

# Alignments in the nobelium isotopes<sup>\*</sup>

ZHENG Shi-Jie(郑世界)<sup>1</sup> XU Fu-Rong(许甫荣)<sup>1,2;1)</sup> YUAN Cen-Xi(袁岑溪)<sup>1</sup> QI Chong(齐冲)<sup>1</sup>

<sup>1</sup> (School of Physics and State Key Laboratory of Nuclear Physics and Technology,  
Peking University, Beijing 100871, China)

<sup>2</sup> (Center of Theoretical Nuclear Physics, National Laboratory of Heavy Ion Accelerator of Lanzhou,  
Lanzhou 730000, China)

**Abstract** Total-Routhian-Surface calculations have been performed to investigate the deformation and alignment properties of the No isotopes. It is found that normal deformed and superdeformed states in these nuclei can coexist at low excitation energies. In neutron-deficient No isotopes, the superdeformed shapes can even become the ground states. Moreover, we plotted the kinematic moments of inertia of the No isotopes, which follow very nicely available experimental data. It is noted that, as the rotational frequency increases, alignments develop at  $\hbar\omega = 0.2 - 0.3$  MeV. Our calculations show that the occupation of the  $\nu j_{15/2}$  orbital plays an important role in the alignments of the No isotopes.

**Key words** superheavy nuclei, TRS, alignments

**PACS** 21.10.-k, 21.60.-n, 25.85.Ca

## 1 Introduction

In the past decade, great progress has been noted in the synthesis of superheavy nuclei. Superheavy elements with charge numbers up to  $Z = 118$  have been produced<sup>[1–3]</sup>, which can be identified through the identification of the alpha decays of the ground states. On the theoretical side, investigations have predicted the shape coexistence of spherical, prolate, oblate and triaxial deformations near the ground states of the superheavy nuclei<sup>[4–6]</sup>.

Such superheavy nuclei are very unstable and very difficult to detect. As a result, only in a few cases excited states have been observed in superheavy nuclei. Recently, the rotational bands of <sup>252</sup>No and <sup>254</sup>No have been identified and extended to the spin-parity of  $20^+$  and  $16^+$ , respectively<sup>[7–10]</sup>. The rotational properties provide a unique opportunity to extract further detailed structure information and test theoretical models in this extreme mass region. In this paper, total-Routhian-Surface (TRS) calculations<sup>[15]</sup> have been performed in the deformation space of  $\hat{\beta} = (\beta_2, \gamma, \beta_4)$  to study the deformations, kinematic

moments of inertia and alignments of the nobelium isotopes.

## 2 The model

The total Routhian  $E^\omega(Z, N, \hat{\beta})$  of a nucleus  $(Z, N)$  at a rotational frequency  $\omega$  and deformation  $\hat{\beta}$  is calculated as<sup>[11]</sup>

$$E^\omega(Z, N, \hat{\beta}) = E^{\omega=0}(Z, N, \hat{\beta}) + [\langle \Psi^\omega | \hat{H}^\omega | \Psi^\omega \rangle - \langle \Psi^\omega | \hat{H}^\omega | \Psi^\omega \rangle_{\omega=0}], \quad (1)$$

where  $E^{\omega=0}(Z, N, \hat{\beta})$  is the total energy at the zero frequency while the last two terms in the bracket represent the change in energy due to the rotation. The energy  $E^{\omega=0}(Z, N, \hat{\beta})$  takes into account the macroscopic liquid-drop energy<sup>[12]</sup>, the microscopic shell correction<sup>[13, 14]</sup> and the pairing energy<sup>[15]</sup>. The total Hamiltonian appearing in the above equation,  $\hat{H}^\omega$ , is written as<sup>[11]</sup>

$$\hat{H}^\omega = \sum_{ij} [(\langle i | h_{ws} | j \rangle - \lambda \delta_{ij}) a_i^\dagger a_j - \omega \langle i | \hat{j}_x | j \rangle a_i^\dagger a_j] - G \sum_{i, i' > 0} a_i^\dagger a_i^\dagger a_{i'} a_{i'} . \quad (2)$$

Received 17 June 2008

<sup>\*</sup> Supported by National Natural Science Foundation of China (10735010, 10525520) and Chinese Major State Basic Research Development Program (2007CB815000)

1) E-mail: frxu@pku.edu.cn

©2009 Chinese Physical Society and the Institute of High Energy Physics of the Chinese Academy of Sciences and the Institute of Modern Physics of the Chinese Academy of Sciences and IOP Publishing Ltd

where we adopt a non-axially deformed Woods-Saxon (WS) potential for the single-particle Hamiltonian,  $h_{ws}$ .

The pairing correlation is treated using the Lipkin-Nogami approach<sup>[15]</sup> in which the particle number is conserved approximately and thus the spurious pairing phase transition encountered in the BCS calculation can be avoided (see Ref. [15] for the detailed formulation of the cranked Lipkin-Nogami TRS method). Both monopole and quadrupole pairings are considered<sup>[16]</sup> with the monopole pairing strength  $G$  being determined by the average gap method<sup>[17]</sup> and the quadrupole strengths obtained by restoring the Galilean invariance broken by the seniority pairing force<sup>[16, 18, 19]</sup>. Pairing correlations are dependent on the rotational frequency and deformation. In order to include such dependences in the TRS, we have to perform a pairing-deformation-frequency self-consistent TRS calculation, i.e., for any given deformation and frequency, the pairing is self-consistently treated by a Hartree-Fock-Bogolyubov-like equation<sup>[15]</sup>. At a given frequency, the deformation of a state is determined by minimizing the calculated TRS. The total collective angular momentum is calculated as follows<sup>[20]</sup>

$$I_x = \sum_{\alpha, \beta > 0} \langle \beta | \hat{j}_x | \alpha \rangle \rho_{\alpha, \beta} + \sum_{\tilde{\alpha}, \tilde{\beta} > 0} \langle \tilde{\beta} | \hat{j}_x | \tilde{\alpha} \rangle \rho_{\tilde{\alpha}, \tilde{\beta}}, \quad (3)$$

where  $\rho$  is the density matrix in the representation of signature basis being denoted explicitly by  $\alpha, \beta$  ( $\tilde{\alpha}$  and  $\tilde{\beta}$  stand for the opposite signatures). The moments of inertia are obtained by  $\mathfrak{J}^{(1)} = I_x / \omega$ .

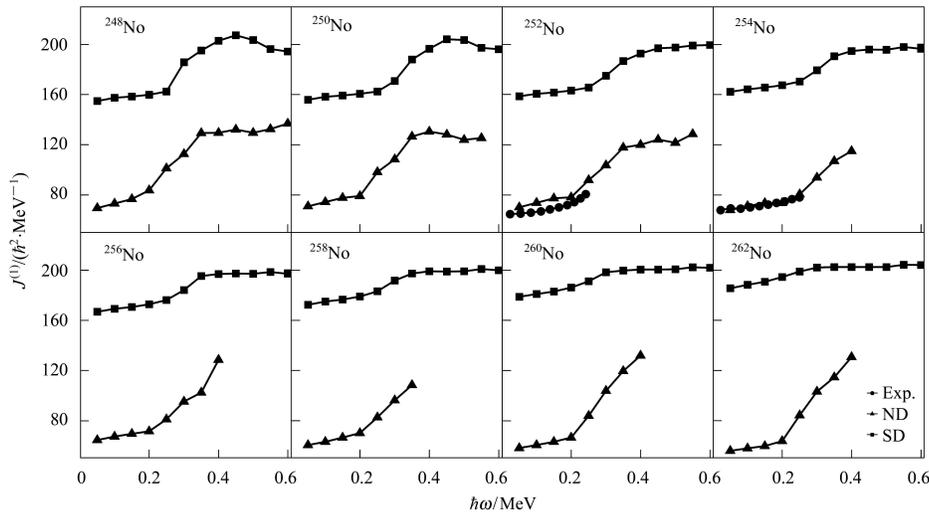


Fig. 2. Experimental<sup>[8–10]</sup> and calculated kinematic moments of inertia for No isotopes. ND and SD represent rotational bands of normal deformed and superdeformed states, respectively.

### 3 Calculations and discussions

TRS calculations for even-even nobelium isotopes have been performed and ground-state deformations are obtained, as shown in Fig. 1. All these nuclei have axial-symmetric deformations. Besides this, the coexistence of normal deformed and superdeformed prolate shapes is observed, consistent with the relativistic mean field calculations<sup>[5, 6]</sup>. The No isotopes with  $N \geq 150$  have well-deformed prolate ground-state shapes with  $\beta_2 \approx 0.24$ . The experimentally deduced ground-state deformations for the nuclei  $^{252}\text{No}$  and  $^{254}\text{No}$  are  $\beta_2 = 0.26$  and  $0.27$ , respectively, and agree well with the present results. Our calculations show that the stable moderate deformations in these nuclei can be related to the large energy gap at around  $\beta_2 = 0.27$  with  $N = 152$  in the single particle diagram. However, for the nuclei  $^{246, 248}\text{No}$ , the superdeformed states with  $\beta_2 = 0.68$  suddenly become ground states.

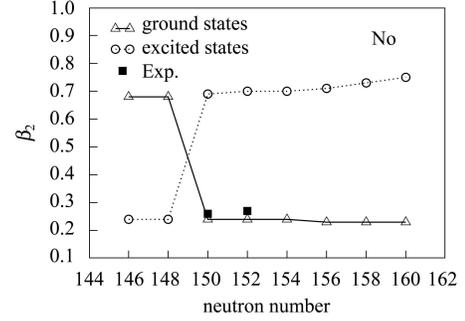


Fig. 1. Calculated quadrupole deformations,  $\beta_2$ , for even-even No isotopes. The open triangles and circles represent ground-state and first excited state deformations, respectively. The filled squares represent the experimental results<sup>[8]</sup>.

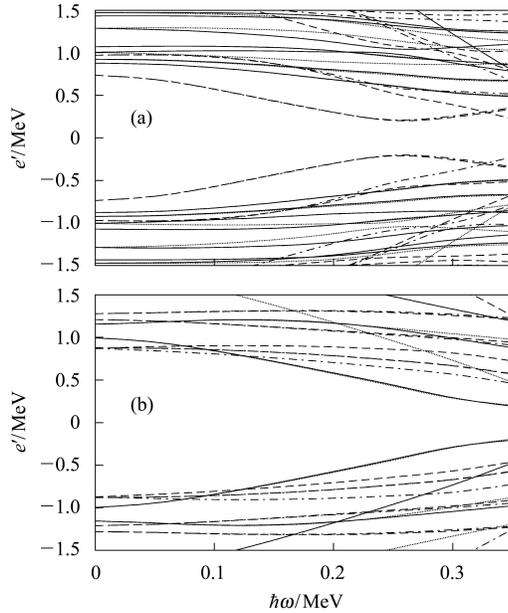


Fig. 3. Quasineutron (a) and quasiproton (b) Routhians of  $^{252}\text{No}$  with deformations  $(\beta_2, \gamma, \beta_4) = (0.238, -1.259^\circ, 0.017)$ .  $(\pi, \alpha)$ : solid =  $(+, +1/2)$ , dotted =  $(+, -1/2)$ , dot-dash =  $(-, +1/2)$ , dashed =  $(-, -1/2)$ .

To investigate the rotational properties of the superheavy nuclei, the kinematic moments of inertia of nobelium isotopes have been calculated and compared with experiments. Results are shown in Fig. 2, as a function of rotational frequency.  $^{252,254}\text{No}$  are the heaviest nuclei whose rotational bands have been observed experimentally up to date. However, there is little information at high rotational frequencies. In our calculations, the alignments of  $^{252}\text{No}$  appear with a rotational frequency greater than 0.2 MeV. For  $^{254}\text{No}$  the alignments appear slower than for  $^{250}\text{No}$ . In our calculations we obtain the kinematic moments of inertia for both, the normal deformations and the shape-coexisting superdeformations. For the nuclei

$^{252,254}\text{No}$ , the calculated kinematic moments of inertia of normal deformed shapes are in good agreement with experiment. It shows an upbending with  $\hbar\omega = 0.20$  for  $^{252}\text{No}$  and a slower one for  $^{254}\text{No}$ . For the other No isotopes our calculations also show an obvious upbending with  $\hbar\omega = 0.2\text{--}0.4$  MeV.

The single-quasiparticle Routhians have been calculated for the deformation of  $(\beta_2, \gamma, \beta_4) = (0.238, -1.259^\circ, 0.017)$  which are obtained from the TRS plots shown in Fig. 3. For the quasi-proton Routhians (lower panel), there are no alignments up to  $\hbar\omega = 0.35$  MeV. However, alignments appear around  $\hbar\omega = 0.25$  MeV for the quasi-neutron Routhians. It is considered to be the alignments of the  $\nu j_{15/2}$  quasineutrons. Thus, the location of the high- $j$  states plays an important role for the alignments. It also should be pointed out that the energy decrement of the  $\nu j_{15/2}$  orbital would result in a faster alignment in  $^{252}\text{No}$  whereas in a slower one in  $^{254}\text{No}$ <sup>[21]</sup>.

## 4 Summary

In conclusion, theoretical investigations have been carried out within the TRS model to study the properties of superheavy nobelium isotopes. The coexistence of normal deformed and superdeformed prolate shapes have been observed in these nuclei. The kinematic moments of inertia for the rotational bands are calculated, which agree well with available experimental results. If the rotational frequency increases, alignments can develop at  $\hbar\omega = 0.2\text{--}0.3$  MeV. It is found that the  $\nu j_{15/2}$  orbital plays an important role in the onset of the alignments. Our calculations for the rotational properties of No isotopes may be helpful for the understanding of collectivity in even heavier nuclei.

## References

- Oganessian Yu Ts, Yeremin A V, Popeko A G et al. Nature(London), 1999, **400**: 242
- Hofmann S, Münzenberg G. Rev. Mod. Phys., 2000, **72**: 733
- Oganessian Yu Ts, Utyonkov V K, Lobanov Yu V et al. Phys. Rev. C, 2006, **74**: 044602
- Ćwiok S, Heenen P H, Nazarewicz W. Nature(London), 2000, **433**: 705
- REN Z. Phys. Rev. C, 2002, **65**: 051304
- REN Z, Toki H. Nucl. Phys. A, 2001, **689**: 691
- Julin R. Nucl. Phys. A, 2001, **685**: 221
- Herzberg R D, Amzal N, Becker F et al. Phys. Rev. C, 2001, **65**: 014303
- Reiter P, Khoo T L, Lister C J et al. Phys. Rev. Lett., 1999, **82**: 509
- Leino M, Kankaanpää H, Herzberg R D et al. Eur. Phys. J. A, 1999, **6**: 63
- Nazarewicz W, Wyss R, Johnson A. Nucl. Phys. A, 1989, **503**: 285
- Myers W D, Swiatecki W J. Nucl. Phys., 1966, **81**: 1
- Nazarewicz W, Riley M A, Garrett J D. Nucl. Phys. A, 1990, **512**: 61
- Strutinsky V M. Yad. Fiz., 1966, **3**: 614; Nucl. Phys. A, 1967, **95**: 420
- Satula W, Wyss R, Magierski P. Nucl. Phys. A, 1994, **578**: 45
- Satula W, Wyss R. Phys. Rev. C, 1994, **50**: 2888
- Möller P, Nix J R. Nucl. Phys. A, 1992, **536**: 20
- Sakamoto H, Kishimoto T. Phys. Lett. B, 1994, **245**: 321
- XU F R, Satula W, Wyss R. Nucl. Phys. A, 2000, **669**: 119
- XU F R, Wyss R, Walker P M. Phys. Rev. C, 1999, **60**: R2888
- Bender M, Bonche P, Duguet T, Heenen P H. Nucl. Phys. A, 2003, **723**: 354