$D^{0}-\overline{D}^{0}$ mixing and CP violation at BES-III^{*}

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Abstract Recently, both BaBar and Belle experiments found evidences of neutral D mixing. In this paper, we discuss the sensitivities of the measurements of D mixing parameters at BES-III. With CP tag technique at $\psi(3770)$ peak, the extraction of the strong phase difference in $D^0 \rightarrow K\pi$ decay at BES-III are discussed. We also make an estimate on the measurements of the mixing rate R_M by using the coherent data at $\psi(3770)$ peak. The CP violation in D system is predicted with an unobserved level in the Standard Model. Any significant CP violation in the D system indicates the existence of new physics. The sensitivity of the measurements of CP violation in the D system is estimated in the coherent D decays. Finally, the search for the rare D decays are discussed, in which some of the forbidden decays are smoking gun of new physics.

Key words D mixing, BES-III experiment, CP violation, rare decay

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1 Introduction

Evidence for mixing in the neutral D meson system has recently been reported by the BaBar and Belle collaborations^[1, 2]. These experiments find nonvanishing width and mass differences between the two neutral D mass eigenstates, which indicates for the first time that the Flavor-changing-Neutral-Current (FCNC) has been observed in the up-type quark sector. As we know, theoretical predictions of x and yin the Standard Model (SM) are very uncertain due to long-distance dilutions^[3, 4]. Fortunately, *CP* violation in mixing is $O(10^{-6})$ in the SM so *CP* violation involving $D^0\overline{D}^0$ oscillations is a reliable probe of New Physics.

Although the time-dependent analyses have been done at B factories, the D mesons produced there are incoherent. The most promising place to produce $D^0\overline{D}^0$ pairs with low backgrounds is the $\psi(3770)$ resonance just above the $D^0\overline{D}^0$ threshold at CLEO-c and BES-III experiments^[5], on which the coherent information can be used. The amplitude for $\psi(3770)$ decaying to $D^0\overline{D}^0$ is $\langle D^0\overline{D}^0|H|\psi(3770)\rangle$, and the $D^0\overline{D}^0$ pair system is in a state with charge parity C = -1, which can be defined as ^[6]

$$|\mathbf{D}^{0}\overline{\mathbf{D}}^{0}\rangle^{C=-1} = \frac{1}{\sqrt{2}} \left[|\mathbf{D}^{0}\rangle|\overline{\mathbf{D}}^{0}\rangle - |\overline{\mathbf{D}}^{0}\rangle|\mathbf{D}^{0}\rangle \right].$$
(1)

Although there is a weak current contribution in $\psi(3770) \rightarrow D^0 \overline{D}^0$ decay, which may not conserve charge parity, the $D^0 \overline{D}^0$ pair can not be in a state with C = +1. The reason is that the relative orbital angular momentum of $D^0 \overline{D}^0$ pair must be l = 1 because of angular momentum conservation. A boson-pair with l = 1 must be in an anti-symmetric state, the anti-symmetric state of particle-anti-particle pair must be in a state with C = -1.

The $\psi(3770)$ decays will provide another opportunity to search for $D^0 - \overline{D}^0$ mixing and understand the source of CP violation in charm system. In this paper, we discuss the measurements of the mixing parameters in D system at the BES-III experiments near the open-charm threshold. With data at $\psi(3770)$ peak, the CP violation can be probed by considering the quantum-correlation between $D^0\overline{D}^0$ in the decay of $\psi(3770) \rightarrow D^0\overline{D}^0$.

2 Basic definitions

With the assumption of CPT invariance, the mass eigenstates of $D^0-\overline{D}^0$ system are $|D_1\rangle = p|D^0\rangle + q|\overline{D}^0\rangle$ and $|D_2\rangle = p|D^0\rangle - q|\overline{D}^0\rangle$ with eigenvalues $\mu_1 = m_1 - \frac{i}{2}\Gamma_1$ and $\mu_2 = m_2 - \frac{i}{2}\Gamma_2$, respectively, where

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the m_1 and Γ_1 (m_2 and Γ_2) are the mass and width of D_1 (D_2). For the method of detecting $D^0-\overline{D}^0$ mixing involving the $D^0 \to K\pi$ decay mentioned above, in order to separate the Doubly-Cabibbo-suppressed (DCS) decay from the mixing signal, one must study the time-dependent decay rate at the B factories. The proper-time evolution of the particle states $|D_{phys}^0(t)\rangle$

and $|\overline{\mathbf{D}}_{\text{phys}}^{0}(t)\rangle$ are given by

$$\begin{aligned} |\mathbf{D}^{0}_{\mathrm{phys}}(t)\rangle &= g_{+}(t)|\mathbf{D}^{0}\rangle - \frac{q}{p}g_{-}(t)|\overline{\mathbf{D}}^{0}\rangle, \\ |\overline{\mathbf{D}}^{0}_{\mathrm{phys}}(t)\rangle &= g_{+}(t)|\overline{\mathbf{D}}^{0}\rangle - \frac{p}{q}g_{-}(t)|\mathbf{D}^{0}\rangle, \end{aligned}$$
(2)

where

$$g_{\pm} = \frac{1}{2} \left(e^{-im_2 t - \frac{1}{2}\Gamma_2 t} \pm e^{-im_1 t - \frac{1}{2}\Gamma_1 t} \right) , \qquad (3)$$

with definitions

$$m \equiv \frac{m_1 + m_2}{2}, \quad \Delta m \equiv m_2 - m_1,$$

$$\Gamma \equiv \frac{\Gamma_1 + \Gamma_2}{2}, \quad \Delta \Gamma \equiv \Gamma_2 - \Gamma_1 , \qquad (4)$$

Note the sign of Δm and $\Delta \Gamma$ is to be determined by experiments. In practice, one defines the following mixing parameters

$$x \equiv \frac{\Delta m}{\Gamma}, \quad y \equiv \frac{\Delta \Gamma}{2\Gamma}$$
 (5)

The time-dependent decay amplitudes for $D^0_{phys}(t) \rightarrow K^+\pi^-$ and $\overline{D}^0_{phys}(t) \rightarrow K^-\pi^+$ are described as

$$\langle \mathbf{K}^{+}\pi^{-} | \mathcal{H} | \mathbf{D}_{\mathrm{phys}}^{0}(t) \rangle = g_{+}(t) A_{\mathbf{K}^{+}\pi^{-}} - \frac{q}{p} g_{-}(t) \overline{A}_{\mathbf{K}^{+}\pi^{-}} = \frac{q}{p} \overline{A}_{\mathbf{K}^{+}\pi^{-}} [\lambda g_{+}(t) - g_{-}(t)],$$
 (6)

$$\langle \mathbf{K}^{-}\pi^{+} | \mathcal{H} | \overline{\mathbf{D}}_{\mathrm{phys}}^{0}(t) \rangle = g_{+}(t) \overline{A}_{\mathbf{K}^{-}\pi^{+}} - \frac{p}{q} g_{-}(t) A_{\mathbf{K}^{-}\pi^{+}} = \frac{p}{q} A_{\mathbf{K}^{-}\pi^{+}} [\overline{\lambda} g_{+}(t) - g_{-}(t)],$$
(7)

where $A_{K^+\pi^-} = \langle K^+\pi^- | \mathcal{H} | D^0 \rangle$, $\overline{A}_{K^+\pi^-} = \langle K^+\pi^- | \mathcal{H} | \overline{D}^0 \rangle$, $A_{K^-\pi^+} = \langle K^-\pi^+ | \mathcal{H} | D^0 \rangle$, and $\overline{A}_{K^-\pi^+} = \langle K^-\pi^+ | \mathcal{H} | \overline{D}^0 \rangle$. Here, λ and $\overline{\lambda}$ are defined as:

$$\lambda \equiv \frac{p}{q} \frac{A_{\mathrm{K}^+\pi^-}}{\overline{A}_{\mathrm{K}^+\pi^-}},\tag{8}$$

$$\overline{\lambda} \equiv \frac{q}{p} \frac{\overline{A}_{\mathrm{K}^-\pi^+}}{A_{\mathrm{K}^-\pi^+}}.$$
(9)

From Eqs. (6) and (7), one can derive the general expression for the time-dependent decay rate, in agree-

ment with [7, 8]:

$$\frac{\mathrm{d}\Gamma(\mathrm{D}^{0}_{\mathrm{phys}}(t) \to \mathrm{K}^{+}\pi^{-})}{\mathrm{d}t\mathcal{N}} = |\overline{A}_{\mathrm{K}^{+}\pi^{-}}|^{2} \left|\frac{q}{p}\right|^{2} \mathrm{e}^{-\Gamma t} \times \\ [(|\lambda|^{2}+1)\mathrm{cosh}(y\Gamma t) + \\ (|\lambda|^{2}-1)\mathrm{cos}(x\Gamma t) + \\ 2\mathrm{Re}(\lambda)\mathrm{sinh}(y\Gamma t) + \\ 2\mathrm{Im}(\lambda)\mathrm{sin}(x\Gamma t)], \quad (10)$$

$$\frac{\mathrm{d}\Gamma(\mathrm{D}^{\circ}_{\mathrm{phys}}(t) \to \mathrm{K}^{-}\pi^{+})}{\mathrm{d}t\mathcal{N}} = |A_{\mathrm{K}^{-}\pi^{+}}|^{2} \left|\frac{p}{q}\right|^{2} \mathrm{e}^{-\Gamma t} \times \\ [(|\overline{\lambda}|^{2}+1)\mathrm{cosh}(y\Gamma t) + \\ (|\overline{\lambda}|^{2}-1)\mathrm{cos}(x\Gamma t) + \\ 2\mathrm{Re}(\overline{\lambda})\mathrm{sinh}(y\Gamma t) + \\ 2\mathrm{Im}(\overline{\lambda})\mathrm{sin}(x\Gamma t)], \quad (11)$$

where \mathcal{N} is a common normalization factor. In order to simplify the above formula, we make the following definition:

$$\frac{q}{p} \equiv (1+A_{\rm M}) \mathrm{e}^{-\mathrm{i}\beta} , \qquad (12)$$

where β is the weak phase in mixing and $A_{\rm M}$ is a real-valued parameter which indicates the magnitude of *CP* violation in the mixing. For $f = K^{-}\pi^{+}$ final state, we define

$$\frac{A_{\mathrm{K}^{+}\pi^{-}}}{\overline{A}_{\mathrm{K}^{+}\pi^{-}}} \equiv -\sqrt{r'} \mathrm{e}^{-\mathrm{i}\alpha'}; \quad \frac{\overline{A}_{\mathrm{K}^{-}\pi^{+}}}{A_{\mathrm{K}^{-}\pi^{+}}} \equiv -\sqrt{r} \mathrm{e}^{-\mathrm{i}\alpha}, \quad (13)$$

where r' and α' (r and α) are the ratio and relative phase of the DCS decay rate and the Cabibbi-favored (CF) decay rate. Then, λ and $\overline{\lambda}$ can be parameterized as

$$\lambda = -\sqrt{r'} \frac{1}{1+A_{\rm M}} \mathrm{e}^{-\mathrm{i}(\alpha'-\beta)},\qquad(14)$$

$$\overline{\lambda} = -\sqrt{r} (1 + A_{\rm M}) \mathrm{e}^{-\mathrm{i}(\alpha + \beta)} \,. \tag{15}$$

In order to demonstrate the *CP* violation in decay, we define $\sqrt{r'} \equiv \sqrt{R_{\rm D}}(1+A_{\rm D})$ and $\sqrt{r} \equiv \sqrt{R_{\rm D}}\frac{1}{1+A_{\rm D}}$. Thus, Eqs. (14) and (15) can be expressed as

$$\lambda = -\sqrt{R_{\rm D}} \frac{1 + A_{\rm D}}{1 + A_{\rm M}} \mathrm{e}^{-\mathrm{i}(\delta - \phi)}, \qquad (16)$$

$$\overline{\lambda} = -\sqrt{R_{\rm D}} \frac{1 + A_{\rm M}}{1 + A_{\rm D}} \mathrm{e}^{-\mathrm{i}(\delta + \phi)}, \qquad (17)$$

where $\delta = \frac{\alpha' + \alpha}{2}$ is the averaged phase difference between DCS and CF processes, and $\phi = \frac{\alpha - \alpha'}{2} + \beta$.

We can characterize the CP violation in the mixing amplitude, the decay amplitude, and the interference between amplitudes with and without mixing, by real-valued parameters $A_{\rm M}$, $A_{\rm D}$, and ϕ as in Refs. [9, 10]. In the limit of CP conservation, $A_{\rm M}$, $A_{\rm D}$ and ϕ are all zero. $A_{\rm M} = 0$ means no CP violation in mixing, namely, |q/p| = 1; $A_{\rm D} = 0$ means no CP violation in decay, for this case, $r = r' = R_{\rm D} = |\overline{A}_{{\rm K}^-\pi^+}/A_{{\rm K}^-\pi^+}|^2 = |A_{{\rm K}^+\pi^-}/\overline{A}_{{\rm K}^+\pi^-}|^2$; $\phi = 0$ means no CP violation in the interference between decay and mixing.

3 $D^0 - \overline{D}^0$ mixing

In experimental searches, one can define CF decay as right-sign (RS) and DCS decay or via mixing followed by a CF decay as wrong-sign (WS). Here, we define the ratio of WS to RS decays as for D^0 :

$$R(t) = \frac{\mathrm{d}\Gamma(\mathrm{D}^{0}_{\mathrm{phys}}(t) \to \mathrm{K}^{+}\pi^{-})}{\mathrm{d}t\mathcal{N} \times \mathrm{e}^{-\Gamma|t|} \times 2|\overline{A}_{\mathrm{K}^{+}\pi^{-}}|^{2}} , \qquad (18)$$

and for \overline{D}^0 :

$$\overline{R}(t) = \frac{\mathrm{d}\Gamma(\overline{\mathrm{D}}_{\mathrm{phys}}^{0}(t) \to \mathrm{K}^{-}\pi^{+})}{\mathrm{d}t\mathcal{N} \times \mathrm{e}^{-\Gamma|t|} \times 2|A_{\mathrm{K}^{-}\pi^{+}}|^{2}} , \qquad (19)$$

Taking into account that $|\lambda|$, $|\overline{\lambda}| \ll 1$ and $x, y \ll 1$, keeping terms up to order x^2 , y^2 and R_D in the expressions, neglecting CP violation in mixing, decay and the interference between decay with and without mixing ($A_{\rm M} = 0$, $A_{\rm D} = 0$, and $\phi = 0$), expanding the time-dependence for $xt, yt \lesssim \Gamma^{-1}$, combining Eqs. (10) and (11), we can write Eqs. (18) and (19) as

$$R(t) = \overline{R}(t) = R_{\rm D} + \sqrt{R_{\rm D}} y' \Gamma t + \frac{{x'}^2 + {y'}^2}{4} (\Gamma t)^2 , (20)$$

where

$$x' = x\cos\delta + y\sin\delta , \qquad (21)$$

$$y' = -x\sin\delta + y\cos\delta \ . \tag{22}$$

In the limit of SU(3) symmetry, $A_{K^+\pi^-}$ and $\overline{A}_{K^+\pi^-}$ $(A_{K^-\pi^+}$ and $\overline{A}_{K^-\pi^+})$ are simply related by CKM factors, $A_{K^+\pi^-} = (V_{cd}V_{us}^*/V_{cs}V_{ud}^*)\overline{A}_{K^+\pi^-}^{[11]}$. In particular, $A_{K^+\pi^-}$ and $\overline{A}_{K^+\pi^-}$ have the same strong phase, leading to $\alpha' = \alpha = 0$ in Eq. (13). But the SU(3) symmetry is broken according to the recent precise measurements from the B factories, the ratio^[9]:

$$\mathcal{R} = \frac{\mathcal{B}\mathcal{R}(\mathbf{D}^0 \to \mathbf{K}^+ \pi^-)}{\mathcal{B}\mathcal{R}(\overline{\mathbf{D}}^0 \to \mathbf{K}^+ \pi^-)} \left| \frac{V_{\rm ud} V_{\rm cs}^*}{V_{\rm us} V_{\rm cd}^*} \right|^2 , \qquad (23)$$

is unity in the SU(3) symmetry limit. But, the world average for this ratio is

$$\mathcal{R}_{\rm exp} = 1.21 \pm 0.03 \;, \tag{24}$$

computed from the individual measurements using the standard method of Ref. [7]. Since the SU(3)is broken in $D \to K\pi$ decays at the level of 20%, in which case the strong phase δ should be non-zero. Recently, a time-dependent analysis in $D \to K\pi$ has been performed based on 384 fb⁻¹ luminosity at $\Upsilon(4S)$ by BaBar experiment^[1]. By assuming *CP* conservation, they obtained the following neutral D mixing results

$$R_{\rm D} = (3.03 \pm 0.16 \pm 0.10) \times 10^{-3},$$

$$x'^{2} = (-0.22 \pm 0.30 \pm 0.21) \times 10^{-3},$$
 (25)

$$y' = (9.7 \pm 4.4 \pm 3.1) \times 10^{-3}.$$

The result is inconsistent with the no-mixing hypothesis with a significance of 3.9 standard deviations. The results from BaBar and Belle are in agreement within 2 standard deviation on the exact analysis of y' measurement by using $D \rightarrow K\pi$ as listed in Table 1.

Table 1. Experimental results used in the paper. Only one error is quoted, we have combined in quadrature statistical and systematic contributions.

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parameter	$BaBar(\times 10^{-3})$	$\text{Belle}(\times 10^{-3})$	technique
x'^2	$-0.22 \pm 0.37^{[1]}$	$0.18^{+0.21}_{-0.23}$ ^[12]	Κπ
y'	$9.7 \pm 5.4^{[1]}$	$0.6^{+4.0}_{-3.9}^{[12]}$	Κπ
$R_{\rm D}$	$3.03 \pm 0.19^{[1]}$	$3.64 \pm 0.17^{[12]}$	Κπ
y_{CP}	_	$13.1 \pm 4.1^{[2]}$	$K^+K^-, \pi^+\pi^-$
x	—	$8.0 \pm 3.4^{[2]}$	$K_S \pi^+ \pi^-$
y	—	$3.3 \pm 2.8^{[2]}$	$K_S \pi^+ \pi^-$

At $\psi(3770)$ peak, to extract the mixing parameter y, one can make use of rates for exclusive $D^0\overline{D}^0$ combination, where both the D^0 and \overline{D}^0 final states are specified (known as double tags or DT), as well as inclusive rates, where either the D^0 or \overline{D}^0 is identified and the other D^0 decays generically (known as single tags or ST)^[13]. With the DT tag technique^[14, 15], one can fully consider the quantum correlation in C = -1and $C = +1 \ D^0\overline{D}^0$ pairs produced in the reaction $e^+e^- \rightarrow D^0\overline{D}^0(n\pi^0)$ and $e^+e^- \rightarrow D^0\overline{D}^0\gamma(n\pi^0)^{[13, 16, 17]}$, respectively.

For the ST, in the limit of CP conservation, the rate of D⁰ decays into a CP eigenstate is given as^[13]:

$$\Gamma_{\rm f_{\eta}} \equiv \Gamma({\rm D}^0 \to {\rm f_{\eta}}) = 2A_{\rm f_{\eta}}^2 \left[1 - \eta y\right], \qquad (26)$$

where f_{η} is a *CP* eigenstate with eigenvalue $\eta = \pm 1$, and $A_{f_{\eta}} = |\langle f_{\eta} | \mathcal{H} | D^0 \rangle|$ is the real-valued decay amplitude.

For the DT case, Gronau et al.^[11] and Xing^[18] have considered time-integrated decays into correlated pairs of states, including the effects of non-zero final state phase difference. As discussed in Ref. [11], the rate of $(D^0\overline{D}^0)^{C=-1} \rightarrow (l^{\pm}X)(f_{\eta})$ is described as^[11]:

$$\Gamma_{l;f_{\eta}} \equiv \Gamma[(l^{\pm}X)(f_{\eta})] = A_{l^{\pm}X}^{2} A_{f_{\eta}}^{2} (1+y^{2}) \approx A_{l^{\pm}X}^{2} A_{f_{\eta}}^{2} , \qquad (27)$$

where $A_{l^{\pm}X} = |\langle l^{\pm}X | \mathcal{H} | D^0 \rangle|$ is real-valued amplitude

for semileptonic decays, here, we neglect y^2 term since $y \ll 1$.

For C = -1 initial $D^0 \overline{D}^0$ state, y can be expressed in term of the ratios of DT rates and the double ratios of ST rates to DT rates^[13]:

$$y = \frac{1}{4} \left(\frac{\Gamma_{l;f_{+}} \Gamma_{f_{-}}}{\Gamma_{l;f_{-}} \Gamma_{f_{+}}} - \frac{\Gamma_{l;f_{-}} \Gamma_{f_{+}}}{\Gamma_{l;f_{+}} \Gamma_{f_{-}}} \right).$$
(28)

For a small y, its error, $\Delta(y)$, is approximately $1/\sqrt{N_{l\pm X}}$, where $N_{l\pm X}$ is the total number of $(l^{\pm}X)$ events tagged with CP-even and CP-odd eigenstates. The number $N_{l\pm X}$ of CP tagged events is related to the total number of $D^0\overline{D}^0$ pairs $N(D^0\overline{D}^0)$ through $N_{l\pm X} \approx N(D^0\overline{D}^0)[\mathcal{BR}(D^0 \to l^{\pm} + X) \times \mathcal{BR}(D^0 \to f_{\pm}) \times \epsilon_{tag}] \approx 1.5 \times 10^{-3} N(D^0\overline{D}^0)$, here we take the branching ratio-times-efficiency factor $(\mathcal{BR}(D^0 \to f_{\pm}) \times \epsilon_{tag})$ for tagging CP eigenstates is to be about 1.1% (the total branching ratio into CP eigenstates is larger than about 5%^[7]). We find

$$\Delta(y) = \frac{\pm 26}{\sqrt{N(D^0 \overline{D}^0)}} = \pm 0.003 .$$
 (29)

If we take the central value of y from the measurement of y_{CP} at Belle experiment^[2], thus, at BES-III experiment^[5], with 20 fb⁻¹ data at $\psi(3770)$ peak, the significance of the measurement of y could be around 4.3σ deviation from zero^[19].

We can also take advantage of the coherence of the D⁰ mesons produced at the $\psi(3770)$ peak to extract the strong phase difference δ between DCS and CF decay amplitudes. Because the *CP* properties of the final states produced in the decay of the $\psi(3770)$ are anti-correlated^[16, 17], one D⁰ state decaying into a final state with definite *CP* properties immediately identifies or tags the *CP* properties of the other side. As discussed in Ref. [11], the process of one D⁰ decaying to K⁻ π^+ , while the other D⁰ decaying to a *CP* eigenstate f_n can be described as

$$\Gamma_{\mathrm{K}\pi;\mathrm{f}_{\eta}} \equiv \Gamma[(\mathrm{K}^{-}\pi^{+})(\mathrm{f}_{\eta})] \approx A^{2}A_{\mathrm{f}_{\eta}}^{2}|1+\eta\sqrt{R_{\mathrm{D}}}\mathrm{e}^{-\mathrm{i}\delta}|^{2} \approx A^{2}A_{\mathrm{f}_{\eta}}^{2}(1+2\eta\sqrt{R_{\mathrm{D}}}\mathrm{cos}\delta) , \qquad (30)$$

where $A = |\langle \mathbf{K}^- \pi^+ | \mathcal{H} | \mathbf{D}^0 \rangle|$ and $A_{\mathbf{f}_{\eta}} = |\langle \mathbf{f}_{\eta} | \mathcal{H} | \mathbf{D}^0 \rangle|$ are the real-valued decay amplitudes, and we have neglected the y^2 terms in Eq. (30). In order to estimate the total sample of events needed to perform a useful measurement of δ , one defined^[11, 20] an asymmetry

$$\mathcal{A} \equiv \frac{\Gamma_{\mathrm{K}\pi;\mathrm{f}_{+}} - \Gamma_{\mathrm{K}\pi;\mathrm{f}_{-}}}{\Gamma_{\mathrm{K}\pi;\mathrm{f}_{+}} + \Gamma_{\mathrm{K}\pi;\mathrm{f}_{-}}} , \qquad (31)$$

where $\Gamma_{\mathrm{K}\pi;\mathrm{f}_{\pm}}$ is defined in Eq. (30), which is the rates for the $\psi(3770) \rightarrow D^0 \overline{D}^0$ configuration to decay into flavor eigenstates and a *CP*-eigenstates f_{\pm} . Eq. (30) implies a small asymmetry, $\mathcal{A} = 2\sqrt{R_{\mathrm{D}}} \mathrm{cos} \delta$. For a small asymmetry, a general result is that its error $\Delta \mathcal{A}$ is approximately $1/\sqrt{N_{\mathrm{K}^-\pi^+}}$, where $N_{\mathrm{K}^-\pi^+}$ is the total number of events tagged with *CP*-even and *CP*odd eigenstates. Thus one obtained

$$\Delta(\cos\delta) \approx \frac{1}{2\sqrt{R_{\rm D}}\sqrt{N_{\rm K^-\pi^+}}} . \tag{32}$$

The expected number $N_{\mathrm{K}^-\pi^+}$ of CP-tagged events can be connected to the total number of $\mathrm{D}^0\overline{\mathrm{D}}^0$ pairs $N(\mathrm{D}^0\overline{\mathrm{D}}^0)$ through $N_{\mathrm{K}^-\pi^+} \approx N(\mathrm{D}^0\overline{\mathrm{D}}^0)\mathcal{BR}(\mathrm{D}^0 \rightarrow \mathrm{K}^-\pi^+) \times \mathcal{BR}(\mathrm{D}^0 \rightarrow \mathrm{f}_{\pm}) \times \epsilon_{\mathrm{tag}} \approx 4.2 \times 10^{-4} N(\mathrm{D}^0\overline{\mathrm{D}}^0)^{[11]}$, here, as in Ref. [11], we take the branching ratiotimes-efficiency factor $\mathcal{BR}(\mathrm{D}^0 \rightarrow \mathrm{f}_{\pm}) \times \epsilon_{\mathrm{tag}} = 1.1\%$. With the measured $R_{\mathrm{D}} = (3.03 \pm 0.19) \times 10^{-3}$ and $\mathcal{BR}(\mathrm{D}^0 \rightarrow \mathrm{K}^-\pi^+) = 3.8\%^{[7]}$, one found^[11]

$$\Delta(\cos\delta) \approx \frac{\pm 444}{\sqrt{N(\mathbf{D}^0 \overline{\mathbf{D}}^0)}} \ . \tag{33}$$

At BES-III, about $72 \times 10^6 \text{ D}^0 \overline{\text{D}}^0$ pairs can be collected with 4 years' running. If considering both $\text{K}^-\pi^+$ and $\text{K}^+\pi^-$ final states, we thus estimate that one may be able to reach an accuracy of about 0.04 for $\cos\delta$.

By combining the measurements of x in $D^0 \rightarrow K_S \pi \pi$ and y_{CP} from Belle, one can obtain $R_M = (1.18 \pm 0.6) \times 10^{-4}$. At the $\psi(3770)$ peak, $D^0 \overline{D}^0$ pair are produced in a state that is quantum-mechanically coherent^[16, 17]. This enables a simple new method to measure D^0 mixing parameters in a way similar to that proposed in Ref. [11]. At BES-III, the measurement of R_M can be performed unambiguously with the following reactions^[16]:

(i)
$$e^+e^- \to \psi(3770) \to D^0\overline{D}^0 \to (K^\pm\pi^\mp)(K^\pm\pi^\mp),$$

(ii) $e^+e^- \to \psi(3770) \to D^0\overline{D}^0 \to (K^-e^+\nu)(K^-e^+\nu),$
(iii) $e^+e^- \to D^-D^{*+} \to (K^+\pi^-\pi^-)(\pi^+_{soft}[K^+e^-\nu]).$
(34)

Reaction (i) in Eq. (34) can be normalized to $D^0\overline{D}^0 \rightarrow (K^-\pi^+)(K^+\pi^-)$, the following time-integrated ratio is obtained by neglecting CP violation:

$$\frac{N[(\mathbf{K}^{-}\pi^{+})(\mathbf{K}^{-}\pi^{+})]}{N[(\mathbf{K}^{-}\pi^{+})(\mathbf{K}^{+}\pi^{-})]} \approx \frac{x^{2} + y^{2}}{2} = R_{\mathrm{M}} .$$
(35)

For the case of semileptonic decay, as (ii) in Eq. (34), we have

$$\frac{N(l^{\pm}l^{\pm})}{N(l^{\pm}l^{\mp})} = \frac{x^2 + y^2}{2} = R_{\rm M} , \qquad (36)$$

The observation of reaction (i) would be definite evidence for the existence of $D^0 \cdot \overline{D}^0$ mixing since the final state $(K^{\pm}\pi^{\mp})(K^{\pm}\pi^{\mp})$ can not be produced from DCS decay due to quantum statistics^[16, 17]. In particular, the initial $D^0\overline{D}^0$ pair is in an odd eigenstate of C which will preclude, in the absence of mixing between the D^0 and \overline{D}^0 over time, the formation of the symmetric state required by Bose statistics if the decays are to be the same final state. This final state is also very appealing experimentally, because it involves a two-body decay of both charm mesons, with energetic charged particles in the final state that form an overconstrained system. Particle identification is crucial in this measurement because if both the kaon and pion are misidentified in one of the two D-meson decays in the event, it becomes impossible to discern whether mixing has occurred. At BES-III, where the data sample is expected to be 20 fb⁻¹ integrated luminosity at $\psi(3770)$ peak, the limit will be 10^{-4} at 95% C.L. for $R_{\rm M}$, but only if the particle identification capabilities are adequate.

Reactions (ii) and (iii) offer unambiguous evidence for the mixing because the mixing is searched for in the semileptonic decays for which there are no DCS decays. Of course since the time-evolution is not measured, observation of Reactions (ii) and (iii) actually would indicate the violation of the selection rule relating the change in charm to the change in leptonic charge which holds true in the SM^[16].

In Table 2, the sensitivity for $R_{\rm M}$ measurements in different decay modes are estimated with 4 years' run at BEPC-II.

Table 2. The sensitivity for $R_{\rm M}$ measurements at BES-III with different decay modes with 4 years' run at BEPC-II.

$D^0\overline{D}^0$ mixing					
	events	sensitivity			
reaction	$RS(\times 10^4)$	$R_{\rm M}(\times 10^{-4})$			
$\psi(3770) \rightarrow (K^{-}\pi^{+})(K^{-}\pi^{+})$	10.4	1.0			
$\psi(3770) \mathop{\rightarrow} (\mathrm{K^-e^+}\nu)(\mathrm{K^-e^+}\nu)$	8.9				
$\psi(3770) \rightarrow (\mathrm{K^-e^+}\nu)(\mathrm{K^-\mu^+}\nu)$	8.1	3.7			
$\psi(3770) \mathop{\rightarrow} (K^-\mu^+\nu)(K^-\mu^+\nu)$	7.3				

In the limit of CP conservation, by combining the measurements of x in $\mathbb{D}^0 \to \mathrm{K}_{\mathrm{S}}\pi\pi$ and y_{CP} from Belle, one can obtain $R_{\mathrm{M}} = (1.18 \pm 0.6) \times 10^{-4}$. With 20 fb⁻¹ data at BES-III, about 12 events for the decay process $\mathbb{D}^0 \overline{\mathbb{D}}^0 \to (\mathrm{K}^{\pm}\pi^{\mp})(\mathrm{K}^{\pm}\pi^{\mp})$ can be produced. One can observe 3.0 events after considering the selection efficiency at BES-III, which could be about 25% for the four charged particles. The background contamination due to double particle misidentification is about 0.6 event with 20 fb⁻¹ data at BES-III^[21].

4 *CP* violation in D system

For the direct CP violation, the SM predictions are as large as 0.1% for D⁰ decays, and 1% level for D⁺ and D_s decays^[22]. At BES-III, one can also look at the CP violation by exploiting the quantum coherence at the $\psi(3770)$. Consider the case where both the D⁰ and the \overline{D}^0 decay into CP eigenstates, then the decays $\psi(3770) \rightarrow f_{+}^i f_{+}^i$ or $f_{-}^i f_{-}^i$ are forbidden, where f₊ (f₋) denotes a CP+ eigenstate (CP- eigenstate). This is because $CP(f_{\pm}f_{\pm}^{i}) = CP(f_{\pm}^{i})CP(f_{\pm}^{i})(-1)^{l} =$ -1, while, for the l = 1 $\psi(3770)$ state, $CP(\psi(3770)) =$ +1. Thus the observation of a final state such as $(K^{+}K^{-})(\pi^{+}\pi^{-})$ constitutes evidence of the CP violation. For $(K^{+}K^{-})(\pi^{+}\pi^{-})$ mode, the sensitivity at BES-III is about 1% level. Moreover, all pairs of the CP eigenstates, where both eigenstates are even or both are odd, can be summed over for the CP violation measurements at BES-III.

5 Dalitz plot analyses

Recent studies of multi-body decays of D mesons provide a direct probe of the final state interactions by looking at the interference between intermediate state resonances on the Dalitz Plot (DP). When D mesons decay into three or more daughters, intermediate resonances dominate the decay rates. These resonances will cause a non-uniform distribution of events in phase space on the DP. Since all events on the DP have the same final states, different resonances at the same location on DP will interfere. This provides the opportunity to measure both the amplitudes and phases of the intermediate decay channels, which in turn allows to deduce their relative branching fractions. These phase differences can even allow details about very broad resonances to be extracted by observing their interferences with other intermediate states.

The most important thing is that recent studies of multi-body decays of D mesons probe a variety of physics including light spectroscopy ($\pi\pi$, K π and KK S-wave states), searches for CP violation and $D^0-\overline{D}^0$ mixing. Currently, the decay $D^0 \to K_S \pi^+\pi^$ plays very important role in the determination of ϕ_3/γ . Recently BaBar and Belle^[23, 24] have reported $\gamma = (70 \pm 31^{+12+14}_{-10-11})^{\circ}$ and $\phi_3 = (77^{+17}_{-19} \pm 13 \pm 11)^{\circ}$, respectively, where the third error is the systematic error due to modelling of DP. The precision of these measurements will eventually be limited by the understanding of the $D^0 \to K_S \pi^+ \pi^-$ decays. Although K-matrix description of the $\pi\pi$ S-wave may yield improved models of the Dalitz Plot and the error on ϕ_3/γ may be decreased from $\pm 10^\circ$ to a few degrees, it is still a model-dependent way to extract the angle. At BES-III, by using the coherence of $D^0\overline{D}^0$ pairs at $\psi(3770)$ peak, one can study the CP-tagged and flavor-tagged Dalitz Plot by doing binned analysis^[25]. This method is a model-independent. According to the estimation in Ref. [25], the proposed super-B factory^[26] with its design integrated luminosity of 50 ab⁻¹, would allow a measurement of ϕ_3/γ with accuracy below 2°. To keep the uncertainty due to D DP decays below that level, around 10^{-4} CP-tagged D decays are needed, corresponding to ~ 10 fb⁻¹ data which can be obtained at BEPC-II with two years' luminosity.

6 Rare charm decays

The searches for the rare-decay processes have played an important role in the development of the SM. The short-distance flavor-changing neutral current (FCNC) processes in the charm decays are much more highly suppressed by the GIM mechanism than the corresponding down-type quark decays because of the large top quark mass. The observation of D^+ FCNC decays $D^+ \rightarrow \pi^+ l^+ l^-$ and $D^+ \rightarrow K^+ l^+ l^$ could therefore provide indication of new physics or of unexpectedly large rates for the long-distance SM processes like $D^+ \rightarrow \pi^+ V$, $V \rightarrow l^+ l^-$, with real or virtual vector meson V. Recently, the CLEO-c reported the branching fraction of the resonant decay $\mathcal{BR}(D^+ \to \pi^+ \phi \to \pi^+ e^+ e^-) = (2.8 \pm 1.9 \pm 0.2) \times 10^{-6}.$ The lepton-number-violating (LNV) or the leptonflavor-violating (LFV) decays $D^+ \rightarrow \pi^- l^+ l^+$, $K^- l^+ l^+$ and $\pi^+\mu^+e^-$ are forbidden in the SM. Past searches have set the upper limits for the dielectron and dimuon decay modes^[7]. In Table 3 and Table 4, the current limits and expected sensitivities at BES-III are summarized for D^+ and D^0 , respectively. Detailed

Table 3. Current and projected 90%-CL upper limits on rare D⁺ decay modes at BES-III with 20 fb⁻¹ data at $\psi(3770)$ peak. We assume the selection efficiencies for all modes are 35%.

mode	reference	best upper	BES-III
	experiment	$limits(10^{-6})$	$(\times 10^{-8})$
$\pi^+ e^+ e^-$	$CLEO-c^{[27]}$	7.4	5.6
$\pi^+\mu^+\mu^-$	$FOCUS^{[28]}$	8.8	8.7
$\pi^+\mu^+\mathrm{e}^-$	$E791^{[29]}$	34	8.3
$\pi^- \mathrm{e^+e^+}$	$CLEO-c^{[27]}$	3.6	5.6
$\pi^-\mu^+\mu^+$	$FOCUS^{[28]}$	4.8	8.7
$\pi^-\mu^+ \mathrm{e}^+$	$E791^{[29]}$	50	5.9
$\mathrm{K^+e^+e^-}$	$CLEO-c^{[27]}$	6.2	6.7
$\mathrm{K}^{+}\mu^{+}\mu^{-}$	$FOCUS^{[28]}$	9.2	10.5
$\mathrm{K}^{+}\mu^{+}\mathrm{e}^{-}$	$E791^{[29]}$	68	8.3
$\rm K^-e^+e^+$	$CLEO-c^{[27]}$	4.5	6.7
$K^-\mu^+\mu^+$	$FOCUS^{[28]}$	13	10.4
$\mathrm{K^-}\mu^+\mathrm{e^+}$	$E687^{[30]}$	130	8.3

description on the rare charm decays can be found in Refs. [20, 35]. The charm meson radiative decays are also very important to understand final state interaction which may enhance the decay rates. In Refs. [20, 35], the decay rates of $D \rightarrow V\gamma$ (V can be ϕ , ω , ρ and K^{*}) had been estimated to be $10^{-5} - 10^{-6}$, which can be reached at BES-III.

Table 4. Current and projected 90%-CL upper limits on rare D^0 decay modes at BES-III with 20 fb⁻¹ data at $\psi(3770)$ peak.

mode	reference	best upper	BES-III
	experiment	$\limits(10^{-6})$	$(\times 10^{-8})$
γγ	$CLEO^{[31]}$	28	5.0
$\mu^+\mu^-$	$D0^{[32]}$	2.4	17
$\mu^+ \mathrm{e}^-$	$E791^{[29]}$	8.1	4.3
e^+e^-	$E791^{[29]}$	6.2	2.4
$\pi^0\mu^+\mu^-$	$E653^{[33]}$	180	12.3
$\pi^0 \mu^+ e^-$	$CLEO^{[34]}$	86	9.7
$\pi^0 e^+ e^-$	$CLEO^{[34]}$	45	7.9
$K_{\rm S}\mu^+\mu^-$	$E653^{[33]}$	260	10.6
${\rm K_S}\mu^+{\rm e}^-$	$CLEO^{[34]}$	100	9.6
$K_{\rm S}e^+e^-$	$CLEO^{[34]}$	110	7.5
$\eta\mu^+\mu^-$	$CLEO^{[34]}$	530	10
$\eta\mu^+ \mathrm{e}^-$	$CLEO^{[34]}$	100	10
$\eta \mathrm{e^+e^-}$	$CLEO^{[34]}$	110	10

7 Summary

In conclusion, we discuss the constraints on the strong phase difference in $D^0 \to K\pi$ decay according to the most recent measurements of y', $y_{\rm CP}$ and xfrom B factories. We estimate the sensitivity of the measurement of mixing parameter y at $\psi(3770)$ peak in BES-III experiment. With 20 fb^{-1} data, the uncertainty $\Delta(y)$ could be 0.003. Thus, assuming y at a percent level, we can make a measurement of y at a significance of 4.3σ deviation from zero. The sensitivity of the strong phase difference at BES-III are obtained by using data near the $D\overline{D}$ threshold with CP tag technique at BES-III experiment. Finally, we estimated the sensitivity of the measurements of the mixing rate $R_{\rm M}$, and find that BES-III experiment may not be able to make a significant measurement of $R_{\rm M}$ with current luminosity by using coherent $D\overline{D}$ state at $\psi(3770)$ peak.

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