Properties of hybrid stars in an extended MIT bag model*

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Abstract The properties of hybrid stars are investigated in the framework of the relativistic mean field theory (RMFT) and an MIT bag model with density-dependent bag constant to describe the hadron phase (HP) and quark phase (QP), respectively. We find that the density-dependent $B(\rho)$ decreases with baryon density ρ; this decrement makes the strange quark matter become more energetically favorable than ever, which makes the threshold densities of the hadron-quark phase transition lower than those of the original bag constant case. In this case, the hyperon degrees of freedom can not be considered. As a result, the equations of state of a star in the mixed phase (MP) become softer whereas those in the QP become stiffer, and the radii of the star obviously decrease. This indicates that the extended MIT bag model is more suitable to describe hybrid stars with small radii.

Key words hybrid stars, density-dependent bag constant, equations of state, mass-radios relations

PACS 12.39.Ba, 26.60.-c, 97.60.Jd

1 Introduction

Since Witten's speculation that strange quark matter (SQM) may be the true ground state of Quantum Chromodynamics (QCD)[1], many investigations have been carried out in order to understand the properties of SQM and their astrophysical significance. If the conjecture is true, namely, the energy per baryon of SQM could be less than that of even ^{56}Fe, some or even all neutron stars may turn out to be strange stars. According to our previous investigation on the energy per baryon for strange quark matter[2], when the bag constant increases to a proper value (it differs for different quark models), SQM may only be meta-stable, and the high pressure in the central regions of neutron stars may lead to the phase transition of strange matter and form hybrid stars[3]. Owing to the extremely high central density and wide density range of the neutron stars, there is no one physics (theory) to depict the whole density range of hybrid stars. We have to use different theories for different phases. Because of the difficulty of QCD in the nonperturbative domain, phenomenological models reflecting the characteristics of the strong interaction are widely used in the study of the stability and properties of SQM. One of them is the MIT bag model[4]. The bag model has also been used to study the thermodynamics of deconfinement phase transition[3]. Prasad et al. studied SQM in the framework of the MIT bag model with density-dependent bag constant[5], in which three models for density-dependent B were employed[6—8]. The properties for hybrid stars were also studied in the framework of the relativistic mean field theory (RMFT)[9] and the MIT bag model with unchanged bag constant to describe the quark phase (QP)[10]. If the MIT bag model with density-dependent bag constant is used in this field, the calculation results will undergo a significant change. In this paper, we shall study hybrid stars by using the density-dependent bag constant in the framework of the MIT bag model to describe the quark phase (QP) of hybrid stars. We use the RMFT

Received 28 August 2008
* Supported by National Natural Science Foundation of China (10275029, 10675054), Natural Science Foundation of Henan Education Department(2008A140009), and The Science Foundation of Nanyang Normal University
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to describe the hadronic phase. It is indicated in our recent work[11] that different equations of state (EOS) for the hadronic phase in the framework of the RMFT have a weak influence on the gross structure of hybrid stars. Therefore, we choose GL85 EOS, with σ meson nonlinear self-interaction[12, 13]. To describe the mixed phase in which hadronic matter and strange quark matter coexist, we use the two-component Gibbs phase equilibrium condition and the global electric charge neutral constraint. We present the particle fractions and the EOS of neutron stars. Utilizing the EOS, we integrate the Telman-Oppenheimer-Volkoff (TOV) equations[14] as an initial value problem for a given choice of the central energy density (ε_c) and obtain stellar mass-radius relations numerically.

This paper is organized as follows. In Sect. 2, we discuss the density-dependent bag parameter for two models. The hadron-quark mixed phase is the topic of Sect. 3. In Sect. 4, our numerical results for stellar properties are presented. Sect. 5 contains a short summary and concluding remarks.

2 Density-dependent bag constant

Of the many models of the nucleon that have been constructed so far, one of the most useful ones is the MIT bag model[4]. The nucleon is assumed to be a "bubble" of perturbative vacuum immersed in the nonperturbative or true vacuum, and quarks are confined to the bubble by means of a net inward pressure B exerted on it by the surrounding vacuum. As the density increases (or temperature rises) above the deconfinement value, there is no difference between the two vacua, and the net inward pressure B must vanish. In other words, B must be viewed as a temperature-dependent quantity[15]. The analogy between the bag constant B and the condensation energy in the Nambu-Jona-Lasinio(NJL) model at finite temperature or density was pointed out in Ref. [16]. It was shown that as the temperature rises, this condensation energy decreases and goes to zero at the transition temperature. This provides a model of the temperature-dependent B. The density dependence of B can be shown in an analogous manner: Since the deconfinement and the concomitant vanishing of B can also be brought about by raising the baryon number density ρ, B should be treated as a density-dependent quantity[17].

There have been a number of attempts in the literature to evaluate the density dependence of B. The results, however, are model dependent, and no consensus seems to have emerged. In this paper, we shall study hybrid stars in the framework of the MIT bag model for the quark phase, making use of the results of the following two density-dependent bag parameters.

(a) Burgio et al.[7] used the CERN SPS data on heavy-ion collisions to justify and determine the density dependence of B. They have presented their $B(\rho)$ in a parametric form:

$$B(\rho)=B_{\rm as}+(B(0)-B_{\rm as})\exp(-\beta x^2)\,, \qquad (1)$$

where $x\equiv\rho/\rho_0$ is the normalized baryon number density, ρ_0 is the baryon number density of ordinary nuclear matter, and $B_{\rm as}=38$ MeV/fm^3, $B_0=200$ MeV/fm$^3=(198$ MeV$)^4$, $\beta=0.14$ (from Ref. [7]).

(b) Aguirre[8] used the NJL model to study the modification of the QCD vacuum with increasing baryonic density and to extract the medium dependence of B. He has calculated $B(\rho)$ for symmetric uds quark matter relevant to a CFL quark star. His $B(\rho)$, in MeV/fm^3, is given by

$$B(\rho) = a+b_1x+b_2x^2+b_3x^3+b_4x^4+b_5x^5,\ x\leqslant 9 = \beta\exp[-\alpha(x-9)],\ x>9 \qquad (2)$$

where x is as defined earlier. Further, $a=291.59096$, $b_1=-142.25581$, $b_2=39.29997$, $b_3=-6.04592$, $b_4=0.46817$, $b_5=-0.01421$, $\alpha=0.253470705$, and $\beta=19.68764$ (from Ref. [8]).

In Fig. 1, $B(\rho)$ versus ρ corresponding to Eqs. (1)—(2) is shown. Firstly, the two curves have opposite slopes and at very small baryon number density, say $\rho<0.2$ fm^{-3}, Aguirre's $B(\rho)$ is bigger than Burgio's owing to its very big vacuum value a. Secondly, the density-dependent bag parameters, $B(\rho)$, are decreased explicitly with the baryon number density ρ and this decrement makes the strange quark matter become more energetically favorable than that of the constant B case, which makes the threshold densities of the onset and end of the hadron-quark phase transition smaller in a hybrid star (see Fig. 2).

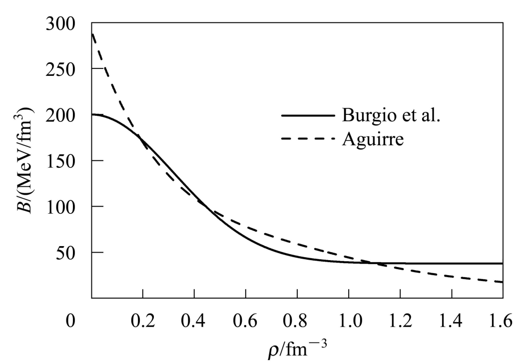

Fig. 1. Bag parameter versus baryon number density ρ for two models considered in this paper.

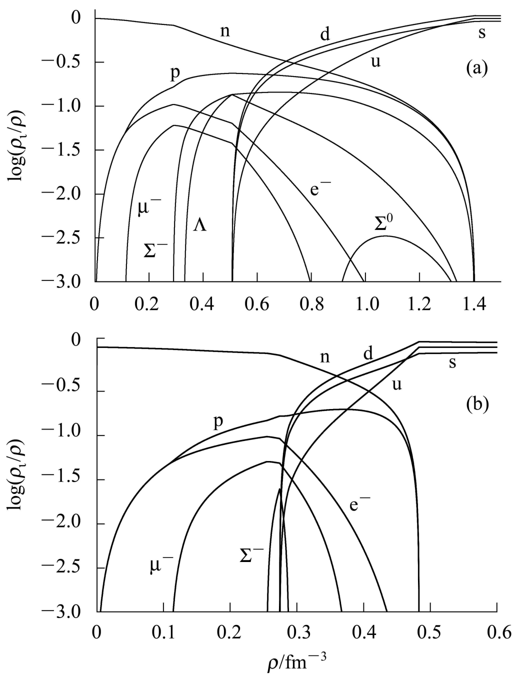

Fig. 2. Particle fractions for hybrid stars vs. baryon number density ρ. Here, (a) and (b) correspond to the constant bag parameter ($B^{1/4}$ = 200 MeV) and Burgio's density-dependent bag parameter, respectively.

3 Hadron-quark mixed phase

In this case SQM is only meta-stable, and high pressure in the central region of the neutron stars may lead to the phase transition of strange matter and formation of hybrid stars. This phase transition from hadronic matter to quark matter belongs to the first-order phase transition. In a thermodynamical system as (a mixed phase of) hybrid star matter, there exist two independent components or conserved charges, namely, the baryon number density and the electric charge[3]. For the mixed phase (MP) consisting of hadronic matter and quark matter, we choose μ_e, μ_n as independent variables and use the Gibbs condition for phase equilibrium and a constraint for global electric charge neutrality to depict the mixed phase. The Gibbs condition for mechanical and chemical equilibrium at zero temperature is

$$p_{\mathrm{HP}}(\mu_n, \mu_e) = p_{\mathrm{Qp}}(\mu_n, \mu_e)\,. \tag{3}$$

In the MP, hadrons and quarks are in chemical equilibrium and satisfy

$$\mu_n = \mu_u + 2\mu_d\,, \tag{4}$$

which is the joint condition of the hadronic phase, hadron-quark MP and pure QP. The MP satisfies the global charge neutral condition

$$q_{\mathrm{MP}} = (1-\gamma)q_{\mathrm{HP}} + \gamma q_{\mathrm{QP}} = 0\,, \tag{5}$$

and the total baryon number conservation

$$\rho_{\mathrm{MP}} = (1-\gamma)\rho_{\mathrm{HP}} + \gamma\rho_{\mathrm{QP}}\,. \tag{6}$$

where $\gamma = V_Q/V$ (and $V = V_Q + V_H$) is the volume fraction of quark matter in the MP ($\gamma = 0$ for the pure hadronic phase, and $\gamma = 1$ for the pure quark phase. In the course of the phase transition, γ increases from 0 to 1). The total energy density of the MP is given by

$$E_{\mathrm{MP}} = (1-\gamma)E_{\mathrm{HP}} + \gamma E_{\mathrm{QP}}\,. \tag{7}$$

Sets of nonlinear transcendental equations of the HP and MP are solved self-consistently at the definite baryon number density, and then the EOS of the hadronic phase, mixed phase and quark phase can be obtained. Linking them together, we obtain the total EOS for neutron star matter in the interior of a hybrid star. In the RMFT, the coupling constants for hyperon-hyperon are chosen in the following way. The ratio of coupling constants for hyperon-hyperon to nucleon-nucleon is defined as

$$\chi_\sigma = \frac{g_{\sigma H}}{g_{\sigma N}}, \chi_\omega = \frac{g_{\omega H}}{g_{\omega N}}, \chi_\rho = \frac{g_{\rho H}}{g_{\rho N}}. \tag{8}$$

Since χ_σ is given by the binding energy for Λ, χ_σ and χ_ω are very close to each other, and χ_ρ is insensitive to EOS, we choose $\chi = \chi_\sigma = \chi_\omega = \chi_\rho$ (=0.6, if not declared) in this paper[12, 13].

4 Numerical results

Using the Gl85 parameter set for hadronic matter and the MIT bag model with density-dependent bag constant for SQM, we calculate the hybrid star particle fractions, the EOS and the corresponding stellar mass-radius relations numerically.

The particle fractions of hybrid star matter versus the total baryon number density are shown in Fig. 2, where (a) is the case for constant B and (b) is for Burgio's density-dependent $B(\rho)$. As seen in Fig. 2(a), hadron-quark phase transition begins in ρ = 0.51 fm^{-3} and ends in ρ = 1.4 fm^{-3}, and three species of hyperons, Λ, Σ^- and Σ^0 appear in this constant B($B^{1/4}$ = 200 MeV) case. Since the density-dependent $B(\rho)$ decreases with baryon number density, which makes SQM more energetically favorable than ever, the hadron-quark phase transition begins in ρ = 0.275 fm^{-3}, and ends in ρ = 0.48 fm^{-3}. These threshold densities are lower than those of the constant B case, namely, the pure QP appears in a lower density region and only Σ^- hyperons appear (see

Fig. 2(b)). From Fig. 2(b), one notes that the abundance of Σ^- hyperons is very small. As is well known, the Σ^- potential depth in normal nuclear matter densities is unclear. The prediction value ranges from -30 MeV to $+30$ MeV. In this paper, we assume that all hyperons in the lightest octet have the same coupling constants and $\chi = 0.6$, which expresses an attractive Σ^- potential depth. If the repulsive Σ^- potential depth is used, Σ^- hyperons will never appear. Therefore, hyperon degrees of freedom can be neglected. The particle fractions of hybrid star matter for Aguirre's density-dependent $B(\rho)$ are similar to those of Burgio's.

We calculate the EOS, pressure versus energy density, of hybrid star matter for two density-dependent $B(\rho)$ models, and the results are shown in Fig. 3. For comparison, the corresponding result of the constant B is also shown in Fig. 3 (labeled constant B). The density-dependent $B(\rho)$ softens the EOS in the mixed phase region and stiffens it in the quark matter region. This would influence the corresponding mass-radius relations of hybrid stars.

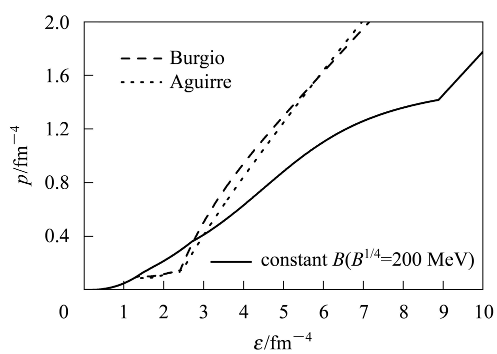

Fig. 3. Hybrid star pressure versus energy density (EOS) for constant ($B^{1/4} = 200$ MeV) and density-dependent bag constant.

In order to calculate the stellar mass-radius relations, we assume that a neutron star (including hybrid or strange star) is a relativistic and spherically symmetric distribution of mass in hydrostatic equilibrium. These equilibrium configurations are obtained by solving the TOV equations. They are

$$\frac{\mathrm{d}P}{\mathrm{d}r} = -\frac{G[E+P][M+4\pi r^3 P]}{r[r-2GM]} \tag{9}$$

and

$$\frac{\mathrm{d}M}{\mathrm{d}r} = 4\pi r^2 E(r)\ , \tag{10}$$

where G is the gravitational constant and $M(r)$ is the enclosed gravitational mass. From Eq. (10) the gravitational mass can be obtained. It is

$$M(R) = 4\pi \int_0^R r^2 E(r)\,\mathrm{d}r\ , \tag{11}$$

with R being the stellar radius. Given an EOS as an input, these equations can be integrated from the origin $\epsilon(r=0) = \epsilon_c$ (which is the central energy density) to the pressure on the stellar surface, where the pressure vanishes. With the EOS shown in Fig. 3 as input, we calculate the mass-radius relations of hybrid stars numerically. The evaluated results are seen in Fig. 4. For the constant bag parameter case, the gravitational mass of the star (maximum point of the mass-radius relation curve) is $1.66\mathrm{M}_\odot$ ($\mathrm{M}_\odot$ is solar mass) and the corresponding radius R is greater than 12 km. However, for the density-dependent B case, the gravitational mass is in the scope of the observation value and the corresponding R is smaller than 11 km. For Burgio's bag parameter case, the gravitational mass equals $1.84\mathrm{M}_\odot$ and the radius R is smaller than 11 km whereas for Aguirre's bag parameter case, M is $1.76\mathrm{M}_\odot$ and R is 10.3 km. Since the density-dependent $B(\rho)$ decreases with baryon number density ρ and this decrement of B would increase the pressure and decrease the energy density of the QP distinctly with ρ, strange quark matter becomes more energetically favorable than ever, which makes the threshold as well as the end densities of the hadron-quark phase transition lower than those of the original bag constant case, namely, the pure quark phase appears at relatively small baryon number density. As a result, the hybrid stars become more compact than ever and therefore, the corresponding radii of the stars become smaller. This indicates that with the density-dependent bag parameters, the extended MIT bag model is more suitable to describe hybrid stars with small radii.

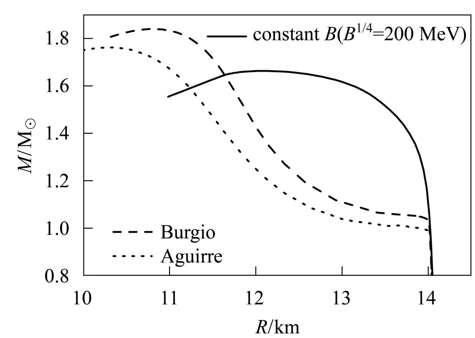

Fig. 4. Hybrid star mass-radius relations corresponding to EOS in Fig. 3.

5 Summary and concluding remarks

We have studied hybrid stars making use of the MIT bag model with density-dependent bag constant for the SQM and GL85 parameter set in the framework of the RMFT for hadronic matter. We study the particle fractions, equations of state of the star and stellar mass-radius relations numerically. We find that the density-dependent $B(\rho)$ decreases with baryon number density and this decrement makes the strange quark matter become more energetically favorable than ever, which makes the threshold densities of hadron-quark phase transition lower than those of the bag constant case in hybrid star matter. In this case, hyperon degrees of freedom can not be considered. As a result, the EOS in the MP region becomes softer whereas the EOS in the QP region becomes stiffer, and consequentially, the radius corresponding to the maximum mass obviously decreases. This indicates that with the density-dependent bag parameters, the extended MIT bag model is more suitable to describe hybrid stars with small radii. This conclusion implies that the recent discovered millisecond X-ray pulsar, SAXJ1808.4—3658[18, 19], may be a hybrid star.

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