Darkon dark matter, unparticle effects and collider physics*

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Abstract In this talk I report recent results on the simplest dark matter model, the Darkon model, and supersymmetric unparticle effects on dark matter, and some implications for collider physics. I first discuss dark matter properties and collider signatures in the Darkon model, and then I discuss some implications for dark matter if a scalar unparticle is introduced to the MSSM.

Key words dark matter, supersymmetry, unparticle, collider

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1 Introduction

Understanding the nature of dark matter is one of the most challenging problems in particle physics and cosmology. Although dark matter contributes about 20% to the energy density of our universe\(^{[1]}\), the identity of the basic constituents of the dark matter is still not known.

One of the popular candidates for dark matter is the Weakly Interacting Massive Particle (WIMP). Detection of WIMP candidate is extremely important in understanding the nature of dark matter and also the fundamental particle physics model providing the candidate. The traditional way is to measure the dark matter flux at earth detectors. It is interesting to see whether WIMP can be produced and detected at collider experiments directly. The Standard Model (SM) does not contain a dark matter candidate. Among the many possible WIMPs, the lightest supersymmetric particle in the Minimal Supersymmetric SM (MSSM) is the most popular one. But so far no direct experimental evidence from collider and dark matter search has been obtained for supersymmetry. It is possible that the susy dark matter is modified by other new physics making it more difficult to detect than expected. It is also entirely possible that dark matter originated in a totally different way than that in the MSSM. Here I report two recent publications with my collaborators related to dark matter concerning the simplest dark matter model, the darkon model\(^{[2]}\), and supersymmetrized unparticles effects on dark matter\(^{[3]}\), and some implications for collider physics.

2 The darkon dark matter and its production at colliders

The darkon model is the simplest model which has a candidate of WIMP. This model contains a real SM singlet field, the darkon D, in addition to the SM particle contents (SM+D). The darkon field as dark matter was first considered by Silveira and Zee\(^{[4]}\), and further studied later by several others groups\(^{[2, 5-8]}\). In the following we provide some details about dark matter properties in this model.

The darkon field D must interact weakly with the standard matter field sector and should not decay rapidly into SM particles to play the role of dark matter. The simplest way of introducing the darkon D is to make it stable against decay and demand that the darkons can only be created or annihilated in pairs. If the interaction of D with the SM particles is required to be renormalizable, it can only couple to the SM Higgs doublet field H. Beside the kinetic energy term\(-\frac{1}{2}\partial_{\mu}D\partial^{\mu}D\), the general form of other terms in
the Lagrangian are given by\[6, 7\]

\[\mathcal{L}_D = -\left(\frac{\lambda_D}{4} D^4 + \frac{1}{2} (m_D^2 + \lambda v^2) D^2 + \frac{1}{2} \lambda h^2 D^2 + \lambda v h D^2\right),\] (1)

Note that the above Lagrangian is invariant under a D → −D Z₂ symmetry. The parameters in the potential should be such that the D field will not develop vacuum expectation value (vev) and the Z₂ symmetry is not broken, after SU(2)l × U(1) spontaneously breaks down to U(1)em, to make sure that darkons can only be produced or annihilated in pairs, and that D will not mix with the standard Higgs field to avoid possible fast decays of the type D → ff and other SM particles. The relic density of D is then decided, to the leading order, by annihilation of a pair of DD into SM particles through Higgs exchange\[5, 6, 8\], DD → h → X where X indicates SM particles.

Eliminating the pseudo-goldstone boson “eaten” by W and Z, we have the physical Higgs h coupling to D as

\[L_D = -\left(\frac{\lambda_D}{4} D^4 + \frac{1}{2} (m_D^2 + \lambda v^2) D^2 + \frac{1}{2} \lambda h^2 D^2 + \lambda v h D^2\right),\] (2)

where \(v = 246\ GeV\) is the vev of H. The D field has a mass \(m_D^2 = m_h^2 + \lambda v^2\). The last term \(\lambda v h D^2\) plays an important role in determining the relic density of the dark matter.

The annihilation of a DD pair into SM particles is through s-channel h exchange. To have some idea how this works, let us consider DD → h → ff. We parameterize Higgs-fermion and Higgs-darkon interactions as

\[L_Y = -\left(a_{ij} \bar{f}_R^i f_L^j h + b h D^2\right),\] (3)

where \(R(L) = (1 + \gamma_b)/2\). In the SM, \(a_{ij} = m_i \delta_{ij}/v\) and \(b = \lambda v\).

The total averaging annihilation rate of a pair DD to fermion pairs is then given by

\[
\langle v, \sigma \rangle = \frac{16 b^2}{32 m_D^2} \frac{1}{(4 m_D^2 - m_b^2)^2 + m_b^2 m_h^2} \times \sum_i N_i^c |a_{ij}|^2 (4 m_D^2 - 4 m_f^2)^{3/2}.
\] (4)

where \(N_i^c\) is the number of colors of the f-fermion. For a quark \(N_i^c = 3\) and a lepton \(N_i^c = 1\). \(i\) sums over the fermions with \(m_i < m_D\). In the above \(v,\) is the average relative velocity of the two D particles. We have used the fact that for cold dark matter D, the velocity is small, therefore to a good approximation the average relative speed of the two D is \(v_s = 2\rho_{DM}/m_D\) and \(s = (p_t + p_l)^2\) is equal to \(4 m_D^2\).

If there are other decay channels, the sum should also include these final states. The above can be rewritten and generalized to\[7\]

\[
\langle v, \sigma \rangle = \frac{8 \beta^2}{(4 m_D^2 - m_b^2)^2 + m_b^2 m_h^2} \frac{\Gamma(\tilde{h} \to X^\prime)}{2 m_D},
\] (5)

where \(\Gamma(\tilde{h} \to X^\prime) = \sum_i \Gamma(\tilde{h} \to X_i)\) with \(\tilde{h}\) being a “virtual” Higgs having the same couplings to other states as the Higgs h, but with a mass of \(2 m_D\). \(X_i\) indicate any possible decay modes of \(\tilde{h}\). For a given model \(\Gamma(\tilde{h} \to X^\prime)\) is obtained by calculating the \(h\) width and then set the mass equal to \(2 m_D\).

To produce the right relic density for dark matter \(\Omega_D\), the annihilation rate needs to satisfy the following\[9\]

\[
\langle v, \sigma \rangle \approx \frac{1.07 \times 10^9 x_t}{\sqrt{g_* m_{pl} (\Omega_D h^2)}},
\]

\[
x_t \approx \ln \frac{0.038 m_{pl} m_D \langle v, \sigma \rangle}{\sqrt{g_* x_t}},
\] (6)

where \(m_{pl} = 1.22 \times 10^{19}\ GeV, x_t = m_D/T_t\) with \(T_t\) being the freezing temperature, and \(g_*\) is the relativistic degrees of freedom with mass less than \(T_t\). Note that the ‘h’ in \(\Omega_D h^2\) is the normalized Hubble constant, not the Higgs field.

For given values of \(m_D\) and \(\Omega_D h^2\), \(x_t\) and \(g_*\) can be determined and therefore also \(\langle v, \sigma \rangle\). Then one can determine the parameter \(b\). In Fig. 1 we show the allowed range for the parameter \(b/v = \lambda\) as a function of the darkon mass \(m_D\) for several values of Higgs mass \(m_h\) with \(\Omega_D h^2\) set in the range \(0.095–0.112\) determined from cosmological observations\[1\]. We see that the darkon mass can be as low as a GeV. Search for low mass dark matter candidate may provide important information about dark matter models\[10\]. We
note that when the darkon mass decreases, \( \lambda \) becomes larger. For small enough \( m_D \), \( \lambda \) can be close to one which may upset applicability of perturbative calculation.

We have seen from the above discussion that the darkon mass can be as low as a GeV. With a low enough mass, darkon can be produced at colliders. The experimental signature of darkon production would be missing energy. Since the darkon directly couple to Higgs boson, the dominate production mechanism of darkon would be associated with the production of Higgs boson. Also since the \( Z_2 \) symmetry is not broken, darkons can only produced in pairs through \( h \to DD \).

In our previous discussions on dark matter density we have seen that the coupling \( \lambda = b/v \) in a wide range of darkon mass is not much smaller than 1, it is clear that the introduction of darkon will affect processes mediated by Higgs exchange and Higgs decay itself. In Fig. 2 we show the branching ratio \( B(h \to DD) \) as a function of \( m_D \) for several values of \( m_h \). We see that the invisible decay \( h \to DD \) dominates over the Higgs decay width if \( m_D \) is significantly below the \( h \to DD \) threshold because the small Yukawa couplings to light fermion which results in a larger \( \lambda \) to account for the dark matter density. However such invisible domination becomes weaker when \( h \to VV \) modes become kinematically allowed.

![Fig. 2. Branching ratio of \( h \to DD \) in SM+D as a function of \( m_D \) for \( m_h = 120 \) GeV (the region extended to the most left), 200 GeV (region in the middle) and 350 GeV (region started from \( m_D \) mass above 50 GeV or so), respectively.](image)

At LHC and ILC, a large number of Higgs bosons may be produced if kinematically accessible\(^{11-13}\). The various production cross sections of Higgs at LHC and ILC are typically a few pb \(^ {13} \). Assuming the integrated luminosities at LHC and ILC to be 200 fb \(^{-1} \), a large number of Higgs can be copiously produced and its properties studied in details. The main effect of the darkon field on the Higgs properties is to add an invisible decay mode \( h \to DD \) to the Higgs particle. Due to this additional mode, the Higgs width will be broader and affects determinations of the Higgs mass, and also decay properties in processes such as \( pp \to Xh \to XX' \) and \( e^+e^- \to Z^+ \to Zh \to ZX' \). Here \( X' \) indicates the final states used to study \( h \) properties. It has been shown that the processes \( pp \to Xh \to XX' \) and \( e^+e^- \to Z^+ \to Zh \to ZX' \) can be used to study invisible decays of Higgs bosons\(^{14, 15}\). Therefore detailed studies of these processes can provide more information about darkon properties.

### 3 Supersymmetrized unparticle effects on dark matter

Among the many possible WIMPs, the lightest supersymmetric particle in the Minimal Supersymmetric SM (MSSM) is the most popular one. But so far no direct experimental evidence has been obtained for supersymmetry. It is possible that the susy dark matter is modified by other new physics making it more difficult to detect thus far escaped from direct dark matter search. Now I discuss a scenario where the MSSM dark matter properties are modified, the unparticle effects on dark matter properties.

The concept of unparticle\(^{16}\) stems from the observation that certain high energy theory with a nontrivial infrared fixed-point at some scale \( A_d \) may develop a scale-invariant degree of freedom below the scale. The notion of mass does not apply to such an identity; instead, its kinematics is mainly determined by its scaling dimension \( d_U \) under scale transformations. The unparticle must interact with particles, however feebly, to be physically relevant; and the interaction can be well described in effective field theory (EFT). The unparticle \( O_d \) interaction with SM particles at low energy has the form

\[
\lambda A_d^{4-d_{SM}} H O_d \ .
\]  

(7)

There has been a burst of activities since the seminal work of Georgi\(^{16}\), on various aspects of unparticle physics. In Ref. [17] a class of operators involving SM particles and unparticles are listed. Using these operators one can study unparticle phenomenology in a systematic way.

When the scale invariant sector has interactions with the SM sector, the scale invariance will be broken. For example an interaction of the form \( \lambda A_d^{4-d_{SM}} H^2 O_d \) will generate a term, after Higgs develops a vacuum expectation value \( v / \sqrt{2} \), since a term of the form \( \lambda A_d v O_d / \sqrt{2} \) will be generated. A Yukawa
type of coupling \( \lambda A_{\text{cut}}^{-d_{\mu}} \tilde{Q}_L HU_R O_{\mu} \), at one loop level can generate a term of the form \( m_{\mu}^2 O_{\mu}^2 \) with \( m_{\mu}^2 \) given by \( m_{\mu}^2 \approx (\lambda \bar{A}_{\text{cut}}^{-d_{\mu}}/16\pi^2) A_{\text{cut}}^2 \). Here \( A_{\text{cut}} \) is a cut off scale of the effective theory. If the cut off scale is large the breaking of scale invariance can be larger. This situation is similar to the hierarchy problem of Higgs mass. One can eliminate such large loop correction maintaining low energy effect of unparticle and stabilize the theory by making the whole theory supersymmetric. This motivates us in Ref. [2] to consider unparticle effects in a supersymmetric theory and build a simple supersymmetrized unparticle model.

The model is a minimal extension to MSSM. Besides the usual MSSM contents with R-parity, we add a complex SM singlet chiral unparticle operator which has a scalar unparticle \( O_{\mu} \) with dimension \( d_{\mu} \) and also a spinorial partner \( \tilde{O}_{\mu} \) with dimension \( d_{\mu} + 1/2 \). Its associated \( F \) term \( F_{\mu} \) has dimension \( d_{\lambda} + 1 \). Normalizing the supersymmetric unparticle operator to a dimension one chiral field, we write super-field \( O_{\mu} \) as

\[
O_{\mu} = (O_{\mu} + \theta \tilde{O}_{\mu} + \theta^2 F_{\mu}),
\]

where \((O_{\mu}, \tilde{O}_{\mu}, F_{\mu}) = A_{\text{cut}}^{-d_{\mu}} (O_{\mu}, \tilde{O}_{\mu}, F_{\mu})\). One then treats the component super-fields similar to the components of usual chiral fields to construct the supersymmetric Lagrangian.

Since the unparticle does not have gauge interaction, its interactions with the MSSM particles arise entirely from the super-potential. The lowest dimension operator involving the unparticle is,

\[
H_O = \lambda H_1 H_2 O_{\mu},
\]

where \( H_1, H_2 \) are the two Higgs doublets in the MSSM. The component fields and vev are written as \( H_{(1,2)}^T = (h_{(1,2)}^+, (v_1 + h_1^0 + i a_1)/\sqrt{2}, \) and \( H_{(1,2)}^T = ((v_2 + h_2^0 + i a_2)/\sqrt{2}, h_2^0) \).

With the introduction of \( H_O \) into the super potential, the SM gauge interactions are not affected. However due to the new term \( H_O \) in the super-potential, there are some interesting consequences. With R-parity, the lightest supersymmetric particle (LSP) in MSSM is stable and can play the role of dark matter. In the model under consideration, the new operator introduces several unparticle interactions to the model which may lead to unstable LSP in the MSSM which will modify the properties of dark matter. We have

\[
L_{\text{cut}} = \lambda^2 A_{\text{cut}}^{-2d_{\mu}} (|H_1 O_{\mu}|^2 + |H_2 O_{\mu}|^2) + \lambda A_{\text{cut}}^{-d_{\mu}} (\tilde{H}_1 \tilde{H}_2 O_{\mu} + H_1 \tilde{H}_2 O_{\mu} + \tilde{H}_1 H_2 O_{\mu}),
\]

where the field (operator) with “tilde” indicates the suer-partner field (operator).

A neutral Higgsino can decay into an spinor unparticle due to the terms \( H_1 \tilde{H}_2 O_{\mu} \). After the Higgs doublets develop non-zero vev’s a matrix element, \( M(\tilde{H}_1 \rightarrow \tilde{H}_2) = \lambda A_{\text{cut}}^{-d_{\mu}} v_1 \tilde{H}_1 O_{\mu} \), will be generated. Here \( i \) and \( j \) take the values 1 and 2 with \( i \neq j \). If the scale invariant property of the unparticle hold down to very low energy, the phase space for the unparticle is proportional \( \theta(p_0)\theta(p - \mu^2) \). Here \( p \) is the momentum of the unparticle. This property allows the Higgsino of any mass to decay into an unparticle with

\[
\Gamma(\tilde{H}_1 \rightarrow \tilde{H}_2) = \frac{\lambda A_{\text{cut}}^{-d_{\mu}} v_1^2 m_{\tilde{H}_1} A_{\text{cut}}}{2} \times (m_{\tilde{H}_1}^2 \theta(p_0)\theta(m_{\tilde{H}_1}^2),
\]

where

\[
A_{\text{cut}} = (16\pi^2/2\pi^{2d_{\mu}}) \Gamma(d_{\mu} + 1/2)/\Gamma(d_{\mu} - 1) \Gamma(2d_{\mu}).
\]

It is clear that if the LSP in the MSSM has finite mixing with Higgsino, it will not be stable in this model. The LSP cannot play the role of usual dark matter. If a significant portion of LSP decays into unparticle, even if one assumes that the unparticle also provide the usual gravitational attractions, the detection would be more difficult since the unparticle is even more inertial than the usual LSP. This may relax the parameter space excluded by direct dark matter search and make direct production at colliders for dark matter more efficient. To keep the usual susy dark matter picture, it is necessary to make some modifications. A possibility is that the LSP contains no Higgsino component. This requires fine tuning and may not be natural. Here we point out that the new unparticle interactions introduced by \( H_O \) may provide another natural way.

The crucial point for this solution is that some of the new interactions, after the Higgs doublets develop vev’s, break scale invariance explicitly, such as \( v_i^2 O_{\mu}^2 \) from \( |H_1 O_{\mu}|^2 \) term. One may also introduce a susy breaking terms, \( \mu_{\text{un}} A_{\text{cut}}^{1-d_{\mu}} H_1 H_2 O_{\mu} \) and \( H_1^\dagger H_2 O_{\mu} \) in the theory. These terms induce terms of the form \( v_i v_j O_{\mu} \) which also breaks the scale invariance. Some implications for such an operator has been discussed in Ref. [18]. Assuming that the scale for these scale invariant breaking effects is \( \mu^2 \), it was suggested in Ref. [18] that the phase space should be changed to be proportional to \( \theta(p_0)\theta(p^2 - \mu^2) \). This implies that the Higgsino cannot decay into an unparticle with mass less than \( \mu \) and be stable. This scale should be proportional to \( v_i v_j \) which is the electroweak scale. If
this is indeed the case, the LSP in MSSM can still be a good candidate for dark matter.

The new interactions due to $L_O$ can also change Higgs boson decay property. For example, the supersymmetric breaking $A$-term, $\mu_{\text{susy}} \lambda^{1-d_{\mu}} \mu H_d H_u$ can induce a term $\mu_{\text{susy}} \lambda^{1-d_{\mu}} (v_1 / \sqrt{2} h_0^0) O_{U\mu}$ leading to Higgs decay into an unparticle if the Higgs boson mass is larger than $\mu$. The decay width is given by

$$\Gamma (h \rightarrow U) = \frac{\mu_{\text{susy}} \lambda^{1-d_{\mu}}}{2 m_h} (v_1 \sin \alpha + v_2 \cos \alpha)^2 \times$$

$$A_{d_{\mu}} (m_h^2)^{d_{\mu}-2} \theta(m_h^2) \theta(m_h^2 - \mu^2). \quad (12)$$

where $\alpha$ is the mixing angle for the neutral Higgs mixing with $h = \cos \alpha h_1^0 - \sin \alpha h_2^0$ and $H = \sin \alpha h_1^0 + \cos \alpha h_2^0$.

One can obtain the decay rate for $H$ by replacing $v_1 \sin \alpha + v_2 \cos \alpha$ by $v_1 \cos \alpha - v_2 \sin \alpha$. The term $\lambda^2 A_{d_{\mu}}^2 (v_1 / \sqrt{2} h_0^0) O_{U\mu}^2$ induced from $[H, O_{U\mu}]^2$, will cause Higgs $h_0^0$ to decay into two unparticles if the Higgs boson mass is larger than $2\mu$. These decays will contribute to the invisible decay width of Higgs particle, and affect Higgs search at LHC and ILC. Using the processes $pp \rightarrow Xh \rightarrow XX'$ and $e^+ e^- \rightarrow Z^* \rightarrow Zh \rightarrow ZX'$, unparticle effects on invisible Higgs decay can also be studied.

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References