Influence of Curvature and Compressibility Energies upon Prescission Neutron Emission of Neutron-Deficient Fissioning Systems*

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Abstract The aim of this paper is to study the influence of the curvature and the compressibility energies on prescission neutron emission of four neutron-deficient systems: ^{188}Pt, ^{200}Pb, ^{213}Fr and ^{224}Th within a modified version of the combined dynamical statistical model. The calculated results show that prescission neutron multiplicities from the modified model are closer to the data compared with the underestimate of the original one. The physics that the compressibility energy needs to be considered in this work is discussed.

Key words curvature energy, compressibility energy, neutron-deficient fissioning system, level-density parameter, prescission neutron multiplicity

1 Introduction

Due to nuclear dissipation, the fission process of hot nuclei becomes slow. As a result, an excess of prescission neutrons[1], light charged particles[2] and giant-dipole γ rays[3] have been observed compared with expectations from statistical model. Theoretically, many models considering the nuclear dissipation have successfully illuminated the phenomena[4]. Besides these, the combined dynamical statistical model (CDSM) has simultaneously reproduced the prescission light particle multiplicity and fission probability in a wide range of excitation energies and fissility of compound nuclei. For some neutron-deficient systems, however, the prescission neutron multiplicities calculated within the CDSM are smaller in comparison with experimental data at low energies[5]. The unique properties of the nucleus determine that the nuclear friction is likely to stem from nuclear collisions with the nuclear surface, or nucleon-nucleon collisions in the nuclear surface region[6]. So it is worth paying particular attention to the influence of those quantities related to nuclear surface properties on the decay of hot nuclei. Recently, nuclear liquid-drop model (LDM) was revisited, and an explicit introduction of the surface-curvature terms was presented[7]. Then the ground-state binding energies and fission barriers were simultaneously reproduced with a reasonable precision. This work gives us a hint to investigate the underestimate of the CDSM for the prescission neutron multiplicity of neutron-deficient systems: ^{188}Pt, ^{200}Pb, ^{213}Fr and ^{224}Th by taking curvature and compressibility energies into consideration.

2 The CDSM

The CDSM is a combination of dynamical Langevin equation and statistical model to describe the fission process of heavy-ion reaction. In the dy-

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namical branch of the model the evaporation of light particles and γ-rays is accounted for by coupling a Monte Carlo technique to the fission dynamics. In the statistical branch of the model a fission width consistent with the dynamical description is used along with the standard way of calculating the light particle emission. In order to elucidate our work, we only briefly review the CDSM, for more detailed introduction, please refer to Ref. [5].

Supposing that we deal with overdamped motion, we use the following one-dimensional reduced Langevin equation to describe the fission process

$$\frac{\mathrm{d}q}{\mathrm{d}t}=\frac{T}{M\beta(q)}\frac{\partial S(q)}{\partial q}+\sqrt{\frac{T}{M\beta(q)}}\Gamma(t), \tag{1}$$

where $q=\mathrm{d}/2R_0$, d is the distance between the centers of mass of the future fission fragments, R_0 is the radius of the compound radius, M is the inertia parameter, and T is the nuclear temperature. $\beta(q)$ is the coordinate-dependent reduced friction parameter. The so-called SPS-friction[8] is used in the present work. $\Gamma(t)$ is the Langevin force, which meets $\langle\Gamma(t)\rangle=0$ and $\langle\Gamma(t)\Gamma(t')\rangle=2\delta_\varepsilon(t-t')$.

The driving force of Eq. (1) is derived from the entropy

$$S(q)=2\sqrt{a(q)[E^*_{\mathrm{tot}}-V(q)]}, \tag{2}$$

where E^*_{tot} is the total excitation energy and $E^*_{\mathrm{tot}}=E_{\mathrm{c.m.}}+Q$ with Q being the energy released in fusion, $V(q)$ is potential energy calculated by the LDM. $a(q)$ is coordinate-dependent level-density parameter (LDP) which only includes volume and surface contributions of single-particle level density.

When the calculation switches into the statistical branch of the model, the fission width $\Gamma_{\mathrm{f}}=\hbar R_{\mathrm{f}}$ is calculated according to

$$\begin{aligned}R_{\mathrm{f}}\;=\;&\frac{T_{\mathrm{gs}}\sqrt{|S''_{\mathrm{gs}}|\,S''_{\mathrm{sd}}}}{2\pi M\beta_{\mathrm{gs}}}\exp[S(q_{\mathrm{gs}})-S(q_{\mathrm{sd}})]\times\\&2\left\{1+\mathrm{erf}\left[(q_{\mathrm{sc}}-q_{\mathrm{sd}})\sqrt{S''_{\mathrm{sd}}/2}\right]\right\}^{-1},\end{aligned} \tag{3}$$

where $\mathrm{erf}(x)=(2/\sqrt{\pi})\int_0^x\mathrm{d}t\exp(-t^2)$ is the error function, the subscripts gs, sd and sc mark the positions of the ground state, the saddle point and the scission point, respectively. The K-interpretation[9] has been used in the present calculation. In the whole simulation, the decay widths Γ_ν of the emitted light particles ($\nu=\mathrm{n,p},\alpha,\mathrm{d}$) are calculated with the parameterizations of Blann[10]. The width for giant-dipole γ rays is given by Lynn formula[11].

3 The modified version of the CDSM (MCDSM)

Compared with the CDSM, we now calculate the one-dimensional potential $V(q)$ by a truncated and linearized droplet model (DM)[12], in which the curvature and the compressibility energies are considered. In addition, the potential $V(q)$ also includes a rotational energy. Such that it can be read as

$$\begin{aligned}V(A,Z,L,q)=&a_2(1-b_3I^2)A^{2/3}(B_{\mathrm{s}}(q)-1)+\\&a_3A^{1/3}(B_{\mathrm{k}}(q)-1)-a_4A^{1/3}(B_{\mathrm{p}}(q)-1)+\\&c_1Z^2A^{-1/3}(B_{\mathrm{C}}(q)-1)+c_{\mathrm{r}}L^2A^{-5/3}(B_{\mathrm{r}}(q)-1),\end{aligned} \tag{4}$$

where the coefficients a_2, a_3, a_4, c_1 and c_{r} correspond to the surface, curvature, Coulomb, compressibility and rotational energy, respectively. Their values are taken from Refs. [12] and [8], respectively. $I=(A-2Z)/A$ means the relative excess of neutron. L is the angular momentum. $B_{\mathrm{s}}(q)$ stands for the shape dependence of the surface energy, namely, the dimensionless surface area of the deformed nucleus, normalized to that of the spherical configuration. Similarly, $B_{\mathrm{k}}(q)$, $B_{\mathrm{C}}(q)$, $B_{\mathrm{p}}(q)$ and $B_{\mathrm{r}}(q)$ represent the shape dependences of curvature, Coulomb, compressibility and rotational energy, respectively, all of which are normalized to the values of a spherical counterpart. In contrast to Hasse's work[12], for simplicity we take the shape dependence of the Coulomb redistribution energy as unity now. By virtue of the Funny-Hill parameterizations $\{c,h,\alpha\}$[13], Gontchar et al. presented the expressions of $B_{\mathrm{s}}(q)$ and $B_{\mathrm{r}}(q)$[14], which are also used in this work. Furthermore, in the spirit of their work, we draw the following ansatz for $B_{\mathrm{k}}(q)$

$$B_{\mathrm{k}}(q)=\begin{cases}3.556q^2-2.667q+1.5, & \text{if } q<0.452,\\ 0.416q^2+0.237q+1.014, & \text{if } q\geqslant 0.452.\end{cases} \tag{5}$$

The relationship $B_{\mathrm{p}}(q)=B^2_{\mathrm{s}}(q)$, owing to Hasse[12], is adopted in the present calculations.

In Ref. [12], an energy unit and a set of dimensionless parameters have been defined as the following

$$\begin{aligned} E^0 &= a_2(1-b_3I^2)A^{2/3}+a_3A^{1/3}-2a_4A^{1/3}, \\ \xi &= (c_1Z^2A^{-1/3}+2c_2Z^2A^{1/3})/2E^0, \\ \eta &= a_3A^{1/3}/E^0, \\ \zeta &= c_2Z^2A^{1/3}/E^0, \\ \chi &= a_4A^{1/3}/E^0. \end{aligned} \tag{6}$$

Here the coefficient c_2 corresponds to the Coulomb redistribution energy, its value is also taken from Ref. [12]. According to the similar procedure in Ref. [8], with Eq. (6) we can arrive at the following formula for $B_{\rm C}(q)$

$$B_{\rm C}(q)=1+\frac{(1-B_{\rm s})+\chi(1+B_{\rm p}-2B_{\rm s})+\eta(B_{\rm k}-B_{\rm s})+B_{\rm Bar}}{2(\xi-\zeta)}, \tag{7}$$

where $B_{\rm Bar}$ is relative fission barrier. The same approximation for $B_{\rm Bar}$ in Ref. [8] is used in this paper.

An example of such a potential $V(q)$ for ^{224}Th with zero angular momentum is shown in Fig. 1.

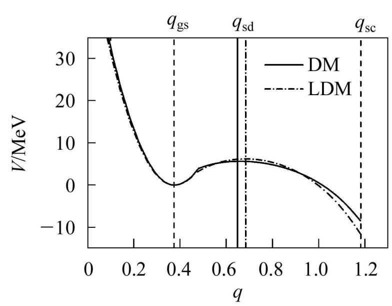

Fig. 1. The potentials calculated respectively within the LDM (dash-dot line) and the DM (solid line) versus the collective coordinate for ^{224}Th. The ground state situation, the saddle point (DM: solid straight line; LDM: dash-dot straight line) and the scission point are also depicted.

In order to consistently investigate the effect of the curvature and the compressibility energies on the decay of hot nuclei, a correct choice of the asymptotic values of the LDPs is necessary. Such that, the coordinate-dependent parameter including a contribution of the curvature component of single-particle level-density may be written as

$$a(q)=a_{\rm v}A+a_{\rm s}A^{2/3}B_{\rm s}(q)+a_{\rm k}A^{1/3}B_{\rm k}(q), \tag{8}$$

where $a_{\rm v}$, $a_{\rm s}$ and $a_{\rm k}$ are the coefficients corresponding to the volume, surface and curvature component of the single-particle level density, respectively. Three sets of the coefficients are listed in Table 1. The definitions of $B_{\rm s}(q)$ and $B_{\rm k}(q)$ are in coincidence with those of Eq. (4). It needs to note that they aren't taken again as unity now, which is different from Töke and Swiatecki's[16] and Reisdorf's[18] works. The absolute values of LDPs are shown as a function of the atomic number in Fig. 2(a). The coordinate dependences of these LDPs for mass number A=216 are displayed in Fig. 2(b).

Table 1. The theoretical values for volume-, surface- and curvature-component of the LDPs for a deformed nucleus in Eq. (8).

author	$a_{\rm v}$/MeV^{-1}	$a_{\rm s}$/MeV^{-1}	$a_{\rm k}$/MeV^{-1}
Ignatyuk et al.[15]	0.073	0.095	
Töke and Swiatecki[16]	0.069	0.213	0.385
Mughabghab and Dunford[17]	0.076	0.180	0.157

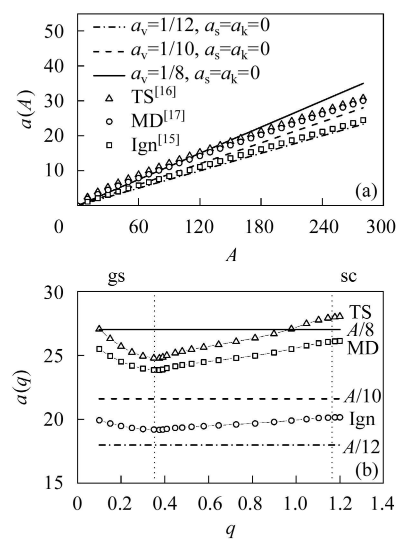

Fig. 2. (a) The different LDP (at the ground state) are shown as functions of mass number A (Ign: squares[15]; TS: up-triangles[16]; MD: circles[17]) in comparison with $A/8$ (solid line), $A/10$ (dash line) and $A/12$ (dot line); (b) The coordinate dependences of LDP are displayed and also compared with the values for $A/8$, $A/10$ and $A/12$ for A=216. The lines are guides to the eyes.

4 Numerical results and discussions

Before the calculated results are displayed, it needs to emphasize that new Standard Parameter Set

(SPS) in the MCDSM is constitute of DM-potential, MD-LDP and SPS-friction compared with the old one including the LDM-potential, Ign-LDP and SPS-friction in the CDSM.

Figure 3 shows the prescission neutron multiplicities calculated within the framework of the MCDSM and the CDSM, respectively, for four neutron-deficient systems: ^{188}Pt, ^{200}Pb, ^{213}Fr and ^{224}Th. The experimental data are taken from Ref. [1]. It is seen that the results calculated within the MCDSM are closer to data compared with those within the CDSM. Besides that, one can find that the multiplicity calculated within the MCDSM is larger than that within the CDSM. In particular, the MCDSM reproduces well in contrast to the underestimation of the CDSM for ^{16}O+^{197}Au $\rightarrow$ ^{213}Fr and ^{16}O+^{208}Pb $\rightarrow$ ^{224}Th. The satisfactory agreement for the reaction ^{19}F+^{169}Tm $\rightarrow$ ^{188}Pt calculated within the MCDSM is also found in Fig. 3(a). While for ^{19}F+^{181}Ta $\rightarrow$ ^{200}Pb system in Fig. 3(b), the MCDSM reproduces well at lower energies. At relatively higher energies, however, the MCDSM overestimates slightly compared with the underestimation of the CDSM. Compared with the identical underestimation from the CDSM, one can see that the agreement for the heavier nuclei is better than for the lighter nuclei from the MCDSM.

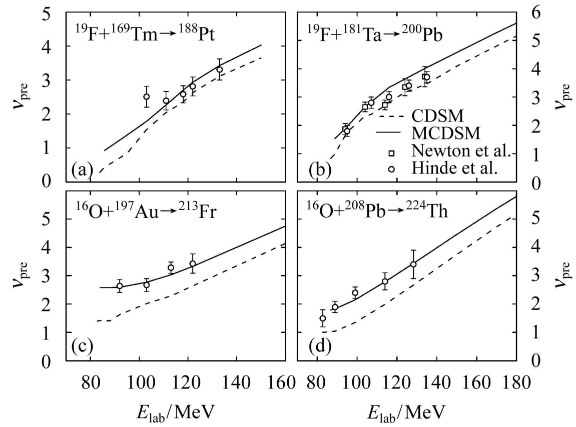

Fig. 3. The calculated prescission neutron multiplicities ν_{pre} for four neutron deficient systems ^{19}F+^{169}Tm $\rightarrow$ ^{188}Pt (a), ^{19}F+^{181}Ta $\rightarrow$ ^{200}Pb (b), ^{16}O+^{197}Au $\rightarrow$ ^{213}Fr (c) and ^{16}O+^{208}Pb $\rightarrow$ ^{224}Th (d) within the modified CDSM model, are displayed and in contrast to that within the CDSM model (dash lines). Experimental data (circles and squares) are taken from Ref. [1].

Table 2. The calculated reaction Q-values (unit: MeV), particle binding energies B_{i} (i = n, p, α) (unit: MeV), the potential energies (unit: MeV) for the four systems mentioned above within the DM, are compared with those correspondents from the LDM.

systems	DM					LDM				
	Q	B_{n}	B_{p}	B_{α}	V	Q	B_{n}	B_{p}	B_{α}	V
^{188}Pt	−9.9	7.38	5.55	−2.89	18.88	−26.07	8.51	5.42	−4.01	17.36
^{200}Pb	−13.08	6.97	5.69	−3.41	13.27	−29.81	8.15	5.42	−4.60	12.90
^{213}Fr	−19.60	6.88	5.24	−4.46	7.46	−38.65	8.06	3.40	−5.66	8.00
^{224}Th	−29.28	6.27	5.83	−4.48	5.63	−46.64	7.56	5.30	−5.81	6.19

Table 2 lists the reaction Q-values, particle binding energies B_{i} (i = n, p, α), potentials calculated within the DM for four systems mentioned above to compare with the corresponding results from the LDM. One can see that the Q-values increase, however, the fission barriers change slightly. These two kinds of actions together with the stronger coordinate-dependent LDP induce the entropy to rise. Due to this, from Eq. (3), we can deduce that the fission width in the MCDSM will get enhanced compared with that in the CDSM. While at the same time, the particle emission width does hardly change according to Blann's and Lynn's formulas. Thus the fission mode gets strengthened, while the particle emission modes will be weakened relatively. However, the increasing neutron binding energies and the decreasing ones for proton and alpha will counteract the behavior to some extent, which will result in facilitating neutron emission and further inhibiting the emission of proton and alpha particle. In addition, we observe from Fig. 1 that for ^{224}Th the length of the descent from saddle to scission becomes longer, furthermore, the slope of the potential becomes flatter, all of which will cause the time traveling from saddle to scission to get longer. The situation is similar for the other three systems. The behavior is in favor of the emission of light particle and will balance the proportion of fission to some extent. Then, on the whole, the prescission neutron multiplicity becomes larger than that calculated by the CDSM, and closer to the data. It is interesting to point out that the phenomenon for ^{213}Fr and ^{224}Th is more evident than ^{188}Pt and ^{200}Pb. On the one hand, it is because that the fission barriers for the former systems descend compared with the rise of the latter. On the other hand, our calculation reveals that the phenomenon also results from the protracted length of the descent from saddle to scission for the former is more evident than for the latter.

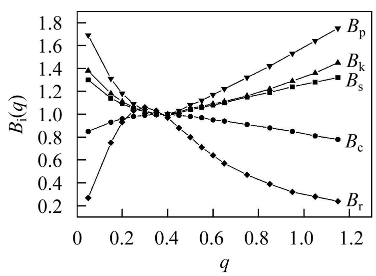

Fig. 4. The deformation dependences for surface energy: squares, Coulomb energy: circles, curvature energy: up-triangles, compressibility energy: down-triangles and rotational energy: diamonds, respectively. The lines are guides to the eyes.

It can be seen from Eq. (4) that the curvature energy and the compressibility energy are all proportional to $A^{1/3}$. But they have different deformation dependences. According to Eqs. (5) and (7) and the relation $B_{\rm p}(q) = B_{\rm s}^2(q)$ as well as the expressions of $B_{\rm s}(q)$ and $B_{\rm r}(q)$ drawn by Gontchar et al.[5, 14], in Fig. 4 we present the deformation dependences for the five kinds of energies. It can be observed that, with increasing q, $B_{\rm s}(q)$, $B_{\rm k}(q)$ and $B_{\rm p}(q)$ become stronger; however $B_{\rm C}(q)$ and $B_{\rm r}(q)$ become weaker. Although the coefficient of the compressibility energy is smaller than that of the curvature energy, we can observe from Fig. 4 that the compressibility energy has the strongest dependence on the shape of nucleus. Hence, in order to completely investigate the influence of the surface properties of nuclei on the dynamical deexcitation process of hot nuclei, it is necessary to consider together the two kinds of energies. As has been pointed out by Pomorski and Dudek, the compressibility energy also offsets the contribution of the curvature energy to binding energy and fission barrier[7], which will place a influence on the prescission neutron multiplicity. Without the compressibility energy, the binding energy will increase a little, which prohibits slightly the emission of the light particle. Besides that, without the energy, it also results in the fission barrier to rise then be in favor of the emission of the light particle. Then it can be deduced that the two opposite behaviors will induce a little increase of prescission light particle multiplicity. Moreover, from Eq. (8) we also learn that the $a(k)$ is a compromise coefficient owing to the inverse contributions of two kinds of DM correction energies. If the compressibility energy isn't taken into accout, it needs a unreasonable $a(k)$ to reproduce the data well, then it also bears up to consider the two kinds of energies at the same time.

5 Summary

Within the framework of the modified CDSM, we calculated the prescission neutron multiplicities for four neutron-deficient systems: ^{188}Pt, ^{200}Pb, ^{213}Fr and ^{224}Th. The present calculated results are closer to the data compared with the underestimate of the CDSM because the curvature energy and the compressibility energy are taken into consideration. Due to the stronger coordinate dependence and its significance in choosing the level density parameter, the compressibility energy must be involved in the calculation.

References

1 Hinde D J, Charity R J, Foote G S et al. Nucl. Phys., 1986, **A452**: 550—572; Newton J O, Hinde D J, Charity R J et al. Nucl. Phys., 1988, **A483**: 126—152; Cabrera J, Keutgen Th, Masri Y El et al. Phys. Rev., 2003, **C68**: 034613

2 Lestone J P, Leigh J R, Newton J O et al. Phys. Rev. Lett., 1991, **67**: 1078—1081; Ikezoe H, Shikazono N, Nagame Y et al. Phys. Rev., 1992, **C46**: 1922—1933; John B, Choudhury R K, Nayak B K et al. Phys. Rev., 2001, **C65**: 054301

3 Hofman D J, Back B B, Diószegi I et al. Phys. Rev. Lett., 1994, **72**: 470—473; Diószegi I, Shaw N P, Mazumdar I et al. Phys. Rev., 2000, **C61**: 024613

4 Charity R J, McMahan M A, Wozniak G J et al. Nucl. Phys., 1988, **A483**: 371—405; Lestone J P. Phys. Rev. Lett., 1993, **70**: 2245—2248; Ye W. Prog. Theor. Phys., 2003, **109**: 933—942; Pomorski K, Bartel J, Richert J et al. Nucl. Phys., 1996, **A605**: 87—119

5 Mavlitov N D, Fröbrich P, Gontchar I I. Z. Phys., 1992, **A342**: 195—198; Fröbrich P, Gontchar I I, Mavlitov N D. Nucl. Phys., 1993, **A556**: 281—306; Fröbrich P, Gontchar I I. Phys. Rep., 1998, **292**: 131—237

6 Nix J R, Sierk A J. Proceedings of the 6th Adriatic Conference on Nuclear Physics: Frontiers of Heavy-Ion Physics. Singapore: World Scientific, 1990. 333; Hinde D J. Phys. Rev., 1989, **C39**: 2268—2284

7 Pomorski K, Dudek J. Phys. Rev., 2003, **C67**: 044316

8 Gontchar I I, Litnevsky L A. Z. Phys., 1997, **A359**: 149—155; Gontchar I I, Litnevsky L A, Fröbrich P. Comput. Phys. Commun., 1997, **107**: 223—245

9 Gettinger A E, Gontchar I I. J. Phys. G: Nucl. Part. Phys., 2000, **26**: 347—356

10 Blann M. Phys. Rev., 1980, **C21**: 1770—1782

11 Lynn J E. Theory of Neutron Resonance Reactions. Clarendon: Oxford, 1968

12 Hasse R W. Ann. Phys., 1971, **68**: 377—461

13 Brack M, Damgaard J, Jensen A S et al. Rev. Mod. Phys., 1972, **44**: 320—405

14 Gontchar I I, Fröbrich P, Pischasov N I. Phys. Rev., 1993, **C47**: 2228—2235

15 Ignatyuk A V, Itkis M G, Okolovich V N et al. Sov. J. Nucl. Phys., 1975, **21**: 612—632

16 Töke J, Swiatecki W J. Nucl. Phys., 1981, **A372**: 141—150

17 Mughabghab S F, Dunford C. Phys. Rev. Lett., 1998, **81**: 4083—4086

18 Reisdorf W. Z. Phys., 1981, **A300**: 227—238

曲率能及压缩能对缺中子裂变系统断前中子发射的影响*

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摘要　用改进了的动力学与统计相结合的模型, 研究了曲率能及压缩能对4个缺中子裂变系统: ^{188}Pt, ^{200}Pb, ^{213}Fr和^{224}Th的断前中子发射的影响. 计算结果显示用改进模型得到的断前中子多重性相比原来模型的低估要更靠近实验数据. 对于目前工作考虑压缩能的原因进行了讨论.

关键词　曲率能　压缩能　缺中子裂变系统　能级密度参数　断前中子多重性

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