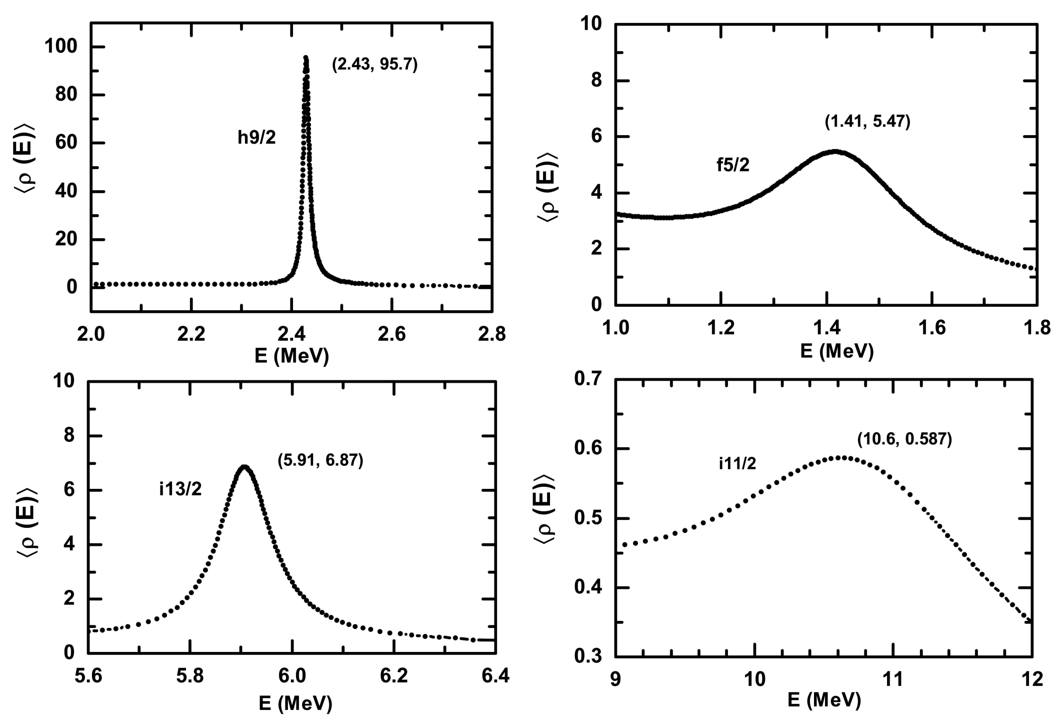

Fig. 2. The averaged density $\langle \rho_{\rm R}(E) \rangle$ of resonance states $\nu h9/2$, $\nu f5/2$, $\nu i13/2$ and $\nu i11/2$ in ^{122}Zr.

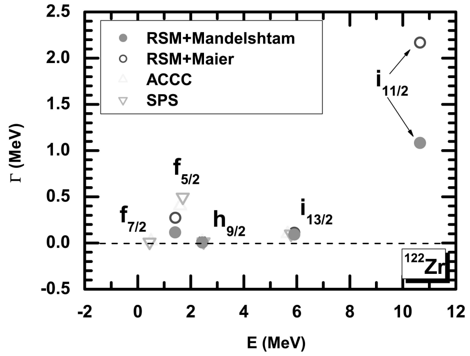

Fig. 3. Energies and widths of single neutron resonant states in ^{122}Zr from different methods. The data labeled with ACCC and SPS are taken from Ref. [6].

The resonance energy E_γ and width Γ for the neutron resonance states in ^{122}Zr are plotted in Fig. 3. One can see that the resonance widths given by Mandelshtam method are always smaller than those given by Maier method. It is due to the statistic error in the calculation of $\rho_{\rm R}(E)$ as shown in Eq. (6). A way to decrease this error is taking a larger ΔR in the Mandelshtam method. Compared with the results by RMF-ACCC and RMF-SPS methods, a wide resonance state of $\nu i11/2$, instead of the narrow resonant state $\nu f7/2$ has been found in present study. Moreover, the calculated resonance parameters of $\nu h9/2$ and $\nu i13/2$ are coincide with those given by RMF-ACCC and RMF-SPS methods.

5 Summary

In conclusion, RMF-RSM approach has been applied to study the resonance states in ^{122}Zr with both Maier method and Mandelshtam method. The averaged density of states, neutron resonance states with energies and widths are studied. Four neutron resonance states: $\nu h9/2$, $\nu i13/2$, $\nu f5/2$ and $\nu i11/2$ have been found, where the former two states are narrow resonances, while the latter ones are wide resonances. The results are compared with those obtained by RMF-ACCC and RMF-SPS approaches.

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