Scaling property in one-nucleon removal reactions induced by exotic nuclei*

FANG De-Qing(方德清)1) FU Yao(付瑶) SUN Xiao-Yan(孙小燕) CAI Xiang-Zhou(蔡翔舟)
GUO Wei(郭威) TIAN Wen-Dong(田文栋) WANG Hong-Wei(王宏伟) MA Yu-Gang(马余刚)

(Shanghai Institute of Applied Physics, Chinese Academy of Sciences, Shanghai 201800, China)

Abstract Assuming a core plus valence nucleon structure, one-nucleon removal reaction is investigated within the framework of few-body Glauber theory. Fermi-type distribution is used for the core density, while the wavefunction of the valence nucleon is calculated by solving the single particle eigenvalue problem of the Schrödinger equation with the Woods-Saxon potential. The parallel momentum distribution ($P_{//}$) of the fragments is calculated for isotopes with $3 < Z < 18$. A remarkable scaling property is observed from the dependence of the dimensionless quantity $R_{\rm v}^2/R_{\rm c}^2$ on the full width at half maximum of the parallel momentum distribution (FWHM$_{P_{//}}$). $R_{\rm v}^2/R_{\rm c}^2$ is a measure of the exotic extent of the nuclear halo. Based on the obtained scaling law, FWHM$_{P_{//}}$ can be used as an experimental observable to extract $R_{\rm v}^2/R_{\rm c}^2$ and measure the exotic extent for the nuclear halo.

Key words fragment momentum distribution, few-body Glauber model

PACS 25.60.Gc, 24.10.-i

1 Introduction

The halo structure has been found to be one of the most important properties of nuclei far from the β-stability line. A lot of research have been done on this subject in the past two decades since the observation of the neutron halo structure in ^{11}Li[1—3]. Necessary conditions for the occurrence of the nuclear halo structure are small separation energy and low angular momentum for the valence nucleon, which are demonstrated in the general scaling laws for nuclei with a two-body structure. The scaling prescriptions have been observed for exotic nuclei through solving the Schrödinger equation with the square well potential. The results for the lowest three angular momenta could be expressed as[4, 5]

$$\frac{R_{\rm v}^2}{R_{\rm cn}^2}=\begin{cases}10.44\ {\rm MeV\ fm}^2/(S_{\rm n}R_{\rm cn}^2), & (l=0)\\ 3.65\ {\rm MeV}^{1/2}\ {\rm fm}/(S_{\rm n}R_{\rm cn}^2)^{1/2}, & (l=1)\\ 1.40, & (l=2)\end{cases} \quad (1)$$

where $R_{\rm v}$ is the root of the mean square (rms) radius of the valence nucleon, $R_{\rm cn}$ is the radius of the interaction potential describing the interaction between the valence nucleon and the core, and $S_{\rm n}$ is the separation energy of the valence nucleon. The scaling laws for the nuclear halo have also been observed and studied by other authors[6, 7]. From these studies, the dimensionless parameter $R_{\rm v}^2/R_{\rm cn}^2$ could be used as a good quantity to describe the exotic extent for one-nucleon halo nuclei. $R_{\rm v}^2/R_{\rm cn}^2 > 2$ has been proposed to be the condition for the occurrence of the halo structure[5]. It indicates that the probability of finding the valence nucleon outside the range of potential from the core is larger than 50%. As shown in Ref. [5], $R_{\rm cn}$ is roughly related to the rms radius of the core by $R_{\rm cn}^2=\frac{5}{3}(R_{\rm c}^2+\Delta)$, where $\Delta = 2\sim 4$ fm^2 accounts for the range of nucleon-nucleon interaction. From the experimental point of view, it is well known that $R_{\rm c}$ can be extracted from the experimental reaction cross

Received 28 May 2008

* Supported by National Natural Science Foundation of China (10775168, 10405032), Shanghai Development Foundation for Science and Technology (06QA14062, 06JC14082, 05XD14021), Major State Basic Research Development Program in China (2007CB815004) and Knowledge Innovation Project of Chinese Academy of Sciences (KJCX3.SYW.N2).

1) E-mail: dqfang@sinap.ac.cn

section of the core with the help of a reaction theory such as the Glauber model[8]. Thus $R^2_{\rm cn}$ can be easily determined by the above equation. But it is quite difficult to determine the rms radius of the valence nucleon experimentally. If we can present a method to determine $R^2_{\rm v}/R^2_{\rm cn}$ experimentally, it will be very significant to the study of the structure for exotic nuclei. Thus it is very important to find some method to extract $R_{\rm v}$ or $R^2_{\rm v}/R^2_{\rm cn}$ from the experimental observable.

The experimental measurements of the reaction cross section ($\sigma_{\rm R}$), fragment momentum distribution ($P_{//}$) after one-nucleon removal, the quadrupole moment and the Coulomb dissociation have been demonstrated to be very effective methods to identify and investigate the structure of halo nuclei. The neutron skin or halo nuclei ^{6,8}He, ^{11}Li, ^{11}Be, ^{19}C, etc.[1—3, 9—12], have been identified by these experimental methods. As we know, the longitudinal momentum distribution of fragments carries nuclear structure information of the projectile, especially the configuration of the valence nucleon. Measuring the momentum distribution has been shown to be an effective method for studing the configuration of the halo nucleon. From the studies we have learnt that the width of the momentum distribution could reflect the exotic structure of the nuclei. If $R^2_{\rm v}/R^2_{\rm cn}$ is used to describe the exotic extent for halo nuclei, the quantitative relation of the above observable with $R^2_{\rm v}/R^2_{\rm cn}$ is still not clear. Due to the simple relation between $R^2_{\rm cn}$ and $R^2_{\rm c}$ as shown previously, we will study the dependence of $R^2_{\rm v}/R^2_{\rm c}$ instead of $R^2_{\rm v}/R^2_{\rm cn}$ on the full width at half maximum of the longitudinal momentum distribution (FWHM$_{P_{//}}$). The method to extract $R^2_{\rm v}/R^2_{\rm c}$ from the experimental FWHM$_{P_{//}}$ data will be discussed.

2 Method description

To study one-nucleon removal reactions for exotic nuclei, a few-body Glauber model (FBGM) is adopted [13]. In this method, a core plus one valence nucleon is assumed for the projectile. The total wave function of the nucleus is expressed as [13]

$$\Psi=\sum_{ij}\psi^i_{\rm core}\phi^j_0\ , \qquad (2)$$

where $\psi_{\rm core}$ and ϕ_0 are the wave functions of the core and valence nucleon; i, j denote the different configurations for the core nucleus and the valence nucleon, respectively. For the core densities, Fermi-type distribution is used for the neutron and the proton

$$\rho_i(r)=\frac{\rho^0_i}{1+\exp\left(\dfrac{r-C_i}{t_i/4.4}\right)},\quad i={\rm n,\,p}, \qquad (3)$$

with C_i the half density radius, t_i the diffuseness and ρ^0_i the normalization constant to ensure the integration of the density distribution equaling neutron (i=n) and proton (i=p) numbers. C_i is determined by the droplet model[14]

$$C_i=R_i[1-(b_i/R_i)^2],\quad i={\rm n,\,p}, \qquad (4)$$

where $b_i=0.413t_i$, R_i is the equivalent sharp surface radius of the neutron and the proton. R_i and t_i are given by the droplet model. The density distribution for the valence nucleon is calculated by solving the eigenvalue problem of the Woods-Saxon potential[15]

$$\frac{{\rm d}^2R(r)}{{\rm d}r^2}+\frac{2\mu}{\hbar^2}\left[E-U(r)-\frac{l(l+1)\hbar^2}{2\mu r^2}\right]R(r)=0,$$
$$U(r)=-V_0f(r)+V_{ls}(l\cdot s)r_0^2\frac{1}{r}\frac{\rm d}{{\rm d}r}f(r)+V_{\rm Coul}, \qquad (5)$$

where $R(r)$ denotes the radial wave function of the valence nucleon, μ is the reduced mass of the valence nucleon, l is the angular quantum number and $\hbar$ is the reduced Planck constant. $f(r)=\left[1{+}\exp\left(\dfrac{r-R}{a}\right)\right]^{-1}$ with $R=r_0A_{\rm c}^{1/3}$ (V_{ls} = 17 MeV). V_0 is the depth of the potential, and $V_{\rm Coul}$ is the Coulomb potential. The separation energy of the valence nucleon is reproduced by adjusting the potential depth. In the calculation the diffuseness (a) and radius parameter (r_0) are chosen to be 0.67 fm and 1.27 fm, respectively.

3 Calculations and discussions

The wavefunction of the valence nucleon is calculated by this method. We have done the calculations for isotopes with $3<Z<18$. In the calculation, the separation energy of the valence nucleon for all nuclei is taken from Ref. [16]. The orbit of the valence nucleon is determined according to the shell levels of the shell model[17]. For neutron-rich nuclei, $l=1$ is considered if the neutron number satisfies $2<N\leqslant 8$. $l=0,2$ is considered if $8<N\leqslant 20$ for the possible orbit mixing of the s and d wave in exotic nuclei. Nuclei with $N>20$ are not studied. The same method is used for proton-rich nuclei. The density distribution of the valence nucleon is obtained by the square of the wavefunction. Its rms radius could be determined from the density distribution. The rms radius of the core is calculated from the sum of the Fermi-type distributions of the neutron and the proton.

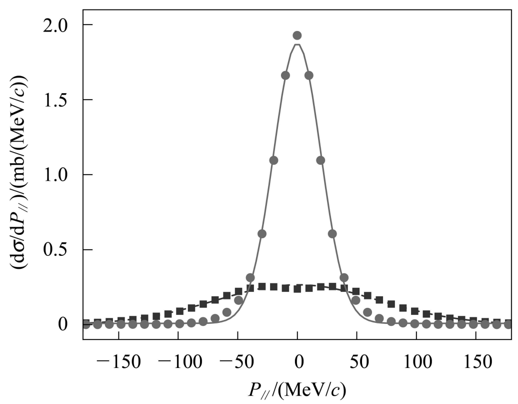

Fig. 1. The momentum distribution of the fragment after one-neutron removal from $^{19}C+^{12}C$ at 70 A MeV. The solid circles and solid squares are for the valence neutron in s and d wave configurations. The solid and dashed lines are gaussian fits to the s and d waves.

With the density distribution of the core and the wavefunction of the valence nucleon, the momentum distribution of the fragment can be calculated by using the few-body Glauber model. In the calculation, the target is ^{12}C and the projectile energy is 70 A MeV. For the calculated momentum distribution, a gaussian function is used to fit the distributions for all nuclei and extract $\text{FWHM}_{P_{//}}$ ($\text{FWHM}_{P_{//}} = 2.35\sigma$ with σ being the fitted width parameter of the gaussian function). As an example, the calculated momentum distribution after one-neutron removal for ^{19}C is shown in Fig. 1. In this case, the s or d wave is assumed for the valence neutron. Even the one-neutron separation energy of ^{19}C is only 0.577 MeV, the gaussian function can describe well the momentum distribution for the s and d wave. From the obtained results, we have found a very simple relation between R_{v} and $1/\text{FWHM}_{P_{//}}$. To show the scaling behavior between $\text{FWHM}_{P_{//}}$ and $R_{\text{v}}^2/R_{\text{c}}^2$, we use $1/w \equiv 1/(\text{FWHM}_{P_{//}} \cdot R_{\text{c}})^2$. The correlation plots between $1/w$ and $R_{\text{v}}^2/R_{\text{c}}^2$ for both neutron and proton rich nuclei are shown in Fig. 2. A remarkable scaling behavior has been seen which can be described by the function ax^b (a,b are free parameters). The fitted functions for the three angular momenta ($l=0,1,2$) could be expressed as

$$\frac{R_{\text{v}}^2}{R_{\text{c}}^2} = 10^{(3.311+0.204l)}/w^{0.729}, \quad (l=0,1,2). \qquad (6)$$

We have found that the same parameter $b = 0.729$ can describe very well the scaling properties of both neutron and proton rich nuclei with different angular momentum. If we show the dependence of $R_{\text{v}}^2/R_{\text{c}}^2$ on a modified quantity $1/\widetilde{w} \equiv 10^{0.28l}/w$, the results for three angular momenta overlap. The compact relation is shown in the inset of Fig. 2, which can be written as

$$\frac{R_{\text{v}}^2}{R_{\text{c}}^2} = 10^{3.311}/\widetilde{w}^{0.729} \ . \qquad (7)$$

The above results show that scaling property exists in the relation between $R_{\text{v}}^2/R_{\text{c}}^2$ and $\text{FWHM}_{P_{//}}$. This scaling phenomemon is due to the uncertainty principle ($\Delta r \Delta p$ =const). For the valence nucleon, the large value of R_{v}^2 means a large value of Δr. Narrow momentum distribution means a small value of Δp. From the uncertainty principle, the large value of Δr indicates a small value of Δp. In our calculation, R_{v}^2 is large when $\text{FWHM}_{P_{//}}$ is small and R_{v}^2 is small when $\text{FWHM}_{P_{//}}$ is large, which are similar with the uncertainty principle. Of course the relation between R_{v}^2 and $\text{FWHM}_{P_{//}}$ is more complicated than that of Δr and Δp. If $\text{FWHM}_{P_{//}}$ is measured experimentally, it is possible to extract $R_{\text{v}}^2/R_{\text{c}}^2$ based on Eq. (6).

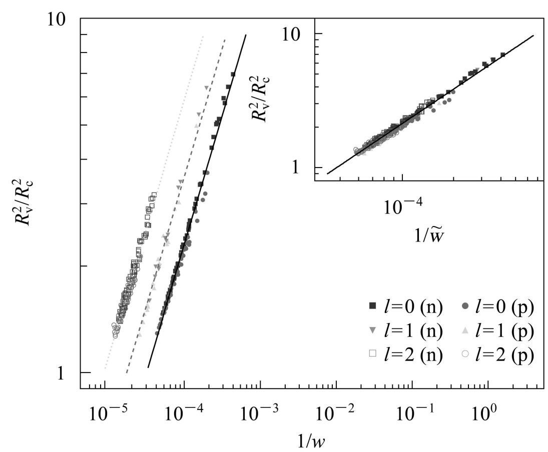

Fig. 2. The dependence of $R_{\text{v}}^2/R_{\text{c}}^2$ on $1/w$ and $1/\widetilde{w}$ (the inset). The solid squares, solid down triangles and open squares are for neutron-rich nuclei with the orbital angular momentum $l = 0,1,2$, respectively. The solid circles, solid up triangles and open circles are for proton-rich nuclei with the orbital angular momentum $l=0,1,2$. For details, see the text.

4 Conclusion

In summary, the fragment momentum distribution ($P_{//}$) is investigated by adopting a core plus one-nucleon structure within the framework of a few-body Glauber theory. Scaling behavior is observed for the dependence of $R_{\text{v}}^2/R_{\text{c}}^2$ with $1/(\text{FWHM}_{P_{//}} \cdot R_{\text{c}})^2$. $1/(\text{FWHM}_{P_{//}} \cdot R_{\text{c}})^2$ has been found to be proportional with R_v^2/R_c^2, which is a measure of the exotic extent of halo nuclei. R_v^2/R_c^2 or R_v^2 could be extracted from the experimental measurement of the fragment momentum distribution. Since R_v^2 is an important observable for exotic nuclei, the scaling property between R_v^2/R_c^2 and $1/(\mathrm{FWHM}_{P_{//}} \cdot R_c)^2$ will provide a good approach for investigating and identifying halo nuclei.

References

1 Tanihata I et al. Phys. Rev. Lett., 1985, **55**: 2676
2 Tanihata I et al. Phys. Lett. B, 1985, **160**: 380
3 Tanihata I et al. Phys. Lett. B, 1992, **287**: 307
4 Fedorov D V, Jesen A S, Riisager K. Phys. Rev. C, 1994, **49**: 201; 1994, **50**: 2372; Phys. Lett. B, 1993, **312**: 1
5 Hansen P G, Jesen A S. Ann. Rev. Nucl. Part. Sci., 1995, **45**: 591
6 LIN C J et al. Phys. Rev. C, 2002, **66**: 067302
7 LIU Z H et al. Phys. Rev. C, 2003, **68**: 024305
8 Ozawa A et al. Nucl. Phys. A, 2001, **693**: 32
9 Fukuda M et al. Phys. Lett. B, 1991, **268**: 339
10 Bazin D et al. Phys. Rev. Lett., 1995, **74**: 3569
11 Nakamura T et al. Phys. Rev. Lett., 1999, **83**: 1112
12 Ozawa A et al. Nucl. Phys. A, 2001, **691**: 599
13 Ogawa Y et al. Nucl. Phys. A, 1994, **571**: 784; Abu-Ibrahim B et al. Comput. Phys. Comm., 2003, **151**: 369
14 Myers W et al. Nucl. Phys. A, 1983, **410**: 61
15 Speth J et al. Phys. Rep., 1977, **33**: 127
16 Audi G, Wapstra A H. Nucl. Phys. A, 1993, **565**: 66
17 Mayer M G, Jensen J H D. Elementary Theory of Nuclear Shell Structure. New York: John Wiley and Sons, 1955. 58