A new gravitational model for dark energy*

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Abstract A new gravitational model for dark energy is presented based on the model of de Sitter gauge theory of gravity. In the model, in addition to the cosmological constant, the homogeneous and isotropic torsion and its coupling with curvature play an important role for dark energy. The model may supply the universe with a natural transit from decelerating expansion to accelerating expansion.

Key words dark energy, de Sitter gauge theory of gravity, torsion

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1 Introduction

The observations on SN Ia[1] show that our universe is expanding acceleratingly. In order to explain the accelerating expansion, many dark energy models have been constructed. For example, some strange fields in some models are introduced to effectively describe the behaviors of the dark energy. An arbitrary function of curvature scalar R in gravitational Lagrangian is also used to explain the dark energy. All theses models are phenomenological ones.

The purpose of this paper is to present a new gravitational model for the dark energy other than the cosmological constant based on the model of de Sitter gauge theory of gravity[2—4], whose formulation is inspired by the dS invariant special relativity[5—7] and the principle of localization[3]. The model of the theory of gravity is constructed from the first principle.

We shall first review the model of dS gauge theory of gravity very briefly, and then present the new gravitational model for dark energy. Finally, we shall give some concluding remarks.

2 A model of de Sitter gauge theory of gravity

Like the Poincaré gauge theory of gravity[8, 9] in which the full Poincaré symmetry of a Minkowski spacetime is localized, the model of dS gauge theory of gravity can be stimulated by that gravity should be based on the idea of the localization of the full dS-symmetry of a dS spacetime and its dynamics is supposed to be governed by a gauge-like one with a dimensionless coupling constant g. At present, the theory is constructed in a special gauge called the dS-Lorentz gauge. In the special gauge, the dS connection reads[2—4, 10—14]

$$(\check{B}_{\mu}^{AB})=\begin{pmatrix} B_{\mu}^{ab} & R^{-1}e_{\mu}^{a} \\ -R^{-1}e_{\mu}^{b} & 0 \end{pmatrix}\in\mathfrak{so}(1,4), \qquad (1)$$

where $\check{B}^{AB}{}_{\mu}=\eta^{BC}\check{B}^{A}_{C\mu}$ with $(\eta^{BC})=\mathrm{diag}(1,-1,-1,-1,-1)$ and $\mu,\nu,a,b=0,\cdots,3$, B_{μ}^{ab} is the Lorentz connection, e_{μ}^{a} is the orthogonal tetrad, and R is the invariant parameter for the local de Sitter space, which is related to cosmological constant by $\Lambda=3/R^2$. The curvature of the connection is

$$\check{\mathcal{F}}_{\mu\nu}=(\check{\mathcal{F}}_{\mu\nu}^{AB})=\begin{pmatrix} F_{\mu\nu}^{ab}+R^{-2}e_{\mu\nu}^{ab} & R^{-1}T_{\mu\nu}^{a} \\ -R^{-1}T_{\mu\nu}^{b} & 0 \end{pmatrix}\in\mathfrak{so}(1,4), \qquad (2)$$

where $e_{b\mu\nu}^{a}=e_{\mu}^{a}e_{b\nu}-e_{\nu}^{a}e_{b\mu}$, $e_{a\mu}=\eta_{ab}e_{\mu}^{b}$, $F_{\mu\nu}^{ab}$ and $T_{\mu\nu}^{a}$ are the curvature and torsion of the Lorentz connec-

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tion, respectively.

The model of dS gauge theory of gravity we consider here has the action

$$S_{\rm T}=S_{\rm GYM}+S_{\bf M} \ , \tag{3}$$

where $S_{\rm M}$ is the action of the source with minimum coupling to the gravitational fields and $S_{\rm GYM}$ is the gauge-like action in Lorentz gauge of the model[2—4]:

$$\begin{aligned} S_{\rm GYM} \ = \ & \frac{\hbar}{4g^2}\int_{\mathcal{M}} {\rm d}^4 x e {\rm Tr}_{{\rm d}S}(\check{\mathcal{F}}_{\mu\nu}\check{\mathcal{F}}^{\mu\nu})= \\ & -\int_{\mathcal{M}} {\rm d}^4 x e\left[\frac{\hbar}{4g^2}F^{ab}_{\mu\nu}F^{\mu\nu}_{ab}- \right. \\ & \left. \chi(F-2\Lambda)-\frac{\chi}{2}T^a_{\mu\nu}T_a^{\mu\nu}\right]. \end{aligned} \tag{4}$$

Here, $e=\det(e^a_\mu)$, a dimensionless constant g should be introduced as usual in the gauge theory to describe the self-interaction of the gauge field, χ a dimensional coupling constant related to g and R, and $F=-\frac{1}{2}F^{ab}_{\mu\nu}e^{\mu\nu}_{ab}$ the scalar curvature of the Cartan connection, the same as the action in the Einstein-Cartan theory. In order to make sense in comparison with the Einstein-Cartan theory, we should take $R=(3/\Lambda)^{1/2}$, $\chi=1/(16\pi G)$ and $\hbar g^{-2}=3\chi\Lambda^{-1}$.

The field equations can be given via the variational principle with respect to e^a_μ, B^{ab}_μ,

$$T_{a\ \ ||\nu}^{\mu\nu}-F_a^\mu+\frac{1}{2}Fe^\mu_a-\Lambda e^\mu_a=8\pi G(T^\mu_{Ma}+T^\mu_{Ga}), \tag{5}$$

$$F_{ab\ \ ||\nu}^{\mu\nu}=R^{-2}(16\pi G S^\mu_{Mab}+S^\mu_{Gab}). \tag{6}$$

In Eqs. (5) and (6), $||$ represents the covariant derivative compatible with Christoffel symbol $\{{}^{\mu}_{\nu\kappa}\}$ and spin connection ${B^a}_{b\mu}$. Besides, $F^\mu_a=-F^{\mu\nu}_{ab}e^b_\nu$, $F=F^\mu_a e^a_\mu$. T^μ_{Ma}, S^μ_{Gab},

$$T^\mu_{Ga} \ = \ \hbar g^{-2}T^\mu_{Fa}+2\chi T^\mu_{Ta}, \tag{7}$$

$$S^\mu_{Gab} \ = \ S^\mu_{Fab}+2S^\mu_{Tab} \ , \tag{8}$$

are the tetrad form of the stress-energy tensors and spin current for matter and gravity, respectively, where

$$T^\mu_{Fa} \ := \ e^\kappa_a{\rm Tr}(F^{\mu\lambda}F_{\kappa\lambda})-\frac{1}{4}e^\mu_a{\rm Tr}(F^{\lambda\sigma}F_{\lambda\sigma}), \tag{9}$$

$$T^\mu_{Ta} \ := \ e^\kappa_a T^{\mu\lambda}_b T^b_{\kappa\lambda}-\frac{1}{4}e^\mu_a T^{\lambda\sigma}_b T^b_{\lambda\sigma}, \tag{10}$$

are the tetrad form of the stress-energy tensors for curvature and torsion, and

$$S^\mu_{Fab} \ := \ -e_{ab\ \ ||\nu}^{\mu\nu}=Y^\mu_{\lambda\nu}e^{\lambda\nu}_{ab}+Y^\nu_{\lambda\nu}e^{\mu\lambda}_{ab}, \tag{11}$$

$$S^\mu_{Tab} \ := \ T^{\mu\lambda}_{[a}e_{b]\lambda}, \tag{12}$$

are the spin currents for curvature F and torsion T, respectively. Here,

$$Y^\lambda_{\mu\nu}=\frac{1}{2}(T^\lambda_{\nu\mu}+T^\lambda_{\mu\nu}+T^\lambda_{\nu\mu}). \tag{13}$$

is the contortion.

3 Contribution of torsion to dark energy

To deal with the cosmological solutions, we suppose, as usual, that the universe is homogeneous and isotropic and that the matter in the universe is filled with the perfect fluid and dominated by dust. Thus, the spacetime is described by the Friedmann-Robertson-Walker (FRW) metirc

$${\rm d}s^2={\rm d}t^2-a^2(t)\left[\frac{{\rm d}r^2}{1-kr^2}+r^2({\rm d}\theta^2+\sin^2\theta{\rm d}\phi^2)\right], \tag{14}$$

and the stress-energy tensor of the matter is given by

$$T^{\mu\nu}=\rho U^\mu U^\nu \ , \tag{15}$$

where $a(t)$ is the scale factor, $k=0$, ±1, U^μ is 4-velocity of comoving observers and ρ is the energy density.

Now, we consider the cosmological solutions with torsion for the above equations. In order for the homogeneity and isotropy to be preserved, we consider the homogeneous and isotropic torsion in the model of dS gauge theory of gravity,

$$\begin{cases} \boldsymbol{T}^0=0 \\ \boldsymbol{T}^i=T_+\boldsymbol{b}^0\wedge\boldsymbol{b}^i, \end{cases} \tag{16}$$

where $\boldsymbol{b}^0$ and $\boldsymbol{b}^i$ are the frame 1-form, T_+ is a function of time t, which is invariant under reflection. The spin currents of matter fields in the universe are supposed to be zero. Then, the Einstein-like equations and Yang-like equations reduce to

$$\begin{aligned} & -\frac{\ddot a^2}{a^2}-\left(\dot T_++2\frac{\dot a}{a}T_+-2\frac{\ddot a}{a}\right)\dot T_++T_+^4-4\frac{\dot a}{a}T_+^3+\left(5\frac{\dot a^2}{a^2}+\right. \\ & \left. 2\frac{k}{a^2}-\frac{3}{R^2}\right)T_+^2+2\frac{\dot a}{a}\left(\frac{\ddot a}{a}-2\frac{\dot a^2}{a^2}-2\frac{k}{a^2}+\frac{3}{R^2}\right)T_++ \\ & \frac{\dot a^2}{a^2}\left(\frac{\dot a^2}{a^2}+2\frac{k}{a^2}-\frac{2}{R^2}\right)+\frac{k^2}{a^4}-\frac{2}{R^2}\frac{k}{a^2}+\frac{2}{R^4}= \\ & -\frac{16\pi G}{3R^2}\rho, \end{aligned} \tag{17}$$

$$\frac{\ddot{a}^2}{a^2}+\left(\dot{T}_+ +2\frac{\dot{a}}{a}T_+ -2\frac{\ddot{a}}{a}+\frac{6}{R^2}\right)\dot{T}_+ -T_+^4+4\frac{\dot{a}}{a}T_+^3- \left(5\frac{\dot{a}^2}{a^2}+2\frac{k}{a^2}+\frac{3}{R^2}\right)T_+^2-2\frac{\dot{a}}{a}\left(\frac{\ddot{a}}{a}-2\frac{\dot{a}^2}{a^2}-2\frac{k}{a^2}- \frac{6}{R^2}\right)T_+ -\frac{4}{R^2}\frac{\ddot{a}}{a}-\frac{\dot{a}^2}{a^2}\left(\frac{\dot{a}^2}{a^2}+2\frac{k}{a^2}+\frac{2}{R^2}\right)-\frac{k^2}{a^4}- \frac{2}{R^2}\frac{k}{a^2}+\frac{6}{R^4}=0, \tag{18}$$

$$\ddot{T}_+ +3\frac{\dot{a}}{a}\dot{T}_+ -\left(2T_+^2-6\frac{\dot{a}}{a}T_+ -\frac{\ddot{a}}{a}+5\frac{\dot{a}^2}{a^2}+2\frac{k}{a^2}- \frac{3}{R^2}\right)T_+ -\frac{\dddot{a}}{a}-\frac{\dot{a}\ddot{a}}{a^2}+2\frac{\dot{a}^3}{a^3}+2\frac{\dot{a}}{a}\frac{k}{a^2}=0. \tag{19}$$

The three equations can determine the three variables T_+, a, and ρ. Eqs. (17) and (18) give rise to

$$\frac{\ddot{a}}{a} = -H^2-\frac{k}{a^2}+\frac{4}{3}\pi G\rho+\frac{2}{R^2}+ \frac{3}{2}(\dot{T}_+ +3HT_+ -{T_+}^2), \tag{20}$$

where $H=\dot{a}/a$ is the Hubble parameter. With the help of Eqs. (20), (19) and (17) can be rewritten as

$$\ddot{T}_+ = -3(H+\frac{3}{2}T_+)\dot{T}_+ +\left[\frac{13}{2}(T_+ -3H)T_+ +6H^2- \frac{8}{R^2}+\frac{3k}{a^2}-\frac{28}{3}\pi G\rho\right]T_+ -\frac{8}{3}\pi G\dot{\rho}, \tag{21}$$

$$\left[\frac{4}{3}\pi G\rho\right]^2+\frac{4}{3}\pi G\rho\left[\dot{T}_+ +7HT_+ -3T_+^2- 2\left(H^2+\frac{k}{a^2}-\frac{2}{R^2}\right)\right]-\frac{16}{3R^2}\pi G\rho+ \left(\frac{2}{R^2}-H^2-\frac{k}{a^2}\right)\dot{T}_+ +\left(\frac{8}{R^2}-3H^2-\frac{3k}{a^2}\right)HT_+ + \left(\frac{37}{4}H^2+\frac{k}{a^2}-\frac{3}{R^2}\right)T_+^2+\frac{7}{2}HT_+\dot{T}_+ - \frac{3}{2}T_+^2\dot{T}_+ -\frac{13}{2}HT_+^3+\frac{1}{4}\dot{T}_+^2+\frac{5}{4}T_+^4- \frac{2}{R^2}\left(H^2+\frac{k}{a^2}-\frac{1}{R^2}\right)=0\ . \tag{22}$$

In principle, one can solve Eqs. (20)—(22) to obtain the cosmological solution for $a(t)$, $\rho(t)$, and $T_+(t)$. In the following, we shall find some numerical solutions for some reasonable initial parameters.

For the convenience to compare our model with the ΛCDM model in GR, we rewrite Eq. (17) as

$$1=\Omega_m+\Omega_\Lambda+\Omega_k+\Omega_{D_r}+\Omega_{D_1}, \tag{23}$$

where

$$\Omega_m=\frac{8\pi G\rho}{3H^2},\ \Omega_\Lambda=\frac{\Lambda}{3H^2},\ \Omega_k=\frac{-k}{a^2H^2}, \quad \Omega_{D_r}=\frac{D_r}{H^2},\ \Omega_{D_1}=\frac{D_1}{H^2}, \tag{24}$$

with

$$D_r := \frac{1}{2}R^2{T_{Ft}}^t+{T_{Tt}}^t=\frac{3}{2}[R^2(B^2-A^2)+{T_+}^2], \tag{25}$$

$$D_1 := -\frac{1}{3}{T_t}^{t\nu}{}_{||\nu}+2HT_+ -{T_+}^2=3HT_+ -2{T_+}^2, \tag{26}$$

where

$$A=\dot{T}_+ +HT_+ -\frac{\ddot{a}}{a},\ B=2HT_+ -{T_+}^2-H^2-\frac{k}{a^2}. \tag{27}$$

$D_r/(8\pi G)$ is the energy density of dark radiation contributed from both curvature and torsion, $D_1/(8\pi G)$ is the dark energy of the first part from the torsion. They are new contributions in comparison with the ΛCDM model in GR, in which $1=\Omega_m+\Omega_\Lambda+\Omega_k$, and play the role of the dynamical dark energy.

By virtue of Eq. (23), Eq. (20) can be rewritten as

$$q=\frac{1}{2}\Omega_m-\Omega_\Lambda+\Omega_{D_r}+\frac{1}{2}(\Omega_{D_1}-\Omega_{D_2}), \tag{28}$$

where $\Omega_{D_2}=D_2/H^2$ and

$$D_2 = -{T_r}^{r\nu}{}_{||\nu}+4HT_+ +2\dot{T}_+ -{T_+}^2= 3\dot{T}_+ +6HT_+ -{T_+}^2\ . \tag{29}$$

$D_2/(8\pi G)$ is the dark energy of the second part from the torsion.

The dark radiation Ω_{D_r} and dark energy Ω_{D_1} and Ω_{D_2} as well as Ω_Λ are unobservable directly at present time. However, the present values of $\Omega_{D_1}+\Omega_{D_r}$ and $\Omega_{D_2}-\Omega_{D_r}$ can be determined from Eq. (23) and (28) if the present values of q, Ω_m, Ω_Λ and Ω_k are known. This gives a chance to estimate 'the density of torsion' in the universe.

The behavior of the scale factor can be obtained by numerically integrating the above equations backward from today. The initial conditions for numerical calculation may be chosen based on the following facts. The kinematical analysis on the data of the SN Ia observations shows that the present deceleration parameter should be about $q_0\approx-0.7$ — -0.81[15—17]. (A subscript 0 denotes the present value as usual.) The density of matter (including baryonic and dark matter) on the scale of galaxy clusters is estimated between $\Omega_{m0}\approx 0.2$—0.3[18], which is consistent with the cosmological estimate from the observation data of WMAP[19], SDSS[20], etc. in the framework of general relativity.

Figure 1 shows the evolution of scale factor for $q_0=-0.81$ as argued in[17] and $\Omega_{m0}=0.24$. When q_0 and Ω_{m0} are fixed, there are still two degrees of freedom among Ω_{k0}, $\Omega_{\Lambda 0}$, $\Omega_{D_r0}+\Omega_{D_10}$ and $\Omega_{D_20}-\Omega_{D_r0}$. Since the model of gravity is stimulated from dS invariant special relativity and since the spatial curvature is argued to be positive in the dS invariant special relativity, we consider here the case $\Omega_{k0}=-0.02$ as an example, which is also consistent with the astronomical observation[19, 20] as long as the cosmological constant is small enough. Now, only one degree of freedom remains. We choose $\Omega_{\Lambda 0}$ as independent one and plot curves for different values of $\Omega_{\Lambda 0}$. In the figure, the horizontal axis is time in the unit of H_0^{-1} and the vertical axis is a/a_0. From the figure, we can find that the larger the cosmological constant, the younger the universe is. Obviously, some models have been ruled out because they cannot explain the ages of the oldest globular clusters[21], which are between 10 and 13 Gyr. But, there are still wide parameter ranges (roughly speaking, $0<\Omega_{\Lambda 0}<0.35$) for the models which might be used to explain the evolution of the universe. It is remarkable that the models supply a natural transit from the decelerating expansion to accelerating expansion without the help of the strange fields such as quintessence, K-essence, phantom, quintom, etc. For example, the model with $\Omega_{\Lambda 0}=0.345$ behaves as $a\rightarrow 0$ as $t\rightarrow 0$. In this case, $\Omega_{D_10}+\Omega_{D_r0}=0.435$, $\Omega_{D_20}-\Omega_{D_r0}=1.605$, $\frac{1}{2}(\Omega_{D_10}-\Omega_{D_20})+\Omega_{D_r0}=-0.585$ due to Eq. (23) and (28). It means that on the large scale, the effect of torsion cannot be ignored. The ratio of the energy density of torsion to the critical energy density is even greater than those for cosmological constant and matter.

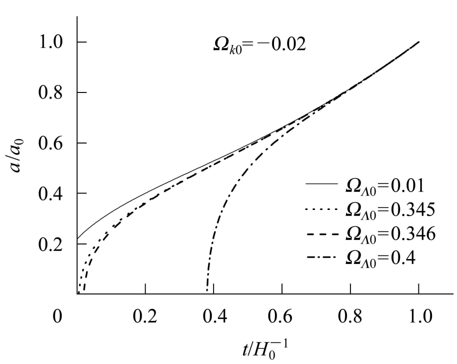

Fig. 1. Plots of the evolution of scale factor subject to different parameters. The horizontal axis is time in the unit of H_0^{-1}, while the vertical axis is the ratio of the scale factor to its present value.

Figure 2 plots the behavior of deceleration parameter versus red shift z. We can see that the transit from the decelerating expansion to the accelerating expansion happens around at $z=0.9$, which is qualitatively consistent with the analysis on the SN Ia observation[15].

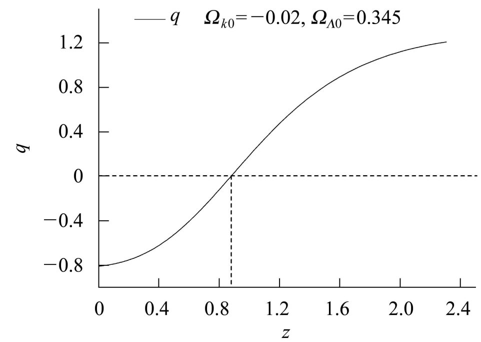

Fig. 2. Plot of the deceleration parameter versus red shift z. The transit from the decelerating expansion to the accelerating expansion occurs at $z<1$ for this model.

4 Concluding Remarks

The astronomical observations show that the universe is probably asymptotic dS. It suggests that there is a need to analyze the observation data based on a theory with local dS symmetry.

We have shown that the torsion and its coupling with curvature may serve as the dark energy in addition to the cosmological constant, which makes the universe transit from decelerating expansion to accelerating expansion. The transit may occur around at $z=0.9$, which is qualitatively consistent with the analysis on the SN Ia observation. The reason that the redshift of the transit is systematically greater than the previous analysis is that the relation between q and z is obviously not a linear one in our model, while the previous analysis is based on the assumption $q=q_0+q_1z$[15]. If we make the linear fitting for the $q-z$ curve and then parallel transport the line so that it goes through q_0 at $z=0$, then we shall get a smaller redshift for the transit. When $0<\Omega_\Lambda<0.345$ the spin-current-free cosmological solutions with torsion are reasonable.

In our cosmological models, the effects of torsion could not be ignored on the large scale, which is even greater than that of the matter density or cosmological constant. Even though, it is very difficult to directly measure the energy density of torsion by local experiments because its order of magnitude is the same as that of the cosmological constant.

Needless to say, to check whether the model can really explain the gravitational phenomena in the universe, much work is needed.

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