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Theoretical and numerical analysis of coherent Smith-Purcell radiation*

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Abstract Coherent enhancement of Smith-Purcell radiation has attracted people's attention not only in adopting a better source but also in beam diagnostics aspect. In this paper, we study the intrinsic mechanism of coherent Smith-Purcell radiation on the basis of the van den Berg model. The emitted power of Smith-Purcell radiation is determined by the bunch profile in transverse and longitudinal directions. For short bunch whose longitudinal pulse length is comparable with the radiation wavelength, it can be concluded approximately that the power is proportional to the square number of electrons per bunch.

Key words Smith-Purcell radiation, van den Berg model, induced surface current model, coherence

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1 Introduction

In 1950s, S. J. Smith and E. M. Purcell[1] suggested that if an electron passed close to the surface of a metal diffraction grating, moving perpendicular to the tooth, there would be radiation emitted from the grating. The experiment was carried out in a Van de Graaff generator and an electron accelerator tube. It is the first time to observe the radiation in the optical band which is the so-called Smith-Purcell radiation or SPR as an abbreviation, named after its discoverers. Furthermore, they derived the famous dispersive relation from the experiment which can be described as

$$\lambda = \frac{D}{n}(\beta_0^{-1} - \sin\eta), \tag{1}$$

where λ is the emission wavelength, D is the period of the grating, n is the diffraction order, β_0 is the relative velocity of the electron beam corresponding to the speed of light, and η is the polar angle, as shown in Fig. 1.

In 1961, Toraldo di Francia[2] concluded that the field of the electron was composed of a set of evanescent waves which would be scattered by the grating as outgoing waves. And we define the waves as far-field Smith-Purcell radiation. By then, the electromagnetic field of SPR can be calculated, although the theory is only suitable for particular grating profiles.

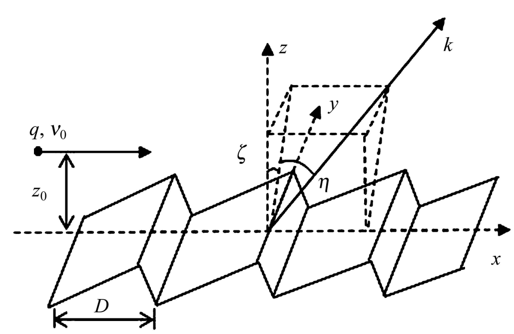

Fig. 1. Diagram defining the parameters.

In 1970s, van den Berg[3, 4] provided the general formulas for the field and the power, based on the diffraction theory for a monochromatic plane wave in his earlier works[5]. He used a Green's function formulation of the 2D problem to calculate the SP radiation for an arbitrary grating profile. Thus, the calculation of radiation factor is very important to get the final solution. To calculate the radiation factor, several methods have been put forward: the Rayleigh method[6], the integral method[3, 5], the improved point-matching method[7, 8], and the modal

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expansion method[4, 9]. In order to achieve the maximum output of power, the grating and electron parameters need to be optimized[10, 11].

Besides the van den Berg model, different approaches of studying the SPR mechanism have then been put forward: an induced surface current model[12, 13], a resonant diffraction radiation model[14], and a model based on the use of integral equations for an electric field of radiation[15].

Recently, coherent SPR is of great interest for its advantage in beam diagnostic system[16, 17]. The first coherent SPR was observed in LNSTU in Japan[18], and the power intensity was enhanced by several orders compared with the incoherent SPR.

As we know, the current theory of coherent SPR is based on the induced surface current model. J. H. Brownell[13] assumes that the origin of SPR comes from the surface currents induced on the grating by the passing electron and accelerated by the periodic profile of the grating. The radiation energy is related to the induced current. And for different electrons, the phase of current is different. So the energy of coherent SPR can be added separately except for the part corresponding to the phase difference, which is determined by the bunch profile. The model is generally applicable to shallow gratings over a broad electron beam energy range. It is particularly effective when the image charge footprint (proportional to the wavelength) is small compared with the period. And in the van den Berg model, the incident and diffracted fields can be expanded in terms of the discrete mode and the boundary conditions are satisfied on the surface. In principle, the grating with an arbitrary profile can be analyzed with this approach.

Moreover, the results of experiments[19, 20] for non-relativistic electron energies show good agreement with the predictions of van den Berg model. For moderate electron energies[21, 22], the experimental results are inconsistent with each other. So the validity of these experimental results may be thought to be an important problem. However, by now it's difficult to achieve the absolute measurements of the SPR power within the demanded accuracy. And for high electron energies as is presented in the experiment[23] for electrons with energy 855 MeV, the measured intensity of the SPR in the optical range agrees with the van den Berg model and is by 6 orders of magnitude lower than the theoretical estimation of the surface current model.

Our theory is based on the van den Berg model. In this paper, we first briefly describe the universal characteristic of coherent radiation of relativistic beam for various radiation types, in Section 2. And in Section 3, the self-consistent solutions of the generic field and power expressions for coherent SPR based on the van den Berg model are derived, which is put forward for the first time. Next in Section 4, the calculation results using our experimental parameters are provided. Finally, we present our conclusions in Section 5.

2 General description of coherent radiation

Coherent radiation occurs when the fields of every particle can be added coherently independent of the phase difference. In most cases, when the wavelength is comparable to or longer than the longitudinal length of the bunch, the coherent radiation is likely to happen. Under this circumstance, the phase of electromagnetic field is independent of the transverse profile of the bunch, but the longitudinal distribution is related to the phase.

So far as we know, synchrotron radiation from bending magnets[24], undulator radiation from periodic alternate magnets[25], transition radiation from thin screens or foils traversed by the electrons[26], diffraction radiation from screens with holes or slits[27] and Smith-Purcell radiation can be classified to coherent radiation, if suitable conditions are satisfied.

Incoherent radiation occurs when the phase difference should be considered while calculating each component of the field. It is often the case, when the wavelength is shorter than the longitudinal length of the bunch.

Figure 2 shows the difference between incoherent radiation and coherent radiation. Surely, we prefer coherent radiation to incoherent one because the power of the former is approximately proportional to the square of beam current and is enhanced by a factor N compared with the latter, where N is the number of electrons per bunch. Moreover, the coherent superiority is not only in adopting a better source but in beam diagnostics aspect.

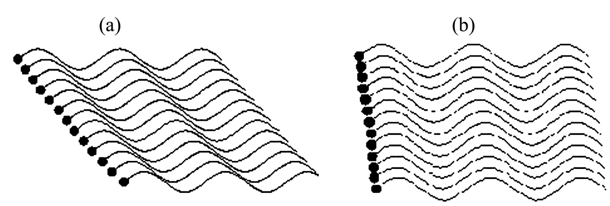

Fig. 2. Schematic of (a) incoherent emission of uncorrelated radiation wave packets and (b) coherent emission of phase correlated wave packets.

3 Theory of coherent SPR

In this section, we adopt the van den Berg model to obtain solutions for coherent SPR. And some equations are similar to those of the van den Berg theory.

First, we adopt Cartesian coordinate here. As is shown in Fig. 1, the grating is periodic in the x direction, and the y direction is parallel to the grating rulings. The charge moves along a trajectory at a constant of $z=z_0$.

The Fourier integrals of the electromagnetic vector field for a point charge can be expressed as[3]

$$\begin{aligned} E^i(x,y,z,t) &= \frac{1}{2\pi^2}\mathrm{Re}\Big[\int_0^\infty \mathrm{d}\omega \int_{-\infty}^{\infty} E^i(x,z;\beta,\omega)\times \\ &\quad \exp(\mathrm{i}\beta y-\mathrm{i}\omega t)\mathrm{d}\beta\Big], \\ H^i(x,y,z,t) &= \frac{1}{2\pi^2}\mathrm{Re}\Big[\int_0^\infty \mathrm{d}\omega \int_{-\infty}^{\infty} H^i(x,z;\beta,\omega)\times \\ &\quad \exp(\mathrm{i}\beta y-\mathrm{i}\omega t)\mathrm{d}\beta\Big], \end{aligned} \tag{2}$$

where the superscript i means the incident field, β and ω represent the parameters in frequency domain with respect to y and t, thus $E^i(x,z;\beta,\omega)$ and $H^i(x,z;\beta,\omega)$ are the Fourier components of electromagnetic field.

Then we obtain from[3] the source-free Maxwell equations the relations

$$\begin{aligned} &(\nabla_\perp+\mathrm{i}\beta i_y)\times H(x,z;\beta,\omega)+\mathrm{i}\omega\varepsilon_0 E(x,z;\beta,\omega)=0 \\ &(\nabla_\perp+\mathrm{i}\beta i_y)\times E(x,z;\beta,\omega)-\mathrm{i}\omega\mu_0 H(x,z;\beta,\omega)=0 \end{aligned}, \tag{3}$$

where ε_0 and μ_0 denote the permittivity and permeability of vacuum.

We next suppose the coordinate of the j-th electron is (x_j,y_j,z_j,t_j). Thus the x component of reflected field emitted by the i-th electron at (x_j,y_j,z_j,t_j) can be written as $E_x^r(x_j,y_j,z_j,t_j)_i$ and $H_x^r(x_j,y_j,z_j,t_j)_i$. We also expand the reflected field in the Fourier integral form as

$$\begin{aligned} E_x^r(x_j,y_j,z_j,t_j)_i &= \frac{1}{2\pi^2}\mathrm{Re}\int_0^\infty \mathrm{d}\omega\int_{-\infty}^{\infty} E_x^r(x_j,z_j;\beta,\omega)_i\times \\ &\quad \exp[\mathrm{i}\beta(y_{j0}-y_{i0})-\mathrm{i}\omega(t_j-t_{i0})]\mathrm{d}\beta, \\ H_x^r(x_j,y_j,z_j,t_j)_i &= \frac{1}{2\pi^2}\mathrm{Re}\int_0^\infty \mathrm{d}\omega\int_{-\infty}^{\infty} H_x^r(x_j,z_j;\beta,\omega)_i\times \\ &\quad \exp[\mathrm{i}\beta(y_{j0}-y_{i0})-\mathrm{i}\omega(t_j-t_{i0})]\mathrm{d}\beta, \end{aligned} \tag{4}$$

in which the subscript '0' means the coordinate at the entrance of the grating.

According to the Floquet theory, when the boundary condition is periodic, the x and y components of the reflected field can be written as an infinite sum of the so-called spatial harmonics, as is called Rayleign expansion. Hence it can also be obtained as

$$\begin{aligned} E_x^r(x_j,z_j;\beta,\omega)_i &= \sum_{n=-\infty}^{\infty} E_n^r(\omega,\beta)_i \exp(\mathrm{i}\alpha_n x_j+\mathrm{i}\gamma_n z_{j0}), \\ H_x^r(x_j,z_j;\beta,\omega)_i &= \sum_{n=-\infty}^{\infty} H_n^r(\omega,\beta)_i \exp(\mathrm{i}\alpha_n x_j+\mathrm{i}\gamma_n z_{j0}), \\ E_y^r(x_j,z_j;\beta,\omega)_i &= \sum_{n=-\infty}^{\infty} \Phi_n^r(\omega,\beta)_i \exp(\mathrm{i}\alpha_n x_j+\mathrm{i}\gamma_n z_{j0}), \\ H_y^r(x_j,z_j;\beta,\omega)_i &= \sum_{n=-\infty}^{\infty} \Psi_n^r(\omega,\beta)_i \exp(\mathrm{i}\alpha_n x_j+\mathrm{i}\gamma_n z_{j0}), \end{aligned} \tag{5}$$

where n is an arbitrary integer which denotes the order of spatial harmonic. And there are relations

$$\alpha_n=\alpha_0+2\pi n/D, \quad \gamma_n=(k_0^2-\beta^2-\alpha_n^2)^{1/2}, \tag{6}$$

in which $\alpha_0=\omega/v_0=k_0c_0/v_0$, v_0 is the velocity of the point charge, c_0 is the velocity of light in vacuum, and D is the period of grating. The two parameters denote the wave number in respective direction. In detail, when γ_n is real, it deals with the propagation wave; accordingly, when γ_n is image, it deals with the evanescent wave.

We conclude from Eq. (2) and (3) that the incident field is composed of a continuous set of plane waves, and for every β and ω, the wave is monochromatic plane wave. So there is such relation as $\beta=k\times\cos(\varphi_0)$, where φ_0 (see Fig. 3) is the angle between the y-axis and the direction of propagation for a single incident plane wave.

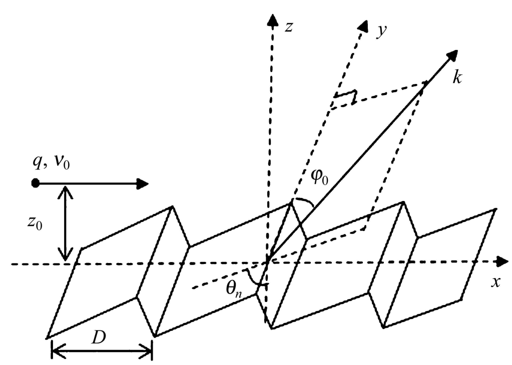

Fig. 3. Diagram defining the angles.

Meanwhile θ_n (see Fig. 3) is introduced which denotes the angle between the negative z-axis and the projection of the direction of propagation for a single reflected plane wave of the n-th order harmonic on the (x,z) plane. Therefore, we obtain from Eq. (5) the relations

$$\alpha_n=k\times\sin(\varphi_0)\sin(\theta_n), \quad \gamma_n=k\times\sin(\varphi_0)\cos(\theta_n). \tag{7}$$

Hence Smith-Purcell formula can easily be given from Eq. (6) and (7) by

$$\sin(\theta_n)=c_0/(v_0\times\sin(\varphi_0))+n\lambda_0/(D\times\sin(\varphi_0)). \tag{8}$$

The x component of the reflected field can also be expressed in terms of $\Phi_n^r(\omega,\beta)$ and $\Psi_n^r(\omega,\beta)$ as

$$(k^2-\beta^2)E_{x,n}^r(\omega,\beta)_i=-\beta\alpha_n\Phi_n^r(\beta,\omega)_i+\omega\mu_0\gamma_n\Psi_n^r(\beta,\omega)_i. \quad (9)$$

From Eq. (4) and (5) we then have

$$E_x^r(x_j,y_j,z_j,t_j)_i=\frac{1}{2\pi^2}\mathrm{Re}\int_0^\infty \exp[-\mathrm{i}\omega(t_j-t_{j0})-\mathrm{i}\omega(t_{j0}-t_{i0})]\mathrm{d}\omega\times \int_{-\infty}^\infty \exp[\mathrm{i}\beta(y_{j0}-y_{i0})\sum_{n=-\infty}^{\infty}E_{n,x}^r(\omega,\beta)_i\times \exp(\mathrm{i}\alpha_n x_j+i\gamma_n z_{j0})\mathrm{d}\beta=\frac{1}{2\pi^2}\mathrm{Re}\sum_{n=-\infty}^{\infty}\int_0^\infty\exp\left[-\mathrm{i}\frac{2\pi nv(t_j-t_{j0})}{D}-\mathrm{i}\omega(t_{j0}-t_{i0})\right]\mathrm{d}\omega\times\int_{-\infty}^\infty E_{n,x}^r(\omega,\beta)_i\exp[\mathrm{i}\beta(y_{j0}-y_{i0})\times\exp(\mathrm{i}\gamma_n z_{j0})\mathrm{d}\beta\ , \quad (10)$$

where

$$\alpha_n=\alpha_0+\frac{2\pi n}{D}=\frac{\omega}{v}+\frac{2\pi n}{D},x_j=v\cdot(t_j-t_{j0}).$$

We now turn our eyes to the energy loss of the point charge due to the reaction of the reflected electric field, traversing the distance of one grating period. Here, we suppose that the transverse motion of charge is not taken into consideration. The mechanical work for the bunch composed of N electrons traversing one grating period can be written as

$$W=-\sum_{j=1}^N\int_{t_{j0}}^{t_{j0}+D/v_0}qv_0E_x^r(x_j,y_j,z_j,t_j)\mathrm{d}t_j=-\sum_{j=1}^N\int_{t_{j0}}^{t_{j0}+D/v_0}\sum_{i=1}^N qv_0E_x^r(x_j,y_j,z_j,t_j)_i\mathrm{d}t_j=-\sum_{j=1}^N\int_{t_{j0}}^{t_{j0}+D/v_0}qv_0\mathrm{d}t_j\sum_{i=1}^N\frac{1}{2\pi^2}\times \mathrm{Re}\sum_{n=-\infty}^{\infty}\int_0^\infty\exp\left[-\mathrm{i}\frac{2\pi nv(t_j-t_{j0})}{D}-\mathrm{i}\omega(t_{j0}-t_{i0})\right]\mathrm{d}\omega\times\int_{-\infty}^\infty E_{n,x}^r(\omega,\beta)_i\exp[\mathrm{i}\beta(y_{j0}-y_{i0})\exp(\mathrm{i}\gamma_n z_{j0})\mathrm{d}\beta. \quad (11)$$

The integral is nonzero only when n equals zero. Thus, it is followed from Eq. (9) and (11)

$$W=-\frac{qD}{2\pi^2}\sum_{j=1}^N\sum_{i=1}^N\mathrm{Re}\int_0^\infty\exp[-\mathrm{i}\omega(t_{j0}-t_{i0})]\mathrm{d}\omega\times\int_{-\infty}^\infty E_{0,x}^r(\omega,\beta)_i\exp[\mathrm{i}\beta(y_{j0}-y_{i0})\exp(\mathrm{i}\gamma_0 z_{j0})\mathrm{d}\beta=-\frac{qD}{2\pi^2}\sum_{j=1}^N\sum_{i=1}^N\mathrm{Re}\int_0^\infty\exp[-\mathrm{i}\omega(t_{j0}-t_{i0})]\mathrm{d}\omega\times\int_{-\infty}^\infty\frac{1}{k^2-\beta^2}[-\beta\alpha_0\Phi_0^r(\omega,\beta)_i+\omega\mu_0\gamma_0\Phi_0^r(\omega,\beta)_i]\times\exp[\mathrm{i}\beta(y_{j0}-y_{i0})\exp(\mathrm{i}\gamma_0 z_{j0})\mathrm{d}\beta=\frac{D}{2\pi^2}\sum_{j=1}^N\sum_{i=1}^N\mathrm{Re}\int_0^\infty\exp[-\mathrm{i}\omega(t_{j0}-t_{i0})]\mathrm{d}\omega\times\int_{-\infty}^\infty\sum_{\mathrm{real}\ \gamma_n}\frac{1}{k^2-\beta^2}\omega[\varepsilon_0\Phi_n^r(\omega,\beta)_i\Phi_n^{r^*}(\omega,\beta)_i+\mu_0\Psi_n^r(\omega,\beta)_i\Psi_n^{r^*}(\omega,\beta)_i]\gamma_n\times\exp[\mathrm{i}\beta(y_{j0}-y_{i0})\exp[\mathrm{i}\gamma_0(z_{j0}-z_{i0})]\mathrm{d}\beta. \quad (12)$$

And the integrand $(k^2-\beta^2)^{-1}\times\omega[\varepsilon_0\Phi_n^r(\omega,\beta)\Phi_n^{r^*}(\omega,\beta)+\mu_0\Psi_n^r(\omega,\beta)\Psi_n^{r^*}(\omega,\beta)]\gamma_n$ can also be written in the form of $\frac{1}{2}[E_n^r(\omega,\beta)\times H_n^{r^*}(\omega,\beta)]\cdot i_z$, which represents the power density radiated in the spectral mode n. If we rewrite the mechanical work in a set of discrete form, we thus obtain

$$W_n=\frac{D}{2\pi^2}\sum_{j=1}^N\sum_{i=1}^N\mathrm{Re}\int_0^\infty\exp[-\mathrm{i}\omega(t_{j0}-t_{i0})]\mathrm{d}\omega\times\int_{-\infty}^\infty\frac{1}{k^2-\beta^2}\omega[\varepsilon_0\Phi_n^r(\omega,\beta)_i\Phi_n^{r^*}(\omega,\beta)_i+\mu_0\Psi_n^r(\omega,\beta)_i\Psi_n^{r^*}(\omega,\beta)_i]\gamma_n\times\exp[\mathrm{i}\beta(y_{j0}-y_{i0})\exp[\mathrm{i}\gamma_0(z_{j0}-z_{i0})]\mathrm{d}\beta, \quad (13)$$

which equals the radiation energy of the n-th mode. If we adopt the polar and azimuth angles as is indicated in Fig. 1, then Eq. (1) can easily be deduced from Eq. (8). As ω, β, k and k_n, can be expressed in η and ζ, we can rewrite Eq. (13) as

$$W_n=\frac{q^2}{2D\varepsilon_0}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\mathrm{d}\eta\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{n^2\cos^2\eta\cos^2\zeta}{(1/\beta_0-\sin\eta)^3}\times\sum_{j=1}^N\sum_{i=1}^N|R_n|^2\times\exp[-\mathrm{i}\omega(t_{j0}-t_{i0})]\times\exp[\mathrm{i}\beta(y_{j0}-y_{i0})\exp\left(-\frac{4\pi|n|(z_{i0}+z_{j0})}{D(1/\beta_0-\sin\eta)}\times\sqrt{1/\beta_0^2-1+\cos^2\eta\sin^2\zeta}\right)\cos\eta\mathrm{d}\zeta, \quad (14)$$

where

$$|R_n|^2 = \frac{4}{e^2}\exp(2|\gamma_0|z_0)(1-\cos^2\eta\sin^2\zeta)^{-1}\times \left[\frac{\varepsilon_0}{\mu_0}\left|\Phi^r_{n,y}(\omega,\beta)\right|^2+\left|\Psi^r_{n,y}(\omega,\beta)\right|^2\right] \qquad (15)$$

is the radiation factor. Then the radiation energy emitted in order n per unit solid angle in direction (η, ζ) is given by

$$\frac{\mathrm{d}W_n}{\mathrm{d}\Omega} = \frac{q^2}{2D\varepsilon_0}\frac{|n|^2\cos^2\eta\cos^2\zeta}{(1/\beta_0-\sin\eta)^3}|R_n|^2\times \sum_{j=1}^{N}\sum_{i=1}^{N}\exp[-\mathrm{i}\omega(t_{j0}-t_{i0})]\exp[\mathrm{i}\beta(y_{j0}-y_{i0})\times \exp\left[-\frac{z_{i0}+z_{j0}}{2h_{\mathrm{int},n}(\eta,\zeta)}\right]= \frac{q^2}{2D\varepsilon_0}\frac{|n|^2\cos^2\eta\cos^2\zeta}{(1/\beta_0-\sin\eta)^3}|R_n|^2\times \left(\sum_{j=i}\sum_{i=1}^{N}+\sum_{j\neq i}\sum_{i=1}^{N}\right)\exp[-\mathrm{i}\omega(t_{j0}-t_{i0})]\times \exp[\mathrm{i}\beta(y_{j0}-y_{i0})\exp\left[-\frac{z_{i0}+z_{j0}}{2h_{\mathrm{int},n}(\eta,\zeta)}\right]. \qquad (16)$$

And there are various methods that deal with the radiation factor for different types of gratings. Here we introduce the interaction height

$$h_{\mathrm{int},n}=\frac{D(1/\beta_0-\sin\eta)}{4\pi|n|\sqrt{1/\beta_0^2-1+\cos^2\eta\sin^2\zeta}},$$

which can indicate how far the electrons away from the grating surface is tolerable, if the value of output power is considered.

Firstly, we deal with the former part of Eq. (16). It can be derived as

$$\sum_{j=i}\sum_{i=1}^{N}\exp[-\mathrm{i}\omega(t_{j0}-t_{i0})]\exp[\mathrm{i}\beta(y_{j0}-y_{i0})\times \exp\left[-\frac{z_{i0}+z_{j0}}{2h_{\mathrm{int},n}(\eta,\zeta)}\right]=\sum_{i=1}^{N}\exp\left[-\frac{z_{i0}}{h_{\mathrm{int},n}(\eta,\zeta)}\right]= N\sum_{m=0}^{\infty}\exp\left[-\frac{m\Delta z}{h_{\mathrm{int},n}(\eta,\zeta)}\right]f_z(m\Delta z)\Delta z= N\int_0^{\infty}\exp\left[-\frac{z}{h_{\mathrm{int},n}(\eta,\zeta)}\right]f_z(z)\mathrm{d}z. \qquad (17)$$

When the bunch is with Gaussian profile, the integral can be written as

$$N\frac{1}{\sqrt{2\pi}\sigma_z}\int_0^{\infty}\exp\left[-\frac{z}{h_{\mathrm{int},n}(\eta,\zeta)}\right]\exp\left[-\frac{(z-z_0)^2}{2\sigma_z}\right]\mathrm{d}z, \qquad (18)$$

where σ_z is the rms bunch radius in the z direction.

Next we deal with the latter part of Eq. (16). It can be obtained as

$$\sum_{j\neq i}\sum_{i=1}^{N}\exp[-\mathrm{i}\omega(t_{j0}-t_{i0})]\exp[\mathrm{i}\beta(y_{j0}-y_{i0})\times \exp\left[-\frac{z_{i0}+z_{j0}}{2h_{\mathrm{int},n}(\eta,\zeta)}\right]=N\sum_{j\neq i}\exp(-\mathrm{i}\omega t_{j0})\exp(\mathrm{i}\beta y_{j0})\times \exp\left[-\frac{z_{j0}}{2h_{\mathrm{int},n}(\eta,\zeta)}\right]\int_{-\infty}^{\infty}\exp(\mathrm{i}\omega t)f_t(t)\mathrm{d}t\times \int_{-\infty}^{\infty}\exp(-\mathrm{i}\beta y)f_y(y)\mathrm{d}y\times \int_0^{\infty}\exp\left[-\frac{z}{2h_{\mathrm{int},n}(\eta,\zeta)}\right]f_z(z)\mathrm{d}z= (N-1)N\left|\int_{-\infty}^{\infty}\exp(-\mathrm{i}\omega t)f_t(t)\mathrm{d}t\right|^2\times \left|\int_{-\infty}^{\infty}\exp(-\mathrm{i}\beta y)f_y(y)\mathrm{d}y\right|^2\times \left|\int_0^{\infty}\exp\left[-\frac{z}{2h_{\mathrm{int},n}(\eta,\zeta)}\right]f_z(z)\mathrm{d}z\right|^2. \qquad (19)$$

From Eqs. (16), (17) and (19) it follows that

$$\frac{\mathrm{d}W_n}{\mathrm{d}\Omega}=\frac{Nq^2}{2D\varepsilon_0}\frac{|n|^2\cos^2\eta\cos^2\zeta}{(1/\beta_0-\sin\eta)^3}|R_n|^2\times[S_{\mathrm{inc}}+(N-1)S_{\mathrm{coh}}], \qquad (20)$$

$$S_{\mathrm{inc}}=\int_0^{\infty}\exp\left[-\frac{z}{h_{\mathrm{int},n}(\eta,\zeta)}\right]f_z(z)\mathrm{d}z$$
$$S_{\mathrm{coh}}=\left|\int_{-\infty}^{\infty}\exp(-\mathrm{i}\omega t)f_t(t)\mathrm{d}t\right|^2\times \left|\int_{-\infty}^{\infty}\exp(-\mathrm{i}\beta y)f_y(y)\mathrm{d}y\right|^2\times \left|\int_0^{\infty}\exp\left[-\frac{z}{2h_{\mathrm{int},n}(\eta,\zeta)}\right]f_z(z)\mathrm{d}z\right|^2 . \qquad (21)$$

In Eq. (21) S_{inc} and S_{coh} represent the incoherent and coherent factor respectively.

Then the power emitted in order n by the beam with a constant current I while passing over N_{w} grating periods per unit solid angle in direction (ζ,η) is given by

$$\frac{\mathrm{d}P_n}{\mathrm{d}\Omega}=\frac{IN_wq|n|^2\beta_0^3\cos^2\eta\cos^2\zeta}{2D\varepsilon_0(1-\beta_0\sin\eta)^3}|R_n|^2[S_{\mathrm{inc}}+(N-1)S_{\mathrm{coh}}]. \qquad (22)$$

The integral equals approximately zero when the bunch length is long enough compared with the radiation wavelength and the coherent portion should not be considered in the emitted power. On the other hand, the integral equals approximately one when the bunch length is short enough. This is the case that the coherent portion has been enhanced ultimately, the incoherent portion can be ignored and the power is proportional to the square of the number of electrons per bunch. These conclusions will be discussed in details in the next section.

4 Numerical calculations

First we should state that the grating and beam parameters adopted in the calculation are based on the design of the Smith-Purcell radiator which will be operated on the Femtosecond-Accelerator in the THz Research Centre of SINAP. The parameters are listed in Table 1.

From Fig. 4, we will find that for bunch with Gaussian profile the coherent factor is symmetric with the x-z plane. Moreover, the value of coherent factor is larger in the backward direction, which represents that the longer the wavelength is the stronger the coherence turns to be. And the value decreases rapidly for larger azimuth angles ζ. Thus we commonly study the Smith-Purcell effect in x-z plane for convenience, which is called the H-polarization diffraction problem.

Various possible bunch profiles in the longitudinal direction such as Gaussian, triangular and exponential temporal profiles are adopted. From Fig. 5, we can conclude that the bunch length affects the value of coherent factor. Besides, bunches with triangular and exponential profiles are more sensitive to the bunch length. That is why Smith-Purcell can be treated as a means of bunch length and profile diagnostics especially for beam with high energy to achieve reasonable resolution. Moreover, the process is noninvasive and causes minimal disruption to the electron beam.

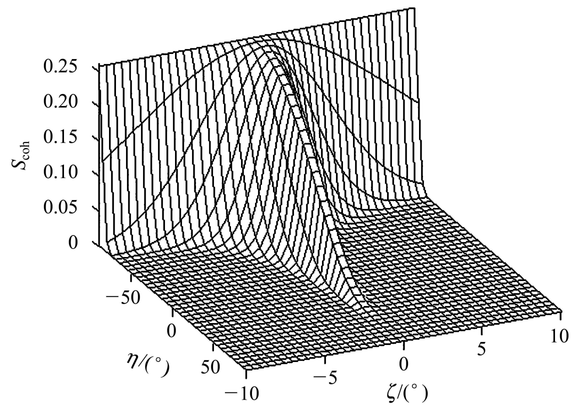

Fig. 4. 3D-plot to show the distribution of coherent factor at any observing angle.

Table 1. Parameters for electron beam and grating.

electron beam parameters						grating parameters	
E/MeV	I_{peak}/A for macrobunch	$\sigma_{x,\text{FWHM}}$/ fs	σ_z/ mm	σ_y/ mm	z_0/ mm	D/ mm	N_w
2.68	0.194	250	0.258	6.4	0.36	0.368	160

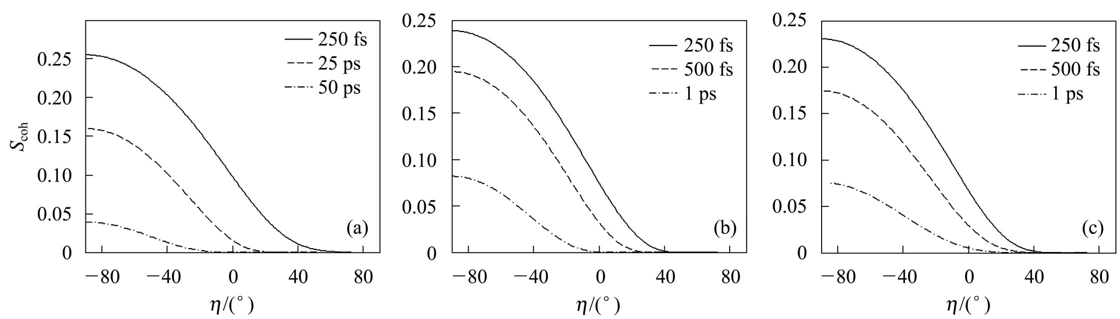

Fig. 5. Comparison of bunches with (a) Gaussian, (b) triangular, and (c) exponential profiles for different bunch lengths.

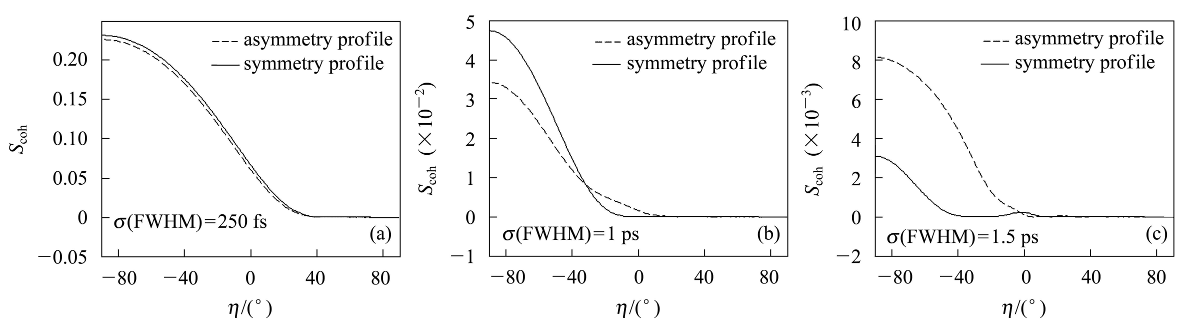

Fig. 6. Coherent factors for asymmetry and symmetry triangular bunch with bunch length (a) 250 fs, (b) 1 ps, and (c) 1.5 ps.

From Fig. 6, we can find that the divergence of coherent factor for asymmetry (dashed line) and symmetry (solid line) bunch profiles is intensified with the increasing of bunch length. And the asymmetry factor is 1.5 uniformly adopted in the calculation.

For bunch with Gaussian profile, the enhancement of coherent factor over incoherent one can be eight orders of magnitude higher seen from Fig. 7. And this performance attracts our attention for the aim to achieve higher power output.

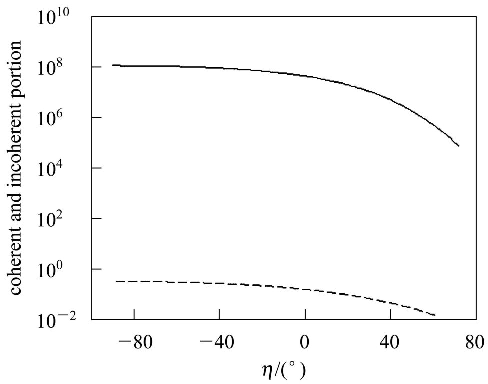

Fig. 7. Comparison of coherent and incoherent portions.

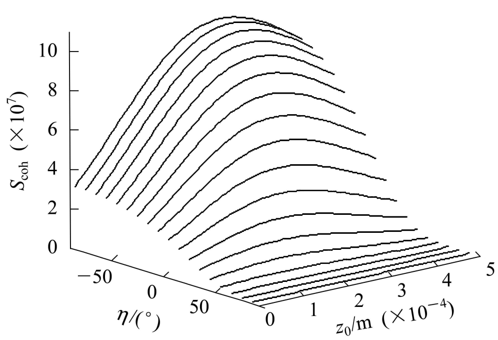

Fig. 8. Plot that shows the optimized the distance z_0 between the beam axis and the grating surface.

From Eq. (20) and (21), we can find that the distance z_0 between the beam axis and the grating surface is only included in the function f_z. Thus we just calculate the portion in the square brackets of Eq. (20) to decide the proper value of distance. Here, we want to remind you that the sign 'N' means the actual number of electrons that pass through the surface of the grating when the distance is shorter than the bunch radius. And it is necessary to put a shielding block before the up end of the grating, to protect the grating from direct impact of electrons. Fig. 8 shows us the influence of z_0 to the coherent factor. And for angle η=0°, the optimized distance is 0.26 mm.

Finally, to calculate the emission power per unit solid angle for ζ=0°, we use a rectangular grating with the grating depth 0.9D and groove wide 0.44D optimized to be suitable for our experiment condition. The result is shown in Fig. 9. And the calculation methods of radiation factor will be detailed in another paper.

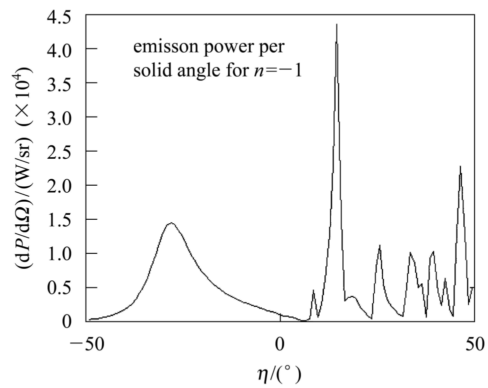

Fig. 9. Radiation power per unit solid angle for ζ=0°.

5 Concluding remarks

The far-field radiation of Smith-Purcell is not directly emitted from the electron beam but diffracted by the grating. Hence the analysis is more complicated compared with other types of coherent radiation listed in Section 2.

We have obtained the general expressions of Smith-Purcell radiation in which the incoherent and coherent effects have been considered simultaneously. Our theory is based on the van den Berg model. The coherent enhancement has been observed in some experiments. And we have provided the theoretical support for further research in this field.

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References

1 Smith S J, Purcell E M. Phys. Rev., 1953, **92**: 1069

2 Toraldo G. di Francia. Nuovo Cimento, 1961, **16**: 1065

3 Berg P M V D. J. Opt. Soc. Am., 1973, **63**: 1588

4 Berg P M V D, TAN T H. J. Opt. Soc. Am., 1974, **64**: 325

5 Berg P M V D. Appl. Sci. Res., 1971, **24**: 261

6 Bachheimer J P. Thesis Ph D. Universite Scientifique et Medicale Grenoble, 1971

7 Ikuno H, Yasuura K. IEEE Trans. Antennas Propag., 1973, **AP-2**: 657

8 Scarlat F, Facina M, Dinca C D et al. Nucl. Instrum. Methods Phys. Res. B, 2001, **173**: 93

9 Kube G. Nucl. Instrum. Methods Phys. Res. B, 2005, **227**: 180

10 Trotz Seth R, Brownell J H, Walsh John E et al. Phys. Rev. E, 2000, **61**: 7057

11 Kube G, Backe H, Euteneuer H et al. Phys. Rev. E, 2002, **65**: 056501

12 Walsh, Woods K, Yeager S. Nucl. Instrum. Methods Phys. Res. A, 1994, **41**: 277

13 Brownell J H, Walsh J, Doucas G. Phys. Rev. E, 1998, **57**: 1075

14 Potylitsyn A P. Nucl. Instrum. Methods Phys. Res. B, 1998, **145**: 60

15 Kesar A S. Phys. Rev. ST Accel. Beams, 2005, **8**: 072801

16 Doucas G, Kimmitt M F, Andrews H L et al. Phys. Rev. ST Accel. Beams, 2002, **5**: 072802

17 Doucas G, Blackmore V, Ottewell B et al. Phys. Rev. ST Accel. Beams, 2006, **9**: 092801

18 Ishi K, Shibata Y, Takahashi T et al. Phys. Rev. E, 1995, **51**: R5212

19 Gover A, Dvorkis P, Elisha U. J. Opt. Soc. Am. B, 1984, **1**: 723

20 Adischev Y N, Vukolov A V, Karlovets D V et al. Pis'ma Zh. Eksp. Teor. Fiz., 2005, **82**: 192; JETP Lett., 2005, **82**: 174

21 Brownell J H, Doucas G, Kimmitt M F et al. J. Phys. D, 1997, **30**: 2478

22 Backe H, Lauth W, Mannweiler H et al. in NATO Workshop: Advanced Radiation Sources and Applications. New York: Springer, 2006. 267—282

23 Kube G, Backe H, Euteneuer H et al. Phys. Rev. E, 2002, **65**: 056501

24 Reiche S, Rosenzweig J B. Phys. Rev. ST Accel. Beams, 2003, **6**: 040702

25 Bonifacio R, Pellegrini C, Narducci L M. Opt. Commun., 1984, **50**: 373

26 Murokh A. Nucl. Instrum. Methods A, 1998, **410**: 549

27 Bolotovskii B M, Voskresenskii G M. Usp. Fiz. Nauk., 1966, **88**: 209