# A note on the mass splitting of $K^*(892)^*$

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**Abstract** Belle Collaboration reported a new observed value of  $K^{*-}(892)$  mass by studying  $\tau^- \to K_S \pi^- \nu_{\tau}$  decay, which is significantly different from the current world average value given by Particle Data Group 2006. Motivated by this new data, we revisit the issue on the  $K^{*0}(892) - K^{*\pm}(892)$  mass splitting. Our theoretical estimation favors the new measurement by Belle Collaboration. Therefore further experimental efforts are urgently needed to improve our understanding of these issues.

Key words  $K^*(892)$ , mass splitting, chiral constituent quark model

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## 1 Introduction

The problem of the mass splitting between the neutral  $K^*(892)$  (i.e.  $K^{*0}$  or  $\bar{K}^{*0}$ ) and the charged  $K^*(892)$  (i.e.  $K^{*\pm}$ ) has a long history. Experimentally, given by Particle Data Group  $2006^{[1]}$ , one has

$$m_{\rm K^{*0}} - m_{\rm K^{*\pm}} = 6.7 \pm 1.2 \text{ MeV}.$$
 (1)

On the other hand, using the observed values of  $K^{*0}$ and  $K^{*\pm}$  masses shown in Ref. [1],

$$m_{\rm K^{*0}} = 896.00 \pm 0.25 \,\,{\rm MeV},$$
 (2)

$$m_{\rm K^{*\pm}} = 891.66 \pm 0.26 \,\,{\rm MeV},$$
 (3)

we obtain

$$m_{\rm K^{*0}} - m_{\rm K^{*\pm}} = 4.34 \pm 0.36 \,\,{\rm MeV},$$
 (4)

with smaller central value and less uncertainty by contrast with Eq. (1). As a conservative estimation, we may thus obtain the experimental value of the mass splitting of  $K^*$ -mesons, denoted by "expt", as

$$(m_{K^{*0}} - m_{K^{*\pm}})_{expt} \sim 4 \text{ to } 8 \text{ MeV.}$$
 (5)

It is well known that, for SU(3) flavor multiplets of hadrons, the mass splittings between their isospin components are caused by two effects: (i)  $m_{\rm u} \neq m_{\rm d}$ (inequality of u-d quark masses); (ii) the electromagnetic interactions inside hadrons. Consequently, the observed mass splitting of K\*-mesons can be ex-

$$(m_{K^{*0}} - m_{K^{*\pm}})_{expt} = (m_{K^{*0}} - m_{K^{*\pm}})_{QM} + (m_{K^{*0}} - m_{K^{*\pm}})_{EM}, \qquad (6)$$

where the subscript QM denotes the contribution due to u-d quark mass difference; EM denotes the one due to electromagnetic interaction, so the second term on the right side of the above equation is called EM-mass difference. Note that these two parts contributions (QM and EM) cannot be measured directly, theoretical calculations for them are therefore necessary.

Usually, the EM-masses of neutral hadrons are smaller than those of their charged partners. For instance, the EM-masses of neutron,  $\pi^0$ , and  $K^0(\bar{K}^0)$ are smaller than the EM-masses of proton,  $\pi^{\pm}$ , and  $K^{\pm [2-5]}$ , respectively. Thus it is reasonable to assume that

$$(m_{K^{*0}} - m_{K^{*\pm}})_{EM} < 0, \tag{7}$$

which leads to

$$(m_{\mathrm{K}^{*0}} - m_{\mathrm{K}^{*\pm}})_{\mathrm{expt}} < (m_{\mathrm{K}^{*0}} - m_{\mathrm{K}^{*\pm}})_{\mathrm{QM}}.$$
 (8)

This implies that, from Eq. (5), a relative large contribution to  $(m_{K^{*0}} - m_{K^{*\pm}})_{QM}$  is required.

Very recently, Belle Collaboration reported a new measurement of  $K^{*-}(892)$  mass by studying  $\tau^- \rightarrow$ 

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 $K_{\rm S}\pi^-\nu_{\tau} \, {\rm decay}^{[6]}$ 

$$m_{K^{*-}} = 895.47 \pm 0.20 (\text{Stat.}) \pm 0.44 (\text{Syst.}) \pm 0.59 (\text{Mod.}) \text{ MeV},$$
 (9)

which is significantly different from the current world average value in Ref. [1]. This will give (here we assume that the neutral  $K^*$  mass remains unchanged),

$$(m_{\rm K^{*0}} - m_{\rm K^{*\pm}})_{\rm expt} = 0.53 \pm 0.80 \,\,{\rm MeV},$$
 (10)

which is quite a small value. If the Belle data are confirmed, there would obviously exist some discrepancy between (10) and (5). In order to clarify this discrepancy, experimentally, it is urgent to confirm or rule out the Belle result (9); and also it is important to carry out a new measurement of the neutral  $K^*$  mass. Meanwhile, further theoretical investigations on this issue will be very helpful.

Unfortunately, it is still an open question to calculate the mass splittings of the low-lying mesons from the first principle of quantum chromodynamics (QCD) due to the non-perturbative feature of QCD. Therefore, at the present stage, one generally appeals to the low energy effective models inspired by QCD. For our purpose, in order to get a consistent evaluation, then to understand the above possible discrepancy, one should adopt the theoretical framework in which both  $(m_{K^{*0}} - m_{K^{*\pm}})_{OM}$  and  $(m_{K^{*0}} - m_{K^{*\pm}})_{EM}$  can be computed systematically. Our previous studies<sup>[4, 7]</sup>, in the framework of chiral constituent quark model, have done such a job already. Actually our analysis<sup>[7]</sup> favors a small value of the  $K^*(892)$  mass splitting, which is consistent with Eq. (10). The purpose of the present note is to make this point more clear.

## 2 Model estimation

Chiral constituent quark model (ChQM) is developed by Manohar and Georgi<sup>[8]</sup>, and the vector meson ( $\omega$ -meson) is firstly introduced into this model in Ref. [9] to study quark spin contents using the chiral soliton approach. The author of Ref. [10] further extended it by including the low-lying 1<sup>-</sup> (vector) and 1<sup>+</sup> (axial-vector) mesons. This model has been investigated extensively<sup>[4, 7, 11—15]</sup> and its theoretical results agree well with the data. The electromagnetic interaction of mesons in ChQM has been well established via vector meson dominance, which makes it possible to evaluate the EM-masses of low-lying mesons in this framework<sup>[4]</sup>.

In Ref. [7], the mass splittings of vector mesons generated from the quark mass effect in ChQM have been derived at the leading order in quark mass expansion, and the explicit mass formulae for the K<sup>\*</sup>- mesons have been obtained, as shown in Eqs. (19) and (20) of the paper. By making a reasonable approximation in numerology, it has been found that

$$(m_{\mathrm{K}^{*0}} - m_{\mathrm{K}^{*\pm}})_{\mathrm{QM}} = \frac{m_{\mathrm{V}}}{4\pi^2 g^2} \frac{m_{\mathrm{d}} - m_{\mathrm{u}}}{m} \approx \frac{1}{2} (m_{\mathrm{d}} - m_{\mathrm{u}}).$$
(11)

As pointed out in Ref. [10], a typical scale or cutoff  $\Lambda$ , which is reflected by an intrinsic parameter g of the model, has been introduced. The value of the quark mass parameters, which is actually scale dependent, should thus be evaluated at this scale. From  $\rho^0 - \omega$ mixing, the author of Ref. [7] further determined (similar studies have been done in Refs. [14, 15])

$$m_{\rm d} - m_{\rm u} = 6.14 \pm 0.36 \,\,{\rm MeV}.$$
 (12)

Consequently, we get [7, 16]

$$(m_{\rm K^{*0}} - m_{\rm K^{*\pm}})_{\rm QM} = 3.07 \pm 0.18 \,\,{\rm MeV}.$$
 (13)

This indicates inequality (8) may not hold for (5); however, it is of no problem for (10). It will be shown below that, when we include the EM-masses contributions, the situation for (5) will be much worse.

In Ref. [4], the electromagnetic mass splittings of  $\pi$ , K, a<sub>1</sub>, K<sub>1</sub>, and K<sup>\*</sup>(892) have been calculated to one-loop order and  $O(\alpha_{\rm EM})$ , which gives

$$(m_{\rm K^{*0}} - m_{\rm K^{*\pm}})_{\rm EM} = -1.76 \pm 0.53 \,\,{\rm MeV}.$$
 (14)

Here we have corrected a sign error for the EM-masses of the vector and axial-vector mesons obtained in Ref. [4] (note that there is no sign error in the case of pseudoscalar mesons), which has been firstly pointed out in Ref. [17]. On the other hand, according to Ref. [10], the large  $N_{\rm C}$  expansion plays an important role in this theory. Model consistency and the phenomenologically successful predictions of the leading order evaluation, as shown in Refs. [4, 7, 10—15], lead us to the reasonable expectation that the theoretical uncertainty from high order corrections cannot exceed ~ 30%, and so we can conservatively estimate the theoretical error in Eq. (14).

Now from Eqs. (13) and (14), we get our estimation in ChQM for the mass splitting of  $K^*(892)$ 

$$(m_{\mathrm{K}^{*0}} - m_{\mathrm{K}^{*\pm}})_{\mathrm{theory}} = (m_{\mathrm{K}^{*0}} - m_{\mathrm{K}^{*\pm}})_{\mathrm{QM}} + (m_{\mathrm{K}^{*0}} - m_{\mathrm{K}^{*\pm}})_{\mathrm{EM}} = 1.31 \pm 0.56 \text{ MeV}.$$
(15)

This is consistent with Eq. (10), in which the new data by Belle Collaboration have been used. However, this is inconsistent with Eq. (5) estimated from the current world average values.

#### **3** Discussions and remarks

Motivated by the new measurement of  $K^{*-}(892)$  mass reported by Belle Collaboration, we reexamine the mass splitting between the neutral  $K^{*}(892)$  and

the charged K<sup>\*</sup>(892). Our analysis shows that there might exist some discrepancy between this new result and the corresponding world average value by Particle Data Group 2006 if the mass of K<sup>\*0</sup>(892) could keep unchanged. In the framework of ChQM, we give a theoretical estimation as  $(m_{\rm K^{*0}} - m_{\rm K^{*\pm}})_{\rm theory} = 1 \sim 2$  MeV, which seems to support the Belle data.

It has been pointed out in Ref. [6] that, none of the previous mass measurements of the charged K<sup>\*</sup>(892) listed in Ref. [1], all of which were performed more than twenty years ago, presented the systematic uncertainties for their measurements; more importantly, all those earlier mass measurements listed there come from the analysis of hadronic reactions and include the effects of final state interaction while Belle Collaboration presents the measurement based on  $\tau^-$  decays, where the decay products of the K<sup>\*-</sup>(892) are the only hadrons involved.

For the neutral K<sup>\*</sup> masses, the situation in Ref. [1] is very different from the charged case (for K<sup>\*±</sup>, only old data obtained more than twenty years ago are adopted there). In 2005 FOCUS Collaboration reported a measurement of the K<sup>\*0</sup> masses as<sup>[18]</sup>

$$m_{\rm K^{*0}} = 895.41 \pm 0.32^{+0.35}_{-0.43} \,{\rm MeV}$$
 (16)

by studying the semileptonic  $D^+ \to K^- \pi^+ \mu^+ \nu$  decay. Similar to the case of  $\tau^- \to K_S \pi^- \nu_\tau$  decay for the  $K^{*-}$  mass measurement, this semileptonic  $D^+$  decay could also provide a nice place to study  $K\pi$  system in the absence of interactions with other hadrons, in which  $K^{*0}$  mesons are the only hadrons produced in the decay final state. It is noteworthy that the recent FOCUS measurement (16) is close to the world average value of the  $K^{*0}(892)$  mass in Ref. [1]. This indicates that the recent data prefer the small value of the mass splitting  $m_{K^{*0}} - m_{K^{*\pm}}$ , which is consis-

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tent with our model estimation; however, which contradicts the world average values by Particle Data Group 2006. Explicitly, if we only consider the values in Eqs. (9) and (16), which are the most recent data for the  $K^{*-}$  and  $K^{*0}$  masses, respectively, one has

$$(m_{\rm K^{*0}} - m_{\rm K^{*\pm}})_{\rm FOCUS-Belle} = -0.06 \pm 0.91 \text{ MeV}.$$
 (17)

Another possible interesting and precise experiment has been proposed in Refs. [16, 19] that K<sup>\*</sup>(892) masses can be measured in BES at BEPC, especially for the neutral K<sup>\*</sup> mass. Since BES II at BEPC has collected about  $5.77 \times 10^7 \text{ J/}\psi$  events, it is practicable to take  $\text{J/}\psi$  as the source of K<sup>\*</sup>(892). The branching ratio for  $\text{J/}\psi \rightarrow \bar{\text{K}}^0\text{K}^{*0}$  is  $4.2 \times 10^{-3}$ ; for  $\text{J/}\psi \rightarrow \text{K}\bar{\text{K}}\pi$ ,  $6 \times 10^{-3}$ ; for K<sup>\*</sup>  $\rightarrow \text{K}\pi$ , about 100%. Therefore, studying three-body decay processes  $\text{J/}\psi \rightarrow \text{K}\bar{\text{K}}\pi$ , one can determine the location of the resonance of K<sup>+</sup> $\pi^{\pm}$  (i.e. K<sup>\*0</sup> or  $\bar{\text{K}}^{*0}$ ), and measure the neutral K<sup>\*</sup> mass with the error below 1 MeV. More accurate measurements can be expected in BESIII after the BES II detector is upgraded.

Our theoretical calculation (15) is not a model independent estimation. However, the discrepancy between the most recent data given by Eq. (10) or (17) and the current world average value by Particle Data Group 2006 does exist. Future dedicated measurements of the K<sup>\*</sup>(892) (including both K<sup>\*0</sup> and K<sup>\*±</sup>) masses with high precision are necessary to clarify this discrepancy. We therefore urge our experimental colleagues to produce more data in order to get a solid and more meaningful conclusion.

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