Rotational to vibrational evolution in some rotational-like nuclei^{*}

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Abstract The excitation spectra of nuclei in the regions 150 < A < 190 and 220 < A < 250 are commonly considered as showing the characteristics of the rotational motion. Whereas in the present work, there is important evidence to indicate that the nuclei discussed transit from rotation to vibration. At present, we encounter two different simple nuclear models being used for the elucidation of different region (lower and higher spin states) of the same nucleus. In addition, in order to study these rotational-like nuclei, as an example, shape calculations using the total Routhian surfaces (TRS) model were carried out for positive-parity states in ¹⁵⁶Gd. Also we have shown the result of the nucleus ¹⁰²Ru which is given as an example of the reverse transition, i.e., vibration to rotation. The TRS plots reveal that, as the spin increases up the band, the former nucleus becomes slightly soft in γ and β direction, whereas the latter becomes rigid in γ direction.

Key words rotation, vibration, phase transition, total Routhian surface(TRS)

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1 Introduction

The rotational and vibrational motions and the transition from one of these motions to another have been the most interesting and significant subjects in the research of nuclear structure. Recently, it has been found in experiment that rotation(or angular momentum) may induce a shape phase transition from the spheroid to the axial ellipsoid^[1]. In the work by Liu et al.^[2], by analyzing the available experimental data they show obviously that, besides the ones identified in the $A \sim 110$ mass region^[1], there exist vibrational to axially rotational phase transition along the yrast line in other nuclei, whereas in the present

work we try to investigate whether the inverse process will occur or not.

2 Calculations and discussions

It has been known that the energy of E2 transition γ -ray over spin (E-GOS) $R = \frac{E_{\gamma}(I \to (I-2))}{I}$ can be taken as a quite good signature to manifest the vibrational to axially rotational shape phase transition along the yrast line^[1], and vice versa one can infer that this simple method can also manifest the evolution from axially rotational to vibrational transition, whereas it was not given examples in that work^[1]. In the present work, we try to give examples for the first

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time that discerns the evolution from rotational to vibrational structure in nuclei as a function of spin. In Fig.1(a) we show the E-GOS curves for a perfect harmonic vibrator and axially symmetric rotor with first 2^+ excitations of 500 and 100 keV, respectively, just the same as the Fig. 1(a) in Ref.[1], and in Fig.1(b), the corresponding E-GOS plot for the axially rotational to vibrational shape phase transition along the yrast line is shown. Obviously it is a simple method for discerning this evolution.



Fig. 1. (a)E-GOS curves for a perfect harmonic vibrator and axially symmetric rotor with first 2^+ excitations of 500 and 100 keV, respectively. (b)The corresponding E-GOS plot for the axially rotational to vibrational shape phase transition along the yrast line.(c)E-GOS curves along the yrast line for the nucleus 156 Gd.

Vibrational excitations of the deformed nuclei are known to be energetically disfavored. They are far from the vrast states^[3]. Recent findings, however, have given evidence for a new type of energetically favored vibrational behavior based on isomers. This effect arises because the maximum-angular-momentum vibrational states have energies proportional to I, whereas rotational energies depend quadratically on I. Thus, as angular momentum increases, rotational excitations become increasingly expensive relative to vibrational excitations. Lee Pattison (University of Manchester, UK) and coworkers have reported a dramatic consequence of this competition^[4]. On top of the $I^{\pi} = 25^+$ isomer of osmium-182, they found a sequence of vibrational excitations that are, in fact, yrast. Their theoretical understanding of this structure is, thus far, based on a model of "tidal waves" that offers a new perspective on nuclear excitation modes.

As an example, in Fig.1(c) we show the empirical E-GOS curve for the yrast sequence of stretched E2 transitions in ¹⁵⁶Gd. The Fig. 1(c) indicates apparently that the yrast states in ¹⁵⁶Gd involve an axially rotational to vibrational shape phase transition. Normally the nucleus 156 Gd(N = 92) are thought belonging to the deformation region, and then one may infer that the energy spectrum of this nucleus is in good rotational, and consequently exhibits a band structure with the ground-state rotational band. So it is interesting to see that this positive-parity yrast band has the vibrational characteristic in the higher-spin region, i.e., the collective motion of a vibratory type have been discovered in the present work which built upon the rotational motion. It is clear now that one and the same nucleus can show features in its excitation spectrum that may require different models for their description. In fact, in low energies, the nucleus in some regions may prefer the spherical shape, and characteristic excitations will involve vibrations and single particle excitations. For higher excitations, the nucleus may prefer a shape that deviates appreciably from spherical symmetry giving rise to characteristic rotational spectra. For other nuclei the situation may be just reversed: the lowest energies may prefer a nonspherical nucleus with its characteristic rotational spectra, and the higher excitations may have spherical symmetry. On the other hand, the experimental energies of the ground-state rotation bands of these nuclei considered in the present work have been studied by Faessler et al. in the framework of their rotation vibration model (RV model)^[5], the basic assumptions of which are the same as in the Bohr-Mottelson theory^[6]; however, rotation vibration is taken into account especially carefully, the agreement between theory and experimental data was found to be excellent^[5]. This indicates that this is an alternative approach in explaining this evolution. In the present work, as an example, shape calculations using the total Routhian surfaces (TRS) model were carried out for positive-parity states in (a) ¹⁵⁶Gd and (b) ¹⁰²Ru. The latter is given just as an example of the other type of transition (vibration-torotation), demonstrated in Ref. [1]. Samples of total Routhian surfaces (TRS) are presented in Fig. 2(a)



Fig. 2. TRS plots in the (β_2, γ) polar coordinate system for the positive-parity states in (a)¹⁵⁶Gd and (b)¹⁰²Ru. The top panels are calculated at $\hbar\omega=0.00$ MeV and 0.10 MeV ($\hbar\omega=0.203$ MeV and 0.304 MeV) and the bottom panels are for $\hbar\omega=0.20$ MeV and 0.30 MeV ($\hbar\omega=0.405$ MeV and 0.506 MeV) for ¹⁵⁶Gd (¹⁰²Gu). A prolate (oblate) shape corresponds to a triaxiality of $\gamma = 0^{\circ}(-60^{\circ})$. The lowest energy in each diagram is indicated by a dot. The contour lines are separated by 200 keV.

and Fig. 2(b), respectively, in the polar coordinate plane (β_2, γ) . At each grid point, the total Routhian was minimized with respect to the hexadecapole deformation β_4 . For ¹⁵⁶Gd, as the spin increases up the band, the calculated TRS's carried out in the present work become slightly soft in both γ and β direction, whereas in the nucleus 102 Ru the calculated TRS's become rigid in γ direction with the increase of spin. According to our total Routhian surfaces (TRS) calculations for the positive-parity states^[7], at very low frequencies, the nucleus ¹⁵⁶Gd is predicted to be prolate with a quadrupole deformation of $\beta_2=0.259$ and a triaxiality parameter of $\gamma = -120^{\circ}$ at rotational frequency $\omega = 0$ (when the rotational frequency $\omega = 0$, i.e., the nucleus is static, the $\gamma = -120^{\circ}$ is equivalent to $\gamma = 0^{\circ}$, namely axial symmetric with prolate shape), which corresponds to ground state in ¹⁵⁶Gd, this is in accordance with the evaluated work of Möller et al.^[8], in which the deformation of ground state in ¹⁵⁶Gd has been calculated to be $\beta_2=0.271$ using a finite-range droplet macroscopic model and a folded-Yukawa single-particle microscopic model. So eventually the present TRS calculations prove it's reasonable to use the rotation vibration model to account for the yrast positive-parity band in ¹⁵⁶Gd.

In Fig. 3 we show the γ -ray energy over spin (E-GOS) plots of a number of nuclei in the mass regions 150 < A < 190 and 220 < A < 250. These data are taken from Refs. [9—26]. In these nuclei, the en-

ergies of various levels are nominally considered to follow very closely the simple formula

$$E_I = \frac{\hbar^2}{2\Im} I(I+1) - BI^2(I+1)^2, \qquad (1)$$

where \Im is an effective moment of inertia. In fact, this formula is just the result of the rotation vibration model(RV model). It is indicated by the observed value of B that the energies can be approximated well enough, especially for the lower values of I, by the simpler formula

$$E_I = \frac{\hbar^2}{2\Im} I(I+1), \qquad (2)$$



Fig. 3. E-GOS curves of some even-even nuclei in the so-called regions of large deformations, 150 < A < 190 and 220 < A < 250. These data are taken from Refs. [9—26].

i.e., these states are rotational collective states. The situation is assumed to be analogous to that of a diatomic molecule where rotational spectra with energies given by equation identical to Eq. (2) are known to exist. But in the current work, when we think it in another way (i.e., when the E-GOS plot for the yrast sequence is applied), a clear transition from rotational to vibrational motion as a function of spin is found. So it shows that the second term of Eq. (1) should not be omitted if we consider all values of I(all spinstates). Therefore this highlight the inherent dangers of simply assuming the rotational model over the



Fig. 4. Using the variation of quantity $\frac{2E(I)}{IE(2)}$ along with spin *I* to manifest the evolution between rotation and vibration.

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entire spin range. In fact, Bohr and Mottelson have used another quantity 2E(I)/IE(2), which varies along with spin I, to manifest the evolution between rotation and vibration skillfully^[27], shown in Fig. 4. The transition from rotation to vibration will occur when the quantity $\frac{2E(I)}{IE(2)}$ is close to the horizontal line from the oblique line, and on the contrary the reverse process occurs.

3 Conclusions

So in this work we tackles the issue of structural evolution as a function of angular momentum, complementing the traditional approach of studying such changes as a function of nucleon number^[28]. Up to now, a theoretical approach to describe the shape phase transition from axial rotation to vibration along the yrast line in individual nucleus has not yet been established, the underlying physics remains to be studied further. However, the purpose of this work is to discern this evolution in some mass regions and we shall not decide to discuss the mechanism (microscopical basis) behind this phenomenon (the evolution of these states) in detail any further.

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