Superfluidity in neutron matter and symmetric nuclear matter and the effect of a microscopic three-body force*

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Abstract The neutron $^3PF_2$ pairing gap in pure neutron matter, neutron $^3PF_2$ gap and neutron-proton $^3SD_1$ gap in symmetric nuclear matter have been studied by using the Brueckner-Hartree-Fock(BHF) approach and the BCS theory. We have concentrated on investigating and discussing the three-body force effect on the nucleon superfluidity. The calculated results indicate that the three-body force enhances remarkably the $^3PF_2$ superfluidity in neutron matter. It also enhances the $^3PF_2$ superfluidity in symmetric nuclear matter and its effect increases monotonically as the Fermi-momentum $k_{\rm F}$ increases, whereas the three-body force is shown to influence only weakly the neutron-proton $^3SD_1$ gap in symmetric nuclear matter.

Key words neutron matter, symmetric nuclear matter, superfluidity, three-body force, BCS theory, BHF approach

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1 Introduction

Superfluidity in nuclear and neutron matter plays an important role in nuclear physics as well as in nuclear astrophysics[1—4]. Theoretical research indicates that[1—3], the neutron fluid in the inner crust of a neutron star is possibly in the $^1S_0$ superfluid state, whereas in the core one expects a coexistence of superfluid neutrons in the $^3PF_2$ state and superconducting protons in the $^1S_0$ channel. It is expected that a number of astrophysical phenomena in neutron stars are rather sensitive to the presence of neutron and proton superfluid phases[4]. Since the neutron and proton superfluidity properties in neutron stars are related only indirectly to the astrophysical observations, reliable and precise theoretical predictions based on microscopic many-body approaches are highly desirable.

The pairing correlations in nuclear medium are essentially related to the underlying nucleon-nucleon (NN) interaction and their magnitude is determined by the competition between the repulsive short-range and attractive long-range parts of the interaction. Three-body forces, which turn out to be crucial for reproducing the empirical saturation properties of nuclear matter in a non-relativistic microscopic approach, are expected to modify strongly the in-vacuum NN interaction, especially the short-range part[5]. In Ref. [6], We investigated the effect of a microscopic three-body force (TBF) on the proton and neutron superfluidity in the $^1S_0$ channel in β-stable neutron star matter. It is found that the TBF has only a small effect on the neutron $^1S_0$ pairing gap, but it suppresses strongly the proton $^1S_0$ superfluidity in β-stable neutron star matter.

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At high density, the nucleon superfluidity is expected to occur mainly in the coupled $^3PF_2$ and $^3SD_1$ channels, the neutron $^3PF_2$ pairing gap in pure neutron matter, neutron $^3PF_2$ gap and neutron-proton $^3SD_1$ gap in symmetric nuclear matter have been studied by using the Brueckner-Hartree-Fock(BHF) approach and the BCS theory in this paper. The effects of the TBF on the superfluidity are well investigated and discussed.

2 BHF approach and BCS theory

The nucleon s.p.energies are calculated by using the BHF approach with a microscopic three-body force for isospin asymmetric nuclear matter[7]. The starting point of the BHF approach is the Brueckner reaction matrix G. The G matrix satisfies the Bethe-Goldstone equation[7—11]

$$G(\rho,\beta,\omega)=V+V\sum_{k_1k_2}\frac{|k_1k_2\rangle Q(k_1k_2)\langle k_1k_2|}{\omega-\varepsilon(k_1)-\varepsilon(k_2)+\mathrm{i}\eta}G(\rho,\beta,\omega), \tag{1}$$

where V is the realistic NN nuclear interaction, $\varepsilon(k)$ stands for the s.p.energies defined as $\varepsilon(k)=\hbar^2k^2/(2m)+U_{\rm BHF}(k)$. Within the BHF approximation the nucleon s.p.potentials are calculated from the real part of the on-shell antisymmetrized G matrix,

$$U(k)=\sum_{k'}n(k')\mathrm{Re}\langle kk'|G[\varepsilon(k)+\varepsilon(k')]|kk'\rangle_{\rm A}, \tag{2}$$

The effect of the TBF is included in the self-consistent Brueckner procedure along the same lines as in Refs. [12,13], where an effective two-body interaction is constructed to avoid the difficulty of solving the full three-body problem. A detailed description and justification of the method are discussed in Refs. [12,13]. The equivalent two-body potential in r-space is written:

$$\langle \boldsymbol{r}_1\boldsymbol{r}_2|V_3|\boldsymbol{r}_1{}'\boldsymbol{r}_2{}'\rangle=\frac{1}{4}\mathrm{Tr}\sum_n\int \mathrm{d}\boldsymbol{r}_3\mathrm{d}\boldsymbol{r}_3{}'\times$$

$$\phi_n^*(\boldsymbol{r}_3{}')(1-\eta(r'_{23}))(1-\eta(r'_{13}))\times$$

$$W_3(\boldsymbol{r}_1{}'\boldsymbol{r}_2{}'\boldsymbol{r}_3{}'|\boldsymbol{r}_1\boldsymbol{r}_2\boldsymbol{r}_3)\phi_n(r_3)(1-\eta(r_{13}))(1-\eta(r_{23})). \tag{3}$$

where the trace is taken with respect to the spin and isospin of the third nucleon, and $\eta(r)$ is the defect function. According to Eq. (3) the effective two-body force is obtained by averaging the TBF over the wave function of the third nucleon taking into account the correlations between this nucleon and the two others. Due to its dependence on the defect function the effective two-body force is calculated self-consistently along with the G-matrix and the s.p. potential at each step of the iterative BHF procedure.

Since our purpose is to investigate the TBF effect, we shall not go beyond the BCS framework[14—16]. In the coupled $^3PF_2$ ($^3SD_1$)channel, the nucleon pairing gap is determined by the standard BCS gap equations[2, 17]

$$\begin{pmatrix}\Delta_L\\ \Delta_{L+2}\end{pmatrix}(k)=-\frac{1}{\pi}\int_0^\infty \mathrm{d}k'k'^2\frac{1}{E_{k'}}\times$$

$$\begin{pmatrix}V_{L,L} & V_{L,L+2}\\ V_{L+2,L} & V_{L+2,L+2}\end{pmatrix}(k,k')\begin{pmatrix}\Delta_L\\ \Delta_{L+2}\end{pmatrix}(k'), \tag{4}$$

$$\rho=\frac{k_{\rm F}^3}{3\pi^2}=2\sum_k\frac{1}{2}\left[1-\frac{\varepsilon(k)-\varepsilon(k_{\rm F})}{E_k}\right]. \tag{5}$$

with $E_k^2=[\varepsilon(k)-\varepsilon(k_{\rm F})]^2+\Delta_L^2(k)+\Delta_{L+2}^2(k)$; $\varepsilon(k)=\hbar^2k^2/(2m)+U_{\rm BHF}(k)$ are the BHF s.p.energies. $\Delta_L(k)$ and $\Delta_{L+2}(k)$ are the gaps for and P and F channels (or S and D channels) respectively. In the BCS gap equation,the most important ingredients are the NN interaction $\begin{pmatrix}V_{L,L} & V_{L,L+2}\\ V_{L+2,L} & V_{L+2,L+2}\end{pmatrix}$ and the s.p.energies. For the NN interaction,we adopt the Argonne V_{18}(AV_{18}) two-body interaction[18] plus a microscopic TBF[12]. The TBF is constructed self-consistently with the two-body force AV_{18} by using the meson-exchange current approach and it contents the contributions from different intermediate virtual processes such as virtual nucleon-antinucleon pair excitations, and nucleon resonances (for details, see Ref. [12]). The TBF effects on the equation of state (EOS) of nuclear matter and its connection to the relativistic effects in the DBHF approach have been reported in Ref. [5].

With the s.p.energies and the microscopic TBF in neutron matter ($\beta=1$) and symmetric nuclear matter($\beta=0$) calculated in the BHF approximation above, we get $\Delta(k)=\sqrt{\Delta_L^2(k)+\Delta_{L+2}^2(k)}$ by solving the secular equation and eigen function of Eq. (4) in momentum space, then couple it with Eq. (5) , finally we can calculate the $^3PF_2$ ($^3SD_1$)pairing gaps $\Delta(k)=\sqrt{\Delta_L^2(k)+\Delta_{L+2}^2(k)}$ by iteration.

3 Numerical results and discussions

3.1 Neutron $^3PF_2$ superfluidity in neutron matter

In order to numerically investigate the effect of the TBF we have solved the gap equation, by adding the effective TBF given in Eq. (3) to the bare AV_{18} two-body force. At the same time the s.p.energy spectrum $\varepsilon(k) = \hbar^2k^2/(2m) + U_{\mathrm{BHF}}(k)$ appearing in the gap equation is computed from the BHF approach by using the AV_{18} plus the same TBF.

Figure 1 shows the neutron $^3PF_2$ pairing gap $\Delta(k)$ in pure neutron matter as a function of the Fermi-momentum k_{F}. The empty circles are obtained by adopting the pure AV_{18} two-body interaction only, while the filled circles are predicted by using the AV_{18} plus the TBF. We can see that without including the TBF, the neutron superfluid phase in the $^3PF_2$ channel can exist in the Fermi-momentum region of 1.2 $\mathrm{fm}^{-1} \leqslant k_{\mathrm{F}} \leqslant 2.5$ fm^{-1} (corresponding to the density range of 0.06 $\mathrm{fm}^{-3}\rho \leqslant 0.56$ fm^{-3}) with a maximal gap value of about 0.22 MeV peaked at a Fermi-momentum 1.9 fm^{-1} (the corresponding density is $\rho \simeq 0.24$ fm^{-3}). We can see the TBF effect by comparing the empty and filled circles. The TBF effect is quite small at relatively low densities ($k_{\mathrm{F}} \leqslant 1.4\,\mathrm{fm}^{-1}$). As the density goes higher, the TBF effect becomes much more pronounced and it enhances strongly the $^3PF_2$ superfluidity in neutron matter. At the Fermi-momentum of 2.1 fm^{-1}, the gap reaches a peak value of about 0.50 MeV, then the TBF effect decreases gradually as the density increases further.

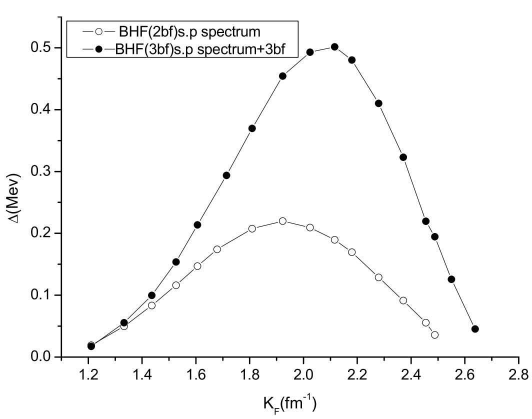

Fig. 1. Neutron $^3PF_2$ pairing gaps in neutron matter with BHF s.p.spectrum.The dot line is predicted by using the AV_{18} plus the TBF, and the circle line by using the AV_{18} two-body force only.

The peak value of the neutron $^3PF_2$ gap is increased by about 127% from 0.22 MeV to 0.50 MeV by inclusion of the TBF. The density region for the superfluid phase is also extended from $k_{\mathrm{F}} \leqslant 2.5$ fm^{-1} to $k_{\mathrm{F}} \leqslant 2.7$ fm^{-1} as compared to the prediction by adopting the pure two-body force.

3.2 Neutron $^3PF_2$ gap and neutron-proton $^3SD_1$ gap in symmetric nuclear matter

We have also calculated the neutron $^3PF_2$ gap and the neutron-proton $^3SD_1$ gap. In Fig. 2, the obtained neutron $^3PF_2$ pairing gap in symmetric nuclear matter is plotted.

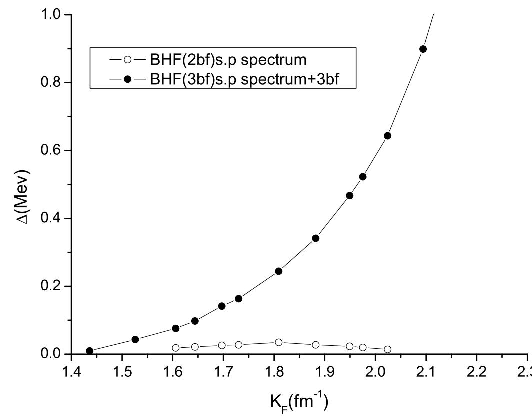

Fig. 2. Neutron $^3PF_2$ pairing gaps insymmetric nuclear matter in the two cases with the TBF and without the TBF.

By using the pure AV_{18} two-body interaction, the predicted neutron superfluidity phase in the $^3PF_2$ channel can exist in the density region of 1.6 $\mathrm{fm}^{-1} \leqslant k_{\mathrm{F}} \leqslant 2.0$ fm^{-1} with a maximal gap value of about 0.035 MeV at $k_{\mathrm{F}} = 1.8\,\mathrm{fm}^{-1}$. By comparing the empty and filled circles, one can see that inclusion of the TBF enhances strongly the neutron $^3PF_2$ superfluidity in symmetric nuclear matter, especially at high densities. The TBF-induced enhancement of the neutron $^3PF_2$ superfluidity is shown to increases monotonically with the increase of Fermi-momentum k_{F}.

The neutron-proton $^3SD_1$ gap in symmetric nuclear matter is shown in Fig. 3. We can see that the predicted neutron-proton superfluidity phase in the $^3SD_1$ channel may exist in the density region of 0.5 $\mathrm{fm}^{-1} \leqslant k_{\mathrm{F}} \leqslant 1.9$ fm^{-1}. By adopting the pure AV_{18} two-body interaction, the peak value of the pairing gap is about 8.5 MeV at $k_{\mathrm{F}} = 0.94\,\mathrm{fm}^{-1}$. The neutron-proton $^3SD_1$ gap with the effect of the TBF is given by the filled circles. By comparing the empty and filled circles we can see that they have close peak values about 8.5 MeV at 0.94 fm^{-1} and at high densities ($k_{\mathrm{F}} \geqslant 1.5\ \mathrm{fm}^{-1}$) the TBF enhances slightly the gap. So we can say the TBF effect on the neutron-proton $^3SD_1$ gap in symmetric nuclear matter is rather weak.

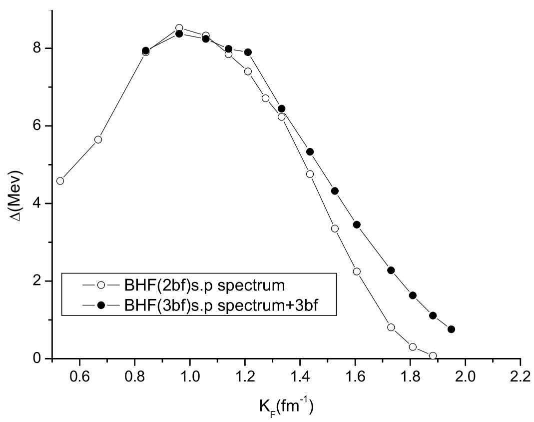

Fig. 3. Neutron-proton $^3SD_1$ pairing gaps in-symmetric nuclear matter in the two cases with the TBF and without the TBF.

The TBF is expected to influence the superfluidity in nuclear matter in two different ways. First it renormalizes the bare nucleon-nucleon interaction in the BCS equation, second it modifies the s.p.energy spectrum $U_{\mathrm{BHF}}(k)$. From further analysis of the computation process, we find that the strongest effect stems from the TBF renormalization of the NN interaction while the change of the s.p.energy spectrum has much weaker influence.

4 Conclusion

In summary,we have explored the influence of the TBF on the neutron pairing gaps in the $^3PF_2$ channel in neutron matter, the neutron $^3PF_2$ gap and the neutron-proton $^3SD_1$ gap in symmetric nuclear matter. It is shown that (1) The TBF enhances remarkably the $^3PF_2$ superfluidity in neutron matter, the peak value of the neutron $^3PF_2$ gap is increased by about 127% from 0.22 MeV to 0.50 MeV and the density region for the superfluid phase is extended from $k_{\mathrm{F}} \leqslant 2.5\ \mathrm{fm}^{-1}$ to $k_{\mathrm{F}} \leqslant 2.7\ \mathrm{fm}^{-1}$ as compared to the predictions by using the pure two-body force. (2) The TBF enhances strongly the neutron $^3PF_2$ superfluidity in symmetric nuclear matter and it makes the corresponding pairing gap increase monotonically as a function of density. The TBF influences only weakly the neutron-proton $^3SD_1$ gap in symmetric nuclear matter.

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