Vacuum condensates of QCD *

QIN Song-Mei(秦松梅)[1] ZHOU Li-Juan(周丽娟)[2]
GU Yun-Ting(顾运厅)[1] MA Wei-Xing(马维兴)[3;1)]

1 (College of Physics Science and Engineering Technology, Guangxi University, Nanning 530004, China)
2 (Collaboration Group of Hadron Physics and Non-perturbative QCD Study, Guangxi University of Technology, Liuzhou 545006, China)
3 (Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100049, China)

Abstract We study the properties of QCD vacuum state in this paper. The values of various local quark vacuum condensates, quark-gluon mixed vacuum condensates, and the structure of non-local quark vacuum condensate are predicted by the solution of Dyson-Schwinger Equations in "rainbow" approximation with three sets of different parameters for effective gluon propagator. The light quark virtuality is also obtained in a consistent way. Our all theoretical results here are in good agreement with the empirical values used widely in literature and many other theoretical calculations.

Key words QCD vacuum condensates, quark propagator, QCD

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1 Introduction

Quantum mechanics dictates that even "empty" space is not empty, but rather filled with quantum fluctuations of all possible kinds. In many contexts, such as in atomic physics, these vacuum fluctuations are subtle effects which can only be observed by precision experiments. In other situations, especially when interactions of sufficient strength are involved, the vacuum fluctuations can be of substantial magnitude and even "condense" into a non-vanishing vacuum expectation value of some quantum fields called vacuum condensate. These vacuum condensates can act as a medium[1], which influences the properties of particles propagating through it.

An important example of such a vacuum condensate is the Higgs vacuum, which is introduced in the Standard Model of particle physics to generate the masses of quarks, leptons, and the gauge bosons of the weak interaction. The vacuum expectation value of the Higgs field, $\langle\phi\rangle = 246$ GeV, is uniquely determined in the Standard Model; The quark and lepton masses differ from one another only due to the different strength of the coupling of each fermion to the Higgs field. At the same time, the quark masses also receive additional contributions from the quark and gluon condensates in the QCD vacuum. In fact, the contribution of the QCD vacuum condensates to the masses for the three light quarks (u, d, s) considerably exceeds the mass believed to be generated by the Higgs field[2]. Therefore, an accurate determination of vacuum condensates is extremely important for studying the quark mass which is the fundamental QCD input parameter of the Standard Model and will give an insight on the flavor physics, revealing the relations between masses and mixed angles, or specific textures of the quark matrix, and provide an indicator if the CP is violating because of quark mass is closely related to the CP- violating observable ϵ'/ϵ — an adjudgement of CP violation[3].

The non-vanishing value of chiral quark vacuum condensates signals the spontaneous breaking of chiral symmetry in QCD, and quantitatively is related to the pseudo-Goldstone bosons mass spectrum[4]. Due to the non-perturbative effect of QCD, the vacuum of QCD has a nontrivial structure. There are nonzero fluctuations of gluon and quark fields in QCD vacuum which manifests itself in the presence of vacuum condensates. The vacuum condensates are very important in the elucidation of the QCD structure

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1) E-mail: mawx@ihep.ac.cn

and in the description of hadron properties. If the vacuum acts as a medium and influences the properties of fundamental particles and their interactions, its properties can conceivably change. This idea has important implications in many aspects of physics. This is another importance to study QCD vacuum.

The properties of non-perturbative QCD vacuum state also determine the quark propagator. The non-perturbative vacuum of QCD is densely populated by long-wave fluctuations of quark and gluon fields. The order parameters of this complicated state are characterized by the vacuum matrix elements of various singlet combinations of quark and gluon fields, such as

$$\langle 0|:\bar{\mathrm{q}}\mathrm{q}:|0\rangle,\quad \langle 0|:G^a_{\mu\nu}G^a_{\mu\nu}:|0\rangle,$$
$$\langle 0|:\bar{\mathrm{q}}[\sigma_{\mu\nu}G^a_{\mu\nu}\frac{\lambda^a}{2}]\mathrm{q}:|0\rangle,\cdots, \quad (1)$$

which are called vacuum condensates of QCD, where $q(x)$ is the quark field, $G^a_{\mu\nu}$ represents the gluon field strength tensor with a being color index ($a=1, 2, \cdots, 8$), and can be expressed as

$$G^a_{\mu\nu}(x)=\partial_\mu A^a_\nu(x)-\partial_\nu A^a_\mu(x)+g_{\mathrm{s}}f^{abc}A^b_\mu(x)A^c_\nu(x). \quad (2)$$

λ^a in Eq. (1) is the Gell-Mann matrix, f^{abc} represents the $SU_c(3)$ structure constants, g_{s} is the coupling constant related to the so-called QCD running coupling constant α_{s} by $\alpha_{\mathrm{s}}(Q)=\dfrac{g_{\mathrm{s}}^2(Q)}{4\pi}$. A^a_μ is gluon field.

As discussed above, in QCD by condensates we mean the vacuum mean values $\langle 0|O_i|0\rangle$ of the local operators $O_i(x)$, which arise due to non-perurbative effects. The latter point is very important and needs clarification. When determining vacuum condensates one implies the averaging only over non-perturbative fluctuations. If for some operators O_i the non-zero vacuum mean value appears also in the perturbative theory, it should not be taken into account in determination of the condensate. In other words, when determining condensates the perturbative vacuum mean values should be subtracted in calculation of the vacuum averages.

Separation of perturbative and non-perturbative contribution into vacuum mean values has some arbitrariness. Usually, this arbitrariness is avoided by introducing some normalization point μ^2 ($\mu^2\sim 1\ \mathrm{GeV}^2$). Integration over momenta of virtual quarks and gluons in the region below μ^2 is referred to condensates, the above μ^2 is referred to perturbative theory. In such a formulation condensates depend on the normalization point μ: $\langle 0\mid O_i\mid 0\rangle=\langle 0\mid O_i\mid 0\rangle\mu$. In perturbation theory, there appear corrections to condensates as series in the coupling constant $\alpha_{\mathrm{s}}(\mu)$:

$$\langle 0|O_i|0\rangle_Q=\langle 0|O_i|0\rangle_\mu\sum_{n=0}^{\infty}C^i_n(Q,\mu)\alpha^n_{\mathrm{s}}(\mu). \quad (3)$$

The running coupling constant α_{s} at the right-hand part of Eq. (3) is normalized at the point μ. The left-hand part of Eq. (3) represents the value of the condensate normalized at the point Q. Coefficients $C^i_n(Q,\mu)$ may have logarithms $\ln(Q^2/\mu^2)$ in powers up to n for C^i_n. Summing up of the terms with highest powers of logarithms leads to the appearance of the so-called anomalous dimension of operators, so that in general form it can be written

$$\langle 0|O_i|0\rangle_Q=\langle 0|O_i|0\rangle_\mu\left[\frac{\alpha_{\mathrm{s}}(\mu)}{\alpha_{\mathrm{s}}(Q)}\right]^{\gamma i}\sum_{n=0}^{\infty}c^i_n(Q,\mu)\alpha^n_{\mathrm{s}}(\mu), \quad (4)$$

where γ_i are the anomalous dimensions (numbers), and c^i_n have already no leading logarithms. If there exist several operators of the given (canonical) dimension, then their mixing is possible in the perturbation theory. Then the relations Eq. (3) and (4) become matrix.

The nonzero local quark vacuum condensate $\langle 0|:\bar{\mathrm{q}}(0)\mathrm{q}(0):|0\rangle$ is responsible surely for the spontaneous breakdown of chiral symmetry. The nonzero local gluon vacuum condensate $\langle 0\ |:G^a_{\mu\nu}G^a_{\mu\nu}:|\ 0\rangle$ defines the mass scale of hadrons through trace anomaly[5]. For example, the nucleon mass M_N depends on the trace of gluon field strength tensor $\mathrm{Tr}G^2$ partly, since $M_N=\langle N|-\dfrac{9\alpha_{\mathrm{s}}}{4\pi}\mathrm{Tr}G^2+m_{\mathrm{u}}\bar{\mathrm{u}}\mathrm{u}+m_{\mathrm{d}}\bar{\mathrm{d}}\mathrm{d}+m_{\mathrm{s}}\bar{\mathrm{s}}\mathrm{s}|N\rangle$[5].

The non-local vacuum condensates $\langle 0|:\bar{q}(x)q(0):|0\rangle$ describes the distribution of quarks in the non-perturbative vacuum[6]. Physically, this means that the vacuum quarks and gluons have a nonzero mean-square momentum called virtuality. Indeed, the quark and gluon average virtualities are connected with the vacuum expectation values[7]. The average virtuality of quarks, λ^2_{q}, is related, by the equation of motion in the chiral limit, to the mixed quark-gluon local vacuum condensate[8]

$$2\lambda^2_{\mathrm{q}}=\frac{\langle 0|:\bar{\mathrm{q}}\mathrm{i}g_{\mathrm{s}}\sigma_{\mu\nu}G^a_{\mu\nu}\dfrac{\lambda^{\mathrm{a}}}{2}\mathrm{q}:|0\rangle}{\langle 0|:\bar{\mathrm{q}}\mathrm{q}:|0\rangle}. \quad (5)$$

Therefore, if one wants to study the quark virtuality λ^2_{q} in vacuum one has to calculate the quark-gluon mixed vacuum condensates and quark vacuum condensate.

Since non-perturbative QCD is insoluble, we have to search for a method to solve this difficulty. One of the acceptable approaches is the QCD sum rule. In the QCD sum rules, we deal with a correlator function which is given by the two-point function

$$\Pi_{\mu\nu}(q)=\mathrm{i}\int_0^\infty \mathrm{d}x\mathrm{e}^{\mathrm{i}qx}\langle 0|T[\chi(x)\bar{\chi}(0)]|0\rangle, \quad (6)$$

where $\chi(x)$ is called interpolator. if we consider hadron, as example, the interpolating field χ is typically constructed from quark field operators combined to give the quantum number of hadron under investigation. At the quark level, one exploits the operator product expansion (OPE)[9] to describe the short distance behavior of the two-point function

$$
\begin{aligned}
T\,[\chi(x),\bar{\chi}(0)] = & \sum_n C_n(x,\mu) O_n(0,\mu) = \\
& C_0(x) I + C_1(x) m_{\rm q} + C_3(x)\bar{\rm q}{\rm q} + C_{4.1}(x) G^a_{\mu\nu} G^{a\mu\nu} + \\
& C_{4.2}(x) m_{\rm q}\bar{\rm q}{\rm q} + C_{5.1}(x)\bar{\rm q}\sigma_{\mu\nu}\frac{\lambda^{\rm a}}{2}{\rm q}G^{a\mu\nu} + \\
& C_{5.2}(x) m_{\rm q} G^a_{\mu\nu} G^{a\mu\nu} + C_{6.1}(x)\bar{\rm q}\Gamma{\rm q}\bar{\rm q}\Gamma{\rm q} + \\
& C_{6.2}(x) m_{\rm q}\bar{\rm q}\sigma_{\mu\nu}\frac{\lambda^{\rm a}}{2}{\rm q}G^{a\mu\nu} + C_{6.3}(x) fabc G^a_{\mu\nu} G^b_{\nu\lambda} G^c_{\lambda\mu} + \\
& C_{7.1}(x)\bar{\rm q}{\rm q}G^a_{\mu\nu} G^{a\mu\nu} + C_{7.2}(x) m_{\rm q}\bar{\rm q}\Gamma{\rm q}\bar{\rm q}\Gamma{\rm q} + \\
& C_{7.3}(x) m_{\rm q} f^{abc} G^a\mu\nu G^b\nu\lambda G^c\lambda\mu + \\
& C_{8.1}(x)\bar{\rm q}\Gamma{\rm q}\bar{\rm q}\sigma_{\mu\nu}\frac{\lambda^{\rm a}}{2}{\rm q}G^{a\mu\nu} + C_{8.2}(x) m_{\rm q}\bar{\rm q}{\rm q}G^a_{\mu\nu} G^{a\mu\nu} + \\
& C_{8.3}(x) G^a_{\mu\nu} G^{a\mu\nu} G^{b\rho\lambda} + \cdots \ , \qquad (7)
\end{aligned}
$$

where μ is the normalization point at which the coefficient functions and the operators are defined. Here we have explicitly included all operators up to dimension eight, to leading order in the quark mass $m_{\rm q}$, having the quantum number of the vacuum. The first digit of the subscript of the Wilson coefficients $C(x)$ denotes the energy dimension of the operator. Γ may take any of the 16 independent Dirac-γ matrices. Thus, to carry out the QCD sum rule calculations one has to have various vacuum condensates of Eq. (7), for instance $\langle 0 \mid \bar{\rm q}{\rm q} \mid 0\rangle$, $\langle 0 \mid G^a_{\mu\nu} G^{a\mu\nu} \mid 0\rangle$, $\langle 0 \mid \bar{\rm q}\sigma^{\mu\nu}\frac{\lambda^{\rm a}}{2}{\rm q}G^{a\mu\nu} \mid 0\rangle$, $\langle 0 \mid \bar{\rm q}\Gamma{\rm q}\bar{\rm q}\Gamma{\rm q} \mid 0\rangle$, $\langle 0 \mid G^a_{\mu\nu} G^b_{\nu\lambda} G^c_{\lambda\mu} \mid 0\rangle$, $\langle 0 \mid G^a_{\mu\nu} G^{a\mu\nu} G^b_{\rho\lambda} G^{b\rho\lambda} \mid 0\rangle$, and so on. This once again shows that the study of QCD vacuum condensates is an important task.

Condensates in QCD are divided into two types: conserving chirality, such as gluonic condensate $\langle 0 \mid \frac{\alpha_{\rm s}}{\pi} G^a_{\mu\nu} G^{a\mu\nu} \mid 0\rangle$, since the term of $G^a_{\mu\nu} G^{a\mu\nu}$ in QCD Lagrangian is invariant with respect to chiral transformation, and violating chirality, for example $\langle 0 \mid \bar{\rm q}{\rm q} \mid 0\rangle$, which is related to quark mass and is of chiral variation. In neglecting quark masses $\hat{m}_f$, the QCD Lagrangian

$$
\mathcal{L} = \sum_f \bar{q}^a_f ({\rm i}\gamma^\mu D\mu - \hat{m}_f) q^a_f - \frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu} \ , \qquad (8)
$$

with $D_\mu = \partial_\mu - {\rm i}g_{\rm s}\frac{\lambda^{\rm a}}{2}A^a_\mu$ is chiral invariant: the left-hand and the right-hand (in chirality) quarks do not interact with each other, both the vector and the axial currents are conserved (except for flavor singlet axial current, non-conservation of which is due to anomaly). In the case of violating chirality, the most important condensates is the quark condensate $\langle 0 \mid \bar{\rm q}{\rm q} \mid 0\rangle$ which can be written in the form

$$
\langle 0 \mid \bar{\rm q}{\rm q} \mid 0\rangle = \langle 0 \mid \bar{\rm q}_{\rm L}{\rm q}_{\rm R} + \bar{\rm q}_{\rm R}{\rm q}_{\rm L}, \mid 0\rangle, \qquad (9)
$$

where ${\rm q}_{\rm L}$, ${\rm q}_{\rm R}$ are the fields of the left-hand and the right-hand (in chirality) quark. Therefore, as follows from Eq. (9), the non-zero value of quark condensate means the transition of the left-hand quark fields into the right-hand quark fields and it is not small would mean the chiral symmetry violation in QCD.

The present paper is organized as the following: In Sect. 2, we briefly introduce the fully dressed quark propagator defined by Dyson-Schwinger Equations (DSEs), and its rainbow approximation form. We solve the DSEs and get their solutions, A_f and B_f, by using a modeling gluon propagator. Since the solutions are related to various QCD vacuum condensates, we set up the relation of various vacuum condensates with function A_f and B_f in Sect. 3 through the operator product expansion of quark propagator. Sect. 4 is devoted to the numerical calculations of various QCD vacuum condensates described by A_f and B_f according to the operator product expansion in Sect. 3. We reserve our summary and conclusion for Sect. 5.

2 Fully dressed quark propagator

2.1 Dyson-Schwinger form of quark propagator

We start from the fully dressed quark propagator, $S_f(p)$, which is defined by Dyson-Schwinger equations[10]

$$
\begin{aligned}
{\rm i}S_f^{-1}(p) = & {\rm i}(S_f^0(p))^{-1} + C_f g_{\rm s}^2 \int \frac{{\rm d}^4 k}{(2\pi)^4} \times \\
& \gamma^\mu S_f(k)\Gamma^\nu(k,p) G_{\mu\nu}(p-k), \qquad (10)
\end{aligned}
$$

with S_f^0 being bare quark propagator, ${\rm i}(S_f^0(p))^{-1} = \not{p} - m_f$. The factor $C_f = 4/3$ stands for the color factor and $g_{\rm s}$ is the strongly coupling constant of QCD which is related to the so-called running coupling constant $\alpha_{\rm s}(Q^2)$ by the equation of $\alpha_{\rm s} = g_{\rm s}^2/4\pi$. The subscript f stands for quark flavors u, d and s. The $\Gamma^\nu(k,p)$ is Bethe-Salpeter (BS) amplitude[11] describing the fully dressed quark-gluon coupling vertex. This renormalized vertex is obtained from the vertex DSEs

$$
\begin{aligned}
\Gamma^\nu(k,p) \ = \ & Z\gamma^\nu + \sum_{b=1}^{N_{\rm c}} \int \frac{{\rm d}^4 q}{(2\pi)^4} S_f^b(k+p) \times \\
& \Gamma_b^\nu(k+p,p+q) S_f^b(p+q) K^b(k,p,q), \qquad (11)
\end{aligned}
$$

where Z is a renormalization constant and $K^b(k,p,q)$ is the kernel related to the fully dressed gluon propagator. Gauge invariance requires that the transverse part of the BS amplitude obey the Ward - Takahashi Identity (WTI)[12]

$$(k-p)_\mu \mathrm{i}\varGamma^\nu(k,p)=S_f^{-1}(k)-S_f^{-1}(p)\ , \qquad (12)$$

which is an extension to non-vanishing momentum transfer of the Ward identity[12], $\varGamma^\nu(k,p) = \frac{\partial}{\partial p_\mu}\Sigma(p)$ with $\Sigma(p)$ being the quark self-energy function formulated[13] by

$$\Sigma(p)=\mathrm{i}g_\mathrm{s}^2\int\frac{\mathrm{d}^4q}{(2\pi)^4}\gamma^\mu\frac{\lambda^\mathrm{a}}{2}S_f(q)\varGamma_b^\nu(q,p)G_{\mu\nu}^{ab}(p-q). \qquad (13)$$

The $G_{\mu\nu}(p-q)$ in Eqs. (10,13) denotes an effective gluon propagator, which is known in the perturbative QCD region but has to be modeled in the non-perturbative region since it is unknown in this region. The general form of $G_{\mu\nu}(q)$ can be written as the following

$$G_{\mu\nu}(q)=\frac{1}{q^2}\left[\left(\delta_{\mu\nu}-\frac{q_\mu q_\nu}{q^2}\right)\frac{1}{1-\varPi}+\xi\frac{q_\mu q_\nu}{q^2}\right], \qquad (14)$$

with ξ being a gauge parameter ($\xi=0$ is for the Landau gauge and $\xi=1$ the Feynman gauge). $\delta_{\mu\nu}=\mathrm{diag}(1,1,1,1)$ is the Euclidean metric. The $\varPi(q)$ is defined by

$$\varPi_{\mu\nu}(q) = \mathrm{i}\left(\delta_{\mu\nu}-\frac{q_\mu q_\nu}{q^2}\right)q^2\varPi(q)= -g_\mathrm{s}^2\int\mathrm{d}^4x\mathrm{e}^{\mathrm{i}qx}\langle 0\,|\,T[J^\mu(x)J^\nu(0)]\,|\,0\rangle_\mathrm{pp}, \qquad (15)$$

where the subscript "pp" represents the proper part of vacuum matrix elements of the time ordering operator product. That is, only one-vector-meson irreducible graphs contribute to $\varPi_{\mu\nu}(q)$.

2.2 Rainbow approximation of DSEs, $\varGamma^\nu=\gamma^\nu$

As it is impossible to solve the complete set of DSEs, Eq. (10), one has to truncate this infinite tower of integral equation in a physically acceptable way to reduce them to something that is soluble. To this end, we make a further simplification by replacing the fully dressed quark-gluon vertex $\varGamma^\nu(k,p)$ in Eq. (11) with its bare one, γ^ν. This is the so-called "rainbow" approximation. Under this approximation, the DSEs comes out to be

$$\mathrm{i}S_f^{-1}(p) = \mathrm{i}(S_f^0(p))^{-1}+\frac{4}{3}g_\mathrm{s}^2\int\frac{\mathrm{d}^4k}{(2\pi)^4}\times \gamma^\mu S_f(k)\gamma^\nu(k,p)G_{\mu\nu}(p-k). \qquad (16)$$

Eq. (16) is our basic equation to study the fully dressed quark propagator and QCD vacuum condensates. However, it should be pointed out that the price to pay for this approximation is the loss of gauge covariance of the DSEs. However, it has been proved that it is a good approximation and used widely in literature.

2.3 Modeling gluon propagator $G(p-q)$

Since the fully dressed gluon propagator $G_{\mu\nu}(q)$ in Eq. (16) is completely unknown at non-perturbative region, we must use a modeling gluon propagator to solve the DSEs. The Feynman-like gauge ($\xi=1$) used here leads to the choice of the following empirical form of the $G_{\mu\nu}(q)$

$$G_{\mu\nu}^{ab}(q)=\delta^{ab}\delta_{\mu\nu}G(q)\ , \qquad (17)$$

for the model gluon propagator with Greek letters representing Lorentz indices and Latin letters standing for color indices. Using the results of Ref. [14] for the propagator $G(q)$ leads to

$$g_\mathrm{s}^2G(s)=\frac{4\pi\alpha(s)}{s}\ , \qquad (18)$$

where $\alpha(s)$ is formulated phenomenally in terms of parameters χ and $\varDelta$ as the following[14]:

$$\alpha(s)=3\pi s\frac{\chi^2}{4\varDelta^2}\mathrm{e}^{-s/\varDelta}+\frac{\pi d}{\ln(s/\varLambda^2+\epsilon)}\ , \qquad (19)$$

which determines the quark-quark interaction through a strength χ and a range parameter $\varDelta$. The first term in Eq. (19), simulates the infrared enhancement and confinement, and the second term matches the leading log re-normalization group results. The QCD scale parameter $\varLambda=0.20$ GeV, and $d=12/(33-2N_f)=12/27$ for flavor number $N_f=3$. The ϵ in the second term of Eq. (19) can be varied in the range of 1.0—2.50. However, we take ϵ to be 2.0 from Ref. [14] in this work. The parameters $\varDelta$ and χ are given by Ref. [14] as shown in Table 1.

Table 1. The values of the strength parameter χ and range parameters $\varDelta$ of the quark-quark interaction used in our present calculations.

set no. of parameters	range parameter $\varDelta$	strength parameter χ
set 1	0.40	1.84
set 2	0.20	1.65
set 3	0.02	1.50

2.4 Solutions of DSEs, self-energy functions A_f and B_f

An important observation is that the general form of the inverse quark propagator $S_f^{-1}(p)$ can be rewritten in Euclidean space[14] as

$$S_f^{-1}(p)=\mathrm{i}\not{p}\cdot A_f(p^2)+B_f(p^2) \qquad (20)$$

in a covariant gauge with $\not{p}=\gamma^\mu p_\mu$. The propagator is renormalized at space - like point μ_p^2 according to $A_f(\mu_p^2)=1$ and $B_f(\mu_p^2)=m_f(\mu_p^2)$ with $m_f(\mu_p^2)$ being the current quark mass whose value is about

$m_{\rm u,d} = 5.1$ MeV for u, d quarks. and $m_{\rm s} = 127.5$ MeV for s quark.

Except for the current quark mass and perturbative corrections, the functions $A_f(p^2)$ and $B_f(p^2)$ are non-perturbative quantities, and satisfy the new set of DSEs in the Feynman gauge[15]

$$[A_f(p^2)-1]p^2 = \frac{8}{3}\int\frac{d^4q}{(2\pi)^4}G(p-q)\times \frac{A_f(q^2)}{q^2A_f^2(q^2)+B_f^2(q^2)}p\cdot q, \quad (21)$$

$$B_f(p^2) = \frac{16}{3}\int\frac{d^4q}{(2\pi)^4}G(p-q)\frac{A_f(q^2)}{q^2A_f^2(q^2)+B_f^2(q^2)}. \quad (22)$$

For practical calculations, we can go a further step to write the DSEs, Eqs. (21, 22), as the following

$$[A_f(s)-1]s = \frac{1}{3\pi^3}\int_0^\infty s'ds'\int_0^\pi \sin^2\phi g_s^2G(s,s')\times \frac{\sqrt{ss'}A_f(s')\cos\phi}{s'A_f^2(s')+B_f^2(s')}d\phi , \quad (23)$$

$$B_f(s) = \frac{2}{3\pi^3}\int_0^\infty s'ds'\int_0^\pi \sin^2\phi g_s^2G(s,s')\times \frac{B_f(s')}{s'A_f^2(s')+B_f^2(s')}d\phi , \quad (24)$$

where $s=p^2$, $g_s^2G(s,s') = g_s^2G(s+s'-2\sqrt{ss'}\cos\phi)$. Therefore, using the modeling gluon propagator $G(p-q)$ in Eq. (18) with $\alpha(s)$ in Eq. (19), we can solve these two coupled integral equations, Eqs. (23, 24), and obtain the self-energy functions A_f and B_f which will be used in the rest parts of this paper to calculate various vacuum condensates of QCD and to predict the structure of the non-local quark vacuum condensates, quark virtuality in the vacuum state, λ_q^2, as well as to study the p^2-dependence of vector self energy functions $[1-A_f(p^2)]$ and scalar self energy function $B_f(p^2)-m_f$. Using the functions A_f and B_f the self energy Σ_f can be written as

$$\Sigma_f(p) = \not{p}[A_f(p)-1]+B_f(p)-m_f . \quad (25)$$

Evidently, while p^2 approaches ∞ the Σ goes to zero and consequently quarks are free, asymptotic freedom.

3 Relation between vacuum condensates and solutions of DSEs, A_f and B_f

In order to set up a closed relation between the various vacuum condensates of QCD and the solutions of DSEs A_f and B_f [see Eqs. (25, 26)], we study the quark propagator in coordinate space and its operator product expansion (OPE) at short Euclidean distance[9]. The OPE is a powerful technique that systematically includes non-perturbative corrections and parameterizes the non-trivial properties of the QCD vacuum in terms of condensates. We extract vacuum condensates by evaluating the quark propagator at short distance via DSEs, and comparing the result with the OPE prediction.

3.1 Operator product expansion of quark propagator

The quark propagator $S_f(p)$ is closely related to the 2-point Green function $G_2(x_1,x_2) = \langle 0\,|\,T[\bar{q}(x_2)q(x_1)]\,|\,0\rangle$ since it represents a propagation of the quark from x_1 to x_2 (or vice versa). Accordingly, the quark propagator in space-time is defined as the correlator function

$$S^{ab}_{\mu\nu}(x) = \langle 0\,|\,T[q^a_\mu(x)\bar{q}^b_\nu(0)]\,|\,0\rangle , \quad (26)$$

where T stands for the time-ordering operator. $T[q(x)\bar{q}(0)]$ can be easily calculated by use of the Wick theorem[16]. Since the propagator is diagonal in color, the propagator can be written as $S(x) = \langle 0\,|\, T[q^a(x)\bar{q}^a(0)]\,|\,0\rangle$, with no sum on the color index a. For the physical vacuum the quark propagator $S_f(x)$ has a perturbative part represented by $S_f^{PT}(x)$ and a non-perturbative part denoted by $S_f^{NPT}(x)$. In the case of vanishing current quark masses ($m_f=0$) one can write

$$S_f(x) = S_f^{PT}(x)+S_f^{NPT}(x) . \quad (27)$$

The perturbative part, $S_f^{PT}(x)$, is given in the configuration space by the Fourier transformation of the free quark propagator $S_f^0(p) = i/(\not{p}-m_f)$ in the momentum space

$$S_f^{PT}(x) = \frac{i}{2\pi^2x^4}\gamma x\delta^{ab} - \frac{m_f}{2^2\pi^2x^2}\delta^{ab}+\cdots . \quad (28)$$

The trouble is in the non-perturbative part $S_f^{NPT}(x)$. One approach to the treatment of the non-perturbative aspects of QCD correlator is to use the operator product expansion (OPE)[6], which implies a continuation to large momenta. The operator product expansion of the non-perturbative sector of quark propagator is given[17] by

$$S_f^{NPT}(x) = -\frac{1}{12}\{\langle 0\,|:\bar{q}(x)q(0):|\,0\rangle + \gamma_\mu\langle 0\,|:\bar{q}(x)\gamma^\mu q(0):|\,0\rangle\}. \quad (29)$$

It should be stressed that normal-ordered products, and therefore S_f^{NPT}, do not vanish in the non-perturbative vacuum. Evidently, if one wants to study the non-perturbative part of the quark propagator, one has to investigate both the scalar part $\langle 0\,|\,\bar{q}(x)q(0)\,|\,0\rangle$ and the vector part $\langle 0\,|\,\bar{q}(x)\gamma^\mu q(x)\,|\,0\rangle$ of Eq. (29). However, for our present purposes here only considering the scalar part of Eq. (29) is sufficiently enough. For short distance, the OPE for the scalar part of $S_f^{NPT}(x)$, $\langle 0\,|:\bar{q}(x)q(0):|\,0\rangle$, gives

$$\langle 0\ |:\bar{q}(x)q(0):|\,0\rangle=\langle 0\,|:\bar{q}(0)q(0):|\,0\rangle-\frac{x^2}{4}\langle 0\,|:\bar{q}(0)ig_s\sigma G(0)q(0):|\,0\rangle+\cdots\ , \qquad (30)$$

where $\langle 0\,|:\bar{q}q:|\,0\rangle$ is the quark vacuum condensate and $\langle 0\,|:\bar{q}(0)\mathrm{i}g_s\sigma\cdot G(0)q(0):|\,0\rangle$ is the quark-gluon mixed vacuum condensate. Here the local operators of the expansion in Eq. (30) are the quark field, the mixture of quark field and gluon field, and so forth. They are various singlet combinations of the quark and gluon field operator.

3.2 Relation between vacuum condensate and functions A_f and B_f

As we said, the vacuum condensates are the vacuum expectation values of local operators, and they are gauge invariant constants. These condensates can be used to constrain the DSEs solutions by the following relationships in Euclidean space and in "rainbow" approximation[17]

$$\langle 0\,|:\bar{q}(x)q(0):|\,0\rangle\ =\ -\frac{12}{16\pi^2}\int_0^\infty \mathrm{d}s\cdot s\frac{B_f(s)}{sA_f^2(s)+B_f^2(s)}\times\left[2\frac{J_1(\sqrt{sx^2})}{\sqrt{sx^2}}\right], \qquad (31)$$

with the notation that $s=p^2$, the Euclidean squared momentum. Therefore, the local ($x=0$) quark vacuum condensate $\langle 0\,|:\bar{q}(0)q(0):|\,0\rangle$ can be expressed by self-energy functions $A_f(s)$ and $B_f(s)$, the solutions of DSEs, as

$$\langle 0\,|:\bar{q}(0)q(0):|\,0\rangle=-\frac{3}{4\pi^2}\int_0^\infty \mathrm{d}s\cdot s\frac{B_f(s)}{sA_f^2(s)+B_f^2(s)}\ . \qquad (32)$$

In a $1/N_{\mathrm{C}}$ expansion of the gluon two point function[15], the quark-gluon mixed vacuum condensate $\langle 0\,|:\bar{q}(0)\mathrm{i}g_s\sigma G(0)q(0):|\,0\rangle$ is derived and can be expressed[13, 15, 17] by

$$\langle 0\,|:\bar{q}(0)\mathrm{i}g_s\sigma G(0)q(0):|\,0\rangle=-\frac{9}{4\pi^2}\int_0^\infty \mathrm{d}s\cdot s\left[s\frac{B_f(s)[2-A_f(s)]}{sA_f^2(s)+B_f^2(s)}+\frac{81\cdot B_f(s)\left\{2sA_f(s)[A_f(s)-1]+B_f^2(s)\right\}}{16[sA_f^2(s)+B_f^2(s)]}\right]. \qquad (33)$$

The non-local four quark vacuum condensates are related to the self-energy functions A_f and B_f by the relationship[13—15]

$$\langle 0\,|:\bar{q}(x)\gamma_\mu\frac{\lambda^a}{2}q(x)\bar{q}(0)\gamma_\mu\frac{\lambda^a}{2}q(0):|\,0\rangle_\mu=-\int\frac{\mathrm{d}^4p}{(2\pi)^4}\int\frac{\mathrm{d}^4q}{(2\pi)^4}\mathrm{e}^{\mathrm{i}x(p-q)}\times\left[4^3\frac{B_f(p^2)}{p^2A_f^2(p^2)+B_f^2(p^2)}\frac{B_f(q^2)}{q^2A_f^2(q^2)+B_f^2(q^2)}+2\times4^2\frac{1}{p^2A_f^2(p^2)+B_f^2(p^2)}\frac{1}{q^2A_f^2(q^2)+B_f^2(q^2)}p\cdot q\right]. \qquad (34)$$

Consequently, the local ($x=0$) four quark vacuum condensates come out from Eq. (34) to be

$$\langle 0\,|:\bar{q}(0)\gamma_\mu\frac{\lambda^a}{2}q(0)\bar{q}(0)\gamma_\mu\frac{\lambda^a}{2}q(0):|\,0\rangle_\mu=-4^3\left[\int\frac{\mathrm{d}^4q}{(2\pi)^4}\frac{B_f(q^2)}{q^2A_f^2(q^2)+B_f^2(q^2)}\right]^2\ . \qquad (35)$$

which has been estimated by factorization approximation. We shall compare their results in the following section.

4 Numerical calculations of various vacuum condensates and results

Using the solutions of DSEs, Eqs. (23,24), with three different sets of quark-quark interaction parameter for effective gluon propagators as given in Table 1, A_f and B_f, leads to our following theoretical predictions for the structure of non-local quark vacuum condensate, p^2-dependence of self-energy functions, various local quark vacuum condensates, and local quark-gluon mixed vacuum condensates as well as quark virtuality in QCD vacuum state. These results are shown in following subsections respectively.

4.1 Violating chirality vacuum condensates

Carrying out the integration over s in Eq. (31), we obtain x — dependence of $\langle 0\,|:\bar{q}(x)q(0):|\,0\rangle$ — the structure of non-local quark vacuum condensate

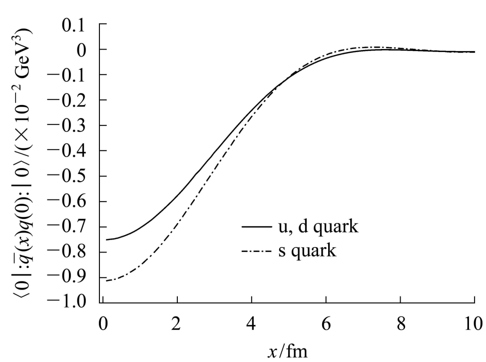

Fig. 1. Structure of non-local vacuum condensates of the light quarks: u, d, and s.

Table 2. The local quark vacuum condensates of QCD, $\langle 0|:\bar{q}q:|0\rangle$ and their ratios.

parameters	$\langle 0|:\bar{q}q:|0\rangle^{u,d}_{\mu^2=1\ \mathrm{GeV}^2}$	$\langle 0|:\bar{q}q:|0\rangle^{s}_{\mu^2=1\ \mathrm{GeV}^2}$	$\dfrac{\langle 0|:\bar{q}q:|0\rangle^{u,d}_{\mu^2=1\ \mathrm{GeV}^2}}{\langle 0|:\bar{q}q:|0\rangle^{s}_{\mu^2=1\ \mathrm{GeV}^2}}$
set No. 1	$-(196\ \mathrm{MeV})^3$	$-(209\ \mathrm{MeV})^3$	0.812
set No. 2	$-(191\ \mathrm{MeV})^3$	$-(205\ \mathrm{MeV})^3$	0.808
set No. 3	$-(179\ \mathrm{MeV})^3$	$-(193\ \mathrm{MeV})^3$	0.797

which describe the distribution of quarks in the non-peturbative vacuum state. The results are plotted in Fig. 1 for u, d and s quarks. As is seen, when $x=0$ Eq. (31) becomes Eq. (32) and the integration of Eq. (32) over s produces values of the local quark vacuum condensates for three different sets of parameters. The results are shown in Table 2.

These results are consistent with the predictions by Gall-Mann- Oakes - Renner relation[18] (GMOR), $(m_u+m_d)\langle 0\mid\bar{q}q\mid 0\rangle=-\frac{1}{2}m_\pi^2 f_\pi^2$, where m_u and m_d are quark masses with value of $m_u+m_d=9.7$ MeV, and $m_\pi=140$ MeV, $f_\pi=93$ MeV are mass and decay constant of pion, respectively. Substituting these values into the GMOR relation produces $\langle 0|\bar{q}q|0\rangle=-0.0087\ \mathrm{GeV}^3$.

From our calculations the ratio of u, d quark vacuum condensate to strange quarks (s) vacuum condensate $\langle 0|\bar{u}u|0\rangle/\langle 0|\bar{s}s|0\rangle\approx 0.8$.

The next in dimension $d=5$ condensate, which violates chiral symmetry, is the quark gluon mixed vacuum condensate. The values for the local quark-gluon mixed vacuum condensates are calculated from Eq. (33) and they are given in Table 3.

Table 3. The mixed quark-gluon local vacuum condensates of QCD, $\langle 0|:\bar{q}ig_s\sigma Gq:|0\rangle$.

para-meters	$\langle 0|:\bar{q}ig_s\sigma Gq:|0\rangle^{u,d}_{\mu^2=1\,\mathrm{GeV}^2}$	$\langle 0|:\bar{q}ig_s\sigma Gq:|0\rangle^{s}_{\mu^2=1\,\mathrm{GeV}^2}$
set No. 1	$-(718\ \mathrm{MeV})^5$	$-(761\ \mathrm{MeV})^5$
set No. 2	$-(719\ \mathrm{MeV})^5$	$-(762\ \mathrm{MeV})^5$
set No. 3	$-(719\ \mathrm{MeV})^5$	$-(759\ \mathrm{MeV})^5$

Our theoretical results are consistent with the empirical values used widely in QCD sum rules[19] and also with the predictions of Lattice calculations[20].

As we mentioned before, the vacuum matrix elements of four fermion operators can be expressed by the square of $\langle 0|:\bar{q}q:|0\rangle$, using Fierz transformations[21] saturating by the vacuum intermediate state and neglecting the contributions of all other states. For example, four quark condensates in vacuum are estimated by the factorization approximation via

$$\langle 0|:\bar{u}\Gamma_1 u\bar{u}\Gamma_2 u:|0\rangle=\frac{1}{16}\langle 0|:\bar{u}u|:0\rangle^2\times\left[\mathrm{Tr}(\Gamma_1)\mathrm{Tr}(\Gamma_2)-\frac{1}{3}\mathrm{Tr}(\Gamma_1\Gamma_2)\right], \tag{36}$$

and

$$\langle 0|:\bar{u}\Gamma_1\lambda^a u\bar{u}\Gamma_2\lambda^a u:|0\rangle=-\frac{1}{9}\langle 0|:\bar{u}u:|0\rangle^2\mathrm{Tr}(\Gamma_1\Gamma_2), \tag{37}$$

and so on, where Γ_1 and Γ_2 are Dirac matrices, and "Tr" denotes a trace over the Dirac indices. Under the factorization approximation, the four quark vacuum condensates come out as being

$$\langle 0|:\bar{q}\sigma_{\mu\nu}\frac{\lambda^a}{2}q\bar{q}\sigma_{\mu\nu}\frac{\lambda^a}{2}q:|0\rangle=-\frac{4}{3}\langle 0|:\bar{q}q:|0\rangle^2, \tag{38}$$

$$\langle 0|:\bar{q}\gamma_\mu\frac{\lambda^a}{2}q\bar{q}\gamma_\mu\frac{\lambda^a}{2}q:|0\rangle=-\frac{4}{9}\langle 0|:\bar{q}q:|0\rangle^2, \tag{39}$$

$$\langle 0|:\bar{q}\gamma_5\frac{\lambda^a}{2}q\bar{q}\gamma_5\frac{\lambda^a}{2}q:|0\rangle=-\frac{1}{9}\langle 0|:\bar{q}q:|0\rangle^2. \tag{40}$$

Our theoretical predictions from Eqs. (38—40) for the dimension $d=6$ condensates, four quark local vacuum condensates built from quark fields, are shown in Tables 4—6.

Table 4. Four quark local vacuum condensates of QCD, $\langle 0|:\bar{q}\gamma_\mu\frac{\lambda_c^a}{2}q\bar{q}\gamma_\mu\frac{\lambda_c^a}{2}q:|0\rangle$.

parameters	$\langle 0|:\bar{q}\gamma_\mu\frac{\lambda_c^a}{2}q\bar{q}\gamma_\mu\frac{\lambda_c^a}{2}q:|0\rangle^{u,d}_{\mu^2=1\ \mathrm{GeV}^2}$	$\langle 0|:\bar{q}\gamma_\mu\frac{\lambda_c^a}{2}q\bar{q}\gamma_\mu\frac{\lambda_c^a}{2}q:|0\rangle^{s}_{\mu^2=1\ \mathrm{GeV}^2}$
set No. 1	$-2.519\times10^{-5}\ \mathrm{GeV}^6$	$-3.704\times10^{-5}\ \mathrm{GeV}^6$
set No. 2	$-2.158\times10^{-5}\ \mathrm{GeV}^6$	$-3.299\times10^{-5}\ \mathrm{GeV}^6$
set No. 3	$-2.462\times10^{-5}\ \mathrm{GeV}^6$	$-2.297\times10^{-5}\ \mathrm{GeV}^6$

Table 5. Four quark local vacuum condensates of QCD, $\langle 0|:\bar{q}\sigma_{\mu\nu}\frac{\lambda_c^a}{2}q\bar{q}\sigma_{\mu\nu}\frac{\lambda_c^a}{2}q:|0\rangle$.

parameters	$\langle 0|:\bar{q}\sigma_{\mu\nu}\frac{\lambda_c^a}{2}q\bar{q}\sigma_{\mu\nu}\frac{\lambda_c^a}{2}q:|0\rangle^{u,d}_{\mu^2=1\ \mathrm{GeV}^2}$	$\langle 0|:\bar{q}\sigma_{\mu\nu}\frac{\lambda_c^a}{2}q\bar{q}\sigma_{\mu\nu}\frac{\lambda_c^a}{2}q:|0\rangle^{s}_{\mu^2=1\ \mathrm{GeV}^2}$
set No. 1	$-7.560\times10^{-5}\ \mathrm{GeV}^6$	$-11.110\times10^{-5}\ \mathrm{GeV}^6$
set No. 2	$-6.473\times10^{-5}\ \mathrm{GeV}^6$	$-9.896\times10^{-5}\ \mathrm{GeV}^6$
set No. 3	$-4.386\times10^{-5}\ \mathrm{GeV}^6$	$-6.891\times10^{-5}\ \mathrm{GeV}^6$

Table 6. Four quark local vacuum condensates of QCD, $\langle 0|:\bar{q}\gamma_5\frac{\lambda_c^a}{2}q\bar{q}\gamma_5\frac{\lambda_c^a}{2}q:|0\rangle$.

parameters	$\langle 0\|:\bar{q}\gamma_5\frac{\lambda_c^a}{2}q\bar{q}\gamma_5\frac{\lambda_c^a}{2}q:\|0\rangle^{\rm u,d}_{\mu^2=1\ \rm GeV^2}$	$\langle 0\|:\bar{q}\gamma_5\frac{\lambda_c^a}{2}q\bar{q}\gamma_5\frac{\lambda_c^a}{2}q:\|0\rangle^{\rm s}_{\mu^2=1\ \rm GeV^2}$
set No. 1	-6.299×10^{-6} GeV^6	-9.261×10^{-6} GeV^6
set No. 2	-5.395×10^{-6} GeV^6	-8.247×10^{-6} GeV^6
set No. 3	-3.655×10^{-6} GeV^6	-5.743×10^{-6} GeV^6

The variation of self-energy functions with s^2 is given in Fig. 2 where the top panel is for scalar function $[B_f-m_f]$ and low one is for vector self-energy $[1-A_f]$. As is seen, while p^2 approaches to ∞ both functions, $[B_f-m_f]$ and $[1-A_f]$, approach to zero.

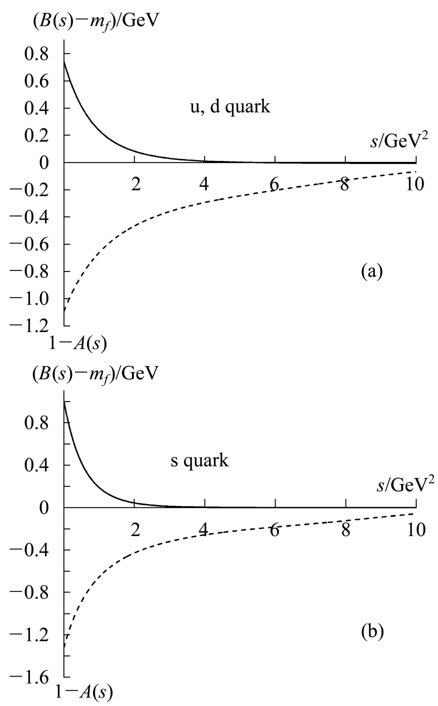

Fig. 2. s^2- dependence of self-energy functions $[1-A_f(s^2)]$ and $B_f(s^2)-m_f$: (a) for u, d quarks; (b) for s quark.

4.2 Quark virtuality in the QCD vacuum state

According to Eq. (5), our theoretical predictions from Eqs. (32,33) for quark virtuality, the nonzero mean square momentum of quarks in QCD vacuum state, is

$$\lambda^2_{\rm u,d} = \frac{1}{2}\frac{\langle 0|:\bar{q}(0)\left[{\rm i}g_{\rm s}\sigma_{\mu\nu}G^a_{\mu\nu}\frac{\lambda^a}{2}\right]q(0):|0\rangle_{\rm u,d}}{\langle 0|:\bar{q}(0)q(0):|0\rangle_{\rm u,d}} = 0.70\ {\rm GeV}^2 \quad (41)$$

for u, d quark, which is in the acceptable range[22] of $\lambda^2_{\rm q}$ between 0.4—1.0 GeV^2. The standard QCD sum rule estimation[23] gives $\lambda^2_{\rm u,d}=0.4\pm0.1$ GeV^2, the QCD sum rule analysis of pion form factor[24] produces $\lambda^2_{\rm u,d} = 0.70$ GeV^2 and lattice QCD calculations[25] $\lambda^2_{\rm u,d} = 0.55$ GeV^2. For s quark, we have

$$\lambda^2_{\rm s} = \frac{1}{2}\frac{\langle 0|:\bar{q}(0)[{\rm i}g_{\rm s}\sigma_{\mu\nu}G^a_{\mu\nu}\frac{\lambda^a}{2}]q(0):|0\rangle_{\rm s}}{\langle 0|:\bar{q}(0)q(0):|0\rangle_{\rm s}} = 1.60\ {\rm GeV}^2 \quad (42)$$

which is consistent with the predictions of lattice QCD[25], $\lambda^2_{\rm s}=2.50$ GeV^2, and the instanton model prediction[26], $\lambda^2_{\rm s}=1.40$ GeV^2.

5 Summary and concluding remarks

We study QCD vacuum in this paper. The values of various local quark vacuum condensates, quark-gluon mixed vacuum condensates, and structure of non-local quark vacuum condensate are predicted by the solution of Dyson-Schwinger Equations in "rainbow" approximation with three sets of parameters for the effective gluon propagator $\alpha(s)$. The light quark virtuality is also obtained in the same way. All theoretical results are in a good agreement with empirical values used widely in literature, and other theoretical approximations.

The QCD vacuum is densely populated by long-wave fluctuations of quark and gluon fields. The order parameters of this complicated vacuum state are characterized by a variety of local vacuum condensates which are vacuum matrix elements of various singlet combinations of quark and gluon fields. The existence of QCD vacuum condensates reflects in a direct way the non-perturbative structure of the QCD vacuum. Studying the non-perturbative structure of QCD vacuum is of decisive importance for strong interaction physics. For instance, in addition to the quark mass originated from coupling to Higgs, the QCD vacuum condensates also make large contributions to light quark masses. An accurate determination of quark masses can give a deep insight on the physics of flavor, revealing relations between masses and mixing angles, or specific textures in the quark mass matrix, which may originate from still uncovered flavor symmetries.

The QCD vacuum acts as a medium and influences the properties of fundamental particles and their interactions, its properties can conceivably change. The nonzero local quark vacuum condensate is responsible for the spontaneous breakdown of

Chiral symmetry. The nonzero local gluon vacuum condensate defines the mass scale of hadrons through trace anomaly. The non-local vacuum condensates describe the distribution of quarks in the non - perturbative QCD vacuum state. Physically, this means that vacuum quark and gluons have a nonzero mean-square momentum called virtuality.

All our predictions on vacuum condensates and quark virtuality are in good agreement with those used widely in literature and other theoretical predictions such as QCD sum rules and lattice QCD calculations.

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